23

Electric Fields

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q23.1 A neutral atom is one that has no net charge. This means that it has the same number of electrons orbiting the nucleus as it has protons in the nucleus. A negatively charged atom has one or more excess electrons.

***Q23.2** (i) Suppose the positive charge has the large value 1 μC. The object has lost some of its conduction electrons, in number 10^{-6} C (1 e/1.60×10^{-19} C) = 6.25×10^{12} and in mass 6.25×10^{12} (9.11×10^{-31} kg) = 5.69×10^{-18} kg. This is on the order of 10^{14} times smaller than the ~1 g mass of the coin, so it is an immeasurably small change. Answer (d).

(ii) The coin gains extra electrons, gaining mass on the order of 10^{-14} times its original mass for the charge -1 μC. Answer (b).

Q23.3 All of the constituents of air are nonpolar except for water. The polar water molecules in the air quite readily "steal" charge from a charged object, as any physics teacher trying to perform electrostatics demonstrations in the summer well knows. As a result—it is difficult to accumulate large amounts of excess charge on an object in a humid climate. During a North American winter, the cold, dry air allows accumulation of significant excess charge, giving the potential (pun intended) for a shocking (pun also intended) introduction to static electricity sparks.

Q23.4 Similarities: A force of gravity is proportional to the product of the intrinsic properties (masses) of two particles, and inversely proportional to the square of the separation distance. An electrical force exhibits the same proportionalities, with charge as the intrinsic property.

Differences: The electrical force can either attract or repel, while the gravitational force as described by Newton's law can only attract. The electrical force between elementary particles is vastly stronger than the gravitational force.

Q23.5 No. The balloon induces polarization of the molecules in the wall, so that a layer of positive charge exists near the balloon. This is just like the situation in Figure 23.4a, except that the signs of the charges are reversed. The attraction between these charges and the negative charges on the balloon is stronger than the repulsion between the negative charges on the balloon and the negative charges in the polarized molecules (because they are farther from the balloon), so that there is a net attractive force toward the wall. Ionization processes in the air surrounding the balloon provide ions to which excess electrons in the balloon can transfer, reducing the charge on the balloon and eventually causing the attractive force to be insufficient to support the weight of the balloon.

***Q23.6** Answer (c). Each charge produces field as if it were alone in the Universe.

***Q23.7** (i) According to the inverse square law, the field is one-fourth as large at twice the distance. The answer is (c), 2×36 cm = 72 cm.

(ii) The field is four times stronger at half the distance away from the charge. Answer (b).

Q23.8 An electric field created by a positive or negative charge extends in all directions from the charge. Thus, it exists in empty space if that is what surrounds the charge. There is no material at point *A* in Figure 23.21(a), so there is no charge, nor is there a force. There would be a force if a charge were present at point *A*, however. A field does exist at point *A*.

***Q23.9** (i) We compute $q_A q_B / r^2$ in each case. In (a) it is 400/4 = 100 (nC/cm)^2. In (b) and (c), 300/4 = 75 (nC/cm)^2. In (d) 600/9 = 67 (nC/cm)^2. In (e) 900/9 = 100 (nC/cm)^2. The ranking is then a = e > b = c > d.

(ii) We compute q_A / r^2 in each case. In (a) it is 20/4 = 5 nC/cm^2. In (b) 30/4 = 7.5 nC/cm^2. In (c) 10/4 = 2.5 nC/cm^2. In (d) 30/9 = 3.3 nC/cm^2. In (e) 45/9 = 5 nC/cm^2. The ranking is then b > a = e > d > c.

***Q23.10** The charge at the upper left creates at the field point electric field to the left, with magnitude we call E_1. The charge at lower right creates downward electric field with an equal magnitude E_1. These two charges together create field $\sqrt{2}E_1$ downward and to the left at 45°. The positive charge is $\sqrt{2}$ times farther from the field point so it creates field $2E_1/(\sqrt{2})^2 = E_1$ upward and to the right. The net field is then $\left(\sqrt{2}-1\right)E_1$ downward and to the left. The answer to question (i) is (d).

(ii) With the positive charge removed, the magnitude of the field becomes $\sqrt{2}E_1$, larger than before, so the answer is (a).

***Q23.11** The certain point must be on the same line as A and B, for otherwise the field components perpendicular to this line would not add to zero. If the certain point is between A and B, it is midway between them, and B's charge is also +40 nC. If the certain point is 4 cm from A and 12 cm from B, then B's charge must be –9(40 nC) = –360 nC. These are the only two possibilities. The answers are (a), (f), and (j).

Q23.12 The direction of the electric field is the direction in which a positive test charge would feel a force when placed in the field. A charge will not experience two electrical forces at the same time, but the vector sum of the two. If electric field lines crossed, then a test charge placed at the point at which they cross would feel a force in two directions. Furthermore, the path that the test charge would follow if released at the point where the field lines cross would be indeterminate.

Q23.13 Both figures are drawn correctly. $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ are the electric fields separately created by the point charges q_1 and q_2 in Figure 23.12 or q and $-q$ in Figure 23.13, respectively. The net electric field is the vector sum of $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$, shown as $\vec{\mathbf{E}}$. Figure 23.19 shows only one electric field line at each point away from the charge. At the point location of an object modeled as a point charge, the direction of the field is undefined, and so is its magnitude.

***Q23.14** Answer (a). The equal-magnitude radially directed field contributions add to zero.

***Q23.15** Answer (c). Contributions to the total field from bits of charge in the disk lie closer together in direction than for the ring.

***Q23.16** (i) Answer (c). Electron and proton have equal-magnitude charges.
(ii) Answer (b). The proton's mass is 1836 times larger than the electron's.

***Q23.17** Answer (b).

Q23.18 Linear charge density, λ, is charge per unit length. It is used when trying to determine the electric field created by a charged rod.

Surface charge density, σ, is charge per unit area. It is used when determining the electric field above a charged sheet or disk.

Volume charge density, ρ, is charge per unit volume. It is used when determining the electric field due to a uniformly charged sphere made of insulating material.

Q23.19 No. Life would be no different if electrons were + charged and protons were – charged. Opposite charges would still attract, and like charges would repel. The naming of + and – charge is merely a convention.

Q23.20 In special orientations the force between two dipoles can be zero or a force of repulsion. In general each dipole will exert a torque on the other, tending to align its axis with the field created by the first dipole. After this alignment, each dipole exerts a force of attraction on the other.

SOLUTIONS TO PROBLEMS

Section 23.1 **Properties of Electric Charges**

P23.1 (a) The mass of an average neutral hydrogen atom is 1.007 9u. Losing one electron reduces its mass by a negligible amount, to

$$1.007\,9\left(1.660\times10^{-27}\text{ kg}\right)-9.11\times10^{-31}\text{ kg}=\boxed{1.67\times10^{-27}\text{ kg}}.$$

Its charge, due to loss of one electron, is

$$0-1\left(-1.60\times10^{-19}\text{ C}\right)=\boxed{+1.60\times10^{-19}\text{ C}}.$$

(b) By similar logic, charge $=\boxed{+1.60\times10^{-19}\text{ C}}$

$$\text{mass }=22.99\left(1.66\times10^{-27}\text{ kg}\right)-9.11\times10^{-31}\text{ kg}=\boxed{3.82\times10^{-26}\text{ kg}}$$

(c) charge of $Cl^- = \boxed{-1.60\times10^{-19}\text{ C}}$

$$\text{mass }=35.453\left(1.66\times10^{-27}\text{ kg}\right)+9.11\times10^{-31}\text{ kg}=\boxed{5.89\times10^{-26}\text{ kg}}$$

(d) charge of $Ca^{++}=-2\left(-1.60\times10^{-19}\text{ C}\right)=\boxed{+3.20\times10^{-19}\text{ C}}$

$$\text{mass }=40.078\left(1.66\times10^{-27}\text{ kg}\right)-2\left(9.11\times10^{-31}\text{ kg}\right)=\boxed{6.65\times10^{-26}\text{ kg}}$$

(e) charge of $N^{3-}=3\left(-1.60\times10^{-19}\text{ C}\right)=\boxed{-4.80\times10^{-19}\text{ C}}$

$$\text{mass }=14.007\left(1.66\times10^{-27}\text{ kg}\right)+3\left(9.11\times10^{-31}\text{ kg}\right)=\boxed{2.33\times10^{-26}\text{ kg}}$$

continued on next page

(f) charge of $N^{4+} = 4\left(1.60\times10^{-19}\text{ C}\right) = \boxed{+6.40\times10^{-19}\text{ C}}$

$$\text{mass} = 14.007\left(1.66\times10^{-27}\text{ kg}\right) - 4\left(9.11\times10^{-31}\text{ kg}\right) = \boxed{2.32\times10^{-26}\text{ kg}}$$

(g) We think of a nitrogen nucleus as a seven-times ionized nitrogen atom.

$$\text{charge} = 7\left(1.60\times10^{-19}\text{ C}\right) = \boxed{1.12\times10^{-18}\text{ C}}$$

$$\text{mass} = 14.007\left(1.66\times10^{-27}\text{ kg}\right) - 7\left(9.11\times10^{-31}\text{ kg}\right) = \boxed{2.32\times10^{-26}\text{ kg}}$$

(h) $\text{charge} = \boxed{-1.60\times10^{-19}\text{ C}}$

$$\text{mass} = \left[2(1.007\,9) + 15.999\right]1.66\times10^{-27}\text{ kg} + 9.11\times10^{-31}\text{ kg} = \boxed{2.99\times10^{-26}\text{ kg}}$$

P23.2 (a) $$N = \left(\frac{10.0\text{ grams}}{107.87\text{ grams/mol}}\right)\left(6.02\times10^{23}\ \frac{\text{atoms}}{\text{mol}}\right)\left(47\ \frac{\text{electrons}}{\text{atom}}\right) = \boxed{2.62\times10^{24}}$$

(b) $$\#\text{ electrons added} = \frac{Q}{e} = \frac{1.00\times10^{-3}\text{ C}}{1.60\times10^{-19}\text{ C/electron}} = 6.25\times10^{15}$$

or $\boxed{2.38 \text{ electrons for every } 10^{9} \text{ already present}}$.

Section 23.2 **Charging Objects by Induction**

Section 23.3 **Coulomb's Law**

P23.3 If each person has a mass of ≈ 70 kg and is (almost) composed of water, then each person contains

$$N \cong \left(\frac{70\,000\text{ grams}}{18\text{ grams/mol}}\right)\left(6.02\times10^{23}\ \frac{\text{molecules}}{\text{mol}}\right)\left(10\ \frac{\text{protons}}{\text{molecule}}\right) \cong 2.3\times10^{28}\text{ protons}$$

With an excess of 1% electrons over protons, each person has a charge

$$q = 0.01\left(1.6\times10^{-19}\text{ C}\right)\left(2.3\times10^{28}\right) = 3.7\times10^{7}\text{ C}$$

So $$F = k_e\frac{q_1q_2}{r^2} = \left(9\times10^{9}\right)\frac{\left(3.7\times10^{7}\right)^2}{0.6^2}\text{ N} = 4\times10^{25}\text{ N}\ \boxed{\sim 10^{26}\text{ N}}$$

This force is almost enough to lift a weight equal to that of the Earth:

$$Mg = 6\times10^{24}\text{ kg}\left(9.8\text{ m/s}^2\right) = 6\times10^{25}\text{ N} \sim 10^{26}\text{ N}$$

***P23.4** In the first situation, $\vec{\mathbf{F}}_{A\text{ on }B,1} = \frac{k_e|q_A||q_B|}{r_1^2}\hat{\mathbf{i}}$. In the second situation, $|q_A|$ and $|q_B|$ are the same.

$$\vec{\mathbf{F}}_{B\text{ on }A,2} = -\vec{\mathbf{F}}_{A\text{ on }B} = \frac{k_e|q_A||q_B|}{r_2^2}\left(-\hat{\mathbf{i}}\right)$$

$$\frac{F_2}{F_1} = \frac{k_e|q_A||q_B|}{r_2^2}\frac{r_1^2}{k_e|q_A||q_B|}$$

$$F_2 = \frac{F_1 r_1^2}{r_2^2} = 2.62\ \mu\text{N}\left(\frac{13.7\text{ mm}}{17.7\text{ mm}}\right)^2 = 1.57\ \mu\text{N}$$

Then $\vec{\mathbf{F}}_{B\text{ on }A,2} = \boxed{1.57\ \mu\text{N to the left}}$.

P23.5 (a) $F_e = \frac{k_e q_1 q_2}{r^2} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{\left(3.80\times10^{-10}\text{ m}\right)^2} = \boxed{1.59\times10^{-9}\text{ N}}$ (repulsion)

(b) $F_g = \frac{Gm_1m_2}{r^2} = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.67\times10^{-27}\text{ kg}\right)^2}{\left(3.80\times10^{-10}\text{ m}\right)^2} = \boxed{1.29\times10^{-45}\text{ N}}$

The electric force is $\boxed{\text{larger by }1.24\times10^{36}\text{ times}}$.

(c) If $k_e\frac{q_1q_2}{r^2} = G\frac{m_1m_2}{r^2}$ with $q_1 = q_2 = q$ and $m_1 = m_2 = m$, then

$$\frac{q}{m} = \sqrt{\frac{G}{k_e}} = \sqrt{\frac{6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2}} = \boxed{8.61\times10^{-11}\text{ C/kg}}$$

P23.6 We find the equal-magnitude charges on both spheres:

$$F = k_e\frac{q_1q_2}{r^2} = k_e\frac{q^2}{r^2} \quad \text{so} \quad q = r\sqrt{\frac{F}{k_e}} = (1.00\text{ m})\sqrt{\frac{1.00\times10^4\text{ N}}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2}} = 1.05\times10^{-3}\text{ C}$$

The number of electron transferred is then

$$N_{\text{xfer}} = \frac{1.05\times10^{-3}\text{ C}}{1.60\times10^{-19}\text{ C}/e^-} = 6.59\times10^{15}\text{ electrons}$$

The whole number of electrons in each sphere is

$$N_{\text{tot}} = \left(\frac{10.0\text{ g}}{107.87\text{ g/mol}}\right)\left(6.02\times10^{23}\text{ atoms/mol}\right)\left(47\ e^-/\text{atom}\right) = 2.62\times10^{24}\ e^-$$

The fraction transferred is then

$$f = \frac{N_{\text{xfer}}}{N_{\text{tot}}} = \left(\frac{6.59\times10^{15}}{2.62\times10^{24}}\right) = \boxed{2.51\times10^{-9}} = 2.51\text{ charges in every billion}$$

P23.7 $F_1 = k_e \frac{q_1 q_2}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(7.00\times10^{-6}\ \text{C})(2.00\times10^{-6}\ \text{C})}{(0.500\ \text{m})^2} = 0.503\ \text{N}$

$$F_2 = k_e \frac{q_1 q_2}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(7.00\times10^{-6}\ \text{C})(4.00\times10^{-6}\ \text{C})}{(0.500\ \text{m})^2} = 1.01\ \text{N}$$

$$F_x = 0.503\cos 60.0° + 1.01\cos 60.0° = 0.755\ \text{N}$$

$$F_y = 0.503\sin 60.0° - 1.01\sin 60.0° = -0.436\ \text{N}$$

$$\vec{\mathbf{F}} = (0.755\ \text{N})\hat{\mathbf{i}} - (0.436\ \text{N})\hat{\mathbf{j}} = \boxed{0.872\ \text{N at an angle of } 330°}$$

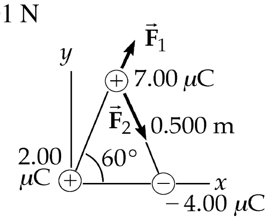


FIG. P23.7

P23.8 Let the third bead have charge Q and be located distance x from the left end of the rod. This bead will experience a net force given by

$$\vec{\mathbf{F}} = \frac{k_e(3q)Q}{x^2}\hat{\mathbf{i}} + \frac{k_e(q)Q}{(d-x)^2}(-\hat{\mathbf{i}})$$

The net force will be zero if $\frac{3}{x^2} = \frac{1}{(d-x)^2}$, or $d - x = \frac{x}{\sqrt{3}}$.

This gives an equilibrium position of the third bead of $x = \boxed{0.634d}$.

The equilibrium is $\boxed{\text{stable if the third bead has positive charge}}$.

P23.9 (a) The force is one of $\boxed{\text{attraction}}$. The distance r in Coulomb's law is the distance between centers. The magnitude of the force is

$$F = \frac{k_e q_1 q_2}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(12.0\times10^{-9}\ \text{C})(18.0\times10^{-9}\ \text{C})}{(0.300\ \text{m})^2} = \boxed{2.16\times10^{-5}\ \text{N}}$$

(b) The net charge of -6.00×10^{-9} C will be equally split between the two spheres, or -3.00×10^{-9} C on each. The force is one of $\boxed{\text{repulsion}}$, and its magnitude is

$$F = \frac{k_e q_1 q_2}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.00\times10^{-9}\ \text{C})(3.00\times10^{-9}\ \text{C})}{(0.300\ \text{m})^2} = \boxed{8.99\times10^{-7}\ \text{N}}$$

P23.10 The top charge exerts a force on the negative charge $\frac{k_e qQ}{(d/2)^2 + x^2}$ which is directed upward and to the left, at an angle of $\tan^{-1}\left(\frac{d}{2x}\right)$ to the x-axis. The two positive charges together exert force

$$\left(\frac{2k_e qQ}{(d^2/4 + x^2)}\right)\left(\frac{(-x)\hat{\mathbf{i}}}{(d^2/4 + x^2)^{1/2}}\right) = m\vec{\mathbf{a}} \text{ or for } x << \frac{d}{2},\quad \vec{\mathbf{a}} \approx \frac{-2k_e qQ}{md^3/8}\vec{\mathbf{x}}$$

(a) The acceleration is equal to a negative constant times the excursion from equilibrium, as in $\vec{\mathbf{a}} = -\omega^2\vec{\mathbf{x}}$, so we have Simple Harmonic Motion with $\omega^2 = \frac{16k_e qQ}{md^3}$.

$T = \frac{2\pi}{\omega} = \boxed{\frac{\pi}{2}\sqrt{\frac{md^3}{k_e qQ}}}$, where m is the mass of the object with charge $-Q$.

(b) $v_{max} = \omega A = \boxed{4a\sqrt{\frac{k_e qQ}{md^3}}}$

P23.11 (a) $F = \frac{k_e e^2}{r^2} = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\frac{\left(1.60 \times 10^{-19} \text{ C}\right)^2}{\left(0.529 \times 10^{-10} \text{ m}\right)^2} = \boxed{8.22 \times 10^{-8} \text{ N}}$ toward the other particle

(b) We have $F = \frac{mv^2}{r}$ from which

$$v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{8.22 \times 10^{-8} \text{ N}\left(0.529 \times 10^{-10} \text{ m}\right)}{9.11 \times 10^{-31} \text{ kg}}} = \boxed{2.19 \times 10^6 \text{ m/s}}$$

Section 23.4 **The Electric Field**

P23.12 The point is designated in the sketch. The magnitudes of the electric fields, E_1 (due to the -2.50×10^{-6} C charge) and E_2 (due to the 6.00×10^{-6} C charge), are

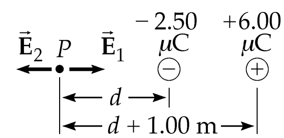


FIG. P23.12

$$E_1 = \frac{k_e q}{r^2} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(2.50 \times 10^{-6} \text{ C}\right)}{d^2} \quad (1)$$

$$E_2 = \frac{k_e q}{r^2} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(6.00 \times 10^{-6} \text{ C}\right)}{\left(d + 1.00 \text{ m}\right)^2} \quad (2)$$

Equate the right sides of (1) and (2)

to get $\left(d + 1.00 \text{ m}\right)^2 = 2.40d^2$

or $d + 1.00 \text{ m} = \pm 1.55d$

which yields $d = 1.82 \text{ m}$

or $d = -0.392 \text{ m}$

The negative value for d is unsatisfactory because that locates a point between the charges where both fields are in the same direction.

Thus, $d = \boxed{1.82 \text{ m to the left of the} - 2.50\ \mu\text{C charge}}$

P23.13 For equilibrium, $\vec{\mathbf{F}}_e = -\vec{\mathbf{F}}_g$

or $q\vec{\mathbf{E}} = -mg\left(-\hat{\mathbf{j}}\right)$

Thus, $\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}}$

(a) $\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}} = \frac{\left(9.11\times10^{-31}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{\left(-1.60\times10^{-19}\text{ C}\right)}\hat{\mathbf{j}} = \boxed{-\left(5.58\times10^{-11}\text{ N/C}\right)\hat{\mathbf{j}}}$

(b) $\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}} = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{\left(1.60\times10^{-19}\text{ C}\right)}\hat{\mathbf{j}} = \boxed{\left(1.02\times10^{-7}\text{ N/C}\right)\hat{\mathbf{j}}}$

P23.14 $F = k_e\frac{q_1q_2}{r^2} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2\left(6.02\times10^{23}\right)^2}{\left[2\left(6.37\times10^6\text{ m}\right)\right]^2} = \boxed{514\text{ kN}}$

***P23.15** The first charge creates at the origin field $\frac{k_eQ}{a^2}$ to the right.

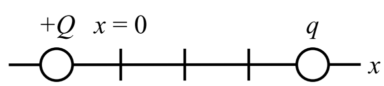


FIG. P23.15

Both charges are on the x axis, so the total field cannot have a vertical comonent, but it can be either to the right or to the left.

$\boxed{\text{If the total field at the origin is to the right, then}}$ q must be negative:

$\frac{k_eQ}{a^2}\hat{\mathbf{i}} + \frac{k_eq}{(3a)^2}\left(-\hat{\mathbf{i}}\right) = \frac{2k_eQ}{a^2}\hat{\mathbf{i}}$ $\quad\boxed{q = -9Q}$

$\boxed{\text{In the alternative, if the total field at the origin is to the left,}}$

$\frac{k_eQ}{a^2}\hat{\mathbf{i}} + \frac{k_eq}{9a^2}\left(-\hat{\mathbf{i}}\right) = \frac{2k_eQ}{a^2}\left(-\hat{\mathbf{i}}\right)$ $\quad\boxed{q = +27Q}$

P23.16 (a) $E = \frac{k_eq}{r^2} = \frac{\left(8.99\times10^9\right)\left(2.00\times10^{-6}\right)}{(1.12)^2} = 14\,400\text{ N/C}$

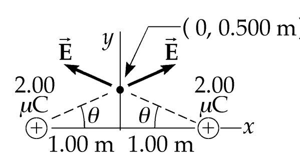


FIG. P23.16

$E_x = 0$ and $E_y = 2(14\,400)\sin 26.6° = 1.29\times10^4\text{ N/C}$

so $\boxed{\vec{\mathbf{E}} = 1.29\times10^4\,\hat{\mathbf{j}}\text{ N/C}}$

(b) $\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \left(-3.00\times10^{-6}\right)\left(1.29\times10^4\,\hat{\mathbf{j}}\right) = \boxed{-3.86\times10^{-2}\,\hat{\mathbf{j}}\text{ N}}$

P23.17 (a) $\vec{\mathbf{E}} = \frac{k_eq_1}{r_1^2}\hat{\mathbf{r}}_1 + \frac{k_eq_2}{r_2^2}\hat{\mathbf{r}}_2 + \frac{k_eq_3}{r_3^2}\hat{\mathbf{r}}_3 = \frac{k_e(2q)}{a^2}\hat{\mathbf{i}} + \frac{k_e(3q)}{2a^2}\left(\hat{\mathbf{i}}\cos 45.0° + \hat{\mathbf{j}}\sin 45.0°\right) + \frac{k_e(4q)}{a^2}\hat{\mathbf{j}}$

$\vec{\mathbf{E}} = 3.06\frac{k_eq}{a^2}\hat{\mathbf{i}} + 5.06\frac{k_eq}{a^2}\hat{\mathbf{j}} = \boxed{5.91\frac{k_eq}{a^2}\text{ at }58.8°}$

(b) $\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \boxed{5.91\frac{k_eq^2}{a^2}\text{ at }58.8°}$

P23.18 The electric field at any point x has the x-component

$$E=-\frac{k_eq}{(x-a)^2}+\frac{k_eq}{(x-(-a))^2}=-\frac{k_eq(4ax)}{(x^2-a^2)^2}$$

When x is much, much greater than a, we find $E\approx\boxed{-\frac{4a(k_eq)}{x^3}}$.

P23.19 (a) One of the charges creates at P a field $\vec{\mathbf{E}}=\frac{k_eQ/n}{R^2+x^2}$ at an angle θ to the x-axis as shown.

When all the charges produce field, for $n>1$, the components perpendicular to the x-axis add to zero.

The total field is $\frac{nk_e(Q/n)\hat{\mathbf{i}}}{R^2+x^2}\cos\theta=\boxed{\frac{k_eQx\hat{\mathbf{i}}}{(R^2+x^2)^{3/2}}}$.

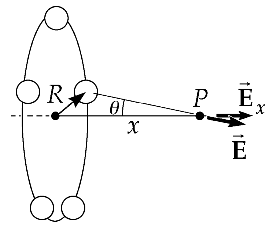


FIG. P23.19

(b) A circle of charge corresponds to letting n grow beyond all bounds, but the result does not depend on n. Smearing the charge around the circle does not change its amount or its distance from the field point, so it $\boxed{\text{does not change the field}}$.

Section 23.5 **Electric Field of a Continuous Charge Distribution**

P23.20 $E=\int\frac{k_edq}{x^2}$, where $dq=\lambda_0dx$

$$E=k_e\lambda_0\int_{x_0}^{\infty}\frac{dx}{x^2}=k_e\lambda_0\left(-\frac{1}{x}\right)\Bigg|_{x_0}^{\infty}=\boxed{\frac{k_e\lambda_0}{x_0}}$$

$\boxed{\text{The direction is} -\hat{\mathbf{i}} \text{ or left for } \lambda_0>0.}$

P23.21 $$E=\frac{k_e\lambda\ell}{d(\ell+d)}=\frac{k_e(Q/\ell)\ell}{d(\ell+d)}=\frac{k_eQ}{d(\ell+d)}=\frac{(8.99\times10^9)(22.0\times10^{-6})}{(0.290)(0.140+0.290)}$$

$\vec{\mathbf{E}}=\boxed{1.59\times10^6\text{ N/C, directed toward the rod}}$.

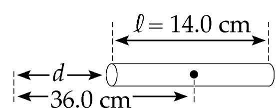


FIG. P23.21

P23.22 $$E=\frac{k_eQx}{(x^2+a^2)^{3/2}}$$

For a maximum, $$\frac{dE}{dx}=Qk_e\left[\frac{1}{(x^2+a^2)^{3/2}}-\frac{3x^2}{(x^2+a^2)^{5/2}}\right]=0$$

$x^2+a^2-3x^2=0$ or $x=\frac{a}{\sqrt{2}}$

Substituting into the expression for E gives

$$E=\frac{k_eQa}{\sqrt{2}\left(\frac{3}{2}a^2\right)^{3/2}}=\frac{k_eQ}{3\frac{\sqrt{3}}{2}a^2}=\boxed{\frac{2k_eQ}{3\sqrt{3}a^2}}=\boxed{\frac{Q}{6\sqrt{3}\pi\,\epsilon_0a^2}}$$

P23.23 $E = \dfrac{k_e x Q}{\left(x^2+a^2\right)^{3/2}} = \dfrac{\left(8.99\times10^9\right)\left(75.0\times10^{-6}\right)x}{\left(x^2+0.100^2\right)^{3/2}} = \dfrac{6.74\times10^5\,x}{\left(x^2+0.010\,0\right)^{3/2}}$ We choose the x axis along the axis of the ring.

(a) At $x = 0.010\,0$ m, $\vec{\mathbf{E}} = 6.64\times10^6\,\hat{\mathbf{i}}$ N/C $=$ $\boxed{6.64\hat{\mathbf{i}}\ \text{MN/C}}$

(b) At $x = 0.050\,0$ m, $\vec{\mathbf{E}} = 2.41\times10^7\,\hat{\mathbf{i}}$ N/C $=$ $\boxed{24.1\hat{\mathbf{i}}\ \text{MN/C}}$

(c) At $x = 0.300$ m, $\vec{\mathbf{E}} = 6.40\times10^6\,\hat{\mathbf{i}}$ N/C $=$ $\boxed{6.40\hat{\mathbf{i}}\ \text{MN/C}}$

(d) At $x = 1.00$ m, $\vec{\mathbf{E}} = 6.64\times10^5\,\hat{\mathbf{i}}$ N/C $=$ $\boxed{0.664\hat{\mathbf{i}}\ \text{MN/C}}$

P23.24 $E = 2\pi k_e \sigma\left(1 - \dfrac{x}{\sqrt{x^2+R^2}}\right)$

$$E = 2\pi\left(8.99\times10^9\right)\left(7.90\times10^{-3}\right)\left(1-\frac{x}{\sqrt{x^2+(0.350)^2}}\right) = 4.46\times10^8\left(1-\frac{x}{\sqrt{x^2+0.123}}\right)$$

(a) At $x = 0.050\,0$ m, $E = 3.83\times10^8$ N/C $=$ $\boxed{383\ \text{MN/C}}$

(b) At $x = 0.100$ m, $E = 3.24\times10^8$ N/C $=$ $\boxed{324\ \text{MN/C}}$

(c) At $x = 0.500$ m, $E = 8.07\times10^7$ N/C $=$ $\boxed{80.7\ \text{MN/C}}$

(d) At $x = 2.000$ m, $E = 6.68\times10^8$ N/C $=$ $\boxed{6.68\ \text{MN/C}}$

P23.25 (a) From the Example in the chapter text, $E = 2\pi k_e \sigma\left(1 - \dfrac{x}{\sqrt{x^2+R^2}}\right)$

$$\sigma = \frac{Q}{\pi R^2} = 1.84\times10^{-3}\ \text{C/m}^2$$

$$E = \left(1.04\times10^8\ \text{N/C}\right)(0.900) = 9.36\times10^7\ \text{N/C} = \boxed{93.6\ \text{MN/C}}$$

approximation: $E = 2\pi k_e \sigma = \boxed{104\ \text{MN/C}\ \text{(about 11\% high)}}$

(b) $E = \left(1.04\times10^8\ \text{N/C}\right)\left(1-\dfrac{30.0\ \text{cm}}{\sqrt{30.0^2+3.00^2}\ \text{cm}}\right) = \left(1.04\times10^8\ \text{N/C}\right)(0.004\,96) = \boxed{0.516\ \text{MN/C}}$

approximation: $E = k_e\dfrac{Q}{r^2} = \left(8.99\times10^9\right)\dfrac{5.20\times10^{-6}}{(0.30)^2} = \boxed{0.519\ \text{MN/C}\ \text{(about 0.6\% high)}}$

P23.26 The electric field at a distance x is $E_x = 2\pi k_e\sigma\left[1-\dfrac{x}{\sqrt{x^2+R^2}}\right]$

This is equivalent to $E_x = 2\pi k_e\sigma\left[1-\dfrac{1}{\sqrt{1+R^2/x^2}}\right]$

For large x, $\dfrac{R^2}{x^2} << 1$ and $\sqrt{1+\dfrac{R^2}{x^2}} \approx 1+\dfrac{R^2}{2x^2}$

so
$$E_x = 2\pi k_e\sigma\left(1-\frac{1}{\left[1+R^2/\left(2x^2\right)\right]}\right)$$
$$= 2\pi k_e\sigma\frac{\left(1+R^2/\left(2x^2\right)-1\right)}{\left[1+R^2/\left(2x^2\right)\right]}$$

Substitute $\sigma = \dfrac{Q}{\pi R^2}$, $E_x = \dfrac{k_eQ\left(1/x^2\right)}{\left[1+R^2/\left(2x^2\right)\right]} = k_eQ\left(x^2+\dfrac{R^2}{2}\right)$

But for $x >> R$, $\dfrac{1}{x^2+R^2/2} \approx \dfrac{1}{x^2}$, so $\boxed{E_x \approx \dfrac{k_eQ}{x^2} \text{ for a disk at large distances}}$

P23.27 Due to symmetry $E_y = \int dE_y = 0$, and $E_x = \int dE\sin\theta = k_e\int\dfrac{dq\sin\theta}{r^2}$

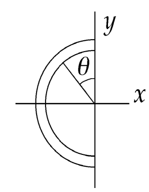


FIG. P23.27

where $dq = \lambda ds = \lambda r d\theta$,

so that $E_x = \dfrac{k_e\lambda}{r}\int_0^{\pi}\sin\theta d\theta = \dfrac{k_e\lambda}{r}(-\cos\theta)\Big|_0^{\pi} = \dfrac{2k_e\lambda}{r}$

where $\lambda = \dfrac{q}{L}$ and $r = \dfrac{L}{\pi}$

Thus, $E_x = \dfrac{2k_eq\pi}{L^2} = \dfrac{2\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(-7.50\times10^{-6} \text{ C}\right)\pi}{(0.140 \text{ m})^2}$

Solving, $E_x = -2.16\times10^7$ N/C

Since the rod has a negative charge, $\vec{\mathbf{E}} = \left(-2.16\times10^7\,\hat{\mathbf{i}}\right)$ N/C $=$ $\boxed{-21.6\hat{\mathbf{i}} \text{ MN/C}}$.

P23.28 (a) We define $x = 0$ at the point where we are to find the field. One ring, with thickness dx, has charge $\frac{Qdx}{h}$ and produces, at the chosen point, a field

$$d\vec{\mathbf{E}} = \frac{k_e x}{(x^2 + R^2)^{3/2}} \frac{Qdx}{h} \hat{\mathbf{i}}$$

The total field is

$$\vec{\mathbf{E}} = \int_{\text{all charge}} d\mathbf{E} = \int_d^{d+h} \frac{k_e Qxdx}{h(x^2 + R^2)^{3/2}} \hat{\mathbf{i}} = \frac{k_e Q\hat{\mathbf{i}}}{2h} \int_{x=d}^{d+h} (x^2 + R^2)^{-3/2} 2xdx$$

$$\vec{\mathbf{E}} = \frac{k_e Q\hat{\mathbf{i}}}{2h} \frac{(x^2 + R^2)^{-1/2}}{(-1/2)}\Bigg|_{x=d}^{d+h} = \boxed{\frac{k_e Q\hat{\mathbf{i}}}{h}\left[\frac{1}{(d^2 + R^2)^{1/2}} - \frac{1}{((d+h)^2 + R^2)^{1/2}}\right]}$$

(b) Think of the cylinder as a stack of disks, each with thickness dx, charge $\frac{Qdx}{h}$, and charge-per-area $\sigma = \frac{Qdx}{\pi R^2 h}$. One disk produces a field

$$d\vec{\mathbf{E}} = \frac{2\pi k_e Qdx}{\pi R^2 h}\left(1 - \frac{x}{(x^2 + R^2)^{1/2}}\right)\hat{\mathbf{i}}$$

So, $$\vec{\mathbf{E}} = \int_{\text{all charge}} d\vec{\mathbf{E}} = \int_{x=d}^{d+h} \frac{2k_e Qdx}{R^2 h}\left(1 - \frac{x}{(x^2 + R^2)^{1/2}}\right)\hat{\mathbf{i}}$$

$$\vec{\mathbf{E}} = \frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[\int_d^{d+h} dx - \frac{1}{2}\int_{x=d}^{d+h} (x^2 + R^2)^{-1/2} 2xdx\right] = \frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[x\Big|_d^{d+h} - \frac{1}{2}\frac{(x^2 + R^2)^{1/2}}{1/2}\Bigg|_d^{d+h}\right]$$

$$\vec{\mathbf{E}} = \frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[d + h - d - ((d+h)^2 + R^2)^{1/2} + (d^2 + R^2)^{1/2}\right]$$

$$\vec{\mathbf{E}} = \boxed{\frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[h + (d^2 + R^2)^{1/2} - ((d+h)^2 + R^2)^{1/2}\right]}$$

P23.29 (a) The electric field at point P due to each element of length dx is $dE = \frac{k_e dq}{x^2 + y^2}$ and is directed along the line joining the element to point P. By symmetry,

$E_x = \int dE_x = 0$ and since $dq = \lambda dx$,

$E = E_y = \int dE_y = \int dE \cos\theta$ where $\cos\theta = \frac{y}{\sqrt{x^2 + y^2}}$

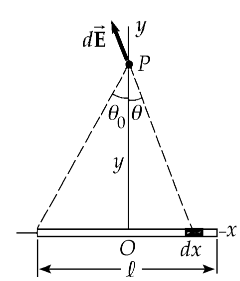


FIG. P23.29

Therefore, $$E = 2k_e \lambda y \int_0^{\ell/2} \frac{dx}{(x^2 + y^2)^{3/2}} = \boxed{\frac{2k_e \lambda \sin\theta_0}{y}}$$

(b) For a bar of infinite length, $\theta_0 = 90°$ and $E_y = \boxed{\frac{2k_e \lambda}{y}}$

P23.30 (a) The whole surface area of the cylinder is $A = 2\pi r^2 + 2\pi rL = 2\pi r(r+L)$.

$$Q = \sigma A = (15.0\times10^{-9}\text{ C/m}^2)2\pi(0.025\,0\text{ m})[0.025\,0\text{ m} + 0.060\,0\text{ m}] = \boxed{2.00\times10^{-10}\text{ C}}$$

(b) For the curved lateral surface only, $A = 2\pi rL$.

$$Q = \sigma A = (15.0\times10^{-9}\text{ C/m}^2)[2\pi(0.025\,0\text{ m})(0.060\,0\text{ m})] = \boxed{1.41\times10^{-10}\text{ C}}$$

(c) $Q = \rho V = \rho\pi r^2 L = (500\times10^{-9}\text{ C/m}^3)[\pi(0.025\,0\text{ m})^2(0.060\,0\text{ m})] = \boxed{5.89\times10^{-11}\text{ C}}$

P23.31 (a) Every object has the same volume, $V = 8(0.030\,0\text{ m})^3 = 2.16\times10^{-4}\text{ m}^3$.

For each, $Q = \rho V = (400\times10^{-9}\text{ C/m}^3)(2.16\times10^{-4}\text{ m}^3) = \boxed{8.64\times10^{-11}\text{ C}}$

(b) We must count the 9.00 cm^2 squares painted with charge:

(i) $6\times4 = 24$ squares

$$Q = \sigma A = (15.0\times10^{-9}\text{ C/m}^2)24.0(9.00\times10^{-4}\text{ m}^2) = \boxed{3.24\times10^{-10}\text{ C}}$$

(ii) 34 squares exposed

$$Q = \sigma A = (15.0\times10^{-9}\text{ C/m}^2)24.0(9.00\times10^{-4}\text{ m}^2) = \boxed{3.24\times10^{-10}\text{ C}}$$

(iii) 34 squares

$$Q = \sigma A = (15.0\times10^{-9}\text{ C/m}^2)34.0(9.00\times10^{-4}\text{ m}^2) = \boxed{4.59\times10^{-10}\text{ C}}$$

(iv) 32 squares

$$Q = \sigma A = (15.0\times10^{-9}\text{ C/m}^2)32.0(9.00\times10^{-4}\text{ m}^2) = \boxed{4.32\times10^{-10}\text{ C}}$$

(c) (i) total edge length: $\ell = 24\times(0.030\,0\text{ m})$

$$Q = \lambda\ell = (80.0\times10^{-12}\text{ C/m})24\times(0.030\,0\text{ m}) = \boxed{5.76\times10^{-11}\text{ C}}$$

(ii) $Q = \lambda\ell = (80.0\times10^{-12}\text{ C/m})44\times(0.030\,0\text{ m}) = \boxed{1.06\times10^{-10}\text{ C}}$

(iii) $Q = \lambda\ell = (80.0\times10^{-12}\text{ C/m})64\times(0.030\,0\text{ m}) = \boxed{1.54\times10^{-10}\text{ C}}$

(iv) $Q = \lambda\ell = (80.0\times10^{-12}\text{ C/m})40\times(0.030\,0\text{ m}) = \boxed{0.960\times10^{-10}\text{ C}}$

Section 23.6 **Electric Field Lines**

P23.32

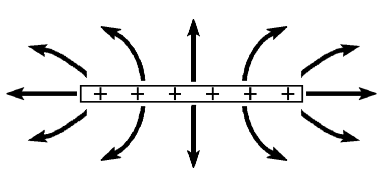


FIG. P23.32

P23.33

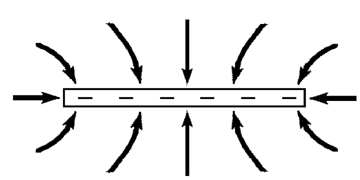


FIG. P23.33

P23.34 (a) $\dfrac{q_1}{q_2} = \dfrac{-6}{18} = \boxed{-\dfrac{1}{3}}$

(b) $\boxed{q_1 \text{ is negative, } q_2 \text{ is positive}}$

P23.35 (a) The electric field has the general appearance shown. It is zero $\boxed{\text{at the center}}$, where (by symmetry) one can see that the three charges individually produce fields that cancel out.

In addition to the center of the triangle, the electric field lines in the second figure to the right indicate three other points near the middle of each leg of the triangle where $E = 0$, but they are more difficult to find mathematically.

(b) You may need to review vector addition in Chapter Three. The electric field at point P can be found by adding the electric field vectors due to each of the two lower point charges: $\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2$.

The electric field from a point charge is $\vec{\mathbf{E}} = k_e \dfrac{q}{r^2}\hat{\mathbf{r}}$.

As shown in the solution figure at right,

$\vec{\mathbf{E}}_1 = k_e \dfrac{q}{a^2}$ to the right and upward at 60°

$\vec{\mathbf{E}}_2 = k_e \dfrac{q}{a^2}$ to the left and upward at 60°

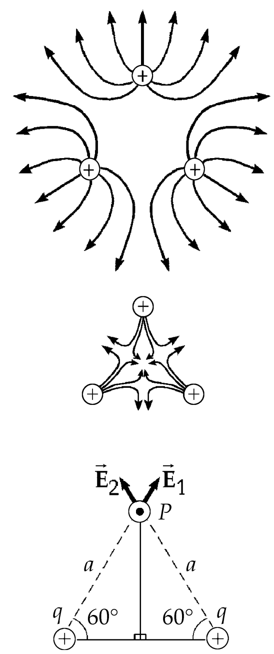


FIG. P23.35

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = k_e \frac{q}{a^2}\left[\left(\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}}\right) + \left(-\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}}\right)\right] = k_e \frac{q}{a^2}\left[2\left(\sin 60°\hat{\mathbf{j}}\right)\right]$$

$$= \boxed{1.73 k_e \frac{q}{a^2}\hat{\mathbf{j}}}$$

Section 23.7 **Motion of a Charged Particle in a Uniform Electric Field**

P23.36 (a) $|a| = \dfrac{qE}{m} = \dfrac{\left(1.602\times10^{-19}\right)\left(6.00\times10^{5}\right)}{\left(1.67\times10^{-27}\right)} = 5.76\times10^{13} \text{ m/s so } \vec{\mathbf{a}} = \boxed{-5.76\times10^{13}\,\hat{\mathbf{i}} \text{ m/s}^2}$

(b) $v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$

$0 = v_i^2 + 2\left(-5.76\times10^{13}\right)\left(0.070\,0\right)$ $\qquad \boxed{\vec{\mathbf{v}}_i = 2.84\times10^{6}\,\hat{\mathbf{i}} \text{ m/s}}$

(c) $v_f = v_i + at$

$0 = 2.84\times10^{6} + \left(-5.76\times10^{13}\right)t$ $\qquad t = \boxed{4.93\times10^{-8} \text{ s}}$

P23.37 (a) $a = \frac{qE}{m} = \frac{1.602\times10^{-19}(640)}{1.67\times10^{-27}} = \boxed{6.14\times10^{10}\ \text{m/s}^2}$

(b) $v_f = v_i + at \qquad 1.20\times10^6 = \left(6.14\times10^{10}\right)t \qquad t = \boxed{1.95\times10^{-5}\ \text{s}}$

(c) $x_f - x_i = \frac{1}{2}\left(v_i + v_f\right)t \qquad x_f = \frac{1}{2}\left(1.20\times10^6\right)\left(1.95\times10^{-5}\right) = \boxed{11.7\ \text{m}}$

(d) $K = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.67\times10^{-27}\ \text{kg}\right)\left(1.20\times10^6\ \text{m/s}\right)^2 = \boxed{1.20\times10^{-15}\ \text{J}}$

P23.38 The particle feels a constant force: $\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \left(1\times10^{-6}\ \text{C}\right)\left(2\ 000\ \text{N/C}\right)\left(-\hat{\mathbf{j}}\right) = 2\times10^{-3}\ \text{N}\left(-\hat{\mathbf{j}}\right)$

and moves with acceleration: $\vec{\mathbf{a}} = \frac{\sum\vec{\mathbf{F}}}{m} = \frac{\left(2\times10^{-3}\ \text{kg}\cdot\text{m/s}^2\right)\left(-\hat{\mathbf{j}}\right)}{2\times10^{-16}\ \text{kg}} = \left(1\times10^{13}\ \text{m/s}^2\right)\left(-\hat{\mathbf{j}}\right)$

Note that the gravitational force is on the order of a trillion times smaller than the electrical force exerted on the particle. The particle's x-component of velocity is constant at $\left(1.00\times10^5\ \text{m/s}\right)\cos 37° = 7.99\times10^4\ \text{m/s}$. Thus it moves in a parabola opening downward. The maximum height it attains above the bottom plate is described by

$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$: $\qquad 0 = \left(6.02\times10^4\ \text{m/s}\right)^2 - \left(2\times10^{13}\ \text{m/s}^2\right)\left(y_f - 0\right)$

$y_f = 1.81\times10^{-4}\ \text{m}$

Since this is less than 10 mm, the particle does not strike the top plate, but moves in a symmetric parabola and strikes the bottom plate after a time given by

$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 \qquad 0 = 0 + \left(6.02\times10^4\ \text{m/s}\right)t + \frac{1}{2}\left(-1\times10^{13}\ \text{m/s}^2\right)t^2$

since $t > 0$, $\qquad t = 1.20\times10^{-8}\ \text{s}$

The particle's range is $\qquad x_f = x_i + v_x t = 0 + \left(7.99\times10^4\ \text{m/s}\right)\left(1.20\times10^{-8}\ \text{s}\right) = 9.61\times10^{-4}\ \text{m}.$

In sum,

The particle strikes the negative plate after moving in a parabola with a height of 0.181 mm and a width of 0.961 mm.

P23.39 The required electric field will be $\boxed{\text{in the direction of motion}}$.

Work done = ΔK

so, $\qquad -Fd = -\frac{1}{2}mv_i^2$ (since the final velocity = 0)

which becomes $\qquad eEd = K$

and $\qquad E = \boxed{\frac{K}{ed}}$

P23.40 $v_i = 9.55\times10^3 \text{ m/s}$

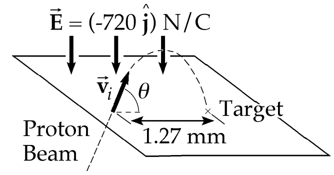


FIG. P23.40

(a) $$a_y = \frac{eE}{m} = \frac{(1.60\times10^{-19})(720)}{(1.67\times10^{-27})} = 6.90\times10^{10}\text{ m/s}^2$$

From the large magnitude of this vertical acceleration, we can note that the gravitational force on the particle is negligible by comparison to the electrical force.

$$R = \frac{v_i^2\sin 2\theta}{a_y} = 1.27\times10^{-3}\text{ m so that}$$

$$\frac{(9.55\times10^3)^2\sin 2\theta}{6.90\times10^{10}} = 1.27\times10^{-3}$$

$\sin 2\theta = 0.961$ $\quad\theta = \boxed{36.9°}$ $\quad 90.0° - \theta = \boxed{53.1°}$

(b) $t = \dfrac{R}{v_{ix}} = \dfrac{R}{v_i\cos\theta}$ If $\theta = 36.9°$, $t = \boxed{167\text{ ns}}$. If $\theta = 53.1°$, $t = \boxed{221\text{ ns}}$.

P23.41 (a) $$t = \frac{x}{v_x} = \frac{0.050\,0}{4.50\times10^5} = 1.11\times10^{-7}\text{ s} = \boxed{111\text{ ns}}$$

(b) $$a_y = \frac{qE}{m} = \frac{(1.602\times10^{-19})(9.60\times10^3)}{(1.67\times10^{-27})} = 9.21\times10^{11}\text{ m/s}^2$$

$$y_f - y_i = v_{yi}t + \frac{1}{2}a_y t^2:\qquad y_f = \frac{1}{2}(9.21\times10^{11})(1.11\times10^{-7})^2 = 5.68\times10^{-3}\text{ m} = \boxed{5.68\text{ mm}}$$

(c) $v_x = \boxed{4.50\times10^5\text{ m/s}}$ $\qquad v_{yf} = v_{yi} + a_y t = (9.21\times10^{11})(1.11\times10^{-7}) = \boxed{1.02\times10^5\text{ m/s}}$

Additional Problems

P23.42 The two given charges exert equal-size forces of attraction on each other. If a third charge, positive or negative, were placed between them they could not be in equilibrium. If the third charge were at a point $x > 15$ cm, it would exert a stronger force on the 45 μC than on the $-12\ \mu$C, and could not produce equilibrium for both. Thus the third charge must be at $x = -d < 0$. Its equilibrium requires

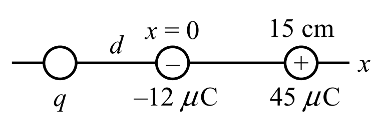


FIG. P23.42

$$\frac{k_e q(12\ \mu\text{C})}{d^2} = \frac{k_e q(45\ \mu\text{C})}{(15\text{ cm} + d)^2}\qquad \left(\frac{15\text{ cm} + d}{d}\right)^2 = \frac{45}{12} = 3.75$$

$15\text{ cm} + d = 1.94d$ $\qquad d = 16.0$ cm

The third charge is at $\boxed{x = -16.0\text{ cm}}$. The equilibrium of the $-12\ \mu$C requires

$$\frac{k_e q(12\ \mu\text{C})}{(16.0\text{ cm})^2} = \frac{k_e(45\ \mu\text{C})12\ \mu\text{C}}{(15\text{ cm})^2}\qquad \boxed{q = 51.3\ \mu\text{C}}$$

All six individual forces are now equal in magnitude, so we have equilibrium as required, and this is the only solution.

P23.43 The proton moves with acceleration $|a_p| = \dfrac{qE}{m} = \dfrac{(1.60\times10^{-19}\ \text{C})(640\ \text{N/C})}{1.673\times10^{-27}\ \text{kg}} = 6.13\times10^{10}\ \text{m/s}^2$

while the e^- has acceleration $|a_e| = \dfrac{(1.60\times10^{-19}\ \text{C})(640\ \text{N/C})}{9.110\times10^{-31}\ \text{kg}} = 1.12\times10^{14}\ \text{m/s}^2 = 1\,836a_p$

(a) We want to find the distance traveled by the proton (i.e., $d = \frac{1}{2}a_p t^2$), knowing:

$$4.00\ \text{cm} = \frac{1}{2}a_p t^2 + \frac{1}{2}a_e t^2 = 1\,837\left(\frac{1}{2}a_p t^2\right)$$

Thus, $d = \dfrac{1}{2}a_p t^2 = \dfrac{4.00\ \text{cm}}{1\,837} = \boxed{21.8\ \mu\text{m}}$

(b) The distance from the positive plate to where the meeting occurs equals the distance the sodium ion travels (i.e., $d_{\text{Na}} = \frac{1}{2}a_{\text{Na}}t^2$). This is found from:

$4.00\ \text{cm} = \dfrac{1}{2}a_{\text{Na}}t^2 + \dfrac{1}{2}a_{\text{Cl}}t^2$: $\quad 4.00\ \text{cm} = \dfrac{1}{2}\left(\dfrac{eE}{22.99\ \text{u}}\right)t^2 + \dfrac{1}{2}\left(\dfrac{eE}{35.45\ \text{u}}\right)t^2$

This may be written as $\quad 4.00\ \text{cm} = \dfrac{1}{2}a_{\text{Na}}t^2 + \dfrac{1}{2}(0.649a_{\text{Na}})t^2 = 1.65\left(\dfrac{1}{2}a_{\text{Na}}t^2\right)$

so $\quad d_{\text{Na}} = \dfrac{1}{2}a_{\text{Na}}t^2 = \dfrac{4.00\ \text{cm}}{1.65} = \boxed{2.43\ \text{cm}}$

P23.44 (a) The field, E_1, due to the 4.00×10^{-9} C charge is in the $-x$ direction.

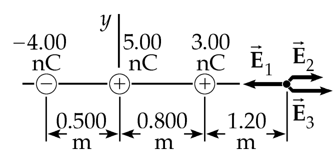


FIG. P23.44 (a)

$$\vec{\mathbf{E}}_1 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-4.00\times10^{-9}\ \text{C})}{(2.50\ \text{m})^2}\hat{\mathbf{i}}$$
$$= -5.75\hat{\mathbf{i}}\ \text{N/C}$$

Likewise, E_2 and E_3, due to the 5.00×10^{-9} C charge and the 3.00×10^{-9} C charge, are

$$\vec{\mathbf{E}}_2 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-9}\ \text{C})}{(2.00\ \text{m})^2}\hat{\mathbf{i}} = 11.2\ \text{N/C}\ \hat{\mathbf{i}}$$

$$\vec{\mathbf{E}}_3 = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-9}\ \text{C})}{(1.20\ \text{m})^2}\hat{\mathbf{i}} = 18.7\ \text{N/C}\ \hat{\mathbf{i}}$$

$\vec{\mathbf{E}}_R = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3 = \boxed{24.2\ \text{N/C}}$ in $+x$ direction.

continued on next page

(b) $\vec{\mathbf{E}}_1 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (-8.46 \text{ N/C})\left(0.243\hat{\mathbf{i}} + 0.970\hat{\mathbf{j}}\right)$

$\vec{\mathbf{E}}_2 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (11.2 \text{ N/C})\left(+\hat{\mathbf{j}}\right)$

$\vec{\mathbf{E}}_3 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (5.81 \text{ N/C})\left(-0.371\hat{\mathbf{i}} + 0.928\hat{\mathbf{j}}\right)$

$E_x = E_{1x} + E_{3x} = -4.21\hat{\mathbf{i}} \text{ N/C}$ $\quad E_y = E_{1y} + E_{2y} + E_{3y} = 8.43\hat{\mathbf{j}} \text{ N/C}$

$E_R = \boxed{9.42 \text{ N/C}}$ $\quad \theta = \boxed{63.4° \text{ above } -x \text{ axis}}$

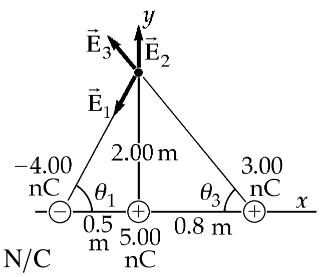


FIG. P23.44 (b)

P23.45 (a) Let us sum force components to find

$\sum F_x = qE_x - T\sin\theta = 0$, and $\sum F_y = qE_y + T\cos\theta - mg = 0$

Combining these two equations, we get

$$q = \frac{mg}{\left(E_x\cot\theta + E_y\right)} = \frac{\left(1.00\times10^{-3}\right)(9.80)}{(3.00\cot 37.0° + 5.00)\times10^5} = 1.09\times10^{-8} \text{ C}$$

$= \boxed{10.9 \text{ nC}}$

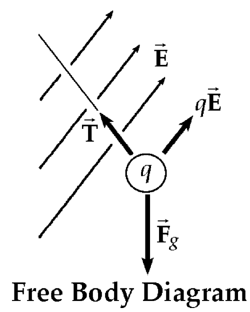


FIG. P23.45

(b) From the two equations for $\sum F_x$ and $\sum F_y$ we also find

$$T = \frac{qEx}{\sin 37.0°} = 5.44\times10^{-3} \text{ N} = \boxed{5.44 \text{ mN}}$$

P23.46 This is the general version of the preceding problem. The known quantities are A, B, m, g, and θ. The unknowns are q and T.

The approach to this problem should be the same as for the last problem, but without numbers to substitute for the variables. Likewise, we can use the free body diagram given in the solution to problem 45.

Again, Newton's second law: $\sum F_x = -T\sin\theta + qA = 0$ (1)

and $\sum F_y = +T\cos\theta + qB - mg = 0$ (2)

(a) Substituting $T = \frac{qA}{\sin\theta}$ into Eq. (2), $\frac{qA\cos\theta}{\sin\theta} + qB = mg$

Isolating q on the left, $\boxed{q = \frac{mg}{(A\cot\theta + B)}}$

(b) Substituting this value into Eq. (1), $\boxed{T = \frac{mgA}{(A\cos\theta + B\sin\theta)}}$

If we had solved this general problem first, we would only need to substitute the appropriate values in the equations for q and T to find the numerical results needed for problem 45. If you find this problem more difficult than problem 45, the little list at the first step is useful. It shows what symbols to think of as known data, and what to consider unknown. The list is a guide for deciding what to solve for in the analysis step, and for recognizing when we have an answer.

P23.47 $F=\dfrac{k_e q_1 q_2}{r^2}$: $\tan\theta=\dfrac{15.0}{60.0}$

$\theta=14.0°$

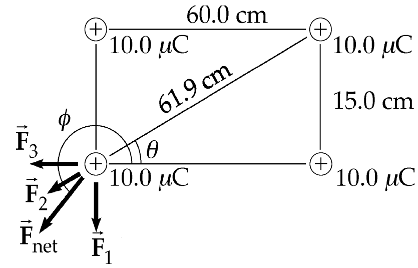


FIG. P23.47

$$F_1=\frac{\left(8.99\times10^{9}\right)\left(10.0\times10^{-6}\right)^2}{(0.150)^2}=40.0\text{ N}$$

$$F_3=\frac{\left(8.99\times10^{9}\right)\left(10.0\times10^{-6}\right)^2}{(0.600)^2}=2.50\text{ N}$$

$$F_2=\frac{\left(8.99\times10^{9}\right)\left(10.0\times10^{-6}\right)^2}{(0.619)^2}=2.35\text{ N}$$

$$F_x=-F_3-F_2\cos14.0°=-2.50-2.35\cos14.0°=-4.78\text{ N}$$

$$F_y=-F_1-F_2\sin14.0°=-40.0-2.35\sin14.0°=-40.6\text{ N}$$

$$F_{\text{net}}=\sqrt{F_x^2+F_y^2}=\sqrt{(4.78)^2+(40.6)^2}=\boxed{40.9\text{ N}}$$

$$\tan\phi=\frac{F_y}{F_x}=\frac{-40.6}{-4.78}$$

$$\phi=\boxed{263°}$$

P23.48 From Figure (a) we have $d\cos30.0°=15.0\text{ cm}$

or $d=\dfrac{15.0\text{ cm}}{\cos30.0°}$

From Figure (b) we have $\theta=\sin^{-1}\left(\dfrac{d}{50.0\text{ cm}}\right)$

$$\theta=\sin^{-1}\left(\frac{15.0\text{ cm}}{50.0\text{ cm}(\cos30.0°)}\right)=20.3°$$

$$\frac{F_q}{mg}=\tan\theta$$

or $F_q=mg\tan20.3°$ (1)

From Figure (c) we have $F_q=2F\cos30.0°$

$$F_q=2\left[\frac{k_e q^2}{(0.300\text{ m})^2}\right]\cos30.0° \quad (2)$$

Combining equations (1) and (2),

$$2\left[\frac{k_e q^2}{(0.300\text{ m})^2}\right]\cos30.0°=mg\tan20.3°$$

$$q^2=\frac{mg(0.300\text{ m})^2\tan20.3°}{2k_e\cos30.0°}$$

$$q^2=\frac{\left(2.00\times10^{-3}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)(0.300\text{ m})^2\tan20.3°}{2\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\cos30.0°}$$

$$q=\sqrt{4.20\times10^{-14}\text{ C}^2}=2.05\times10^{-7}\text{ C}=\boxed{0.205\ \mu\text{C}}$$

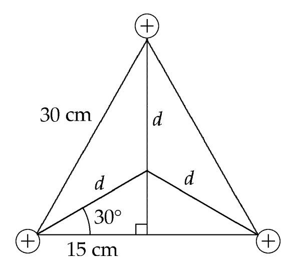


Figure (a)

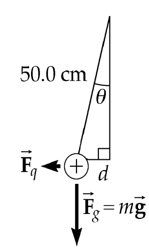


Figure (b)

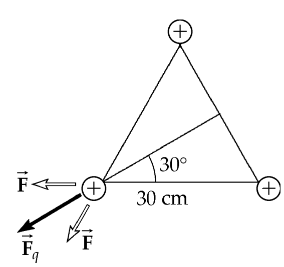


Figure (c)

FIG. P23.48

P23.49 Charge $\frac{Q}{2}$ resides on each of the blocks, which repel as point charges:

$$F = \frac{k_e(Q/2)(Q/2)}{L^2} = k(L - L_i)$$

Solving for Q, $$Q = \boxed{2L\sqrt{\frac{k(L-L_i)}{k_e}}}$$

P23.50 If we place one more charge q at the 29th vertex, the total force on the central charge will add up to zero: $\vec{\mathbf{F}}_{28\text{ charges}} + \frac{k_e qQ}{a^2}$ away from vertex 29 $= 0$ $\quad \vec{\mathbf{F}}_{28\text{ charges}} = \boxed{\frac{k_e qQ}{a^2} \text{ toward vertex 29}}$

P23.51 According to the result of an Example in the chapter text, the left-hand rod creates this field at a distance d from its right-hand end:

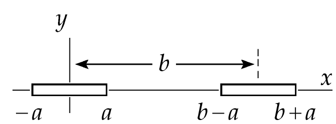


FIG. P23.51

$$E = \frac{k_e Q}{d(2a+d)}$$

$$dF = \frac{k_e QQ}{2a}\frac{dx}{d(d+2a)}$$

$$F = \frac{k_e Q^2}{2a}\int_{x=b-2a}^{b}\frac{dx}{x(x+2a)} = \frac{k_e Q^2}{2a}\left(-\frac{1}{2a}\ln\frac{2a+x}{x}\right)_{b-2a}^{b}$$

$$F = \frac{+k_e Q^2}{4a^2}\left(-\ln\frac{2a+b}{b} + \ln\frac{b}{b-2a}\right) = \frac{k_e Q^2}{4a^2}\ln\frac{b^2}{(b-2a)(b+2a)} = \boxed{\left(\frac{k_e Q^2}{4a^2}\right)\ln\left(\frac{b^2}{b^2-4a^2}\right)}$$

***P23.52** We model the spheres as particles. They have different charges. They exert on each other forces of equal magnitude. They have equal masses, so their strings make equal angles θ with the vertical. The distance r between them is described by $\sin\theta = (r/2)/40$ cm,

so $r = 80\text{ cm}\sin\theta$

Let T represent the string tension. We have

$\Sigma F_x = 0$: $k_e q_1 q_2/r^2 = T\sin\theta$

$\Sigma F_y = 0$: $mg = T\cos\theta$

Divide to eliminate T. $$\frac{k_e q_1 q_2}{r^2 mg} = \tan\theta = \frac{r/2}{\sqrt{(40\text{ cm})^2 - r^2/4}}$$

Cleared of fractions, $$k_e q_1 q_2\sqrt{(80\text{ cm})^2 - r^2} = mgr^3$$

$$8.99\times10^9(\text{N}\cdot\text{m}^2/\text{C}^2)\,300\times10^{-9}\text{C}\,(200\times10^{-9}\text{C})\,\sqrt{(0.8\text{ m})^2 - r^2} = 2.4\times10^{-3}(9.8)\text{ N }r^3$$

$$(0.8\text{ m})^2 - r^2 = 1901\,r^6$$

We home in on a solution by trying values.

r	$0.64 - r^2 - 1901\,r^6$
0	+0.64
0.5	−29.3
0.2	+0.48
0.3	−0.84
0.24	+0.22
0.27	−0.17
0.258	+0.013
0.259	−0.001

Thus the distance to three digits is $\boxed{0.259\text{ m}}$.

***P23.53** $Q = \int \lambda d\ell = \int_{-90.0°}^{90.0°} \lambda_0 \cos\theta R d\theta = \lambda_0 R \sin\theta \Big|_{-90.0°}^{90.0°} = \lambda_0 R[1-(-1)] = 2\lambda_0 R$

$Q = 12.0\ \mu\text{C} = (2\lambda_0)(0.600)\ \text{m} = 12.0\ \mu\text{C} \quad \text{so} \quad \lambda_0 = 10.0\ \mu\text{C/m}$

$$dF_y = \frac{1}{4\pi\epsilon_0}\left(\frac{(3.00\ \mu\text{C})(\lambda d\ell)}{R^2}\right)\cos\theta = \frac{1}{4\pi\epsilon_0}\left(\frac{(3.00\ \mu\text{C})(\lambda_0 \cos^2\theta R d\theta)}{R^2}\right)$$

$$F_y = \int_{-90.0°}^{90.0°} (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.00\times10^{-6}\ \text{C})(10.0\times10^{-6}\ \text{C/m})}{(0.600\ \text{m})}\cos^2\theta d\theta$$

$$F_y = \frac{8.99(30.0)}{0.600}(10^{-3}\ \text{N})\int_{-\pi/2}^{\pi/2}\left(\frac{1}{2}+\frac{1}{2}\cos 2\theta\right)d\theta$$

$$F_y = (0.450\ \text{N})\left(\frac{1}{2}\theta+\frac{1}{4}\sin 2\theta\right)\Big|_{-\pi/2}^{\pi/2} = \boxed{0.707\ \text{N downward}}$$

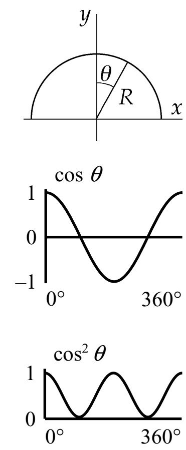


FIG. P23.53

Since the leftward and rightward forces due to the two halves of the semicircle cancel out, $F_x = 0$.

***P23.54** (a) The two charges create fields of equal magnitude, both with outward components along the x axis and with upward and downward y components that add to zero. The net field is then

$$\frac{k_e q}{r^2}\frac{x}{r}\hat{\mathbf{i}} + \frac{k_e q}{r^2}\frac{x}{r}\hat{\mathbf{i}} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2)52\times10^{-9}\ \text{C}\ x\ \hat{\mathbf{i}}}{\text{C}^2((0.25\ \text{m})^2+x^2)^{3/2}}$$

$$= \boxed{\frac{935\ \text{N}\cdot\text{m}^2\ \ x\ \hat{\mathbf{i}}}{\text{C}(0.0625\ \text{m}^2+x^2)^{3/2}}}$$

(b) At $x = 0.36$ m,

$$\vec{\mathbf{E}} = \frac{935\ \text{N}\cdot\text{m}^2\ \ 0.36\ \text{m}\ \hat{\mathbf{i}}}{\text{C}\ (0.0625\ \text{m}^2+(0.36\ \text{m})^2)^{3/2}} = 4.00\ \text{kN/C}\ \hat{\mathbf{i}}$$

(c) We solve $1\,000 = 935\ x\ (0.0625 + x^2)^{-3/2}$ by tabulating values for the field function:

x	$935\ x\ (0.0625 + x^2)^{-3/2}$
0	0
0.01	597
0.02	1 185
0.1	4 789
0.2	5 698
0.36	4 000
0.9	1 032
1	854
∞	0

We see that there are two points where $E = 1\,000$. We home in on them to determine their coordinates as (to three digits) $x = 0.016\,8$ m and $x = 0.916$ m.

(d) The table in part (c) shows that the field is nowhere so large as 16 000 N/C.

(e) The field of a single charge in Question 7 takes on all values from zero to infinity, each at just one point along the positive x axis. The vector sum of the field of two charges, in this problem, is zero at the origin, rises to a maximum at 17.7 cm, and then decreases asymptotically to zero. In the question and the problem, the fields at $x = 36$ cm happen to take similar values. For large x the field of the two charges in this problem shows just the same inverse proportionality to x^2 as the field in the question, being larger by the factor $2(52\ \text{nC})/(57.7\ \text{nC}) = 1.80$ times.

P23.55 (a) From the $2Q$ charge we have $F_e - T_2\sin\theta_2 = 0$ and $mg - T_2\cos\theta_2 = 0$

Combining these we find $\dfrac{F_e}{mg} = \dfrac{T_2\sin\theta_2}{T_2\cos\theta_2} = \tan\theta_2$

From the Q charge we have $F_e = T_1\sin\theta_1 = 0$ and $mg - T_1\cos\theta_1 = 0$

Combining these we find $\dfrac{F_e}{mg} = \dfrac{T_1\sin\theta_1}{T_1\cos\theta_1} = \tan\theta_1$ or $\boxed{\theta_2 = \theta_1}$

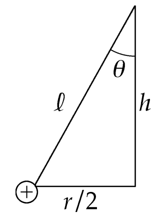


FIG. P23.55

(b) $F_e = \dfrac{k_e 2QQ}{r^2} = \dfrac{2k_eQ^2}{r^2}$

If we assume θ is small then $\tan\theta \approx \dfrac{r/2}{\ell}$

Substitute expressions for F_e and $\tan\theta$ into either equation found in part (a) and solve for r.

$\dfrac{F_e}{mg} = \tan\theta$, then $\dfrac{2k_eQ^2}{r^2}\left(\dfrac{1}{mg}\right) \approx \dfrac{r}{2\ell}$ and solving for r we find $r \approx \left(\dfrac{4k_eQ^2\ell}{mg}\right)^{1/3}$.

P23.56 The bowl exerts a normal force on each bead, directed along the radius line or at 60.0° above the horizontal. Consider the free-body diagram shown for the bead on the left side of the bowl:

$$\sum F_y = n\sin 60.0° - mg = 0,$$

or $n = \dfrac{mg}{\sin 60.0°}$

Also, $\sum F_x = -F_e + n\cos 60.0° = 0,$

or $\dfrac{k_eq^2}{R^2} = n\cos 60.0° = \dfrac{mg}{\tan 60.0°} = \dfrac{mg}{\sqrt{3}}$

Thus, $q = \boxed{R\left(\dfrac{mg}{k_e\sqrt{3}}\right)^{1/2}}$

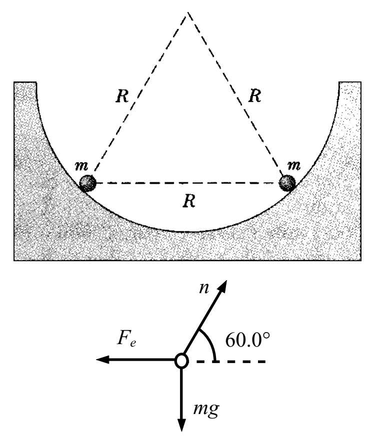


FIG. P23.56

P23.57 (a) The total non-contact force on the cork ball is: $F = qE + mg = m\left(g + \dfrac{qE}{m}\right)$,

which is constant and directed downward. Therefore, it behaves like a simple pendulum in the presence of a modified uniform gravitational field with a period given by:

$$T = 2\pi\sqrt{\frac{L}{g + qE/m}} = 2\pi\sqrt{\frac{0.500\text{ m}}{9.80\text{ m/s}^2 + \left[(2.00\times10^{-6}\text{ C})(1.00\times10^5\text{ N/C})/1.00\times10^{-3}\text{ kg}\right]}}$$

$= \boxed{0.307\text{ s}}$

(b) $\boxed{\text{Yes}}$. Without gravity in part (a), we get $T = 2\pi\sqrt{\dfrac{L}{qE/m}}$

$$T = 2\pi\sqrt{\frac{0.500\text{ m}}{(2.00\times10^{-6}\text{ C})(1.00\times10^5\text{ N/C})/1.00\times10^{-3}\text{ kg}}} = 0.314\text{ s (a 2.28\% difference).}$$

***P23.58** (a) At A the top charge makes the dominant contribution to the field and the net field is downward. At B the total electric field is zero. Between B and C the main change is weakening of the downward electric field of the top charge, so the net field at C is upward. At E the fields of the two bottom charges cancel out and the total field is downward. At F the total field is downward.

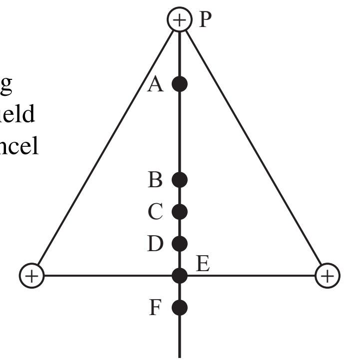


FIG. P23.58

(b) The field is zero at B as it changes from downward at A to upward at C. As a continuous function, the field must pass through the value zero near D as it changes from upward at C to downward at E and F.

(c) Let y represent the distance from E up to the zero-field point. The distance from P to E is $(3^2 - 1.5^2)^{1/2}$ cm = 2.60 cm. Then the requirement that the field be zero is

$$\frac{k_e q}{(2.60\text{ cm} - y)^2} = \frac{k_e q}{(1.5\text{ cm})^2 + y^2}\frac{y}{\sqrt{(1.5\text{ cm})^2 + y^2}} + \frac{k_e q y}{\left[(1.5\text{ cm})^2 + y^2\right]^{3/2}}$$

$$\frac{k_e q}{(2.60\text{ cm} - y)^2} = \frac{2k_e q y}{\left[(1.5\text{ cm})^2 + y^2\right]^{3/2}}$$

$$(1.5^2 + y^2)^{3/2} - 2\,y\,(2.60 - y)^2 = 0$$

As a check on our algebra, we note that $y = (1/3)2.60$ cm = 0.866 cm should be a solution, corresponding to point B. Substituting 0.866 gives 5.20 − 5.20 = 0 as it should. We home in on the smaller answer:

y	$(1.5^2 + y^2)^{3/2} - 2y(2.60 - y)^2$
0	+3.375
0.3	+0.411
0.4	−0.124
0.37	+0.014
0.373	−0.000 6

To three digits the answer is $\boxed{0.373\text{ cm}}$.

P23.59 (a) There are 7 terms which contribute:

3 are s away (along sides)

3 are $\sqrt{2}s$ away (face diagonals) and $\sin\theta = \frac{1}{\sqrt{2}} = \cos\theta$

1 is $\sqrt{3}s$ away (body diagonal) and $\sin\phi = \frac{1}{\sqrt{3}}$

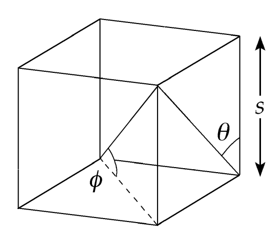


FIG. P23.59

The component in each direction is the same by symmetry.

$$\vec{\mathbf{F}} = \frac{k_e q^2}{s^2}\left[1 + \frac{2}{2\sqrt{2}} + \frac{1}{3\sqrt{3}}\right]\left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right) = \boxed{\frac{k_e q^2}{s^2}(1.90)\left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right)}$$

(b) $F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \boxed{3.29\frac{k_e q^2}{s^2}\text{ away from the origin}}$

P23.60 (a) Zero contribution from the same face due to symmetry, opposite face contributes

$$4\left(\frac{k_e q}{r^2}\sin\phi\right) \quad \text{where} \quad r=\sqrt{\left(\frac{s}{2}\right)^2+\left(\frac{s}{2}\right)^2+s^2}=\sqrt{1.5}s=1.22s$$

$$\sin\phi=\frac{s}{r} \qquad E=4\frac{k_e qs}{r^3}=\frac{4}{(1.22)^3}\frac{k_e q}{s^2}=\boxed{2.18\frac{k_e q}{s^2}}$$

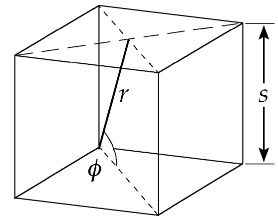


FIG. P23.60

(b) $\boxed{\text{The direction is the } \hat{\mathbf{k}} \text{ direction.}}$

P23.61 The field on the axis of the ring is calculated in an Example in the chapter text as

$$E=E_x=\frac{k_e xQ}{\left(x^2+a^2\right)^{3/2}}$$

The force experienced by a charge $-q$ placed along the axis of the ring is

$$F=-k_e Qq\left[\frac{x}{\left(x^2+a^2\right)^{3/2}}\right] \text{ and when } x \ll a, \text{ this becomes } F=-\left(\frac{k_e Qq}{a^3}\right)x.$$

This expression for the force is in the form of Hooke's law, with an effective spring constant of $k=\frac{k_e Qq}{a^3}$.

Since $\omega=2\pi f=\sqrt{\frac{k}{m}}$, we have $f=\boxed{\frac{1}{2\pi}\sqrt{\frac{k_e Qq}{ma^3}}}$.

P23.62

$$d\vec{\mathbf{E}}=\frac{k_e dq}{x^2+(0.150\text{ m})^2}\left(\frac{-x\hat{\mathbf{i}}+0.150\text{ m}\hat{\mathbf{j}}}{\sqrt{x^2+(0.150\text{ m})^2}}\right)$$

$$=\frac{k_e\lambda\left(-x\hat{\mathbf{i}}+0.150\text{ m}\hat{\mathbf{j}}\right)dx}{\left[x^2+(0.150\text{ m})^2\right]^{3/2}}$$

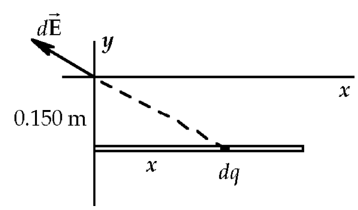


FIG. P23.62

$$\vec{\mathbf{E}}=\int_{\text{all charge}} d\vec{\mathbf{E}}=k_e\lambda\int_{x=0}^{0.400\text{ m}}\frac{\left(-x\hat{\mathbf{i}}+0.150\text{ m}\hat{\mathbf{j}}\right)dx}{\left[x^2+(0.150\text{ m})^2\right]^{3/2}}$$

$$\vec{\mathbf{E}}=k_e\lambda\left[\frac{+\hat{\mathbf{i}}}{\sqrt{x^2+(0.150\text{ m})^2}}\Bigg|_0^{0.400\text{ m}}+\frac{(0.150\text{ m})\hat{\mathbf{j}}x}{(0.150\text{ m})^2\sqrt{x^2+(0.150\text{ m})^2}}\Bigg|_0^{0.400\text{ m}}\right]$$

$$\vec{\mathbf{E}}=\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(35.0\times10^{-9}\text{ C/m}\right)\left[\hat{\mathbf{i}}(2.34-6.67)\text{ m}^{-1}+\hat{\mathbf{j}}(6.24-0)\text{ m}^{-1}\right]$$

$$\vec{\mathbf{E}}=\left(-1.36\hat{\mathbf{i}}+1.96\hat{\mathbf{j}}\right)\times10^3\text{ N/C}=\boxed{\left(-1.36\hat{\mathbf{i}}+1.96\hat{\mathbf{j}}\right)\text{kN/C}}$$

P23.63 The electrostatic forces exerted on the two charges result in a net torque $\tau = -2Fa\sin\theta = -2Eqa\sin\theta$.

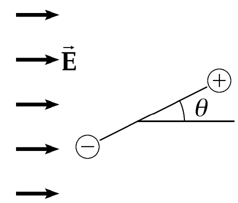


FIG. P23.63

For small θ, $\sin\theta \approx \theta$ and using $p = 2qa$, we have $\tau = -Ep\theta$

The torque produces an angular acceleration given by $\tau = I\alpha = I\dfrac{d^2\theta}{dt^2}$

Combining these two expressions for torque, we have $\dfrac{d^2\theta}{dt^2} + \left(\dfrac{Ep}{I}\right)\theta = 0$

This equation can be written in the form $\dfrac{d^2\theta}{dt^2} = -\omega^2\theta$ which is the standard equation characterizing simple harmonic motion, with $\omega^2 = \dfrac{Ep}{I}$

Then the frequency of oscillation is $f = \omega/2\pi$, or $f = \dfrac{1}{2\pi}\sqrt{\dfrac{pE}{I}} = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{2qaE}{I}}}$

P23.64 $\vec{\mathbf{E}} = \sum \dfrac{k_e q}{r^2}\hat{\mathbf{r}} = \dfrac{k_e q}{a^2}\left(-\hat{\mathbf{i}}\right) + \dfrac{k_e q}{(2a)^2}\left(-\hat{\mathbf{i}}\right) + \dfrac{k_e q}{(3a)^2}\left(-\hat{\mathbf{i}}\right) + \cdots = \dfrac{-k_e q\hat{\mathbf{i}}}{a^2}\left(1 + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \cdots\right) = \boxed{-\dfrac{\pi^2 k_e q}{6a^2}\hat{\mathbf{i}}}$

P23.65 $\vec{\mathbf{E}} = \int d\vec{\mathbf{E}} = \int_{x_0}^{\infty}\left[\dfrac{k_e\lambda_0 x_0 dx\left(-\hat{\mathbf{i}}\right)}{x^3}\right] = -k_e\lambda_0 x_0\hat{\mathbf{i}}\int_{x_0}^{\infty} x^{-3}dx = -k_e\lambda_0 x_0\hat{\mathbf{i}}\left(-\dfrac{1}{2x^2}\Bigg|_{x_0}^{\infty}\right) = \boxed{\dfrac{k_e\lambda_0}{2x_0}\left(-\hat{\mathbf{i}}\right)}$

ANSWERS TO EVEN PROBLEMS

P23.2 (a) 2.62×10^{24} (b) 2.38 electrons for every 10^9 present

P23.4 1.57 μN to the left

P23.6 2.51×10^{-9}

P23.8 $x = 0.634d$. The equilibrium is stable if the third bead has positive charge.

P23.10 (a) period $= \dfrac{\pi}{2}\sqrt{\dfrac{md^3}{k_e qQ}}$ where m is the mass of the object with charge $-Q$ (b) $4a\sqrt{\dfrac{k_e qQ}{md^3}}$

P23.12 1.82 m to the left of the negative charge

P23.14 514 kN

P23.16 (a) $12.9\hat{\mathbf{j}}$ kN/C (b) $-38.6\hat{\mathbf{j}}$ mN

P23.18 See the solution.

P23.20 $\dfrac{k_e\lambda_0}{x_0}\left(-\hat{\mathbf{i}}\right)$

P23.22 See the solution.

P23.24 (a) 383 MN/C away (b) 324 MN/C away (c) 80.7 MN/C away (d) 6.68 MN/C away

P23.26 See the solution.

P23.28 (a) $\frac{k_e Q\hat{\mathbf{i}}}{h}\left[\left(d^2+R^2\right)^{-1/2}-\left((d+h)^2+R^2\right)^{-1/2}\right]$ (b) $\frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[h+\left(d^2+R^2\right)^{1/2}-\left((d+h)^2+R^2\right)^{1/2}\right]$

P23.30 (a) 200 pC (b) 141 pC (c) 58.9 pC

P23.32 See the solution.

P23.34 (a) $-\frac{1}{3}$ (b) q_1 is negative and q_2 is positive

P23.36 (a) $-57.6\hat{\mathbf{i}}$ Tm/s^2 (b) $2.84\hat{\mathbf{i}}$ Mm/s (c) 49.3 ns

P23.38 The particle strikes the negative plate after moving in a parabola 0.181 mm high and 0.961 mm wide.

P23.40 (a) 36.9°, 53.1° (b) 167 ns, 221 ns

P23.42 It is possible in just one way: at $x = -16.0$ cm place a charge of +51.3 μC.

P23.44 (a) 24.2 N/C at 0° (b) 9.42 N/C at 117°

P23.46 (a) $\frac{mg}{A\cot\theta + B}$ (b) $\frac{mgA}{A\cos\theta + B\sin\theta}$

P23.48 0.205 μC

P23.50 $\frac{k_e qQ}{a^2}$ toward the 29th vertex

P23.52 25.9 cm

P23.54 (a) $\vec{\mathbf{E}} = \frac{935\ \mathrm{N\cdot m^2}\ x\ \hat{\mathbf{i}}}{\mathrm{C}\left(0.0625\ \mathrm{m}^2 + x^2\right)^{3/2}}$ (b) 4.00 $\hat{\mathbf{i}}$ kN/C (c) At $x = 0.016\ 8$ m and at $x = 0.916$ m
(d) Nowhere is the field so large. (e) The field of a single charge in Question 7 takes on all values from zero to infinity, each at just one point along the positive x-axis. The vector sum of the field of two charges, in this problem, is zero at the origin, rises to a maximum at 17.7 cm, and then decreases asymptotically to zero. In the question and the problem, the fields at $x = 36$ cm happen to take similar values. For large x the field of the two charges in this problem shows just the same inverse proportionality to x^2 as the field in the question, being larger by the factor 2(52 nC)/(57.7 nC) = 1.8 times.

P23.56 $R\left(\frac{mg}{k_e\sqrt{3}}\right)^{1/2}$

P23.58 (a) At A downward. At B zero. At C upward. At E downward. At F downward. (b) See the solution. (c) 0.373 cm

P23.60 (a) See the solution. (b) $\hat{\mathbf{k}}$.

P23.62 $\left(-1.36\hat{\mathbf{i}} + 1.96\hat{\mathbf{j}}\right)$ kN/C

P23.64 $-\frac{\pi^2 k_e q}{6a^2}\hat{\mathbf{i}}$

24

Gauss's Law

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q24.1 The luminous flux on a given area is less when the sun is low in the sky, because the angle between the rays of the sun and the local area vector, $d\vec{\mathbf{A}}$, is greater than zero. The cosine of this angle is reduced. The decreased flux results, on the average, in colder weather.

Q24.2 The surface must enclose a positive total charge.

Q24.3 The net flux through any gaussian surface is zero. We can argue it two ways. Any surface contains zero charge, so Gauss's law says the total flux is zero. The field is uniform, so the field lines entering one side of the closed surface come out the other side and the net flux is zero.

***Q24.4** (i) Equal amounts of flux pass through each of the six faces of the cube. Answer (e).

(ii) Move the charge to very close below the center of one face, through which the flux is then $q/2\epsilon_0$. Answer (c).

(iii) Move the charge onto one of the cube faces. Then the field has no component perpendicular to this face and the flux is zero. Answer (a).

***Q24.5** (i) Answer (a).

(ii) the flux is zero through the two faces pierced by the filament. Answer (b).

***Q24.6** (i) Answer (a).

(ii) The flux is nonzero through the top and bottom faces, and zero through the other four faces. Answer (c).

***Q24.7** (i) Both spheres create equal fields at exterior points, like particles at the centers of the spheres. Answer (c).

(ii) The field within the conductor is zero. The field within the insulator is 4/5 of its surface value. Answer (f).

Q24.8 Gauss's law cannot tell the different values of the electric field at different points on the surface. When E is an unknown number, then we can say $\int E\cos\theta dA = E\int \cos\theta dA$. When $E(x, y, z)$ is an unknown function, then there is no such simplification.

Q24.9 The electric flux through a sphere around a point charge is independent of the size of the sphere. A sphere of larger radius has a larger area, but a smaller field at its surface, so that the product of field strength and area is independent of radius. If the surface is not spherical, some parts are closer to the charge than others. In this case as well, smaller projected areas go with stronger fields, so that the net flux is unaffected.

Q24.10 Inject some charge at arbitrary places within a conducting object. Every bit of the charge repels every other bit, so each bit runs away as far as it can, stopping only when it reaches the outer surface of the conductor.

***Q24.11** (a) Let q represent the charge of the insulating sphere. The field at A is $(4/5)^3 q/[4\pi(4\text{ cm})^2\epsilon_0]$. The field at B is $q/[4\pi(8\text{ cm})^2\epsilon_0]$. The field at C is zero. The field at D is $q/[4\pi(16\text{ cm})^2\epsilon_0]$. The ranking is A > B > D > C.

(b) The flux through the 4-cm sphere is $(4/5)^3 q/\epsilon_0$. The flux through the 8-cm sphere and through the 16-cm sphere is q/ϵ_0. The flux through the 12-cm sphere is 0. The ranking is B = D > A > C.

***Q24.12** The outer wall of the conducting shell will become polarized to cancel out the external field. The interior field is the same as before. Answer (c).

Q24.13 If the person is uncharged, the electric field inside the sphere is zero. The interior wall of the shell carries no charge. The person is not harmed by touching this wall. If the person carries a (small) charge q, the electric field inside the sphere is no longer zero. Charge $-q$ is induced on the inner wall of the sphere. The person will get a (small) shock when touching the sphere, as all the charge on his body jumps to the metal.

***Q24.14** (i) The shell becomes polarized. Answer (e).
(ii) The net charge on the shell's inner and outer surfaces is zero. Answer (a).
(iii) Answer (c).
(iv) Answer (c).
(v) Answer (a).

Q24.15 There is zero force. The huge charged sheet creates a uniform field. The field can polarize the neutral sheet, creating in effect a film of opposite charge on the near face and a film with an equal amount of like charge on the far face of the neutral sheet. Since the field is uniform, the films of charge feel equal-magnitude forces of attraction and repulsion to the charged sheet. The forces add to zero.

SOLUTIONS TO PROBLEMS

Section 24.1 **Electric Flux**

P24.1 $\Phi_E = EA\cos\theta \qquad A = \pi r^2 = \pi(0.200)^2 = 0.126\text{ m}^2$

$5.20\times 10^5 = E(0.126)\cos 0° \qquad E = 4.14\times 10^6\text{ N/C} = \boxed{4.14\text{ MN/C}}$

P24.2 $\Phi_E = EA\cos\theta = (2.00\times 10^4\text{ N/C})(18.0\text{ m}^2)\cos 10.0° = \boxed{355\text{ kN}\cdot\text{m}^2/\text{C}}$

P24.3 (a) $\Phi_E = \vec{\mathbf{E}}\cdot\vec{\mathbf{A}} = (a\hat{\mathbf{i}} + b\hat{\mathbf{j}})\cdot A\hat{\mathbf{i}} = \boxed{aA}$

(b) $\Phi_E = (a\hat{\mathbf{i}} + b\hat{\mathbf{j}})\cdot A\hat{\mathbf{j}} = \boxed{bA}$

(c) $\Phi_E = (a\hat{\mathbf{i}} + b\hat{\mathbf{j}})\cdot A\hat{\mathbf{k}} = \boxed{0}$

P24.4 (a) $A' = (10.0 \text{ cm})(30.0 \text{ cm})$

$A' = 300 \text{ cm}^2 = 0.030\,0 \text{ m}^2$

$\Phi_{E,A'} = EA'\cos\theta$

$\Phi_{E,A'} = (7.80\times10^4)(0.030\,0)\cos180°$

$\Phi_{E,A'} = \boxed{-2.34 \text{ kN}\cdot\text{m}^2/\text{C}}$

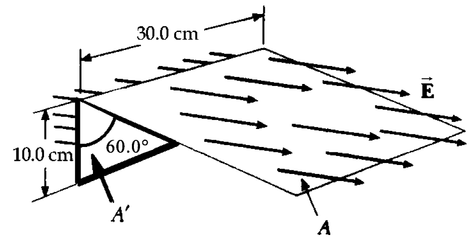


FIG. P24.4

(b) $\Phi_{E,A} = EA\cos\theta = (7.80\times10^4)(A)\cos60.0°$

$A = (30.0 \text{ cm})(w) = (30.0 \text{ cm})\left(\dfrac{10.0 \text{ cm}}{\cos60.0°}\right) = 600 \text{ cm}^2 = 0.060\,0 \text{ m}^2$

$\Phi_{E,A} = (7.80\times10^4)(0.060\,0)\cos60.0° = \boxed{+2.34 \text{ kN}\cdot\text{m}^2/\text{C}}$

(c) The bottom and the two triangular sides all lie *parallel* to $\vec{\mathbf{E}}$, so $\Phi_E = 0$ for each of these. Thus,

$\Phi_{E,\text{total}} = -2.34 \text{ kN}\cdot\text{m}^2/\text{C} + 2.34 \text{ kN}\cdot\text{m}^2/\text{C} + 0 + 0 + 0 = \boxed{0}$

P24.5 $\Phi_E = EA\cos\theta$ through the base

$\Phi_E = (52.0)(36.0)\cos180° = -1.87 \text{ kN}\cdot\text{m}^2/\text{C}$

Note that the same number of electric field lines go through the base as go through the pyramid's surface (not counting the base).

For the slanting surfaces, $\boxed{\Phi_E = +1.87 \text{ kN}\cdot\text{m}^2/\text{C}}$.

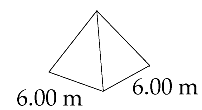


FIG. P24.5

Section 24.2 **Gauss's Law**

P24.6 (a) $E = \dfrac{k_e Q}{r^2}$; $\quad 8.90\times10^2 = \dfrac{(8.99\times10^9)Q}{(0.750)^2}$

But Q is negative since $\vec{\mathbf{E}}$ points inward. $\quad Q = -5.57\times10^{-8} \text{ C} = \boxed{-55.7 \text{ nC}}$

(b) The $\boxed{\text{negative}}$ charge has a $\boxed{\text{spherically symmetric}}$ charge distribution, concentric with the spherical shell.

P24.7 (a) $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0} = \dfrac{(+5.00\ \mu\text{C} - 9.00\ \mu\text{C} + 27.0\ \mu\text{C} - 84.0\ \mu\text{C})}{8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2} = -6.89\times10^6 \text{ N}\cdot\text{m}^2/\text{C}^2$

$\Phi_E = \boxed{-6.89 \text{ MN}\cdot\text{m}^2/\text{C}}$

(b) Since the net electric flux is negative, more lines enter than leave the surface.

P24.8 (a) One-half of the total flux created by the charge q goes through the plane. Thus,

$$\Phi_{E,\text{plane}} = \frac{1}{2}\Phi_{E,\text{total}} = \frac{1}{2}\left(\frac{q}{\epsilon_0}\right) = \boxed{\frac{q}{2\epsilon_0}}$$

(b) The square looks like an infinite plane to a charge *very close* to the surface. Hence,

$$\Phi_{E,\text{square}} \approx \Phi_{E,\text{plane}} = \boxed{\frac{q}{2\epsilon_0}}$$

(c) The plane and the square look the same to the charge.

P24.9 $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0}$

Through S_1 $\quad \Phi_E = \dfrac{-2Q+Q}{\epsilon_0} = \boxed{-\dfrac{Q}{\epsilon_0}}$

Through S_2 $\quad \Phi_E = \dfrac{+Q-Q}{\epsilon_0} = \boxed{0}$

Through S_3 $\quad \Phi_E = \dfrac{-2Q+Q-Q}{\epsilon_0} = \boxed{-\dfrac{2Q}{\epsilon_0}}$

Through S_4 $\quad \Phi_E = \boxed{0}$

P24.10 (a) $\Phi_{E,\text{shell}} = \dfrac{q_{\text{in}}}{\epsilon_0} = \dfrac{12.0\times10^{-6}}{8.85\times10^{-12}} = 1.36\times10^{6}\ \text{N}\cdot\text{m}^2/\text{C} = \boxed{1.36\,\text{MN}\cdot\text{m}^2/\text{C}}$

(b) $\Phi_{E,\text{half shell}} = \dfrac{1}{2}\left(1.36\times10^{6}\ \text{N}\cdot\text{m}^2/\text{C}\right) = 6.78\times10^{5}\ \text{N}\cdot\text{m}^2/\text{C} = \boxed{678\,\text{kN}\cdot\text{m}^2/\text{C}}$

(c) No, the same number of field lines will pass through each surface, no matter how the radius changes.

P24.11 (a) With δ very small, all points on the hemisphere are nearly at a distance R from the charge, so the field everywhere on the curved surface is $\dfrac{k_e Q}{R^2}$ radially outward (normal to the surface). Therefore, the flux is this field strength times the area of half a sphere:

$$\Phi_{\text{curved}} = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E_{\text{local}}A_{\text{hemisphere}}$$

$$\Phi_{\text{curved}} = \left(k_e\frac{Q}{R^2}\right)\left(\frac{1}{2}4\pi R^2\right) = \frac{1}{4\pi\epsilon_0}Q(2\pi) = \boxed{\frac{+Q}{2\epsilon_0}}$$

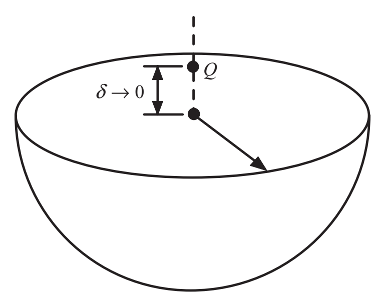


FIG. P24.11

(b) The closed surface encloses zero charge so Gauss's law gives

$$\Phi_{\text{curved}} + \Phi_{\text{flat}} = 0 \qquad \text{or} \qquad \Phi_{\text{flat}} = -\Phi_{\text{curved}} = \boxed{\frac{-Q}{2\epsilon_0}}$$

P24.12 Consider as a gaussian surface a box with horizontal area A, lying between 500 and 600 m elevation.

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0}: \quad (+120\,\text{N/C})A + (-100\,\text{N/C})A = \frac{\rho A(100\ \text{m})}{\epsilon_0}$$

$$\rho = \frac{(20\,\text{N/C})\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}{100\ \text{m}} = \boxed{1.77\times10^{-12}\ \text{C/m}^3}$$

The charge is positive, to produce the net outward flux of electric field.

P24.13 The total charge is $Q-6|q|$. The total outward flux from the cube is $\dfrac{Q-6|q|}{\epsilon_0}$, of which one-sixth goes through each face:

$$(\Phi_E)_{\text{one face}}=\frac{Q-6|q|}{6\epsilon_0}$$

$$(\Phi_E)_{\text{one face}}=\frac{Q-6|q|}{6\epsilon_0}=\frac{(5.00-6.00)\times10^{-6}\ \text{C}\cdot\text{N}\cdot\text{m}^2}{6\times8.85\times10^{-12}\ \text{C}^2}=\boxed{-18.8\ \text{kN}\cdot\text{m}^2/\text{C}}$$

.

P24.14 The total charge is $Q-6|q|$. The total outward flux from the cube is $\dfrac{Q-6|q|}{\epsilon_0}$, of which one-sixth goes through each face:

$$(\Phi_E)_{\text{one face}}=\boxed{\frac{Q-6|q|}{6\epsilon_0}}$$

P24.15 If $R\le d$, the sphere encloses no charge and $\Phi_E=\dfrac{q_{\text{in}}}{\epsilon_0}=\boxed{0}$.

If $R>d$, the length of line falling within the sphere is $2\sqrt{R^2-d^2}$

so $\quad\Phi_E=\boxed{\dfrac{2\lambda\sqrt{R^2-d^2}}{\epsilon_0}}$

P24.16 $$\Phi_{E,\text{hole}}=\vec{\mathbf{E}}\cdot\vec{\mathbf{A}}_{\text{hole}}=\left(\frac{k_eQ}{R^2}\right)(\pi r^2)=\left(\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10.0\times10^{-6}\ \text{C})}{(0.100\ \text{m})^2}\right)\pi(1.00\times10^{-3}\ \text{m})^2$$

$$\Phi_{E,\text{hole}}=\boxed{28.2\ \text{N}\cdot\text{m}^2/\text{C}}$$

P24.17 $$\Phi_E=\frac{q_{\text{in}}}{\epsilon_0}=\frac{170\times10^{-6}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}=1.92\times10^7\ \text{N}\cdot\text{m}^2/\text{C}$$

(a) $(\Phi_E)_{\text{one face}}=\dfrac{1}{6}\Phi_E=\dfrac{1.92\times10^7\ \text{N}\cdot\text{m}^2/\text{C}}{6}\qquad(\Phi_E)_{\text{one face}}=\boxed{3.20\ \text{MN}\cdot\text{m}^2/\text{C}}$

(b) $\Phi_E=\boxed{19.2\ \text{MN}\cdot\text{m}^2/\text{C}}$

(c) The answer to (a) would change because the flux through each face of the cube would not be equal with an asymmetric charge distribution. The sides of the cube nearer the charge would have more flux and the ones further away would have less. The answer to (b) would remain the same, since the overall flux would remain the same.

Section 24.3 **Application of Gauss's Law to Various Charge Distributions**

P24.18 (a) $E = \frac{k_e Q r}{a^3} = \boxed{0}$

(b) $E = \frac{k_e Q r}{a^3} = \frac{(8.99\times10^9)(26.0\times10^{-6})(0.100)}{(0.400)^3} = \boxed{365\ \text{kN/C}}$

(c) $E = \frac{k_e Q}{r^2} = \frac{(8.99\times10^9)(26.0\times10^{-6})}{(0.400)^2} = \boxed{1.46\ \text{MN/C}}$

(d) $E = \frac{k_e Q}{r^2} = \frac{(8.99\times10^9)(26.0\times10^{-6})}{(0.600)^2} = \boxed{649\ \text{kN/C}}$

The direction for each electric field is $\boxed{\text{radially outward}}$.

P24.19 The charge distributed through the nucleus creates a field at the surface equal to that of a point charge at its center: $E = \frac{k_e q}{r^2}$.

$$E = \frac{(8.99\times10^9\ \text{Nm}^2/\text{C}^2)(82\times1.60\times10^{-19}\ \text{C})}{\left[(208)^{1/3}1.20\times10^{-15}\ \text{m}\right]^2}$$

$E = \boxed{2.33\times10^{21}\ \text{N/C}}$ away from the nucleus

P24.20 Note that the electric field in each case is directed radially inward, toward the filament.

(a) $E = \frac{2k_e\lambda}{r} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(90.0\times10^{-6}\ \text{C/m})}{0.100\ \text{m}} = \boxed{16.2\ \text{MN/C}}$

(b) $E = \frac{2k_e\lambda}{r} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(90.0\times10^{-6}\ \text{C/m})}{0.200\ \text{m}} = \boxed{8.09\ \text{MN/C}}$

(c) $E = \frac{2k_e\lambda}{r} = \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(90.0\times10^{-6}\ \text{C/m})}{1.00\ \text{m}} = \boxed{1.62\ \text{MN/C}}$

P24.21 $E = \frac{\sigma}{2\epsilon_0} = \frac{9.00\times10^{-6}\ \text{C/m}^2}{2(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = \boxed{508\ \text{kN/C, upward}}$

P24.22 (a) $E = \frac{2k_e\lambda}{r}$ $\quad 3.60\times10^4 = \frac{2(8.99\times10^9)(Q/2.40)}{0.190}$

$Q = +9.13\times10^{-7}\ \text{C} = \boxed{+913\ \text{nC}}$

(b) $\vec{\mathbf{E}} = \boxed{0}$

P24.23 $mg = qE = q\left(\frac{\sigma}{2\epsilon_0}\right) = q\left(\frac{Q/A}{2\epsilon_0}\right)$ $\quad \frac{Q}{A} = \frac{2\epsilon_0 mg}{q} = \frac{2(8.85\times10^{-12})(0.01)(9.8)}{-0.7\times10^{-6}}$

$= \boxed{-2.48\ \mu\text{C/m}^2}$

***P24.24** (a) A long cylindrical plastic rod 2.00 cm in radius carries charge uniformly distributed throughout its volume, with density 5.00 μC/m^3. Find the magnitude of the electric field it creates at a point P, 3.00 cm from its axis. As a gaussian surface choose a concentric cylinder with its curved surface passing through the point P and with length 8.00 cm.

(b) We solve for

$$E = \frac{(0.02\ \text{m})^2 0.08\ \text{m}\ (5\times10^{-6}\ \text{C/m}^3)}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)2(0.03\ \text{m})\ 0.08\ \text{m}} = \boxed{3.77\ \text{kN/C}}$$

P24.25 The volume of the spherical shell is

$$\frac{4}{3}\pi\left[(0.25\text{ m})^3-(0.20\text{ m})^3\right]=3.19\times10^{-2}\text{ m}^3$$

Its charge is

$$\rho V=\left(-1.33\times10^{-6}\text{ C/m}^3\right)\left(3.19\times10^{-2}\text{ m}^3\right)=-4.25\times10^{-8}\text{ C}$$

The net charge inside a sphere containing the proton's path as its equator is

$$-60\times10^{-9}\text{ C}-4.25\times10^{-8}\text{ C}=-1.02\times10^{-7}\text{ C}$$

The electric field is radially inward with magnitude

$$\frac{k_e|q|}{r^2}=\frac{|q|}{\in_0 4\pi r^2}=\frac{8.99\times10^9\text{Nm}^2\left(1.02\times10^{-7}\text{C}\right)}{\text{C}^2\left(0.25\text{m}\right)^2}=1.47\times10^4\text{ N/C}$$

For the proton

$$\sum F=ma\qquad eE=\frac{mv^2}{r}$$

$$v=\left(\frac{eEr}{m}\right)^{1/2}=\left(\frac{1.60\times10^{-19}\text{ C}\left(1.47\times10^4\text{ N/C}\right)0.25\text{ m}}{1.67\times10^{-27}\text{kg}}\right)^{1/2}=\boxed{5.94\times10^5\text{ m/s}}$$

***P24.26** $\sigma=\left(8.60\times10^{-6}\text{ C/cm}^2\right)\left(\frac{100\text{ cm}}{\text{m}}\right)^2=8.60\times10^{-2}\text{ C/m}^2$

$$E=\frac{\sigma}{2\in_0}=\frac{8.60\times10^{-2}}{2\left(8.85\times10^{-12}\right)}=\boxed{4.86\times10^9\text{ N/C away from the wall}}$$

So long as the distance from the wall is small compared to the width and height of the wall, the distance does not affect the field.

P24.27 If ρ is positive, the field must be radially outward. Choose as the gaussian surface a cylinder of length L and radius r, contained inside the charged rod. Its volume is $\pi r^2 L$ and it encloses charge $\rho\pi r^2 L$. Because the charge distribution is long, no electric flux passes through the circular end caps; $\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}=EdA\cos 90.0°=0$. The curved surface has $\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}=EdA\cos 0°$, and E must be the same strength everywhere over the curved surface.

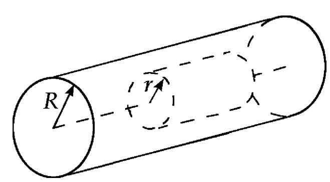


FIG. P24.27

Gauss's law, $\oint\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}=\frac{q}{\in_0}$, becomes $E\int_{\text{Curved Surface}}dA=\frac{\rho\pi r^2 L}{\in_0}$.

Now the lateral surface area of the cylinder is $2\pi rL$:

$$E(2\pi r)L=\frac{\rho\pi r^2 L}{\in_0}\qquad\text{Thus,}\qquad\vec{\mathbf{E}}=\boxed{\frac{\rho r}{2\in_0}\text{ radially away from the cylinder axis}}.$$

P24.28 The distance between centers is $2\times5.90\times10^{-15}$ m. Each produces a field as if it were a point charge at its center, and each feels a force as if all its charge were a point at its center.

$$F=\frac{k_e q_1 q_2}{r^2}=\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{(46)^2\left(1.60\times10^{-19}\text{ C}\right)^2}{\left(2\times5.90\times10^{-15}\text{ m}\right)^2}=3.50\times10^3\text{ N}=\boxed{3.50\text{ kN}}$$

P24.29 (a) $\vec{\mathbf{E}} = \boxed{0}$

(b) $E = \dfrac{k_e Q}{r^2} = \dfrac{(8.99 \times 10^9)(32.0 \times 10^{-6})}{(0.200)^2} = 7.19 \text{ MN/C} \qquad \vec{\mathbf{E}} = \boxed{7.19 \text{ MN/C radially outward}}$

P24.30 Consider two balloons of diameter 0.2 m, each with mass 1 g, hanging apart with a 0.05 m separation on the ends of strings making angles of 10° with the vertical.

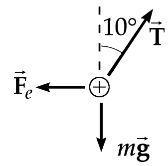


FIG. P24.30

(a) $\sum F_y = T\cos 10° - mg = 0 \Rightarrow T = \dfrac{mg}{\cos 10°}$

$\sum F_x = T\sin 10° - F_e = 0 \Rightarrow F_e = T\sin 10°$, so

$F_e = \left(\dfrac{mg}{\cos 10°}\right)\sin 10° = mg\tan 10° = (0.001 \text{ kg})(9.8 \text{ m/s}^2)\tan 10°$

$F_e \approx 2 \times 10^{-3} \text{ N} \boxed{\sim 10^{-3} \text{ N or 1 mN}}$

(b) $F_e = \dfrac{k_e q^2}{r^2}$

$2 \times 10^{-3} \text{ N} \approx \dfrac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) q^2}{(0.25 \text{ m})^2}$

$q \approx 1.2 \times 10^{-7} \text{ C} \boxed{\sim 10^{-7} \text{ C or 100 nC}}$

(c) $E = \dfrac{k_e q}{r^2} \approx \dfrac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.2 \times 10^{-7} \text{ C})}{(0.25 \text{ m})^2} \approx 1.7 \times 10^4 \text{ N/C} \boxed{\sim 10 \text{ kN/C}}$

(d) $\Phi_E = \dfrac{q}{\epsilon_0} \approx \dfrac{1.2 \times 10^{-7} \text{ C}}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = 1.4 \times 10^4 \text{ N} \cdot \text{m}^2/\text{C} \boxed{\sim 10 \text{ kN} \cdot \text{m}^2/\text{C}}$

P24.31 (a) $E = \dfrac{2k_e\lambda}{r} = \dfrac{2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)\left[(2.00 \times 10^{-6} \text{ C})/7.00 \text{ m}\right]}{0.100 \text{ m}}$

$E = \boxed{51.4 \text{ kN/C, radially outward}}$

(b) $\Phi_E = EA\cos\theta = E(2\pi r\ell)\cos 0°$

$\Phi_E = (5.14 \times 10^4 \text{ N/C})2\pi(0.100 \text{ m})(0.020\,0 \text{ m})(1.00) = \boxed{646 \text{ N} \cdot \text{m}^2/\text{C}}$

Section 24.4 **Conductors in Electrostatic Equilibrium**

P24.32 The fields are equal. The equation $E = \dfrac{\sigma_{\text{conductor}}}{\epsilon_0}$ suggested in the chapter for the field outside the aluminum looks different from the equation $E = \dfrac{\sigma_{\text{insulator}}}{2\epsilon_0}$ for the field around glass. But its charge will spread out to cover both sides of the aluminum plate, so the density is $\sigma_{\text{conductor}} = \dfrac{Q}{2A}$. The glass carries charge only on area A, with $\sigma_{\text{insulator}} = \dfrac{Q}{A}$. The two fields are $\dfrac{Q}{2A\epsilon_0}$, the same in magnitude, and both are perpendicular to the plates, vertically upward if Q is positive.

P24.33 $\oint E\,dA = E(2\pi r l) = \dfrac{q_{\text{in}}}{\epsilon_0}$ $\qquad E = \dfrac{q_{\text{in}}/l}{2\pi\epsilon_0 r} = \dfrac{\lambda}{2\pi\epsilon_0 r}$

(a) $r = 3.00\text{ cm}$ $\qquad \vec{\mathbf{E}} = \boxed{0}$

(b) $r = 10.0\text{ cm}$ $\qquad \vec{\mathbf{E}} = \dfrac{30.0\times10^{-9}}{2\pi(8.85\times10^{-12})(0.100)} = \boxed{5\,400\text{ N/C, outward}}$

(c) $r = 100\text{ cm}$ $\qquad \vec{\mathbf{E}} = \dfrac{30.0\times10^{-9}}{2\pi(8.85\times10^{-12})(1.00)} = \boxed{540\text{ N/C, outward}}$

***P24.34** (a) All of the charge sits on the surface of the copper sphere at radius 15 cm. The field inside is $\boxed{\text{zero}}$.

(b) The charged sphere creates field at exterior points as if it were a point charge at the center:

$$\vec{\mathbf{E}} = \frac{k_e q}{r^2}\text{ away} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2)(40\times10^{-9}\text{ C})}{\text{C}^2(0.17\text{ m})^2}\text{ outward} = \boxed{1.24\times10^4\text{ N/C outward}}$$

(c) $$\vec{\mathbf{E}} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2)(40\times10^{-9}\text{ C})}{\text{C}^2(0.75\text{ m})^2}\text{ outward} = \boxed{639\text{ N/C outward}}$$

(d) All three answers would be the same. The solid copper sphere carries charge only on its outer surface.

P24.35 (a) $E = \dfrac{\sigma}{\epsilon_0}$ $\qquad \sigma = (8.00\times10^4)(8.85\times10^{-12}) = 7.08\times10^{-7}\text{ C/m}^2$

$\sigma = \boxed{708\text{ nC/m}^2}$, positive on one face and negative on the other.

(b) $\sigma = \dfrac{Q}{A}$ $\qquad Q = \sigma A = (7.08\times10^{-7})(0.500)^2\text{ C}$

$Q = 1.77\times10^{-7}\text{ C} = \boxed{177\text{ nC}}$, positive on one face and negative on the other.

***P24.36** Let the flat box have face area A perpendicular to its thickness dx. The flux at $x = 0.3$ m is into the box $-EA = -(6\,000\text{ N/C}\cdot\text{m}^2)(0.3\text{ m})^2 A = -(540\text{ N/C})A$

The flux out of the box at $x = 0.3\text{ m} + dx$

$+EA = -(6\,000\text{ N/C}\cdot\text{m}^2)(0.3\text{ m} + dx)^2 A = +(540\text{ N/C})A + (3\,600\text{ N/C}\cdot\text{m})\,dx\,A$

(The term in $(dx)^2$ is negligible.)

The charge in the box is $\rho A\,dx$ where ρ is the unknown. Gauss's law is

$-(540\text{ N/C})A + (540\text{ N/C})A + (3\,600\text{ N/C}\cdot\text{m})\,dx\,A = \rho A\,dx/\epsilon_0$

Then $\rho = (3600\text{ N/C}\cdot\text{m})\epsilon_0 = (3600\text{ N/C}\cdot\text{m})(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2) = \boxed{31.9\text{ nC/m}^3}$

P24.37 The charge divides equally between the identical spheres, with charge $\dfrac{Q}{2}$ on each. Then they repel like point charges at their centers:

$$F = \frac{k_e(Q/2)(Q/2)}{(L+R+R)^2} = \frac{k_e Q^2}{4(L+2R)^2} = \frac{8.99\times10^9\text{ N}\cdot\text{m}^2(60.0\times10^{-6}\text{ C})^2}{4\text{C}^2(2.01\text{ m})^2} = \boxed{2.00\text{ N}}$$

***P24.38** The surface area is $A = 4\pi a^2$. The field is then

$$E = \frac{k_e Q}{a^2} = \frac{Q}{4\pi \in_0 a^2} = \frac{Q}{A \in_0} = \frac{\sigma}{\in_0}$$

It is not equal to $\sigma/2\in_0$. At a point just outside, the uniformly charged surface looks just like a uniform flat sheet of charge. The distance to the field point is negligible compared to the radius of curvature of the surface.

P24.39 (a) Inside surface: consider a cylindrical surface within the metal. Since E inside the conducting shell is zero, the total charge inside the gaussian surface must be zero, so the inside charge/length $= -\lambda$.

$0 = \lambda\ell + q_{\text{in}}$ so $\dfrac{q_{\text{in}}}{\ell} = \boxed{-\lambda}$

Outside surface: The total charge on the metal cylinder is $2\lambda\ell = q_{\text{in}} + q_{\text{out}}$

$q_{\text{out}} = 2\lambda\ell + \lambda\ell$ so the outside charge/length is $\boxed{3\lambda}$

(b) $$E = \frac{2k_e(3\lambda)}{r} = \frac{6k_e\lambda}{r} = \boxed{\frac{3\lambda}{2\pi\in_0 r}\text{ radially outward}}$$

P24.40 An approximate sketch is given at the right. Note that the electric field lines should be perpendicular to the conductor both inside and outside.

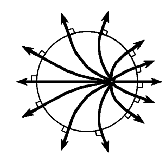


FIG. P24.40

P24.41 (a) The charge density on each of the surfaces (upper and lower) of the plate is:

$$\sigma = \frac{1}{2}\left(\frac{q}{A}\right) = \frac{1}{2}\frac{(4.00\times10^{-8}\text{ C})}{(0.500\text{ m})^2} = 8.00\times10^{-8}\text{ C/m}^2 = \boxed{80.0\text{ nC/m}^2}$$

(b) $$\vec{\mathbf{E}} = \left(\frac{\sigma}{\in_0}\right)\hat{\mathbf{k}} = \left(\frac{8.00\times10^{-8}\text{ C/m}^2}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2}\right)\hat{\mathbf{k}} = \boxed{(9.04\text{ kN/C})\hat{\mathbf{k}}}$$

(c) $$\vec{\mathbf{E}} = \boxed{(-9.04\text{ kN/C})\hat{\mathbf{k}}}$$

Additional Problems

P24.42 In general, $\vec{\mathbf{E}} = ay\hat{\mathbf{i}} + bz\hat{\mathbf{j}} + cx\hat{\mathbf{k}}$

In the xy plane, $z = 0$ and $\vec{\mathbf{E}} = ay\hat{\mathbf{i}} + cx\hat{\mathbf{k}}$

$$\Phi_E = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \int\left(ay\hat{\mathbf{i}} + cx\hat{\mathbf{k}}\right)\cdot\hat{\mathbf{k}}dA$$

$$\Phi_E = ch\int_{x=0}^{w} x\,dx = ch\frac{x^2}{2}\bigg|_{x=0}^{w} = \boxed{\frac{chw^2}{2}}$$

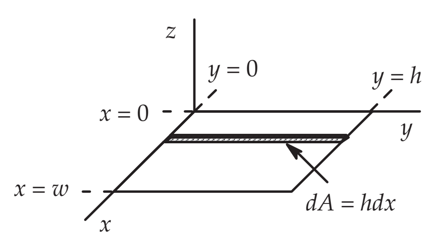


FIG. P24.42

P24.43 (a) Uniform $\vec{\mathbf{E}}$, pointing radially outward, so $\Phi_E = EA$. The arc length is $ds = Rd\theta$, and the circumference is $2\pi r = 2\pi R\sin\theta$.

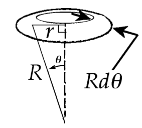


FIG. P24.43

$$A = \int 2\pi r\,ds = \int_0^\theta (2\pi R\sin\theta) R\,d\theta = 2\pi R^2 \int_0^\theta \sin\theta\,d\theta$$

$$= 2\pi R^2 (-\cos\theta)\Big|_0^\theta = 2\pi R^2 (1-\cos\theta)$$

$$\Phi_E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\cdot 2\pi R^2(1-\cos\theta) = \boxed{\frac{Q}{2\epsilon_0}(1-\cos\theta)} \text{ [independent of } R!]$$

(b) For $\theta = 90.0°$ (hemisphere): $\Phi_E = \dfrac{Q}{2\epsilon_0}(1-\cos 90°) = \boxed{\dfrac{Q}{2\epsilon_0}}$.

(c) For $\theta = 180°$ (entire sphere): $\Phi_E = \dfrac{Q}{2\epsilon_0}(1-\cos 180°) = \boxed{\dfrac{Q}{\epsilon_0}}$ [Gauss's Law].

***P24.44** (a) $q_{\text{in}} = +3\ \mu\text{C} - 1\ \mu\text{C} = \boxed{+2.00\ \mu\text{C}}$

(b) The charge distribution is spherically symmetric and $q_{\text{in}} > 0$. Thus, the field is directed $\boxed{\text{radially outward}}$ or to the right at point D.

(c) $E = \dfrac{k_e q_{\text{in}}}{r^2} = \dfrac{8.99\times10^9\ \ 2.00\times10^{-6}\ \text{N/C}}{(0.16)^2} = \boxed{702\ \text{kN/C}}$

(d) Since all points within this region are located inside conducting material, $\boxed{E = 0}$.

(e) $\Phi_E = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = 0 \quad\Rightarrow\quad q_{\text{in}} = \epsilon_0\,\Phi_E = \boxed{0}$

(f) $q_{\text{in}} = \boxed{+3.00\ \mu\text{C}}$

(g) $E = \dfrac{k_e q_{\text{in}}}{r^2} = \dfrac{8.99\times10^9\ \ 3.00\times10^{-6}}{(0.08)^2} = \boxed{4.21\ \text{MN/C to the right}}$ (radially outward).

(h) $q_{\text{in}} = \rho V = \left(\dfrac{+3\ \mu\text{C}}{\frac{4}{3}\pi 5^3}\right)\left(\dfrac{4}{3}\pi 4^3\right) = \boxed{+1.54\ \mu\text{C}}$

(i) $E = \dfrac{k_e q_{\text{in}}}{r^2} = \dfrac{8.99\times10^9\ \ 1.54\times10^{-6}}{(0.04)^2} = \boxed{8.63\ \text{MN/C to the right}}$ (radially outward)

(j) As in part (d), $E = 0$ for $10\ \text{cm} < r < 15\ \text{cm}$. Thus, for a spherical gaussian surface with $10\ \text{cm} < r < 15\ \text{cm}$, $q_{\text{in}} = +3\ \mu\text{C} + q_{\text{inner}} = 0$ where q_{inner} is the charge on the inner surface of the conducting shell. This yields $q_{\text{inner}} = \boxed{-3.00\ \mu\text{C}}$.

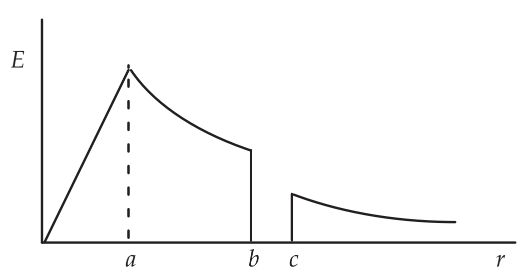


FIG. P24.44(l)

(k) Since the total charge on the conducting shell is $q_{\text{net}} = q_{\text{outer}} + q_{\text{inner}} = -1\ \mu\text{C}$, we have

$$q_{\text{outer}} = -1\ \mu\text{C} - q_{\text{inner}} = -1\ \mu\text{C} - (-3\ \mu\text{C}) = \boxed{+2.00\ \mu\text{C}}$$

(l) This is shown in the figure to the right.

***P24.45** (a) The field is zero within the metal of the shell. The exterior electric field lines end at equally spaced points on the outer surface. The charge on the outer surface is distributed uniformly. Its amount is given by

$$EA = Q/\epsilon_0$$

$$Q = -(890 \text{ N/C})\, 4\pi (0.75 \text{ m})^2\, 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2 = -55.7 \text{ nC}$$

(b) and (c) For the net charge of the shell to be zero, the shell must carry +55.7 nC on its inner surface, induced there by −55.7 nC in the cavity within the shell. The charge in the cavity could have any distribution and give any corresponding distribution to the charge on the inner surface of the shell. For example, a large positive charge might be within the cavity close to its topmost point, and a slightly larger negative charge near its easternmost point. The inner surface of the shell would then have plenty of negative charge near the top and even more positive charge centered on the eastern side.

P24.46 The sphere with large charge creates a strong field to polarize the other sphere. That means it pushes the excess charge over to the far side, leaving charge of the opposite sign on the near side. This patch of opposite charge is smaller in amount but located in a stronger external field, so it can feel a force of attraction that is larger than the repelling force felt by the larger charge in the weaker field on the other side.

P24.47 (a) $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \dfrac{q_{\text{in}}}{\epsilon_0}$

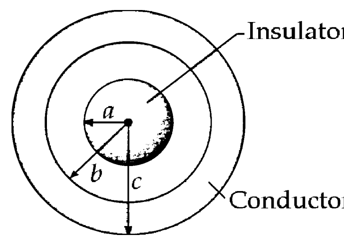


FIG. P24.47

For $r < a$, $\quad q_{\text{in}} = \rho\left(\dfrac{4}{3}\pi r^3\right)$

so $\quad \boxed{E = \dfrac{\rho r}{3\epsilon_0}}$

For $a < r < b$ and $c < r$, $\quad q_{\text{in}} = Q$

So $\quad \boxed{E = \dfrac{Q}{4\pi r^2 \epsilon_0}}$

For $b \le r \le c$, $\quad E = 0$, since $\boxed{E = 0}$ inside a conductor.

(b) Let $q_1 =$ induced charge on the inner surface of the hollow sphere. Since $E = 0$ inside the conductor, the total charge enclosed by a spherical surface of radius $b \le r \le c$ must be zero.

Therefore, $\quad q_1 + Q = 0 \quad$ and $\quad \sigma_1 = \dfrac{q_1}{4\pi b^2} = \boxed{\dfrac{-Q}{4\pi b^2}}$

Let $q_2 =$ induced charge on the outside surface of the hollow sphere. Since the hollow sphere is uncharged, we require

$$q_1 + q_2 = 0 \quad \text{and} \quad \sigma_2 = \frac{q_1}{4\pi c^2} = \boxed{\frac{Q}{4\pi c^2}}$$

P24.48 First, consider the field at distance $r < R$ from the center of a uniform sphere of positive charge $(Q = +e)$ with radius *R*.

$$\left(4\pi r^2\right)E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\rho V}{\epsilon_0} = \frac{\left(\frac{+e}{\frac{4}{3}\pi R^3}\right)\frac{4}{3}\pi r^3}{\epsilon_0} \qquad \text{so} \qquad E = \left(\frac{e}{4\pi\epsilon_0 R^3}\right) r \text{ directed outward}$$

(a) The force exerted on a point charge $q = -e$ located at distance *r* from the center is then

$$F = qE = -e\left(\frac{e}{4\pi\epsilon_0 R^3}\right)r = -\left(\frac{e^2}{4\pi\epsilon_0 R^3}\right)r = \boxed{-Kr}$$

(b) $$K = \frac{e^2}{4\pi\epsilon_0 R^3} = \boxed{\frac{k_e e^2}{R^3}}$$

(c) $$F_r = m_e a_r = -\left(\frac{k_e e^2}{R^3}\right)r, \qquad \text{so} \qquad a_r = -\left(\frac{k_e e^2}{m_e R^3}\right)r = -\omega^2 r$$

Thus, the motion is simple harmonic with frequency $f = \dfrac{\omega}{2\pi} = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{k_e e^2}{m_e R^3}}}$

(d) $$f = 2.47 \times 10^{15} \text{ Hz} = \frac{1}{2\pi}\sqrt{\frac{\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{\left(9.11\times10^{-31}\text{ kg}\right)R^3}}$$

which yields $R^3 = 1.05 \times 10^{-30} \text{ m}^3$, or $R = 1.02\times10^{-10} \text{ m} = \boxed{102 \text{ pm}}$

P24.49 The vertical velocity component of the moving charge increases according to

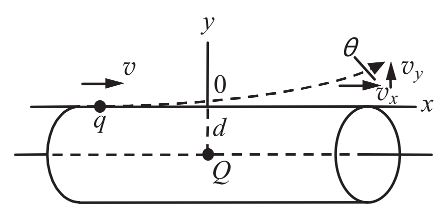


FIG. P24.49

$$m\frac{dv_y}{dt} = F_y \qquad m\frac{dv_y}{dx}\frac{dx}{dt} = qE_y$$

Now $\dfrac{dx}{dt} = v_x$ has the nearly constant value *v*. So

$$dv_y = \frac{q}{mv}E_y dx \qquad v_y = \int_0^{v_y} dv_y = \frac{q}{mv}\int_{-\infty}^{\infty} E_y dx$$

The radially outward component of the electric field varies along the *x* axis, but is described by

$$\int_{-\infty}^{\infty} E_y dA = \int_{-\infty}^{\infty} E_y (2\pi d)dx = \frac{Q}{\epsilon_0}$$

So $\displaystyle\int_{-\infty}^{\infty} E_y dx = \frac{Q}{2\pi d\,\epsilon_0}$ and $v_y = \dfrac{qQ}{mv2\pi d\epsilon_0}$. The angle of deflection is described by

$$\tan\theta = \frac{v_y}{v} = \frac{qQ}{2\pi\epsilon_0\, dmv^2} \qquad \boxed{\theta = \tan^{-1}\frac{qQ}{2\pi\epsilon_0\, dmv^2}}$$

P24.50 Consider the field due to a single sheet and let E_+ and E_- represent the fields due to the positive and negative sheets. The field at any distance from each sheet has a magnitude given by the textbook equation

$$|E_+| = |E_-| = \frac{\sigma}{2\epsilon_0}$$

(a) To the left of the positive sheet, E_+ is directed toward the left and E_- toward the right and the net field over this region is $\vec{\mathbf{E}} = \boxed{0}$.

(b) In the region between the sheets, E_+ and E_- are both directed toward the right and the net field is

$$\vec{\mathbf{E}} = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the right}}$$

(c) To the right of the negative sheet, E_+ and E_- are again oppositely directed and $\vec{\mathbf{E}} = \boxed{0}$.

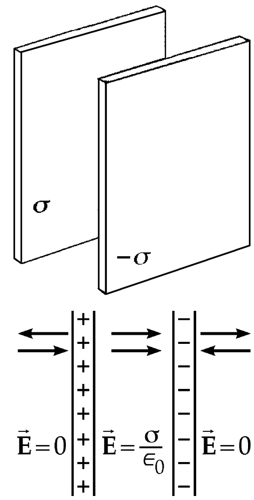


FIG. P24.50

P24.51 The magnitude of the field due to each sheet given by Equation 24.8 is

$$E = \frac{\sigma}{2\epsilon_0} \text{ directed perpendicular to the sheet}$$

(a) In the region to the left of the pair of sheets, both fields are directed toward the left and the net field is

$$\vec{\mathbf{E}} = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the left}}$$

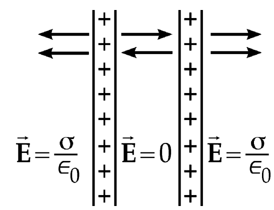


FIG. P24.51

(b) In the region between the sheets, the fields due to the individual sheets are oppositely directed and the net field is

$$\vec{\mathbf{E}} = \boxed{0}$$

(c) In the region to the right of the pair of sheets, both fields are directed toward the right and the net field is

$$\vec{\mathbf{E}} = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the right}}$$

P24.52 The resultant field within the cavity is the superposition of two fields, one $\vec{\mathbf{E}}_+$ due to a uniform sphere of positive charge of radius $2a$, and the other $\vec{\mathbf{E}}_-$ due to a sphere of negative charge of radius a centered within the cavity.

$$\frac{4}{3}\left(\frac{\pi r^3 \rho}{\epsilon_0}\right) = 4\pi r^2 E_+ \qquad \text{so} \qquad \vec{\mathbf{E}}_+ = \frac{\rho r}{3\epsilon_0}\hat{\mathbf{r}} = \frac{\rho \vec{\mathbf{r}}}{3\epsilon_0}$$

$$-\frac{4}{3}\left(\frac{\pi r_1^3 \rho}{\epsilon_0}\right) = 4\pi r_1^2 E_- \qquad \text{so} \qquad \vec{\mathbf{E}}_- = \frac{\rho r_1}{3\epsilon_0}(-\hat{\mathbf{r}}_1) = \frac{-\rho}{3\epsilon_0}\vec{\mathbf{r}}_1$$

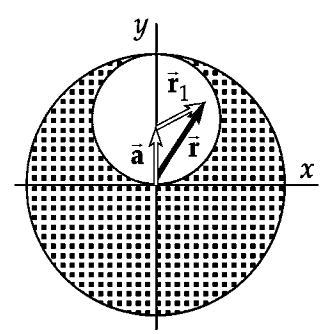


FIG. P24.52

continued on next page

Since $\vec{\mathbf{r}} = \vec{\mathbf{a}} + \vec{\mathbf{r}}_1$, $\quad \vec{\mathbf{E}}_- = \dfrac{-\rho\left(\vec{\mathbf{r}} - \vec{\mathbf{a}}\right)}{3\epsilon_0}$

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_+ + \vec{\mathbf{E}}_- = \frac{\rho\vec{\mathbf{r}}}{3\epsilon_0} - \frac{\rho\vec{\mathbf{r}}}{3\epsilon_0} + \frac{\rho\vec{\mathbf{a}}}{3\epsilon_0} = \frac{\rho\vec{\mathbf{a}}}{3\epsilon_0} = 0\hat{\mathbf{i}} + \frac{\rho a}{3\epsilon_0}\hat{\mathbf{j}}$$

Thus, $\boxed{E_x = 0}$

and $\boxed{E_y = \dfrac{\rho a}{3\epsilon_0}}$ at all points within the cavity

P24.53 Consider the charge distribution to be an unbroken charged spherical shell with uniform charge density σ and a circular disk with charge per area $-\sigma$. The total field is that due to the whole sphere, $\dfrac{Q}{4\pi\epsilon_0 R^2} = \dfrac{4\pi R^2\sigma}{4\pi\epsilon_0 R^2} = \dfrac{\sigma}{\epsilon_0}$ outward plus the field of the disk $-\dfrac{\sigma}{2\epsilon_0} = \dfrac{\sigma}{2\epsilon_0}$ radially inward. The total field is $\dfrac{\sigma}{\epsilon_0} - \dfrac{\sigma}{2\epsilon_0} = \boxed{\dfrac{\sigma}{2\epsilon_0}\text{ outward}}$.

P24.54 The electric field throughout the region is directed along x; therefore, $\vec{\mathbf{E}}$ will be perpendicular to dA over the four faces of the surface which are perpendicular to the yz plane, and E will be parallel to dA over the two faces which are parallel to the yz plane. Therefore,

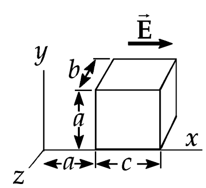


FIG. P24.54

$$\Phi_E = -\left(E_x\big|_{x=a}\right)A + \left(E_x\big|_{x=a+c}\right)A = -\left(3+2a^2\right)ab + \left(3+2(a+c)^2\right)ab$$
$$= 2abc(2a+c)$$

Substituting the given values for a, b, and c, we find $\Phi_E = \boxed{0.269\ \text{N}\cdot\text{m}^2/\text{C}}$.

$Q = \epsilon_0\,\Phi_E = 2.38\times10^{-12}\ \text{C} = \boxed{2.38\ \text{pC}}$

P24.55 $\oint\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \dfrac{q_{\text{in}}}{\epsilon_0}$

(a) For $r > R$, $\quad q_{\text{in}} = \displaystyle\int_0^R Ar^2\left(4\pi r^2\right)dr = 4\pi\frac{AR^5}{5}$

and $\quad E = \boxed{\dfrac{AR^5}{5\epsilon_0 r^2}}$

(b) For $r < R$, $\quad q_{\text{in}} = \displaystyle\int_0^r Ar^2\left(4\pi r^2\right)dr = \frac{4\pi Ar^5}{5}$

and $\quad E = \boxed{\dfrac{Ar^3}{5\epsilon_0}}$

P24.56 The total flux through a surface enclosing the charge Q is $\frac{Q}{\epsilon_0}$. The flux through the disk is

$$\Phi_{\text{disk}} = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

where the integration covers the area of the disk. We must evaluate this integral and set it equal to $\frac{\frac{1}{4}Q}{\epsilon_0}$ to find how b and R are related. In the figure, take $d\vec{\mathbf{A}}$ to be the area of an annular ring of radius s and width ds. The flux through $d\vec{\mathbf{A}}$ is $\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EdA\cos\theta = E(2\pi s ds)\cos\theta$.

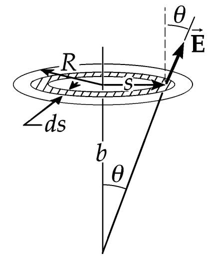


FIG. P24.56

The magnitude of the electric field has the same value at all points within the annular ring,

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{s^2+b^2} \qquad \text{and} \qquad \cos\theta = \frac{b}{r} = \frac{b}{\left(s^2+b^2\right)^{1/2}}.$$

Integrate from $s = 0$ to $s = R$ to get the flux through the entire disk.

$$\Phi_{E,\text{disk}} = \frac{Qb}{2\epsilon_0}\int_0^R \frac{s\,ds}{\left(s^2+b^2\right)^{3/2}} = \frac{Qb}{2\epsilon_0}\left[-\left(s^2+b^2\right)^{1/2}\right]\Bigg|_0^R = \frac{Q}{2\epsilon_0}\left[1-\frac{b}{\left(R^2+b^2\right)^{1/2}}\right]$$

The flux through the disk equals $\frac{Q}{4\epsilon_0}$ provided that $\frac{b}{\left(R^2+b^2\right)^{1/2}} = \frac{1}{2}$.

This is satisfied if $\boxed{R = \sqrt{3}b}$.

P24.57 $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0} = \frac{1}{\epsilon_0}\int_0^r \frac{a}{r}4\pi r^2 dr$

$$E4\pi r^2 = \frac{4\pi a}{\epsilon_0}\int_0^r r\,dr = \frac{4\pi a}{\epsilon_0}\frac{r^2}{2}$$

$\boxed{E = \frac{a}{2\epsilon_0}}$ = constant magnitude

(The direction is radially outward from center for positive a; radially inward for negative a.)

P24.58 In this case the charge density is *not uniform,* and Gauss's law is written as $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{1}{\epsilon_0}\int \rho\,dV$. We use a gaussian surface which is a cylinder of radius r, length ℓ, and is coaxial with the charge distribution.

(a) When $r < R$, this becomes $E(2\pi r\ell) = \frac{\rho_0}{\epsilon_0}\int_0^r \left(a - \frac{r}{b}\right)dV$. The element of volume is a cylindrical shell of radius r, length ℓ, and thickness dr so that $dV = 2\pi r\ell dr$.

$$E(2\pi r\ell) = \left(\frac{2\pi r^2\ell\rho_0}{\epsilon_0}\right)\left(\frac{a}{2} - \frac{r}{3b}\right) \text{ so inside the cylinder, } \boxed{E = \frac{\rho_0 r}{2\epsilon_0}\left(a - \frac{2r}{3b}\right)}$$

(b) When $r > R$, Gauss's law becomes

$$E(2\pi r\ell) = \frac{\rho_0}{\epsilon_0}\int_0^R \left(a - \frac{r}{b}\right)(2\pi r\ell dr) \text{ or outside the cylinder, } \boxed{E = \frac{\rho_0 R^2}{2\epsilon_0 r}\left(a - \frac{2R}{3b}\right)}$$

P24.59 (a) Consider a cylindrical shaped gaussian surface perpendicular to the *yz* plane with one end in the *yz* plane and the other end containing the point *x*:

Use Gauss's law: $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$

By symmetry, the electric field is zero in the *yz* plane and is perpendicular to $d\vec{\mathbf{A}}$ over the wall of the gaussian cylinder. Therefore, the only contribution to the integral is over the end cap containing the point *x*:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0} \quad \text{or} \quad EA = \frac{\rho(Ax)}{\epsilon_0}$$

so that at distance *x* from the mid-line of the slab, $\boxed{E = \frac{\rho x}{\epsilon_0}}$.

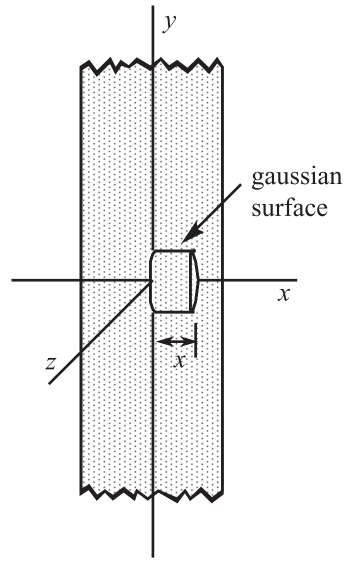


FIG. P24.59

(b) $$a = \frac{F}{m_e} = \frac{(-e)E}{m_e} = -\left(\frac{\rho e}{m_e \epsilon_0}\right)x$$

The acceleration of the electron is of the form $a = -\omega^2 x$ with $\omega = \sqrt{\frac{\rho e}{m_e \epsilon_0}}$

Thus, the motion is simple harmonic with frequency $f = \frac{\omega}{2\pi} = \boxed{\frac{1}{2\pi}\sqrt{\frac{\rho e}{m_e \epsilon_0}}}$

P24.60 Consider the gaussian surface described in the solution to problem 59.

(a) For $x > \frac{d}{2}$, $\quad dq = \rho\, dV = \rho A dx = CAx^2 dx$

$$\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{1}{\epsilon_0}\int dq$$

$$EA = \frac{CA}{\epsilon_0}\int_0^{d/2} x^2 dx = \frac{1}{3}\left(\frac{CA}{\epsilon_0}\right)\left(\frac{d^3}{8}\right)$$

$$E = \frac{Cd^3}{24\epsilon_0} \quad \text{or} \quad \boxed{\vec{\mathbf{E}} = \frac{Cd^3}{24\epsilon_0}\hat{\mathbf{i}} \text{ for } x > \frac{d}{2}; \quad \vec{\mathbf{E}} = -\frac{Cd^3}{24\epsilon_0}\hat{\mathbf{i}} \text{ for } x < -\frac{d}{2}}$$

(b) For $-\frac{d}{2} < x < \frac{d}{2}$ $\quad \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{1}{\epsilon_0}\int dq = \frac{CA}{\epsilon_0}\int_0^x x^2 dx = \frac{CAx^3}{3\epsilon_0}$

$$\boxed{\vec{\mathbf{E}} = \frac{Cx^3}{3\epsilon_0}\hat{\mathbf{i}} \text{ for } x > 0; \quad \vec{\mathbf{E}} = -\frac{Cx^3}{3\epsilon_0}\hat{\mathbf{i}} \text{ for } x < 0}$$

P24.61 (a) A point mass m creates a gravitational acceleration $\vec{\mathbf{g}} = -\frac{Gm}{r^2}\hat{\mathbf{r}}$ at a distance r

The flux of this field through a sphere is $\oint \vec{\mathbf{g}} \cdot d\vec{\mathbf{A}} = -\frac{Gm}{r^2}(4\pi r^2) = -4\pi Gm$

Since the r has divided out, we can visualize the field as unbroken field lines. The same flux would go through any other closed surface around the mass. If there are several or no masses inside a closed surface, each creates field to make its own contribution to the net flux according to

$$\boxed{\oint \vec{\mathbf{g}} \cdot d\vec{\mathbf{A}} = -4\pi Gm_{\text{in}}}$$

(b) Take a spherical gaussian surface of radius r. The field is inward so

$$\oint \vec{\mathbf{g}} \cdot d\vec{\mathbf{A}} = g4\pi r^2 \cos 180° = -g4\pi r^2$$

and $-4\pi Gm_{\text{in}} = -4\pi G\frac{4}{3}\pi r^3\rho$

Then, $-g4\pi r^2 = -4\pi G\frac{4}{3}\pi r^3\rho$ and $g = \frac{4}{3}\pi r\rho G$

Or, since $\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3}$, $g = \frac{M_E Gr}{R_E^3}$ or $\boxed{\vec{\mathbf{g}} = \frac{M_E Gr}{R_E^3} \text{ inward}}$

P24.62 The charge density is determined by $Q = \frac{4}{3}\pi a^3\rho$ $\rho = \frac{3Q}{4\pi a^3}$

(a) The flux is that created by the enclosed charge within radius r:

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{4\pi r^3\rho}{3\epsilon_0} = \frac{4\pi r^3 3Q}{3\epsilon_0\, 4\pi a^3} = \boxed{\frac{Qr^3}{\epsilon_0 a^3}}$$

(b) $\Phi_E = \boxed{\frac{Q}{\epsilon_0}}$. Note that the answers to parts (a) and (b) agree at $r = a$.

(c)

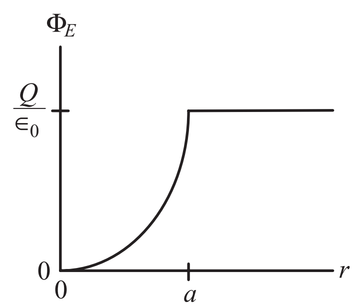


FIG. P24.62(c)

P24.63 $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{q_{in}}{\epsilon_0}$

(a) $$\left(-3.60\times10^3 \text{ N/C}\right)4\pi\left(0.100 \text{ m}\right)^2 = \frac{Q}{8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2} \qquad (a<r<b)$$

$$Q = -4.00\times10^{-9} \text{ C} = \boxed{-4.00 \text{ nC}}$$

(b) We take Q' to be the net charge on the hollow sphere. Outside c,

$$\left(+2.00\times10^2 \text{ N/C}\right)4\pi\left(0.500 \text{ m}\right)^2 = \frac{Q+Q'}{8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2} \qquad (r>c)$$

$$Q+Q' = +5.56\times10^{-9} \text{ C}, \text{ so } Q' = +9.56\times10^{-9} \text{ C} = \boxed{+9.56 \text{ nC}}$$

(c) For $b<r<c$: $E=0$ and $q_{in} = Q+Q_1 = 0$ where Q_1 is the total charge on the inner surface of the hollow sphere. Thus, $Q_1 = -Q = \boxed{+4.00 \text{ nC}}$.

Then, if Q_2 is the total charge on the outer surface of the hollow sphere,

$Q_2 = Q' - Q_1 = 9.56 \text{ nC} - 4.0 \text{ nC} = \boxed{+5.56 \text{ nC}}$.

P24.64 The field direction is radially outward perpendicular to the axis. The field strength depends on r but not on the other cylindrical coordinates θ or z. Choose a gaussian cylinder of radius r and length L. If $r<a$,

$$\Phi_E = \frac{q_{in}}{\epsilon_0} \quad \text{and} \quad E\left(2\pi rL\right) = \frac{\lambda L}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi r\epsilon_0} \quad \text{or} \quad \boxed{\vec{\mathbf{E}} = \frac{\lambda}{2\pi r\epsilon_0}\hat{\mathbf{r}} \qquad (r<a)}$$

If $a<r<b$,
$$E\left(2\pi rL\right) = \frac{\lambda L + \rho\pi\left(r^2-a^2\right)L}{\epsilon_0}$$

$$\boxed{\vec{\mathbf{E}} = \frac{\lambda + \rho\pi\left(r^2-a^2\right)}{2\pi r\epsilon_0}\hat{\mathbf{r}} \qquad (a<r<b)}$$

If $r>b$,
$$E\left(2\pi rL\right) = \frac{\lambda L + \rho\pi\left(b^2-a^2\right)L}{\epsilon_0}$$

$$\boxed{\vec{\mathbf{E}} = \frac{\lambda + \rho\pi\left(b^2-a^2\right)}{2\pi r\epsilon_0}\hat{\mathbf{r}} \qquad (r>b)}$$

***P24.65** (a) Consider a gaussian surface in the shape of a rectangular box with two faces perpendicular to the direction of the field. It encloses some charge, so the net flux out of the box is nonzero. The field must be stronger on one side than on the other. The field cannot be uniform in magnitude.

(b) Now the volume contains no charge. The net flux out of the box is zero. The flux entering is equal to the flux exiting. The field magnitude is uniform at points along one field line. The field magnitude can vary over the faces of the box perpendicular to the field.

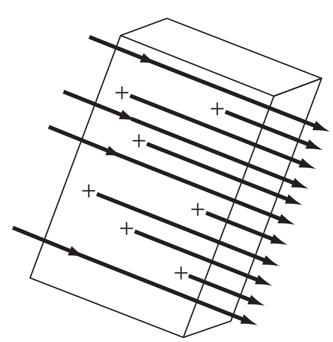


FIG. P24.65(a)

ANSWERS TO EVEN PROBLEMS

P24.2 $355\ \text{kN}\cdot\text{m}^2/\text{C}$

P24.4 (a) $-2.34\ \text{kN}\cdot\text{m}^2/\text{C}$ (b) $+2.34\ \text{kN}\cdot\text{m}^2/\text{C}$ (c) 0

P24.6 (a) -55.7 nC (b) The negative charge has a spherically symmetric distribution concentric with the shell.

P24.8 (a) $\dfrac{q}{2\epsilon_0}$ (b) $\dfrac{q}{2\epsilon_0}$ (c) Plane and square both subtend a solid angle of a hemisphere at the charge.

P24.10 (a) $1.36\ \text{MN}\cdot\text{m}^2/\text{C}$ (b) $678\ \text{kN}\cdot\text{m}^2/\text{C}$ (c) No; see the solution.

P24.12 $1.77\ \text{pC}/\text{m}^3$ positive

P24.14 $\dfrac{Q-6|q|}{6\epsilon_0}$

P24.16 $28.2\ \text{N}\cdot\text{m}^2/\text{C}$

P24.18 (a) 0 (b) $365\ \text{kN}/\text{C}$ (c) $1.46\ \text{MN}/\text{C}$ (d) $649\ \text{kN}/\text{C}$

P24.20 (a) $16.2\ \text{MN}/\text{C}$ toward the filament (b) $8.09\ \text{MN}/\text{C}$ toward the filament (c) $1.62\ \text{MN}/\text{C}$ toward the filament

P24.22 (a) 913 nC (b) 0

P24.24 (a) A long cylindrical plastic rod 2.00 cm in radius carries charge uniformly distributed throughout its volume, with density 5.00 μC/m^3. Find the magnitude of the electric field it creates at a point *P*, 3.00 cm from its axis. As a gaussian surface choose a concentric cylinder with its curved surface passing through the point *P* and with length 8.00 cm. (b) 3.77 kN/C

P24.26 $4.86\ \text{GN}/\text{C}$ away from the wall. It is constant close to the wall.

P24.28 3.50 kN

P24.30 (a) ~1 mN (b) ~100 nC (c) ~10 kN/C (d) ~10 kN · m^2/C

P24.32 $\vec{\mathbf{E}} = Q/2\epsilon_0 A$ vertically upward in each case if $Q > 0$

P24.34 (a) 0 (b) $12.4\ \text{kN}/\text{C}$ radially outward (c) $639\ \text{N}/\text{C}$ radially outward (d) No answer changes. The solid copper sphere carries charge only on its outer surface.

P24.36 31.9 nC/m^3

P24.38 The electric field just outside the surface is given by σ/ϵ_0. At this point the uniformly charged surface of the sphere looks just like a uniform flat sheet of charge.

P24.40 See the solution.

P24.42 $\dfrac{chw^2}{2}$

P24.44 (a) 2.00 μC (b) to the right (c) 702 kN/C (d) 0 (e) 0 (f) 3.00 μC (g) 4.21 MN/C radially outward (h) 1.54 μC (i) 8.63 MN/C radially outward (j) −3.00 μC (k) 2.00 μC (l) See the solution.

P24.46 See the solution.

P24.48 (a, b) See the solution. (c) $\frac{1}{2\pi}\sqrt{\frac{k_e e^2}{m_e R^3}}$ (d) 102 pm

P24.50 (a) 0 (b) $\frac{\sigma}{\epsilon_0}$ to the right (c) 0

P24.52 See the solution.

P24.54 $0.269\ \text{N}\cdot\text{m}^2/\text{C}$; 2.38 pC

P24.56 See the solution.

P24.58 (a) $\frac{\rho_0 r}{2\epsilon_0}\left(a-\frac{2r}{3b}\right)$ (b) $\frac{\rho_0 R^2}{2\epsilon_0\, r}\left(a-\frac{2R}{3b}\right)$

P24.60 (a) $\vec{\mathbf{E}}=\frac{Cd^3}{24\epsilon_0}\hat{\mathbf{i}}$ for $x>\frac{d}{2}$; $\vec{\mathbf{E}}=-\frac{Cd^3}{24\epsilon_0}\hat{\mathbf{i}}$ for $x<-\frac{d}{2}$ (b) $\vec{\mathbf{E}}=\frac{Cx^3}{3\epsilon_0}\hat{\mathbf{i}}$ for $x>0$;

$\vec{\mathbf{E}}=-\frac{Cx^3}{3\epsilon_0}\hat{\mathbf{i}}$ for $x<0$

P24.62 (a) $\frac{Qr^3}{\epsilon_0\, a^3}$ (b) $\frac{Q}{\epsilon_0}$ (c) See the solution

P24.64 For $r<a$, $\vec{\mathbf{E}}=\lambda/2\pi\epsilon_0 r$ radially outward.

For $a<r<b$, $\vec{\mathbf{E}}=[\lambda+\rho\pi(r^2-a^2)]/2\pi\epsilon_0 r$ radially outward.

For $r>b$, $\vec{\mathbf{E}}=[\lambda+\rho\pi(b^2-a^2)]/2\pi\epsilon_0 r$ radially outward.

25

Electric Potential

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q25.1 When one object B with electric charge is immersed in the electric field of another charge or charges A, the system possesses electric potential energy. The energy can be measured by seeing how much work the field does on the charge B as it moves to a reference location. We choose not to visualize A's effect on B as an action-at-a-distance, but as the result of a two-step process: Charge A creates electric potential throughout the surrounding space. Then the potential acts on B to inject the system with energy.

***Q25.2** (i) The particle feels an electric force in the negative x direction. An outside agent pushes it uphill against this force, increasing the potential energy. Answer (a).
(ii) The potential decreases in the direction of the electric field. Answer (c).

***Q25.3** The potential is decreasing toward the bottom of the page, so the electric field is downward. Answer (f).

***Q25.4** (i) At points off the x axis the electric field has a nonzero y component. At points on the negative x axis the field is to the right and positive. At points to the right of $x = 500$ mm the field is to the left and nonzero. The field is zero at one point between $x = 250$ mm and $x = 500$ mm. Answer (b).
(ii) The electric potential is negative at this and at all points. Answer (c). (iii) Answer (d).
(iv) Answer (d).

Q25.5 To move like charges together from an infinite separation, at which the potential energy of the system of two charges is zero, requires *work* to be done on the system by an outside agent. Hence energy is stored, and potential energy is positive. As charges with opposite signs move together from an infinite separation, energy is released, and the potential energy of the set of charges becomes negative.

Q25.6 (a) The equipotential surfaces are nesting coaxial cylinders around an infinite line of charge.
(b) The equipotential surfaces are nesting concentric spheres around a uniformly charged sphere.

***Q25.7** Answer (b). The potential could have any value.

***Q25.8** The same charges at the same distance away create the same contribution to the total potential. Answer (b).

***Q25.9** The change in kinetic energy is the negative of the change in electric potential energy, so we work out $-q\Delta V = -q(V_f - V_i)$ in each case.
(a) $-(-e)(60\text{ V} - 40\text{ V}) = +20$ eV (b) $-(-e)(20\text{ V} - 40\text{ V}) = -20$ eV
(c) $-(e)(20\text{ V} - 40\text{ V}) = +20$ eV (d) $-(e)(10\text{ V} - 40\text{ V}) = +30$ eV
(e) $-(-2e)(50\text{ V} - 40\text{ V}) = +20$ eV (f) $-(-2e)(60\text{ V} - 40\text{ V}) = +40$ eV
With also (g) 0 and (h) +10 eV, the ranking is f > d > c = e = a > h > g > b.

Q25.10 The main factor is the radius of the dome. One often overlooked aspect is also the humidity of the air—drier air has a larger dielectric breakdown strength, resulting in a higher attainable electric potential. If other grounded objects are nearby, the maximum potential might be reduced.

***Q25.11** (i) The two spheres come to the same potential, so q/R is the same for both. With twice the radius, B has twice the charge. Answer (d).
(ii) All the charge runs away from itself to the outer surface of B. Answer (a).

Q25.12 The grounding wire can be touched equally well to any point on the sphere. Electrons will drain away into the ground and the sphere will be left positively charged. The ground, wire, and sphere are all conducting. They together form an equipotential volume at zero volts during the contact. However close the grounding wire is to the negative charge, electrons have no difficulty in moving within the metal through the grounding wire to ground. The ground can act as an infinite source or sink of electrons. In this case, it is an electron sink.

SOLUTIONS TO PROBLEMS

Section 25.1 Electric Potential and Potential Difference

P25.1 (a) Energy of the proton-field system is conserved as the proton moves from high to low potential, which can be defined for this problem as moving from 120 V down to 0 V.

$$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f \qquad 0 + qV + 0 = \frac{1}{2}mv_p^2 + 0$$

$$\left(1.60\times10^{-19}\text{ C}\right)(120\text{ V})\left(\frac{1\text{ J}}{1\text{ V}\cdot\text{C}}\right) = \frac{1}{2}\left(1.67\times10^{-27}\text{ kg}\right)v_p^2$$

$$v_p = \boxed{1.52\times10^{5}\text{ m/s}}$$

(b) The electron will gain speed in moving the other way,

from $V_i = 0$ to $V_f = 120$ V: $\quad K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$

$$0 + 0 + 0 = \frac{1}{2}mv_e^2 + qV$$

$$0 = \frac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)v_e^2 + \left(-1.60\times10^{-19}\text{ C}\right)(120\text{ J/C})$$

$$v_e = \boxed{6.49\times10^{6}\text{ m/s}}$$

P25.2 $\Delta V = -14.0$ V and $Q = -N_A e = -\left(6.02\times10^{23}\right)\left(1.60\times10^{-19}\right) = -9.63\times10^{4}$ C

$\Delta V = \dfrac{W}{Q}$, so $W = Q\Delta V = \left(-9.63\times10^{4}\text{ C}\right)(-14.0\text{ J/C}) = \boxed{1.35\text{ MJ}}$

Section 25.2 **Potential Difference in a Uniform Electric Field**

P25.3 $E = \frac{|\Delta V|}{d} = \frac{25.0 \times 10^3 \text{ J/C}}{1.50 \times 10^{-2} \text{ m}} = 1.67 \times 10^6 \text{ N/C} = \boxed{1.67 \text{ MN/C}}$

P25.4 $V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_A^C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} - \int_C^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$

$V_B - V_A = (-E\cos 180°) \int_{-0.300}^{0.500} dy - (E\cos 90.0°) \int_{-0.200}^{0.400} dx$

$V_B - V_A = (325)(0.800) = \boxed{+260 \text{ V}}$

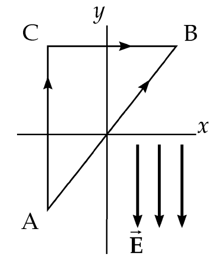


FIG. P25.4

P25.5 $\Delta U = -\frac{1}{2}m\left(v_f^2 - v_i^2\right) = -\frac{1}{2}\left(9.11 \times 10^{-31} \text{ kg}\right)\left[\left(1.40 \times 10^5 \text{ m/s}\right)^2 - \left(3.70 \times 10^6 \text{ m/s}\right)^2\right]$

$= 6.23 \times 10^{-18} \text{ J}$

$\Delta U = q\Delta V$: $\quad +6.23 \times 10^{-18} = \left(-1.60 \times 10^{-19}\right)\Delta V$

$\Delta V = \boxed{-38.9 \text{ V. The origin is at highest potential.}}$

P25.6 Assume the opposite. Then at some point A on some equipotential surface the electric field has a nonzero component E_p in the plane of the surface. Let a test charge start from point A and move some distance on the surface in the direction of the field component. Then $\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ is nonzero. The electric potential charges across the surface and it is not an equipotential surface. The contradiction shows that our assumption is false, that $E_p = 0$, and that the field is perpendicular to the equipotential surface.

P25.7 (a) Arbitrarily choose $V = 0$ at 0. Then at other points

$V = -Ex \quad$ and $\quad U_e = QV = -QEx$

Between the endpoints of the motion,

$\left(K + U_s + U_e\right)_i = \left(K + U_s + U_e\right)_f$

$0 + 0 + 0 = 0 + \frac{1}{2}kx_{\text{max}}^2 - QEx_{\text{max}} \quad$ so $\quad x_{\text{max}} = \boxed{\frac{2QE}{k}}$

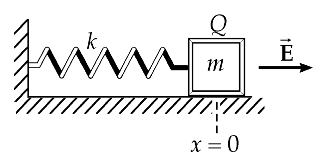


FIG. P25.7

(b) At equilibrium,

$\sum F_x = -F_s + F_e = 0 \quad$ or $\quad kx = QE$

So the equilibrium position is at $x = \boxed{\frac{QE}{k}}$.

(c) The block's equation of motion is $\sum F_x = -kx + QE = m\frac{d^2x}{dt^2}$.

Let $x' = x - \frac{QE}{k}$, $\quad$ or $\quad x = x' + \frac{QE}{k}$,

continued on next page

so the equation of motion becomes:

$$-k\left(x'+\frac{QE}{k}\right)+QE=m\frac{d^2\left(x+QE/k\right)}{dt^2}, \quad \text{or} \quad \frac{d^2x'}{dt^2}=-\left(\frac{k}{m}\right)x'$$

This is the equation for simple harmonic motion $a_{x'}=-\omega^2 x'$

with $\omega=\sqrt{\frac{k}{m}}$

The period of the motion is then $T=\frac{2\pi}{\omega}=\boxed{2\pi\sqrt{\frac{m}{k}}}$

(d) $\left(K+U_s+U_e\right)_i+\Delta E_{\text{mech}}=\left(K+U_s+U_e\right)_f$

$$0+0+0-\mu_k mgx_{\max}=0+\frac{1}{2}kx_{\max}^2-QEx_{\max}$$

$$x_{\max}=\boxed{\frac{2\left(QE-\mu_k mg\right)}{k}}$$

P25.8 Arbitrarily take $V=0$ at point P. Then the potential at the original position of the charge is $-\vec{\mathbf{E}}\cdot\vec{\mathbf{s}}=-EL\cos\theta$. At the final point a, $V=-EL$. Because the table is frictionless we have $(K+U)_i=(K+U)_f$

$$0-qEL\cos\theta=\frac{1}{2}mv^2-qEL$$

$$v=\sqrt{\frac{2qEL(1-\cos\theta)}{m}}=\sqrt{\frac{2\left(2.00\times10^{-6}\text{ C}\right)(300\text{ N/C})(1.50\text{ m})(1-\cos 60.0°)}{0.010\,0\text{ kg}}}=\boxed{0.300\text{ m/s}}$$

P25.9 Arbitrarily take $V=0$ at the initial point. Then at distance d downfield, where L is the rod length, $V=-Ed$ and $U_e=-\lambda LEd$.

(a) $(K+U)_i=(K+U)_f$

$$0+0=\frac{1}{2}\mu Lv^2-\lambda LEd$$

$$v=\sqrt{\frac{2\lambda Ed}{\mu}}=\sqrt{\frac{2\left(40.0\times10^{-6}\text{ C/m}\right)(100\text{ N/C})(2.00\text{ m})}{(0.100\text{ kg/m})}}=\boxed{0.400\text{ m/s}}$$

(b) $\boxed{\text{The same.}}$ Each bit of the rod feels a force of the same size as before.

Section 25.3 **Electric Potential and Potential Energy Due to Point Charges**

P25.10 (a) Since the charges are equal and placed symmetrically, $\boxed{F=0}$.

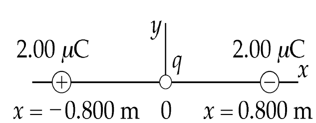


FIG. P25.10

(b) Since $F=qE=0$, $\boxed{E=0}$.

(c) $V=2k_e\frac{q}{r}=2\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{2.00\times10^{-6}\text{ C}}{0.800\text{ m}}\right)$

$V=4.50\times10^4\text{ V}=\boxed{45.0\text{ kV}}$

P25.11 (a) The potential at 1.00 cm is

$$V_1 = k_e \frac{q}{r} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})}{1.00 \times 10^{-2} \text{ m}} = \boxed{1.44 \times 10^{-7} \text{ V}}$$

(b) The potential at 2.00 cm is

$$V_2 = k_e \frac{q}{r} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})}{2.00 \times 10^{-2} \text{ m}} = 0.719 \times 10^{-7} \text{ V}$$

Thus, the difference in potential between the two points is $\Delta V = V_2 - V_1 = \boxed{-7.19 \times 10^{-8} \text{ V}}$.

(c) The approach is the same as above except the charge is -1.60×10^{-19} C. This changes the sign of each answer, with its magnitude remaining the same.

That is, the potential at 1.00 cm is $\boxed{-1.44 \times 10^{-7} \text{ V}}$.

The potential at 2.00 cm is -0.719×10^{-7} V, so $\Delta V = V_2 - V_1 = \boxed{7.19 \times 10^{-8} \text{ V}}$

P25.12 (a) $E_x = \frac{k_e q_1}{x^2} + \frac{k_e q_2}{(x-2.00)^2} = 0$ becomes $E_x = k_e\left(\frac{+q}{x^2} + \frac{-2q}{(x-2.00)^2}\right) = 0$

Dividing by k_e, $2qx^2 = q(x-2.00)^2$ $x^2 + 4.00x - 4.00 = 0$

Therefore $E = 0$ when $x = \frac{-4.00 \pm \sqrt{16.0 + 16.0}}{2} = \boxed{-4.83 \text{ m}}$

(Note that the positive root does not correspond to a physically valid situation.)

(b) $V = \frac{k_e q_1}{x} + \frac{k_e q_2}{2.00 - x} = 0$ or $V = k_e\left(\frac{+q}{x} - \frac{2q}{2.00 - x}\right) = 0$

Again solving for *x*, $2qx = q(2.00 - x)$

For $0 \le x \le 2.00$ $V = 0$ when $x = \boxed{0.667 \text{ m}}$

and $\frac{q}{|x|} = \frac{-2q}{|2-x|}$ For $x < 0$ $x = \boxed{-2.00 \text{ m}}$

P25.13 (a) $E = \frac{|Q|}{4\pi \epsilon_0 r^2}$

$V = \frac{Q}{4\pi \epsilon_0 r}$

$r = \frac{|V|}{|E|} = \frac{3\,000 \text{ V}}{500 \text{ V/m}} = \boxed{6.00 \text{ m}}$

(b) $V = -3\,000 \text{ V} = \frac{Q}{4\pi \epsilon_0 (6.00 \text{ m})}$

$$Q = \frac{-3\,000 \text{ V}}{(8.99 \times 10^9 \text{ V}\cdot\text{m/C})}(6.00 \text{ m}) = \boxed{-2.00\ \mu\text{C}}$$

P25.14 (a) $U = \dfrac{qQ}{4\pi \in_0 r} = \dfrac{\left(5.00\times10^{-9}\text{ C}\right)\left(-3.00\times10^{-9}\text{ C}\right)\left(8.99\times10^{9}\text{ V}\cdot\text{m/C}\right)}{(0.350\text{ m})} = \boxed{-3.86\times10^{-7}\text{ J}}$

The minus sign means it takes 3.86×10^{-7} J to pull the two charges apart from 35 cm to a much larger separation.

(b) $V = \dfrac{Q_1}{4\pi \in_0 r_1} + \dfrac{Q_2}{4\pi \in_0 r_2}$

$$= \frac{\left(5.00\times10^{-9}\text{ C}\right)\left(8.99\times10^{9}\text{ V}\cdot\text{m/C}\right)}{0.175\text{ m}} + \frac{\left(-3.00\times10^{-9}\text{ C}\right)\left(8.99\times10^{9}\text{ V}\cdot\text{m/C}\right)}{0.175\text{ m}}$$

$V = \boxed{103\text{ V}}$

P25.15 $V = \sum_i k\dfrac{q_i}{r_i}$

$$V = \left(8.99\times10^{9}\right)\left(7.00\times10^{-6}\right)\left[\frac{-1}{0.010\,0} - \frac{1}{0.010\,0} + \frac{1}{0.038\,7}\right]$$

$V = \boxed{-1.10\times10^{7}\text{ V} = -11.0\text{ MV}}$

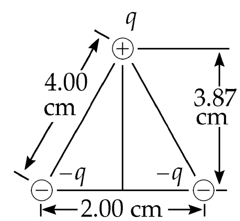


FIG. P25.15

P25.16 (a) $V = \dfrac{k_e q_1}{r_1} + \dfrac{k_e q_2}{r_2} = 2\left(\dfrac{k_e q}{r}\right)$

$$V = 2\left(\frac{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(2.00\times10^{-6}\text{ C}\right)}{\sqrt{(1.00\text{ m})^2 + (0.500\text{ m})^2}}\right)$$

$V = 3.22\times10^{4}\text{ V} = \boxed{32.2\text{ kV}}$

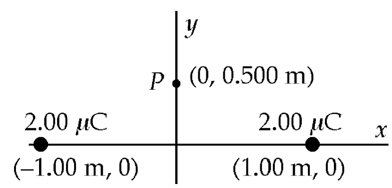


FIG. P25.16

(b) $U = qV = \left(-3.00\times10^{-6}\text{ C}\right)\left(3.22\times10^{4}\text{ J/C}\right) = \boxed{-9.65\times10^{-2}\text{ J}}$

P25.17 $U_e = q_4V_1 + q_4V_2 + q_4V_3 = q_4\left(\dfrac{1}{4\pi \in_0}\right)\left(\dfrac{q_1}{r_1} + \dfrac{q_2}{r_2} + \dfrac{q_3}{r_3}\right)$

$$U_e = \left(10.0\times10^{-6}\text{ C}\right)^2\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1}{0.600\text{ m}} + \frac{1}{0.150\text{ m}} + \frac{1}{\sqrt{(0.600\text{ m})^2 + (0.150\text{ m})^2}}\right)$$

$U_e = \boxed{8.95\text{ J}}$

***P25.18** (a) The first expression, with distances squared, describes an electric field. The second expression describes an electric potential. Then a $\boxed{\text{positive 7 nC charge}}$ is 7 cm from the origin. To create field that is to the left and downward, it must be in the first quadrant, with position vector $\boxed{7\text{ cm at }70°}$. A $\boxed{\text{negative 8 nC charge}}$ 3 cm from the origin creates an upward electric field at the origin, so it must be at $\boxed{3\text{ cm at }90°}$. We evaluate the given expressions:

$$\vec{\mathbf{E}} = -4.39\ \text{kN/C}\,\hat{\mathbf{i}} + 67.8\ \text{kN/C}\,\hat{\mathbf{j}}$$
$$V = -1.50\ \text{kV}$$

(b) $\vec{\mathbf{F}} = q\vec{\mathbf{E}} = -16\times10^{-9}\ \text{C}\left(-4.39\hat{\mathbf{i}} + 67.8\hat{\mathbf{j}}\right)10^{3}\ \text{N/C} = \boxed{\left(7.03\hat{\mathbf{i}} - 109\hat{\mathbf{j}}\right)\times10^{-5}\ \text{N}}$

(c) $U_e = qV = -16\times10^{-9}\ \text{C}\left(-1.50\times10^{3}\ \text{J/C}\right) = \boxed{+2.40\times10^{-5}\ \text{J}}$

P25.19

$$U = U_1 + U_2 + U_3 + U_4$$
$$U = 0 + U_{12} + \left(U_{13} + U_{23}\right) + \left(U_{14} + U_{24} + U_{34}\right)$$
$$U = 0 + \frac{k_eQ^2}{s} + \frac{k_eQ^2}{s}\left(\frac{1}{\sqrt{2}} + 1\right) + \frac{k_eQ^2}{s}\left(1 + \frac{1}{\sqrt{2}} + 1\right)$$
$$U = \frac{k_eQ^2}{s}\left(4 + \frac{2}{\sqrt{2}}\right) = \boxed{5.41\frac{k_eQ^2}{s}}$$

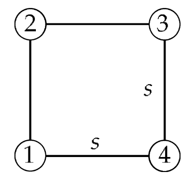


FIG. P25.19

We can visualize the term $\left(4 + \frac{2}{\sqrt{2}}\right)$ as arising directly from the 4 side pairs and 2 face diagonal pairs.

P25.20 Each charge creates equal potential at the center. The total potential is:

$$V = 5\left[\frac{k_e(-q)}{R}\right] = \boxed{-\frac{5k_eq}{R}}$$

P25.21 (a) Each charge separately creates positive potential everywhere. The total potential produced by the three charges together is then the sum of three positive terms. There is $\boxed{\text{no point}}$, at a finite distance from the charges, at which this total potential is zero.

(b) $V = \frac{k_eq}{a} + \frac{k_eq}{a} = \boxed{\frac{2k_eq}{a}}$

P25.22 (a) $V(x) = \dfrac{k_e Q_1}{r_1} + \dfrac{k_e Q_2}{r_2} = \dfrac{k_e(+Q)}{\sqrt{x^2+a^2}} + \dfrac{k_e(+Q)}{\sqrt{x^2+(-a)^2}}$

$$V(x) = \frac{2k_e Q}{\sqrt{x^2+a^2}} = \frac{k_e Q}{a}\left(\frac{2}{\sqrt{(x/a)^2+1}}\right)$$

$$\frac{V(x)}{(k_e Q/a)} = \boxed{\frac{2}{\sqrt{(x/a)^2+1}}}$$

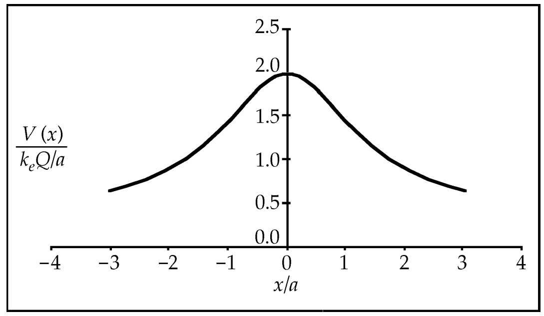


FIG. P25.22(a)

(b) $V(y) = \dfrac{k_e Q_1}{r_1} + \dfrac{k_e Q_2}{r_2} = \dfrac{k_e(+Q)}{|y-a|} + \dfrac{k_e(-Q)}{|y+a|}$

$$V(y) = \frac{k_e Q}{a}\left(\frac{1}{|y/a-1|} - \frac{1}{|y/a+1|}\right)$$

$$\frac{V(y)}{(k_e Q/a)} = \boxed{\left(\frac{1}{|y/a-1|} - \frac{1}{|y/a+1|}\right)}$$

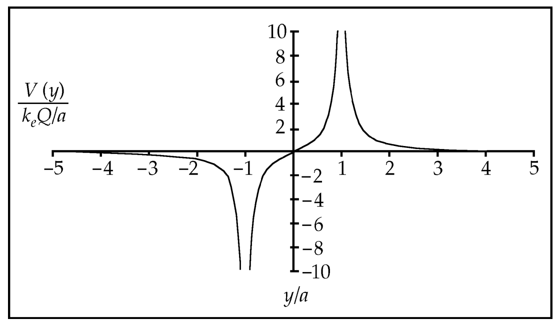


FIG. P25.22(b)

P25.23 Consider the two spheres as a system.

(a) Conservation of momentum: $0 = m_1 v_1 \hat{\mathbf{i}} + m_2 v_2\left(-\hat{\mathbf{i}}\right)$ or $v_2 = \dfrac{m_1 v_1}{m_2}$

By conservation of energy, $0 = \dfrac{k_e(-q_1)q_2}{d} = \dfrac{1}{2}m_1 v_1^2 + \dfrac{1}{2}m_2 v_2^2 + \dfrac{k_e(-q_1)q_2}{r_1+r_2}$

and $\dfrac{k_e q_1 q_2}{r_1+r_2} - \dfrac{k_e q_1 q_2}{d} = \dfrac{1}{2}m_1 v_1^2 + \dfrac{1}{2}\dfrac{m_1^2 v_1^2}{m_2}$

$$v_1 = \sqrt{\frac{2m_2 k_e q_1 q_2}{m_1(m_1+m_2)}\left(\frac{1}{r_1+r_2} - \frac{1}{d}\right)}$$

$$v_1 = \sqrt{\frac{2(0.700\text{ kg})(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2\times10^{-6}\text{ C})(3\times10^{-6}\text{ C})}{(0.100\text{ kg})(0.800\text{ kg})}\left(\frac{1}{8\times10^{-3}\text{ m}} - \frac{1}{1.00\text{ m}}\right)}$$

$$= \boxed{10.8\text{ m/s}}$$

$$v_2 = \frac{m_1 v_1}{m_2} = \frac{0.100\text{ kg}(10.8\text{ m/s})}{0.700\text{ kg}} = \boxed{1.55\text{ m/s}}$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving $\boxed{\text{faster than calculated in (a)}}$.

P25.24 Consider the two spheres as a system.

(a) Conservation of momentum: $0 = m_1 v_1 \hat{\mathbf{i}} + m_2 v_2 \left(-\hat{\mathbf{i}}\right)$

or $v_2 = \dfrac{m_1 v_1}{m_2}$.

By conservation of energy, $0 = \dfrac{k_e(-q_1)q_2}{d} = \dfrac{1}{2}m_1 v_1^2 + \dfrac{1}{2}m_2 v_2^2 + \dfrac{k_e(-q_1)q_2}{r_1 + r_2}$

and $\dfrac{k_e q_1 q_2}{r_1 + r_2} - \dfrac{k_e q_1 q_2}{d} = \dfrac{1}{2}m_1 v_1^2 + \dfrac{1}{2}\dfrac{m_1^2 v_1^2}{m_2}$.

$$v_1 = \boxed{\sqrt{\frac{2m_2 k_e q_1 q_2}{m_1(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}}$$

$$v_2 = \left(\frac{m_1}{m_2}\right)v_1 = \boxed{\sqrt{\frac{2m_1 k_e q_1 q_2}{m_2(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}}$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving $\boxed{\text{faster than calculated in (a)}}$.

P25.25 The original electrical potential energy is

$$U_e = qV = q\frac{k_e q}{d}$$

In the final configuration we have mechanical equilibrium. The spring and electrostatic forces on each charge are $-k(2d) + q\dfrac{k_e q}{(3d)^2} = 0$. Then $k = \dfrac{k_e q^2}{18d^3}$. In the final configuration the total potential energy is $\dfrac{1}{2}kx^2 + qV = \dfrac{1}{2}\dfrac{k_e q^2}{18d^3}(2d)^2 + q\dfrac{k_e q}{3d} = \dfrac{4}{9}\dfrac{k_e q^2}{d}$. The missing energy must have become internal energy, as the system is isolated: $\dfrac{k_e q^2}{d} = \dfrac{4k_e q^2}{9d} + \Delta E_{\text{int}}$.

$$\boxed{\Delta E_{\text{int}} = \frac{5}{9}\frac{k_e q^2}{d}}$$

P25.26 Using conservation of energy for the alpha particle-nucleus system,

we have $K_f + U_f = K_i + U_i$

But $U_i = \dfrac{k_e q_\alpha q_{\text{gold}}}{r_i}$ and $r_i \approx \infty$ Thus, $U_i = 0$

Also $K_f = 0$ ($v_f = 0$ at turning point),

so $U_f = K_i$

or $\dfrac{k_e q_\alpha q_{\text{gold}}}{r_{\min}} = \dfrac{1}{2}m_\alpha v_\alpha^2$

$$r_{\min} = \frac{2k_e q_\alpha q_{\text{gold}}}{m_\alpha v_\alpha^2} = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2)(79)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(6.64\times10^{-27}\ \text{kg}\right)\left(2.00\times10^7\ \text{m/s}\right)^2} = 2.74\times10^{-14}\ \text{m}$$

$$= \boxed{27.4\ \text{fm}}$$

P25.27 Each charge moves off on its diagonal line. All charges have equal speeds.

$$\sum(K+U)_i = \sum(K+U)_f$$

$$0+\frac{4k_eq^2}{L}+\frac{2k_eq^2}{\sqrt{2}L} = 4\left(\frac{1}{2}mv^2\right)+\frac{4k_eq^2}{2L}+\frac{2k_eq^2}{2\sqrt{2}L}$$

$$\left(2+\frac{1}{\sqrt{2}}\right)\frac{k_eq^2}{L} = 2mv^2$$

$$\boxed{v=\sqrt{\left(1+\frac{1}{\sqrt{8}}\right)\frac{k_eq^2}{mL}}}$$

P25.28 A cube has 12 edges and 6 faces. Consequently, there are 12 edge pairs separated by s, $2\times 6 = 12$ face diagonal pairs separated by $\sqrt{2}s$, and 4 interior diagonal pairs separated by $\sqrt{3}s$.

$$U=\frac{k_eq^2}{s}\left[12+\frac{12}{\sqrt{2}}+\frac{4}{\sqrt{3}}\right] = \boxed{22.8\frac{k_eq^2}{s}}$$

Section 25.4 **Obtaining the Value of the Electric Field from the Electric Potential**

P25.29 $V = a+bx = 10.0\text{ V}+(-7.00\text{ V/m})x$

(a) At $x = 0$, $V = \boxed{10.0\text{ V}}$

At $x = 3.00$ m, $V = \boxed{-11.0\text{ V}}$

At $x = 6.00$ m, $V = \boxed{-32.0\text{ V}}$

(b) $E = -\frac{dV}{dx} = -b = -(-7.00\text{ V/m}) = \boxed{7.00\text{ N/C in the }+x\text{ direction}}$

P25.30 (a) For $r < R$ $V = \frac{k_eQ}{R}$

$$E_r = -\frac{dV}{dr} = \boxed{0}$$

(b) For $r \geq R$ $V = \frac{k_eQ}{r}$

$$E_r = -\frac{dV}{dr} = -\left(-\frac{k_eQ}{r^2}\right) = \boxed{\frac{k_eQ}{r^2}}$$

P25.31 $V = 5x - 3x^2y + 2yz^2$

Evaluate E at $(1,\ 0,\ -2)$.

$$E_x = -\frac{\partial V}{\partial x} = \boxed{-5+6xy} = -5+6(1)(0) = -5$$

$$E_y = -\frac{\partial V}{\partial y} = \boxed{+3x^2-2z^2} = 3(1)^2-2(-2)^2 = -5$$

$$E_z = -\frac{\partial V}{\partial z} = \boxed{-4yz} = -4(0)(-2) = 0$$

$$E = \sqrt{E_x^2+E_y^2+E_z^2} = \sqrt{(-5)^2+(-5)^2+0^2} = \boxed{7.07\text{ N/C}}$$

P25.32 (a) $\boxed{E_A > E_B \text{ since } E = \frac{\Delta V}{\Delta s}}$

(b) $E_B = -\frac{\Delta V}{\Delta s} = -\frac{(6-2)\text{ V}}{2\text{ cm}} = \boxed{200\text{ N/C}}$ down

(c) The figure is shown to the right, with sample field lines sketched in.

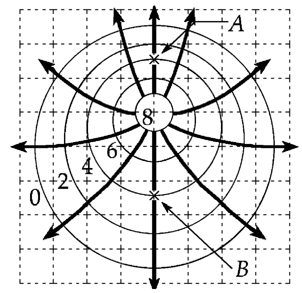


FIG. P25.32

P25.33 $E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left[\frac{k_eQ}{\ell}\ln\left(\frac{\ell+\sqrt{\ell^2+y^2}}{y}\right)\right]$

$$E_y = \frac{k_eQ}{\ell y}\left[1-\frac{y^2}{\ell^2+y^2+\ell\sqrt{\ell^2+y^2}}\right] = \boxed{\frac{k_eQ}{y\sqrt{\ell^2+y^2}}}$$

Section 25.5 **Electric Potential Due to Continuous Charge Distributions**

P25.34 $\Delta V = V_{2R} - V_0 = \frac{k_eQ}{\sqrt{R^2+(2R)^2}} - \frac{k_eQ}{R} = \frac{k_eQ}{R}\left(\frac{1}{\sqrt{5}}-1\right) = \boxed{-0.553\frac{k_eQ}{R}}$

P25.35 (a) $[\alpha] = \left[\frac{\lambda}{x}\right] = \frac{\text{C}}{\text{m}}\cdot\left(\frac{1}{\text{m}}\right) = \boxed{\frac{\text{C}}{\text{m}^2}}$

(b) $V = k_e\int\frac{dq}{r} = k_e\int\frac{\lambda dx}{r} = k_e\alpha\int_0^L\frac{xdx}{d+x} = \boxed{k_e\alpha\left[L-d\ln\left(1+\frac{L}{d}\right)\right]}$

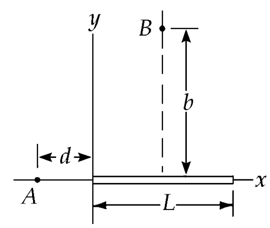


FIG. P25.35

P25.36 $V = \int\frac{k_e dq}{r} = k_e\int\frac{\alpha x dx}{\sqrt{b^2+(L/2-x)^2}}$

Let $z = \frac{L}{2} - x.$

Then $x = \frac{L}{2} - z$, and $dx = -dz$

$$V = k_e\alpha\int\frac{(L/2-z)(-dz)}{\sqrt{b^2+z^2}} = -\frac{k_e\alpha L}{2}\int\frac{dz}{\sqrt{b^2+z^2}} + k_e\alpha\int\frac{zdz}{\sqrt{b^2+z^2}}$$

$$= -\frac{k_e\alpha L}{2}\ln\left(z+\sqrt{z^2+b^2}\right) + k_e\alpha\sqrt{z^2+b^2}$$

$$V = -\frac{k_e\alpha L}{2}\ln\left[\left(\frac{L}{2}-x\right)+\sqrt{\left(\frac{L}{2}-x\right)^2+b^2}\right]\Bigg|_0^L + k_e\alpha\sqrt{\left(\frac{L}{2}-x\right)^2+b^2}\Bigg|_0^L$$

$$V = -\frac{k_e\alpha L}{2}\ln\left[\frac{L/2-L+\sqrt{(L/2)^2+b^2}}{L/2+\sqrt{(L/2)^2+b^2}}\right] + k_e\alpha\left[\sqrt{\left(\frac{L}{2}-L\right)^2+b^2}-\sqrt{\left(\frac{L}{2}\right)^2+b^2}\right]$$

$$V = \boxed{-\frac{k_e\alpha L}{2}\ln\left[\frac{\sqrt{b^2+(L^2/4)}-L/2}{\sqrt{b^2+(L^2/4)}+L/2}\right]}$$

P25.37 $V = \int dV = \frac{1}{4\pi \in_0} \int \frac{dq}{r}$

All bits of charge are at the same distance from O.

So $V = \frac{1}{4\pi \in_0}\left(\frac{Q}{R}\right) = \left(8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{-7.50\times 10^{-6}\text{ C}}{0.140 \text{ m}/\pi}\right) = \boxed{-1.51 \text{ MV}}$.

P25.38 $V = k_e \int_{\text{all charge}} \frac{dq}{r} = k_e \int_{-3R}^{-R} \frac{\lambda dx}{-x} + k_e \int_{\text{semicircle}} \frac{\lambda ds}{R} + k_e \int_{R}^{3R} \frac{\lambda dx}{x}$

$V = -k_e\lambda \ln(-x)\Big|_{-3R}^{-R} + \frac{k_e\lambda}{R}\pi R + k_e\lambda \ln x\Big|_{R}^{3R}$

$V = k_e \ln\frac{3R}{R} + k_e\lambda\pi + k_e \ln 3 = \boxed{k_e\lambda(\pi + 2\ln 3)}$

Section 25.6 **Electric Potential Due to a Charged Conductor**

P25.39 (a) $E = \boxed{0}$;

$V = \frac{k_e q}{R} = \frac{\left(8.99\times 10^9\right)\left(26.0\times 10^{-6}\right)}{0.140} = \boxed{1.67 \text{ MV}}$

(b) $E = \frac{k_e q}{r^2} = \frac{\left(8.99\times 10^9\right)\left(26.0\times 10^{-6}\right)}{(0.200)^2} = \boxed{5.84 \text{ MN/C}}$ away

$V = \frac{k_e q}{R} = \frac{\left(8.99\times 10^9\right)\left(26.0\times 10^{-6}\right)}{0.200} = \boxed{1.17 \text{ MV}}$

(c) $E = \frac{k_e q}{R^2} = \frac{\left(8.99\times 10^9\right)\left(26.0\times 10^{-6}\right)}{(0.140)^2} = \boxed{11.9 \text{ MN/C}}$ away

$V = \frac{k_e q}{R} = \boxed{1.67 \text{ MV}}$

P25.40 Substituting given values into $V = \frac{k_e q}{r}$

$$7.50\times 10^3 \text{ V} = \frac{\left(8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)q}{0.300 \text{ m}}$$

Substituting $q = 2.50\times 10^{-7}\text{C}$,

$$N = \frac{2.50\times 10^{-7}\text{ C}}{1.60\times 10^{-19}\text{ C}/e^-} = \boxed{1.56\times 10^{12} \text{ electrons}}$$

P25.41 The electric field on the surface of a conductor varies inversely with the radius of curvature of the surface. Thus, the field is most intense where the radius of curvature is smallest and vice-versa. The local charge density and the electric field intensity are related by

$$E = \frac{\sigma}{\epsilon_0} \qquad \text{or} \qquad \sigma = \epsilon_0 E$$

(a) Where the radius of curvature is the greatest,

$$\sigma = \epsilon_0 E_{\text{min}} = \left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.80 \times 10^4\ \text{N/C}\right) = \boxed{248\ \text{nC/m}^2}$$

(b) Where the radius of curvature is the smallest,

$$\sigma = \epsilon_0 E_{\text{max}} = \left(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(5.60 \times 10^4\ \text{N/C}\right) = \boxed{496\ \text{nC/m}^2}$$

P25.42 (a) Both spheres must be at the same potential according to $\frac{k_e q_1}{r_1} = \frac{k_e q_2}{r_2}$

where also $q_1 + q_2 = 1.20 \times 10^{-6}\ \text{C}$

Then $q_1 = \frac{q_2 r_1}{r_2}$

$$\frac{q_2 r_1}{r_2} + q_2 = 1.20 \times 10^{-6}\ \text{C}$$

$$q_2 = \frac{1.20 \times 10^{-6}\ \text{C}}{1 + 6\ \text{cm}/2\ \text{cm}} = 0.300 \times 10^{-6}\ \text{C on the smaller sphere}$$

$$q_1 = 1.20 \times 10^{-6}\ \text{C} - 0.300 \times 10^{-6}\ \text{C} = 0.900 \times 10^{-6}\ \text{C}$$

$$V = \frac{k_e q_1}{r_1} = \frac{\left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(0.900 \times 10^{-6}\ \text{C}\right)}{6 \times 10^{-2}\ \text{m}} = \boxed{1.35 \times 10^5\ \text{V}}$$

(b) Outside the larger sphere,

$$\vec{\mathbf{E}}_1 = \frac{k_e q_1}{r_1^2}\hat{\mathbf{r}} = \frac{V_1}{r_1}\hat{\mathbf{r}} = \frac{1.35 \times 10^5\ \text{V}}{0.06\ \text{m}}\hat{\mathbf{r}} = \boxed{2.25 \times 10^6\ \text{V/m away}}$$

Outside the smaller sphere,

$$\vec{\mathbf{E}}_2 = \frac{1.35 \times 10^5\ \text{V}}{0.02\ \text{m}}\hat{\mathbf{r}} = \boxed{6.74 \times 10^6\ \text{V/m away}}$$

The smaller sphere carries less charge but creates a much stronger electric field than the larger sphere.

Section 25.7 The Millikan Oil Drop Experiment

Section 25.8 Application of Electrostatics

P25.43 (a) $E_{\max} = 3.00\times10^6 \text{ V/m} = \dfrac{k_e Q}{r^2} = \dfrac{k_e Q}{r}\left(\dfrac{1}{r}\right) = V_{\max}\left(\dfrac{1}{r}\right)$

$$V_{\max} = E_{\max} r = 3.00\times10^6(0.150) = \boxed{450 \text{ kV}}$$

(b) $\dfrac{k_e Q_{\max}}{r^2} = E_{\max}$ $\left\{\text{or } \dfrac{k_e Q_{\max}}{r} = V_{\max}\right\}$ $Q_{\max} = \dfrac{E_{\max} r^2}{k_e} = \dfrac{3.00\times10^6(0.150)^2}{8.99\times10^9} = \boxed{7.51\ \mu\text{C}}$

P25.44 (a) $V_B - V_A = -\int_A^B \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$ and the field at distance r from a uniformly charged rod (where $r >$ radius of charged rod) is

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k_e\lambda}{r}$$

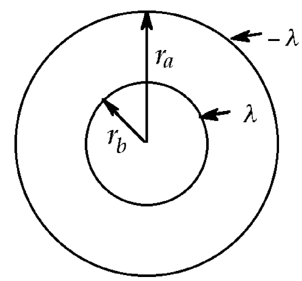


FIG. P25.44

In this case, the field between the central wire and the coaxial cylinder is directed perpendicular to the line of charge so that

$$V_B - V_A = -\int_{r_a}^{r_b} \frac{2k_e\lambda}{r}\,dr = 2k_e\lambda\ln\left(\frac{r_a}{r_b}\right)$$

or $\boxed{\Delta V = 2k_e\lambda\ln\left(\dfrac{r_a}{r_b}\right)}$

(b) From part (a), when the outer cylinder is considered to be at zero potential, the potential at a distance r from the axis is

$$V = 2k_e\lambda\ln\left(\frac{r_a}{r}\right)$$

The field at r is given by

$$E = -\frac{\partial V}{\partial r} = -2k_e\lambda\left(\frac{r}{r_a}\right)\left(-\frac{r_a}{r^2}\right) = \frac{2k_e\lambda}{r}$$

But, from part (a), $2k_e\lambda = \dfrac{\Delta V}{\ln(r_a/r_b)}$.

Therefore, $\boxed{E = \dfrac{\Delta V}{\ln(r_a/r_b)}\left(\dfrac{1}{r}\right)}$.

P25.45 (a) From the previous problem,

$$E = \frac{\Delta V}{\ln(r_a/r_b)} \frac{1}{r}$$

We require just outside the central wire

$$5.50 \times 10^6 \text{ V/m} = \frac{50.0 \times 10^3 \text{ V}}{\ln(0.850 \text{ m}/r_b)} \left(\frac{1}{r_b}\right)$$

or $$\left(110 \text{ m}^{-1}\right) r_b \ln\left(\frac{0.850 \text{ m}}{r_b}\right) = 1$$

We solve by homing in on the required value

r_b (m)	0.0100	0.00100	0.00150	0.00145	0.00143	0.00142
$\left(110 \text{ m}^{-1}\right) r_b \ln\left(\frac{0.850 \text{ m}}{r_b}\right)$	4.89	0.740	1.05	1.017	1.005	0.999

Thus, to three significant figures,

$$r_b = \boxed{1.42 \text{ mm}}$$

(b) At r_a,

$$E = \frac{50.0 \text{ kV}}{\ln(0.850 \text{ m}/0.001\,42 \text{ m})} \left(\frac{1}{0.850 \text{ m}}\right) = \boxed{9.20 \text{ kV/m}}$$

Additional Problems

***P25.46** (a) The two particles exert forces of repulsion on each other. As the projectile approaches the target particle, the projectile slows. The target starts to move in the *x* direction. As long as the projectile is moving faster than the second particle, the two will be approaching. Kinetic energy will be being converted into electric potential energy. When both particles move with equal speeds, the distance between them will momentarily not be changing: this is the instant of closest approach. Thereafter, the target particle, still feeling a forward force, will move faster than the projectile. The particles will separate again. The particles exert forces on each other but never touch. The particles will eventually be very far apart, with zero electric potential energy. All of the U_e they had at closest approach is converted back into kinetic energy. The whole process is an elastic collision. Compare this problem with Problem 9.49 in Chapter 9.

(b) Momentum is constant throughout the process. We equate it at the large-separation initial point and the point *b* of closest approach.

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1b} + m_2\vec{\mathbf{v}}_{2b}$$

$$(2\text{ g})\left(21\hat{\mathbf{i}}\text{ m/s}\right) + 0 = (2\text{ g} + 5\text{ g})\vec{\mathbf{v}}_b$$

$$\vec{\mathbf{v}}_b = \boxed{6.00\hat{\mathbf{i}}\text{ m/s}}$$

(c) Energy conservation between the same two points:

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 + 0 = \frac{1}{2}(m_1 + m_2)v_b^2 + \frac{k_eq_1q_2}{r_b}$$

$$\frac{1}{2}0.002\text{ kg}(21\text{ m/s})^2 + \frac{1}{2}0.005\text{ kg}(0)^2 + 0 = \frac{1}{2}0.007\text{ kg}(6\text{ m/s})^2 + \frac{8.99\times10^9\text{ Nm}^2}{\text{C}^2}\,\frac{15\times10^{-6}\text{ C }8.5\times10^{-6}\text{ C}}{r_b}$$

$$0.441\text{ J} - 0.126\text{ J} = \frac{1.15\text{ J}\cdot\text{m}}{r_b}$$

$$r_b = \frac{1.15\text{ m}}{0.315} = \boxed{3.64\text{ m}}$$

(d) The overall elastic collision is described by conservation of momentum:

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1d} + m_2\vec{\mathbf{v}}_{2d}$$

$$(2\text{ g})\left(21\hat{\mathbf{i}}\text{ m/s}\right) + 0 = 2\text{ g}\vec{\mathbf{v}}_{1d}\hat{\mathbf{i}} + 5\text{ g}\vec{\mathbf{v}}_{2d}\hat{\mathbf{i}}$$

and by the relative velocity equation:

$$v_{1i} - v_{2i} = v_{2d} - v_{1d}$$

$$21\text{ m/s} - 0 = v_{2d} - v_{1d}$$

we substitute

$$v_{2d} = 21\text{ m/s} + v_{1d}$$

$$42\text{ g}\cdot\text{m/s} = 2\text{ g}v_{1d} + 5\text{ g}(21\text{ m/s} + v_{1d}) = 2\text{ g}v_{1d} + 105\text{ g}\cdot\text{m/s} + 5\text{ g}v_{1d}$$

$$-63\text{ g}\cdot\text{m/s} = 7\text{ g}v_{1d}$$

$$v_{1d} = -9.00\text{ m/s}$$

$$\boxed{\vec{\mathbf{v}}_{1d} = -9.00\hat{\mathbf{i}}\text{ m/s}}$$

$$v_{2d} = 21\text{ m/s} - 9\text{ m/s} = 12.0\text{ m/s}$$

$$\boxed{\vec{\mathbf{v}}_{2d} = 12.0\hat{\mathbf{i}}\text{ m/s}}$$

P25.47 $U = qV = k_e \frac{q_1 q_2}{r_{12}} = \left(8.99\times10^9\right)\frac{(38)(54)\left(1.60\times10^{-19}\right)^2}{(5.50+6.20)\times10^{-15}} = 4.04\times10^{-11}\text{ J} = \boxed{253\text{ MeV}}$

***P25.48** (a) The field within the conducting Earth is zero. $E = \sigma/\epsilon_0$

$\sigma = E\epsilon_0 = (-120\text{ N/C})(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2) = \boxed{-1.06\text{ nC/m}^2}$

(b) $QE = \sigma A = \sigma 4\pi r^2 = (-1.06\times10^{-9}\text{ C/m}^2)\,4\pi\,(6.37\times10^6\text{ m})^2 = \boxed{-542\text{ kC}}$

(c) $V = \frac{k_e Q}{R} = \frac{8.99\times10^9\text{ C}^2(-5.42\times10^5\text{ C})}{\text{N}\cdot\text{m}^2(6.37\times10^6\text{ m})} = \boxed{-764\text{ MV}}$

(d) $V_{head} - V_{feet} = Ed = (120\text{ N/C})1.75\text{ m} = \boxed{210\text{ V}}$

(e) $F = \frac{k_e q_1 q_2}{r^2} = \frac{8.99\times10^9\text{ N}\cdot\text{m}^2(5.42\times10^5\text{ C})^2(0.273)}{\text{C}^2(3.84\times10^8\text{ m})^2} = \boxed{4.88\times10^3\text{ N away from Earth}}$

(f) The gravitational force is

$$F = \frac{GM_E M_M}{r^2} = \frac{6.67\times10^{-11}\text{ N}\cdot\text{m}^2(5.98\times10^{24}\text{ kg})(7.36\times10^{22}\text{ kg})}{\text{kg}^2(3.84\times10^8\text{ m})^2} = 1.99\times10^{20}\text{ N}$$

toward the Earth

The gravitational force is larger by $1.99\times10^{20}/4.88\times10^3 = 4.08\times10^{16}$ times and in the opposite direction.

Electrical forces are negligible in accounting for planetary motion.

(g) We require $m(-g) + qE = 0$

$6\times10^{-6}\text{ kg}(-9.8\text{ m/s}^2) + q(-120\text{ N/C}) = 0$

$q = 5.88\times10^{-5}\text{ N}/(-120\text{ N/C}) = \boxed{-490\text{ nC}}$

(h) Less charge to be suspended at the equator. The gravitational force is weaker at a greater distance from the Earth's center. The suspended particle is not quite in equilibrium, but accelerating downward to participate in the daily rotation. At uniform potential, the planet's surface creates a stronger electric field at the equator, where its radius of curvature is smaller.

P25.49 (a) $U = \frac{k_e q_1 q_2}{r} = \frac{-\left(8.99\times10^9\right)\left(1.60\times10^{-19}\right)^2}{0.052\,9\times10^{-9}} = -4.35\times10^{-18}\text{ J} = \boxed{-27.2\text{ eV}}$

(b) $U = \frac{k_e q_1 q_2}{r} = \frac{-\left(8.99\times10^9\right)\left(1.60\times10^{-19}\right)^2}{2^2\left(0.052\,9\times10^{-9}\right)} = \boxed{-6.80\text{ eV}}$

(c) $U = \frac{k_e q_1 q_2}{r} = \frac{-k_e e^2}{\infty} = \boxed{0}$

P25.50 (a) To make a spark 5 mm long in dry air between flat metal plates requires potential difference

$\Delta V = Ed = \left(3\times10^6\text{ V/m}\right)\left(5\times10^{-3}\text{ m}\right) = 1.5\times10^4\text{ V}\ \boxed{\sim10^4\text{ V}}$

(b) The area of your skin is perhaps 1.5 m^2, so model your body as a sphere with this surface area. Its radius is given by $1.5\text{ m}^2 = 4\pi r^2$, $r = 0.35\text{ m}$. We require that you are at the potential found in part (a):

$V = \frac{k_e q}{r}$ $\quad q = \frac{Vr}{k_e} = \frac{1.5\times10^4\text{ V}(0.35\text{ m})}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2}\left(\frac{\text{J}}{\text{V}\cdot\text{C}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right)$

$q = 5.8\times10^{-7}\text{ C}\ \boxed{\sim10^{-6}\text{ C}}$

***P25.51** (a) We have $k_e Q/R = 200$ V and $k_e Q/(R + 10 \text{ cm}) = 150$ V

Then $200\,R = 150(R + 10 \text{ cm})$. The $\boxed{\text{information is sufficient}}$ to determine the charge and original distance, according to $50\,R = 1\,500$ cm.

$R = 30.0$ cm and $Q = 200 \text{ V}(0.3 \text{ m})/(8.9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) = 6.67$ nC

(b) We have $k_e Q/R = 210$ V and $k_e Q/(R + 10 \text{ cm})^2 = 400$ V/m.
Then $210 \text{ V}R = (400 \text{ V/m})(R + 0.1 \text{ m})^2$.

$210 \text{ m}R = 400\,R^2 + 80 \text{ m}R = 4 \text{ m}^2$

The $\boxed{\text{information is not quite sufficient}}$. There are two possibilities, according to

$400\,R^2 - 130 \text{ m}R + 4 \text{ m}^2 = 0$

$$R = \frac{+130 \pm \sqrt{130^2 - 4(400)4}}{800} \text{ m} = \text{ either } 0.291 \text{ m or } 0.034\,4 \text{ m}$$

If the radius is 29.1 cm, the charge is $210(0.291)$ C$/8.99 \times 10^9 = 6.79$ nC. If the radius is 3.44 cm, the charge is $210(0.034\,4)$ C$/8.99 \times 10^9 = 804$ pC.

P25.52 The plates create uniform electric field to the right in the picture, with magnitude $\frac{V_0 - (-V_0)}{d} = \frac{2V_0}{d}$. Assume the ball swings a small distance x to the right. It moves to a place where the voltage created by the plates is lower by $-Ex = -\frac{2V_0}{d}x$. Its ground connection maintains it at $V = 0$ by allowing charge q to flow from ground onto the ball, where $-\frac{2V_0 x}{d} + \frac{k_e q}{R} = 0 \qquad q = \frac{2V_0 xR}{k_e d}$. Then the ball feels electric force $F = qE = \frac{4V_0^2 xR}{k_e d^2}$ to the right. For equilibrium this must be balanced by the horizontal component of string tension according to $T\cos\theta = mg \qquad T\sin\theta = \frac{4V_0^2 xR}{k_e d^2} \quad \tan\theta = \frac{4V_0^2 xR}{k_e d^2 mg} = \frac{x}{L}$ for small x.

Then $V_0 = \left(\frac{k_e d^2 mg}{4RL}\right)^{1/2}$. If V_0 is less than this value, the only equilibrium position of the ball is hanging straight down. If V_0 exceeds this value the ball will swing over to one plate or the other.

***P25.53** For a charge at ($x = -1$ m, $y = 0$), the radial distance away is given by $\sqrt{(x+1)^2 + y^2}$. So the first term will be the potential it creates if

$(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)Q_1 = 36 \text{ V}\cdot\text{m} \qquad Q_1 = 4.00$ nC

The second term is the potential of a charge at ($x = 0$, $y = 2$ m) with

$(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)Q_2 = -45 \text{ V}\cdot\text{m} \qquad Q_2 = -5.01$ nC

Thus we have $\boxed{4.00 \text{ nC at } (-1.00 \text{ m}, 0) \text{ and } -5.01 \text{ nC at } (0, 2.00 \text{ m})}$.

P25.54 (a) Take the origin at the point where we will find the potential. One ring, of width dx, has charge $\frac{Qdx}{h}$ and, according to Example 25.5, creates potential

$$dV = \frac{k_e Qdx}{h\sqrt{x^2+R^2}}$$

The whole stack of rings creates potential

$$V = \int_{\text{all charge}} dV = \int_d^{d+h} \frac{k_e Qdx}{h\sqrt{x^2+R^2}} = \frac{k_e Q}{h}\ln\left(x+\sqrt{x^2+R^2}\right)\Big|_d^{d+h}$$

$$= \boxed{\frac{k_e Q}{h}\ln\left(\frac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)}$$

(b) A disk of thickness dx has charge $\frac{Qdx}{h}$ and charge-per-area $\frac{Qdx}{\pi R^2 h}$. According to Example 25.6, it creates potential

$$dV = 2\pi k_e \frac{Qdx}{\pi R^2 h}\left(\sqrt{x^2+R^2}-x\right)$$

Integrating,

$$V = \int_d^{d+h} \frac{2k_e Q}{R^2 h}\left(\sqrt{x^2+R^2}\,dx - x\,dx\right) = \frac{2k_e Q}{R^2 h}\left[\frac{1}{2}x\sqrt{x^2+R^2}+\frac{R^2}{2}\ln\left(x+\sqrt{x^2+R^2}\right)-\frac{x^2}{2}\right]_d^{d+h}$$

$$V = \boxed{\frac{k_e Q}{R^2 h}\left[(d+h)\sqrt{(d+h)^2+R^2}-d\sqrt{d^2+R^2}-2dh-h^2+R^2\ln\left(\frac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)\right]}$$

P25.55 $W = \int_0^Q V\,dq$ where $V = \frac{k_e q}{R}$. Therefore, $W = \boxed{\frac{k_e Q^2}{2R}}$.

***P25.56** (a) The exact potential is

$$+\frac{k_e q}{r+a}-\frac{k_e q}{r-a} = +\frac{k_e q}{3a+a}-\frac{k_e q}{3a-a} = \frac{k_e q}{4a}-\frac{2k_e q}{4a} = \boxed{-\frac{k_e q}{4a}}$$

(b) The approximate expression $-2k_e qa/x^2$ gives $-2k_e qa/(3a)^2 = -k_e q/4.5a$.
Even though $3a$ is hardly a large distance compared to the $2a$ separation of the charges, the $\boxed{\text{approximate answer is different by only}}$

$$\frac{1/4-1/4.5}{1/4} = \frac{0.5}{4.5} = \boxed{11.1\%}$$

P25.57 $V_2 - V_1 = -\int_{r_1}^{r_2} \vec{\mathbf{E}}\cdot d\vec{\mathbf{r}} = -\int_{r_1}^{r_2} \frac{\lambda}{2\pi\epsilon_0 r}dr$

$$V_2 - V_1 = \boxed{\frac{-\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_2}{r_1}\right)}$$

P25.58 Take the illustration presented with the problem as an initial picture. No external horizontal forces act on the set of four balls, so its center of mass stays fixed at the location of the center of the square. As the charged balls 1 and 2 swing out and away from each other, balls 3 and 4 move up with equal y-components of velocity. The maximum-kinetic-energy point is illustrated. System energy is conserved:

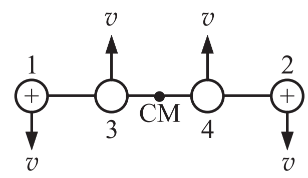


FIG. P25.58

$$\frac{k_e q^2}{a} = \frac{k_e q^2}{3a} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$\frac{2k_e q^2}{3a} = 2mv^2 \qquad \boxed{v = \sqrt{\frac{k_e q^2}{3am}}}$$

P25.59 From an Example in the chapter text, the potential at the center of the ring is $V_i = \frac{k_e Q}{R}$ and the potential at an infinite distance from the ring is $V_f = 0$. Thus, the initial and final potential energies of the point charge-ring system are:

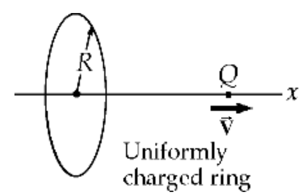


FIG. P25.59

$$U_i = QV_i = \frac{k_e Q^2}{R}$$

and $$U_f = QV_f = 0$$

From conservation of energy,

$$K_f + U_f = K_i + U_i$$

or $$\frac{1}{2}Mv_f^2 + 0 = 0 + \frac{k_e Q^2}{R}$$

giving $$v_f = \boxed{\sqrt{\frac{2k_e Q^2}{MR}}}$$

P25.60 $$V = k_e \int_a^{a+L} \frac{\lambda dx}{\sqrt{x^2+b^2}} = k_e\lambda \ln\left[x + \sqrt{(x^2+b^2)}\right]_a^{a+L} = \boxed{k_e\lambda \ln\left[\frac{a+L+\sqrt{(a+L)^2+b^2}}{a+\sqrt{a^2+b^2}}\right]}$$

P25.61 (a) $$V = \frac{k_e q}{r_1} - \frac{k_e q}{r_2} = \frac{k_e q}{r_1 r_2}(r_2 - r_1)$$

From the figure, for $r >> a$, $\quad r_2 - r_1 \approx 2a\cos\theta$

Then $$v \approx \frac{k_e q}{r_1 r_2} 2a\cos\theta \approx \frac{k_e p\cos\theta}{r^2}$$

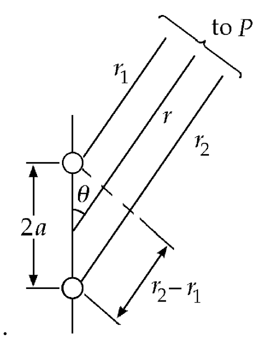


FIG. P25.61

(b) $$E_r = -\frac{\partial V}{\partial r} = \boxed{\frac{2k_e p\cos\theta}{r^3}}$$

In spherical coordinates, the θ component of the gradient is $-\frac{1}{r}\left(\frac{\partial}{\partial\theta}\right)$.

Therefore, $$E_\theta = -\frac{1}{r}\left(\frac{\partial V}{\partial\theta}\right) = \boxed{\frac{k_e p\sin\theta}{r^3}}$$

For $r >> a$ $$E_r(0°) = \frac{2k_e p}{r^3}$$

continued on next page

and $E_r(90°) = 0,$

$E_\theta(0°) = 0$

and $E_\theta(90°) = \frac{k_e p}{r^3}$

These results are $\boxed{\text{reasonable for } r >> a}$. Their directions are as shown in Figure 25.13 (c). However, for $\boxed{r \to 0,\ E(0) \to \infty. \text{ This is unreasonable,}}$ since r is not much greater than a if it is 0.

(c) $\boxed{V = \frac{k_e p y}{\left(x^2 + y^2\right)^{3/2}}}$

and $\boxed{E_x = -\frac{\partial V}{\partial x} = \frac{3k_e p x y}{\left(x^2 + y^2\right)^{5/2}}}$

$\boxed{E_y = -\frac{\partial V}{\partial y} = \frac{k_e p\left(2y^2 - x^2\right)}{\left(x^2 + y^2\right)^{5/2}}}$

P25.62 $dU = Vdq$ where the potential $V = \frac{k_e q}{r}$.

The element of charge in a shell is $dq = \rho$ (volume element) or $dq = \rho\left(4\pi r^2 dr\right)$ and the charge q in a sphere of radius r is

$$q = 4\pi\rho\int_0^r r^2 dr = \rho\left(\frac{4\pi r^3}{3}\right)$$

Substituting this into the expression for dU, we have

$$dU = \left(\frac{k_e q}{r}\right)dq = k_e\rho\left(\frac{4\pi r^3}{3}\right)\left(\frac{1}{r}\right)\rho\left(4\pi r^2 dr\right) = k_e\left(\frac{16\pi^2}{3}\right)\rho^2 r^4 dr$$

$$U = \int dU = k_e\left(\frac{16\pi^2}{3}\right)\rho^2\int_0^R r^4 dr = k_e\left(\frac{16\pi^2}{15}\right)\rho^2 R^5$$

But the *total* charge, $Q = \rho\frac{4}{3}\pi R^3$. Therefore, $\boxed{U = \frac{3}{5}\frac{k_e Q^2}{R}}$.

P25.63 For an element of area which is a ring of radius r and width dr, $dV = \frac{k_e dq}{\sqrt{r^2 + x^2}}$.

$dq = \sigma dA = Cr\left(2\pi r dr\right)$ and

$$V = C\left(2\pi k_e\right)\int_0^R \frac{r^2 dr}{\sqrt{r^2 + x^2}} = \boxed{C\left(\pi k_e\right)\left[R\sqrt{R^2 + x^2} + x^2 \ln\left(\frac{x}{R + \sqrt{R^2 + x^2}}\right)\right]}$$

***P25.64** (a) $k_e Q/r = 8.99 \times 10^9\,(1.6 \times 10^{-9})\text{ V}/2 = \boxed{7.19\text{ V}}$

(b) $$\frac{8.99\;\;1.6}{2(1+\frac{1}{2})} + \frac{8.99\;\;1.6}{2(1+\frac{3}{2})} = \boxed{7.67\text{ V}}$$

(c) $$\frac{8.99\;\;1.6}{4}\left(\frac{1}{1+\frac{1}{4}} + \frac{1}{1+\frac{3}{4}} + \frac{1}{1+\frac{5}{4}} + \frac{1}{1+\frac{7}{4}}\right) = \boxed{7.84\text{ V}}$$

(d) We find $\dfrac{8.99\;\;1.6}{32}\left(\dfrac{1}{1+\frac{1}{32}} + \dfrac{1}{1+\frac{3}{32}} + \cdots + \dfrac{1}{1+\frac{63}{32}}\right) = \boxed{7.900\text{ 2 V}}$

(e) We find $\dfrac{8.99\;\;1.6}{64}\left(\dfrac{1}{1+\frac{1}{64}} + \dfrac{1}{1+\frac{3}{64}} + \cdots + \dfrac{1}{1+\frac{127}{64}}\right) = \boxed{7.901\text{ 0 V}}$

(f) We represent the exact result as

$$V = \frac{k_e Q}{\ell}\ln\left(\frac{\ell + a}{a}\right) = \frac{8.99\;\;1.6\text{ V}}{2}\ln\left(\frac{3}{1}\right) = 7.901\text{ 2 V}$$

Modeling the line as a set of points works nicely. The exact result, represented as 7.901 2 V, is approximated to within 0.8% by the four-particle version. The 16-particle approximation gives a result accurate to three digits, to within 0.05%. The 64-charge approximation gives a result accurate to four digits, differing by only 0.003% from the exact result.

P25.65 The positive plate by itself creates a field $E = \dfrac{\sigma}{2\epsilon_0} = \dfrac{36.0 \times 10^{-9}\text{ C/m}^2}{2\left(8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)} = 2.03\text{ kN/C}$

away from the + plate. The negative plate by itself creates the same size field and between the plates it is in the same direction. Together the plates create a uniform field 4.07 kN/C in the space between.

(a) Take $V = 0$ at the negative plate. The potential at the positive plate is then

$$V - 0 = -\int_0^{12.0\text{ cm}} (-4.07\text{ kN/C})\,dx$$

The potential difference between the plates is $V = \left(4.07 \times 10^3\text{ N/C}\right)(0.120\text{ m}) = \boxed{488\text{ V}}$.

(b) $$\left(\frac{1}{2}mv^2 + qV\right)_i = \left(\frac{1}{2}mv^2 + qV\right)_f$$

$$qV = \left(1.60 \times 10^{-19}\text{ C}\right)(488\text{ V}) = \frac{1}{2}mv_f^2 = \boxed{7.81 \times 10^{-17}\text{ J}}$$

(c) $v_f = \boxed{306\text{ km/s}}$

(d) $$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$$

$$\left(3.06 \times 10^5\text{ m/s}\right)^2 = 0 + 2a(0.120\text{ m})$$

$a = \boxed{3.90 \times 10^{11}\text{ m/s}^2}$ toward the negative plate

(e) $\sum F = ma = \left(1.67 \times 10^{-27}\text{ kg}\right)\left(3.90 \times 10^{11}\text{ m/s}^2\right) = \boxed{6.51 \times 10^{-16}\text{ N}}$ toward the negative plate

(f) $$E = \frac{F}{q} = \frac{6.51 \times 10^{-16}\text{ N}}{1.60 \times 10^{-19}\text{ C}} = \boxed{4.07\text{ kN/C}}$$

P25.66 For the given charge distribution, $V(x,\ y,\ z)=\frac{k_e(q)}{r_1}+\frac{k_e(-2q)}{r_2}$

where $r_1=\sqrt{(x+R)^2+y^2+z^2}$ and $r_2=\sqrt{x^2+y^2+z^2}$

The surface on which $V(x,\ y,\ z)=0$

is given by $k_e q\left(\frac{1}{r_1}-\frac{2}{r_2}\right)=0$, or $2r_1=r_2$

This gives: $4(x+R)^2+4y^2+4z^2=x^2+y^2+z^2$

which may be written in the form: $x^2+y^2+z^2+\left(\frac{8}{3}R\right)x+(0)y+(0)z+\left(\frac{4}{3}R^2\right)=0$ [1]

The general equation for a sphere of radius a centered at $(x_0,\ y_0,\ z_0)$ is:

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2-a^2=0$$

or $x^2+y^2+z^2+(-2x_0)x+(-2y_0)y+(-2z_0)z+\left(x_0^2+y_0^2+z_0^2-a^2\right)=0$ [2]

Comparing equations [1] and [2], it is seen that the equipotential surface for which $V=0$ is indeed a sphere and that:

$$-2x_0=\frac{8}{3}R;\ -2y_0=0;\ -2z_0=0;\quad x_0^2+y_0^2+z_0^2-a^2=\frac{4}{3}R^2$$

Thus, $x_0=-\frac{4}{3}R,\ y_0=z_0=0$, and $a^2=\left(\frac{16}{9}-\frac{4}{3}\right)R^2=\frac{4}{9}R^2$

The equipotential surface is therefore a sphere centered at $\boxed{\left(-\frac{4}{3}R,\ 0,\ 0\right)}$, having a radius $\boxed{\frac{2}{3}R}$.

P25.67 Inside the sphere, $\boxed{E_x=E_y=E_z=0}$.

Outside, $E_x=-\frac{\partial V}{\partial x}=-\frac{\partial}{\partial x}\left(V_0-E_0z+E_0a^3z\left(x^2+y^2+z^2\right)^{-3/2}\right)$

So $E_x=-\left[0+0+E_0a^3z\left(-\frac{3}{2}\right)\left(x^2+y^2+z^2\right)^{-5/2}(2x)\right]=\boxed{3E_0a^3xz\left(x^2+y^2+z^2\right)^{-5/2}}$

$$E_y=-\frac{\partial V}{\partial y}=-\frac{\partial}{\partial y}\left(V_0-E_0z+E_0a^3z\left(x^2+y^2+z^2\right)^{-3/2}\right)$$

$$E_y=-E_0a^3z\left(-\frac{3}{2}\right)\left(x^2+y^2+z^2\right)^{-5/2}2y=\boxed{3E_0a^3yz\left(x^2+y^2+z^2\right)^{-5/2}}$$

$$E_z=-\frac{\partial V}{\partial z}=E_0-E_0a^3z\left(-\frac{3}{2}\right)\left(x^2+y^2+z^2\right)^{-5/2}(2z)-E_0a^3\left(x^2+y^2+z^2\right)^{-3/2}$$

$$E_z=\boxed{E_0+E_0a^3\left(2z^2-x^2-y^2\right)\left(x^2+y^2+z^2\right)^{-5/2}}$$

ANSWERS TO EVEN PROBLEMS

P25.2 1.35 MJ

P25.4 +260 V

P25.6 See the solution.

P25.8 0.300 m/s

P25.10 (a) 0 (b) 0 (c) 45.0 kV

P25.12 (a) -4.83 m (b) 0.667 m and -2.00 m

P25.14 (a) –386 nJ (b) 103 V

P25.16 (a) 32.2 kV (b) –96.5 mJ

P25.18 (a) +7.00 nC with position vector 7.00 cm at 70.0° and –8.00 nC with position vector 3.00 cm at 90.0° (b) $(0.070\,3\,\hat{\mathbf{i}} - 1.09\,\hat{\mathbf{j}})$mN (c) +24.0 μJ

P25.20 $-\dfrac{5k_e q}{R}$

P25.22 See the solution.

P25.24 (a) $v_1 = \sqrt{\dfrac{2m_2 k_e q_1 q_2}{m_1(m_1+m_2)}\left(\dfrac{1}{r_1+r_2}-\dfrac{1}{d}\right)}$ $v_2 = \sqrt{\dfrac{2m_1 k_e q_1 q_2}{m_2(m_1+m_2)}\left(\dfrac{1}{r_1+r_2}-\dfrac{1}{d}\right)}$ (b) Faster than calculated in (a)

P25.26 27.4 fm

P25.28 $22.8\dfrac{k_e q^2}{s}$

P25.30 (a) 0 (b) $\dfrac{k_e Q}{r^2}$ radially outward

P25.32 (a) larger at *A* (b) 200 N/C down (c) See the solution.

P25.34 $-0.553\dfrac{k_e Q}{R}$

P25.36 $-\dfrac{k_e \alpha L}{2}\ln\left[\dfrac{\sqrt{b^2+(L^2/4)}-L/2}{\sqrt{b^2+(L^2/4)}+L/2}\right]$

P25.38 $k_e\lambda(\pi + 2\ln 3)$

P25.40 1.56×10^{12} electrons

P25.42 (a) 135 kV (b) 2.25 MV/m away from the large sphere and 6.74 MV/m away from the small sphere

P25.44 See the solution.

P25.46 (a) The velocity of one particle relative to the other is first a velocity of approach, then zero at closest approach, and then a velocity of recession. (b) 6.00 $\hat{\mathbf{i}}$ m/s (c) 3.64 m (d) –9.00 $\hat{\mathbf{i}}$ m/s for the incident particle and 12.0 $\hat{\mathbf{i}}$ m/s for the target particle.

P25.48 (a) negative 1.06 nC/m^2 (b) –542 kC (c) 764 MV (d) His head is higher in potential by 210 V. (e) 4.88 kN away from the Earth (f) The gravitational force is in the opposite direction and 4.08×10^{16} times larger. Electrical forces are negligible in accounting for planetary motion. (g) –490 nC (h) Less charge to be suspended at the equator. The gravitational force is weaker at a greater distance from the Earth's center. The suspended particle is not quite in equilibrium, but accelerating downward to participate in the daily rotation. At uniform potential, the planet's surface creates a stronger electric field at the equator, where its radius of curvature is smaller.

P25.50 (a) $\sim 10^4$ V (b) $\sim 10^{-6}$ C

P25.52 See the solution.

P25.54 (a) $\dfrac{k_eQ}{h}\ln\left(\dfrac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)$

(b) $\dfrac{k_eQ}{R^2h}\left[(d+h)\sqrt{(d+h)^2+R^2}-d\sqrt{d^2+R^2}-2dh-h^2+R^2\ln\left(\dfrac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)\right]$

P25.56 (a) $-k_eq/4a$ (b) The approximate expression $-2k_eqa/x^2$ gives $-k_eq/4.5a$, which is different by only 11.1%.

P25.58 $\left(\dfrac{k_eq^2}{3am}\right)^{1/2}$

P25.60 $k_e\lambda\ln\left[\dfrac{a+L+\sqrt{(a+L)^2+b^2}}{a+\sqrt{a^2+b^2}}\right]$

P25.62 $\dfrac{3}{5}\dfrac{k_eQ^2}{R}$

P25.64 (a) 7.19 V (b) 7.67 V (c) 7.84 V (d) 7.900 2 V (e) 7.901 0 V (f) Modeling the line as a set of points works nicely. The exact result, represented as 7.901 2 V, is approximated to within 0.8% by the four-particle version. The 16-particle approximation gives a result accurate to three digits, to within 0.05%. The 64-charge approximation gives a result accurate to four digits, differing by only 0.003% from the exact result.

P25.66 See the solution.

26

Capacitance and Dielectrics

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q26.1** (a) False. (b) True. In $Q = C\Delta V$, the capacitance is the proportionality constant relating the variables Q and ΔV.

Q26.2 Seventeen combinations:

Individual $C_1,\ C_2,\ C_3$

Parallel $C_1 + C_2 + C_3,\ C_1 + C_2,\ C_1 + C_3,\ C_2 + C_3$

Series-Parallel
$$\left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1}+C_3,\ \left(\frac{1}{C_1}+\frac{1}{C_3}\right)^{-1}+C_2,\ \left(\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1}+C_1$$
$$\left(\frac{1}{C_1+C_2}+\frac{1}{C_3}\right)^{-1},\ \left(\frac{1}{C_1+C_3}+\frac{1}{C_2}\right)^{-1},\ \left(\frac{1}{C_2+C_3}+\frac{1}{C_1}\right)^{-1}$$

Series
$$\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1},\ \left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1},\ \left(\frac{1}{C_2}+\frac{1}{C_3}\right)^{-1},\ \left(\frac{1}{C_1}+\frac{1}{C_3}\right)^{-1}$$

***Q26.3** Volume is proportional to radius cubed. Increasing the radius by a factor of $3^{1/3}$ will triple the volume. Capacitance is proportional to radius, so it increases by a factor of $3^{1/3}$. Answer (d).

***Q26.4** Let $C_2 = NC_1$ be the capacitance of the large capacitor and C_1 that of the small one. The equivalent capacitance is

$$C_{eq} = C_{eq} = \frac{1}{1/C_1 + 1/NC_1} = \frac{1}{(N+1)/NC_1} = \frac{N}{N+1}C_1$$

This is slightly less than C_1, answer (d).

***Q26.5** We find the capacitance, voltage, charge, and energy for each capacitor.

(a) $C = 20\ \mu$F $\Delta V = 4$ V $Q = C\Delta V = 80\ \mu$C $U = (1/2)Q\Delta V = 160\ \mu$J
(b) $C = 30\ \mu$F $\Delta V = Q/C = 3$ V $Q = 90\ \mu$C $U = 135\ \mu$J
(c) $C = Q/\Delta V = 40\ \mu$F $\Delta V = 2$ V $Q = 80\ \mu$C $U = 80\ \mu$J
(d) $C = 10\ \mu$F $\Delta V = (2U/C)^{1/2} = 5$ V $Q = 50\ \mu$C $U = 125\ \mu$J
(e) $C = 2U/\Delta V^2 = 5\ \mu$F $\Delta V = 10$ V $Q = 50\ \mu$C $U = 250\ \mu$J
(f) $C = Q^2/2U = 20\ \mu$F $\Delta V = 6$ V $Q = 120\ \mu$C $U = 360\ \mu$J

Then (i) the ranking by capacitance is c > b > a = f > d > e.
(ii) The ranking by voltage ΔV is e > f > d > a > b > c.
(iii) The ranking by charge Q is f > b > a = c > d = e.
(iv) The ranking by energy U is f > e > a > b > d > c.

Q26.6 A capacitor stores energy in the electric field between the plates. This is most easily seen when using a "dissectible" capacitor. If the capacitor is charged, carefully pull it apart into its component pieces. One will find that very little residual charge remains on each plate. When reassembled, the capacitor is suddenly "recharged"—by induction—due to the electric field set up and "stored" in the dielectric. This proves to be an instructive classroom demonstration, especially when you ask a student to reconstruct the capacitor without supplying him/her with any rubber gloves or other insulating material. (Of course, this is *after* they sign a liability waiver.)

***Q26.7** (i) According to $Q = C\Delta V$, the answer is (b).
(ii) From $U = (1/2)C\Delta V^2$, the answer is (a).

***Q26.8** The charge stays constant as C is cut in half, so $U = Q^2/2C$ doubles: answer (b).

***Q26.9** (i) Answer (b). (ii) Answer (c). (iii) Answer (c). (iv) Answer (a). (v) Answer (a).

***Q26.10** (i) Answer (b). (ii) Answer (b). (iii) Answer (b). (iv) Answer (c). (v) Answer (b).

Q26.11 The work you do to pull the plates apart becomes additional electric potential energy stored in the capacitor. The charge is constant and the capacitance decreases but the potential difference increases to drive up the potential energy $\frac{1}{2}Q\Delta V$. The electric field between the plates is constant in strength but fills more volume as you pull the plates apart.

Q26.12 The work done, $W = Q\Delta V$, is the work done by an external agent, like a battery, to move a charge through a potential difference, ΔV. To determine the energy in a charged capacitor, we must add the work done to move bits of charge from one plate to the other. Initially, there is no potential difference between the plates of an uncharged capacitor. As more charge is transferred from one plate to the other, the potential difference increases as shown in the textbook graph of ΔV versus Q, meaning that more work is needed to transfer each additional bit of charge. The total work is the area under the curve on this graph, and thus $W = \frac{1}{2}Q\Delta V$.

***Q26.13** Let C = the capacitance of an individual capacitor, and C_s represent the equivalent capacitance of the group in series. While being charged in parallel, each capacitor receives charge

$$Q = C\Delta V_{\text{charge}} = (5.00\times10^{-4}\text{ F})(800\text{ V}) = 0.400\text{ C}$$

While being discharged in series, $$\Delta V_{\text{discharge}} = \frac{Q}{C_s} = \frac{Q}{C/10} = \frac{0.400\text{ C}}{5.00\times10^{-5}\text{ F}} = 8.00\text{ kV}$$

(or 10 times the original voltage). Answer (b).

Q26.14 The potential difference must decrease. Since there is no external power supply, the charge on the capacitor, Q, will remain constant—that is, assuming that the resistance of the meter is sufficiently large. Adding a dielectric *increases* the capacitance, which must therefore *decrease* the potential difference between the plates.

***Q26.15** (i) Answer (a). (ii) Because ΔV is constant, $Q = C\Delta V$ increases, answer (a).
(iii) Answer (c). (iv) Answer (c). (v) $U = (1/2)C\Delta V^2$ increases, answer (a).

Q26.16 Put a material with higher dielectric strength between the plates, or evacuate the space between the plates. At very high voltages, you may want to cool off the plates or choose to make them of a different chemically stable material, because atoms in the plates themselves can ionize, showing *thermionic emission* under high electric fields.

Q26.17 The primary choice would be the dielectric. You would want to choose a dielectric that has a large dielectric constant and dielectric strength, such as strontium titanate, where $\kappa \approx 233$ (Table 26.1). A convenient choice could be thick plastic or Mylar. Secondly, geometry would be a factor. To maximize capacitance, one would want the individual plates as close as possible, since the capacitance is proportional to the inverse of the plate separation—hence the need for a dielectric with a high dielectric strength. Also, one would want to build, instead of a single parallel plate capacitor, several capacitors in parallel. This could be achieved through "stacking" the plates of the capacitor. For example, you can alternately lay down sheets of a conducting material, such as aluminum foil, sandwiched between your sheets of insulating dielectric. Making sure that none of the conducting sheets are in contact with their next neighbors, connect every other plate together. Figure Q26.17 illustrates this idea.

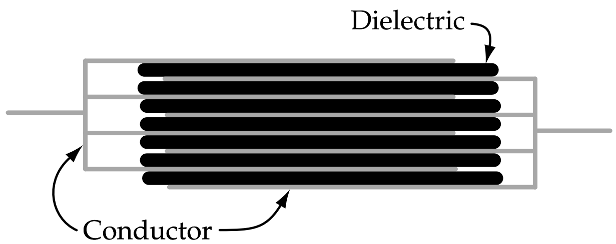


FIG. Q26.17

This technique is often used when "home-brewing" signal capacitors for radio applications, as they can withstand huge potential differences without flashover (without either discharge between plates around the dielectric or dielectric breakdown). One variation on this technique is to sandwich together flexible materials such as aluminum roof flashing and thick plastic, so the whole product can be rolled up into a "capacitor burrito" and placed in an insulating tube, such as a PVC pipe, and then filled with motor oil (again to prevent flashover).

SOLUTIONS TO PROBLEMS

Section 26.1 **Definition of Capacitance**

P26.1 (a) $Q = C\Delta V = \left(4.00\times10^{-6}\text{ F}\right)\left(12.0\text{ V}\right) = 4.80\times10^{-5}\text{ C} = \boxed{48.0\ \mu\text{C}}$

(b) $Q = C\Delta V = \left(4.00\times10^{-6}\text{ F}\right)\left(1.50\text{ V}\right) = 6.00\times10^{-6}\text{ C} = \boxed{6.00\ \mu\text{C}}$

P26.2 (a) $C = \dfrac{Q}{\Delta V} = \dfrac{10.0\times10^{-6}\text{ C}}{10.0\text{ V}} = 1.00\times10^{-6}\text{ F} = \boxed{1.00\ \mu\text{F}}$

(b) $\Delta V = \dfrac{Q}{C} = \dfrac{100\times10^{-6}\text{ C}}{1.00\times10^{-6}\text{ F}} = \boxed{100\text{ V}}$

Section 26.2 **Calculating Capacitance**

P26.3 $E = \dfrac{k_e q}{r^2}$: $\quad q = \dfrac{\left(4.90\times10^{4}\text{ N/C}\right)\left(0.210\text{ m}\right)^2}{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)} = 0.240\ \mu\text{C}$

(a) $\sigma = \dfrac{q}{A} = \dfrac{0.240\times10^{-6}}{4\pi\left(0.120\right)^2} = \boxed{1.33\ \mu\text{C/m}^2}$

(b) $C = 4\pi\epsilon_0 r = 4\pi\left(8.85\times10^{-12}\right)\left(0.120\right) = \boxed{13.3\text{ pF}}$

P26.4 $C = \dfrac{\kappa \in_0 A}{d} = \dfrac{(1.00)(8.85 \times 10^{-12}\ \text{C}^2)(1.00 \times 10^3\ \text{m})^2}{\text{N} \cdot \text{m}^2 (800\ \text{m})} = \boxed{11.1\ \text{nF}}$

The potential between ground and cloud is

$$\Delta V = Ed = (3.00 \times 10^6\ \text{N/C})(800\ \text{m}) = 2.40 \times 10^9\ \text{V}$$
$$Q = C(\Delta V) = (11.1 \times 10^{-9}\ \text{C/V})(2.40 \times 10^9\ \text{V}) = \boxed{26.6\ \text{C}}$$

P26.5 (a) $\Delta V = Ed$ so $E = \dfrac{20.0\ \text{V}}{1.80 \times 10^{-3}\ \text{m}} = \boxed{11.1\ \text{kV/m}}$ toward the negative plate

(b) $E = \dfrac{\sigma}{\in_0}$ so $\sigma = (1.11 \times 10^4\ \text{N/C})(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2) = \boxed{98.3\ \text{nC/m}^2}$

(c) $C = \dfrac{\in_0 A}{d} = \dfrac{(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2)(7.60\ \text{cm}^2)(1.00\ \text{m}/100\ \text{cm})^2}{1.80 \times 10^{-3}\ \text{m}} = \boxed{3.74\ \text{pF}}$

(d) $\Delta V = \dfrac{Q}{C}$ so $Q = (20.0\ \text{V})(3.74 \times 10^{-12}\ \text{F}) = \boxed{74.7\ \text{pC}}$

P26.6 With $\theta = \pi$, the plates are out of mesh and the overlap area is zero.

With $\theta = 0$, the overlap area is that of a semi-circle, $\dfrac{\pi R^2}{2}$. By proportion, the effective area of a single sheet of charge is $\dfrac{(\pi - \theta) R^2}{2}$.

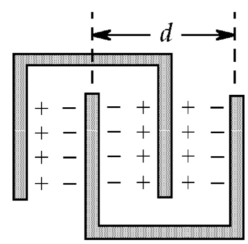


FIG. P26.6

When there are two plates in each comb, the number of adjoining sheets of positive and negative charge is 3, as shown in the sketch. When there are N plates on each comb, the number of parallel capacitors is $2N - 1$ and the total capacitance is

$$C = (2N-1)\frac{\in_0 A_{\text{effective}}}{\text{distance}} = \frac{(2N-1) \in_0 (\pi - \theta) R^2/2}{d/2} = \boxed{\frac{(2N-1) \in_0 (\pi - \theta) R^2}{d}}.$$

P26.7 $Q = \dfrac{\in_0 A}{d}(\Delta V)$ $\qquad \dfrac{Q}{A} = \sigma = \dfrac{\in_0 (\Delta V)}{d}$

$$d = \frac{\in_0 (\Delta V)}{\sigma} = \frac{(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2)(150\ \text{V})}{(30.0 \times 10^{-9}\ \text{C/cm}^2)(1.00 \times 10^4\ \text{cm}^2/\text{m}^2)} = \boxed{4.42\ \mu\text{m}}$$

P26.8 $\sum F_y = 0$: $\qquad T\cos\theta - mg = 0$

$\sum F_x = 0$: $\qquad T\sin\theta - Eq = 0$

Dividing, $\qquad \tan\theta = \dfrac{Eq}{mg}$

so $\qquad E = \dfrac{mg}{q}\tan\theta$

and $\qquad \Delta V = Ed = \boxed{\dfrac{mgd\tan\theta}{q}}$

P26.9 (a) $C = \dfrac{\ell}{2k_e \ln(b/a)} = \dfrac{50.0}{2(8.99\times10^9)\ln(7.27/2.58)} = \boxed{2.68 \text{ nF}}$

(b) Method 1: $\Delta V = 2k_e\lambda \ln\left(\dfrac{b}{a}\right)$

$$\lambda = \frac{q}{\ell} = \frac{8.10\times10^{-6}\text{ C}}{50.0\text{ m}} = 1.62\times10^{-7}\text{ C/m}$$

$$\Delta V = 2(8.99\times10^9)(1.62\times10^{-7})\ln\left(\frac{7.27}{2.58}\right) = \boxed{3.02 \text{ kV}}$$

Method 2: $\Delta V = \dfrac{Q}{C} = \dfrac{8.10\times10^{-6}}{2.68\times10^{-9}} = \boxed{3.02 \text{ kV}}$

***P26.10** The original kinetic energy of the particle is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(2\times10^{-16}\text{ kg})(2\times10^6\text{ m/s})^2 = 4.00\times10^{-4}\text{ J}$$

The potential difference across the capacitor is $\Delta V = \dfrac{Q}{C} = \dfrac{1\,000\ \mu\text{C}}{10\ \mu\text{F}} = 100\text{ V}$.

For the particle to reach the negative plate, the particle-capacitor system would need energy

$$U = q\Delta V = (-3\times10^{-6}\text{ C})(-100\text{ V}) = 3.00\times10^{-4}\text{ J}$$

Since its original kinetic energy is greater than this, $\boxed{\text{the particle will reach the negative plate}}$.

As the particle moves, the system keeps constant total energy

$$(K+U)_{\text{at +plate}} = (K+U)_{\text{at −plate}}: \qquad 4.00\times10^{-4}\text{ J} + (-3\times10^{-6}\text{ C})(+100\text{ V}) = \frac{1}{2}(2\times10^{-16})v_f^2 + 0$$

$$v_f = \sqrt{\frac{2(1.00\times10^{-4}\text{ J})}{2\times10^{-16}\text{ kg}}} = \boxed{1.00\times10^6\text{ m/s}}$$

P26.11 (a) $C = \dfrac{ab}{k_e(b-a)} = \dfrac{(0.070\,0)(0.140)}{(8.99\times10^9)(0.140-0.070\,0)} = \boxed{15.6 \text{ pF}}$

(b) $C = \dfrac{Q}{\Delta V} \qquad \Delta V = \dfrac{Q}{C} = \dfrac{4.00\times10^{-6}\text{ C}}{15.6\times10^{-12}\text{ F}} = \boxed{256 \text{ kV}}$

Section 26.3 **Combinations of Capacitors**

P26.12 (a) Capacitors in parallel add. Thus, the equivalent capacitor has a value of

$$C_{eq} = C_1 + C_2 = 5.00\ \mu\text{F} + 12.0\ \mu\text{F} = \boxed{17.0\ \mu\text{F}}$$

(b) The potential difference across each branch is the same and equal to the voltage of the battery.

$$\Delta V = \boxed{9.00 \text{ V}}$$

(c) $Q_5 = C\Delta V = (5.00\ \mu\text{F})(9.00\text{ V}) = \boxed{45.0\ \mu\text{C}}$

and $Q_{12} = C\Delta V = (12.0\ \mu\text{F})(9.00\text{ V}) = \boxed{108\ \mu\text{C}}$

P26.13 (a) In series capacitors add as $\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{5.00\ \mu\text{F}}+\frac{1}{12.0\ \mu\text{F}}$

and $C_{eq}=\boxed{3.53\ \mu\text{F}}$

(c) The charge on the equivalent capacitor is $Q_{eq}=C_{eq}\Delta V=(3.53\ \mu\text{F})(9.00\ \text{V})=31.8\ \mu\text{C}$

Each of the series capacitors has this same charge on it.

So $Q_1=Q_2=\boxed{31.8\ \mu\text{C}}$

(b) The potential difference across each is $\Delta V_1=\frac{Q_1}{C_1}=\frac{31.8\ \mu\text{C}}{5.00\ \mu\text{F}}=\boxed{6.35\ \text{V}}$

and $\Delta V_2=\frac{Q_2}{C_2}=\frac{31.8\ \mu\text{C}}{12.0\ \mu\text{F}}=\boxed{2.65\ \text{V}}$

P26.14 (a) Capacitors 2 and 3 are in parallel and present equivalent capacitance $6C$. This is in series with capacitor 1, so the battery sees capacitance $\left[\frac{1}{3C}+\frac{1}{6C}\right]^{-1}=\boxed{2C}$.

(b) If they were initially uncharged, C_1 stores the same charge as C_2 and C_3 together. With greater capacitance, C_3 stores more charge than C_2. Then $\boxed{Q_1>Q_3>Q_2}$.

(c) The $(C_2 \parallel C_3)$ equivalent capacitor stores the same charge as C_1. Since it has greater capacitance, $\Delta V=\frac{Q}{C}$ implies that it has smaller potential difference across it than C_1. In parallel with each other, C_2 and C_3 have equal voltages: $\boxed{\Delta V_1>\Delta V_2=\Delta V_3}$.

(d) If C_3 is increased, the overall equivalent capacitance increases. More charge moves through the battery and Q increases. As ΔV_1 increases, ΔV_2 must decrease so Q_2 decreases. Then Q_3 must increase even more: $\boxed{Q_3 \text{ and } Q_1 \text{ increase; } Q_2 \text{ decreases}}$.

P26.15 $C_p=C_1+C_2$ $\qquad \frac{1}{C_s}=\frac{1}{C_1}+\frac{1}{C_2}$

Substitute $C_2=C_p-C_1$ $\qquad \frac{1}{C_s}=\frac{1}{C_1}+\frac{1}{C_p-C_1}=\frac{C_p-C_1+C_1}{C_1(C_p-C_1)}$

Simplifying, $\qquad C_1^2-C_1C_p+C_pC_s=0$

$$C_1=\frac{C_p\pm\sqrt{C_p^2-4C_pC_s}}{2}=\frac{1}{2}C_p\pm\sqrt{\frac{1}{4}C_p^2-C_pC_s}$$

We choose arbitrarily the + sign. (This choice can be arbitrary, since with the case of the minus sign, we would get the same two answers with their names interchanged.)

$$C_1=\frac{1}{2}C_p+\sqrt{\frac{1}{4}C_p^2-C_pC_s}=\frac{1}{2}(9.00\ \text{pF})+\sqrt{\frac{1}{4}(9.00\ \text{pF})^2-(9.00\ \text{pF})(2.00\ \text{pF})}=\boxed{6.00\ \text{pF}}$$

$$C_2=C_p-C_1=\frac{1}{2}C_p-\sqrt{\frac{1}{4}C_p^2-C_pC_s}=\frac{1}{2}(9.00\ \text{pF})-1.50\ \text{pF}=\boxed{3.00\ \text{pF}}$$

P26.16 $C_p = C_1 + C_2$

and $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}.$

Substitute $C_2 = C_p - C_1$: $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_p - C_1} = \frac{C_p - C_1 + C_1}{C_1(C_p - C_1)}$

Simplifying, $C_1^2 - C_1 C_p + C_p C_s = 0$

and $C_1 = \frac{C_p \pm \sqrt{C_p^2 - 4C_p C_s}}{2} = \boxed{\frac{1}{2}C_p + \sqrt{\frac{1}{4}C_p^2 - C_p C_s}}$

where the positive sign was arbitrarily chosen (choosing the negative sign gives the same values for the capacitances, with the names reversed).

Then, from $C_2 = C_p - C_1$

$$C_2 = \boxed{\frac{1}{2}C_p - \sqrt{\frac{1}{4}C_p^2 - C_p C_s}}$$

P26.17 (a) $\frac{1}{C_s} = \frac{1}{15.0} + \frac{1}{3.00}$

$C_s = 2.50\ \mu\text{F}$

$C_p = 2.50 + 6.00 = 8.50\ \mu\text{F}$

$C_{eq} = \left(\frac{1}{8.50\ \mu\text{F}} + \frac{1}{20.0\ \mu\text{F}}\right)^{-1} = \boxed{5.96\ \mu\text{F}}$

(b) $Q = C\Delta V = (5.96\ \mu\text{F})(15.0\ \text{V}) = \boxed{89.5\ \mu\text{C}}$ on $20.0\ \mu\text{F}$

$\Delta V = \frac{Q}{C} = \frac{89.5\ \mu\text{C}}{20.0\ \mu\text{F}} = 4.47\ \text{V}$

$15.0 - 4.47 = 10.53\ \text{V}$

$Q = C\Delta V = (6.00\ \mu\text{F})(10.53\ \text{V}) = \boxed{63.2\ \mu\text{C}}$ on $6.00\ \mu\text{F}$

$89.5 - 63.2 = \boxed{26.3\ \mu\text{C}}$ on $15.0\ \mu\text{F}$ and $3.00\ \mu\text{F}$

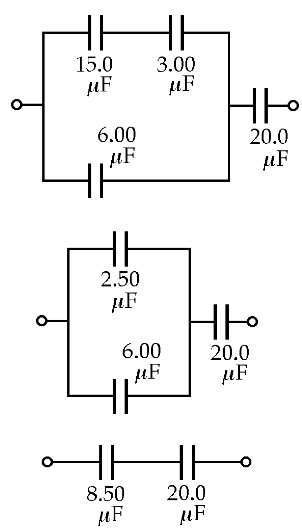


FIG. P26.17

P26.18 (a) In $\boxed{\text{series}}$, to reduce the effective capacitance:

$\frac{1}{32.0\ \mu\text{F}} = \frac{1}{34.8\ \mu\text{F}} + \frac{1}{C_s}$

$C_s = \frac{1}{2.51 \times 10^{-3}/\mu\text{F}} = \boxed{398\ \mu\text{F}}$

(b) In $\boxed{\text{parallel}}$, to increase the total capacitance:

$29.8\ \mu\text{F} + C_p = 32.0\ \mu\text{F}$

$C_p = \boxed{2.20\ \mu\text{F}}$

P26.19 $C = \dfrac{Q}{\Delta V}$ so $6.00 \times 10^{-6} = \dfrac{Q}{20.0}$

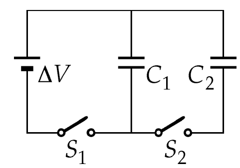


FIG. P26.19

and $Q = \boxed{120\ \mu\text{C}}$

$Q_1 = 120\ \mu\text{C} - Q_2$

and $\Delta V = \dfrac{Q}{C}$: $\dfrac{120 - Q_2}{C_1} = \dfrac{Q_2}{C_2}$

or $\dfrac{120 - Q_2}{6.00} = \dfrac{Q_2}{3.00}$

$(3.00)(120 - Q_2) = (6.00)Q_2$

$Q_2 = \dfrac{360}{9.00} = \boxed{40.0\ \mu\text{C}}$

$Q_1 = 120\ \mu\text{C} - 40.0\ \mu\text{C} = \boxed{80.0\ \mu\text{C}}$

P26.20 For C_1 connected by itself, $C_1 \Delta V = 30.8\ \mu\text{C}$ where ΔV is the battery voltage: $\Delta V = \dfrac{30.8\ \mu\text{C}}{C_1}$.

For C_1 and C_2 in series:

$$\left(\frac{1}{1/C_1 + 1/C_2}\right)\Delta V = 23.1\ \mu\text{C}$$

substituting, $\dfrac{30.8\ \mu\text{C}}{C_1} = \dfrac{23.1\ \mu\text{C}}{C_1} + \dfrac{23.1\ \mu\text{C}}{C_2}$ $\qquad C_1 = 0.333C_2$

For C_1 and C_3 in series:

$$\left(\frac{1}{1/C_1 + 1/C_3}\right)\Delta V = 25.2\ \mu\text{C}$$

$\dfrac{30.8\ \mu\text{C}}{C_1} = \dfrac{25.2\ \mu\text{C}}{C_1} + \dfrac{25.2\ \mu\text{C}}{C_3}$ $\qquad C_1 = 0.222C_3$

For all three:

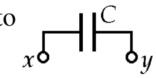
$$Q = \left(\frac{1}{1/C_1 + 1/C_2 + 1/C_3}\right)\Delta V = \frac{C_1 \Delta V}{1 + C_1/C_2 + C_1/C_3} = \frac{30.8\ \mu\text{C}}{1 + 0.333 + 0.222} = \boxed{19.8\ \mu\text{C}}$$

This is the charge on each one of the three.

P26.21 $nC = \dfrac{100}{\underbrace{\frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \cdots}_{n \text{ capacitors}}} = \dfrac{100}{n/C}$

$nC = \dfrac{100C}{n}$ so $n^2 = 100$ and $n = \boxed{10}$

P26.22 According to the suggestion, the combination of capacitors shown is equivalent to



Then
$$\frac{1}{C}=\frac{1}{C_0}+\frac{1}{C+C_0}+\frac{1}{C_0}$$
$$=\frac{C+C_0+C_0+C+C_0}{C_0\left(C+C_0\right)}$$
$$C_0C+C_0^2=2C^2+3C_0C$$
$$2C^2+2C_0C-C_0^2=0$$
$$C=\frac{-2C_0\pm\sqrt{4C_0^2+4\left(2C_0^2\right)}}{4}$$

Only the positive root is physical.

$$\boxed{C=\frac{C_0}{2}\left(\sqrt{3}-1\right)}$$

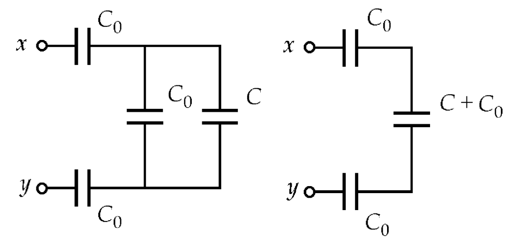


FIG. P26.22

P26.23
$$C_s=\left(\frac{1}{5.00}+\frac{1}{10.0}\right)^{-1}=3.33\ \mu\text{F}$$
$$C_{p1}=2\left(3.33\right)+2.00=8.66\ \mu\text{F}$$
$$C_{p2}=2\left(10.0\right)=20.0\ \mu\text{F}$$
$$C_{eq}=\left(\frac{1}{8.66}+\frac{1}{20.0}\right)^{-1}=\boxed{6.04\ \mu\text{F}}$$

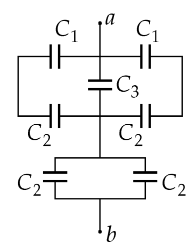


FIG. P26.23

P26.24
$$Q_{eq}=C_{eq}\left(\Delta V\right)=\left(6.04\times10^{-6}\ \text{F}\right)\left(60.0\ \text{V}\right)=3.62\times10^{-4}\ \text{C}$$
$$Q_{p1}=Q_{eq},\quad\text{so}\quad \Delta V_{p1}=\frac{Q_{eq}}{C_{p1}}=\frac{3.62\times10^{-4}\ \text{C}}{8.66\times10^{-6}\ \text{F}}=41.8\ \text{V}$$
$$Q_3=C_3\left(\Delta V_{p1}\right)=\left(2.00\times10^{-6}\ \text{F}\right)\left(41.8\ \text{V}\right)=\boxed{83.6\ \mu\text{C}}$$

P26.25
$$C_s=\left(\frac{1}{5.00}+\frac{1}{7.00}\right)^{-1}=2.92\ \mu\text{F}$$
$$C_p=2.92+4.00+6.00=\boxed{12.9\ \mu\text{F}}$$

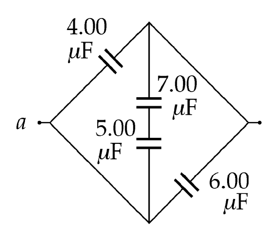


FIG. P26.25

Section 26.4 **Energy Stored in a Charged Capacitor**

P26.26 $U = \frac{1}{2} C\Delta V^2$

$$\Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2(300\text{ J})}{30 \times 10^{-6}\text{ C/V}}} = \boxed{4.47 \times 10^3\text{ V}}$$

P26.27 (a) $U = \frac{1}{2} C(\Delta V)^2 = \frac{1}{2}(3.00\ \mu\text{F})(12.0\text{ V})^2 = \boxed{216\ \mu\text{J}}$

(b) $U = \frac{1}{2} C(\Delta V)^2 = \frac{1}{2}(3.00\ \mu\text{F})(6.00\text{ V})^2 = \boxed{54.0\ \mu\text{J}}$

P26.28 $U = \frac{1}{2} C(\Delta V)^2$

The circuit diagram is shown at the right.

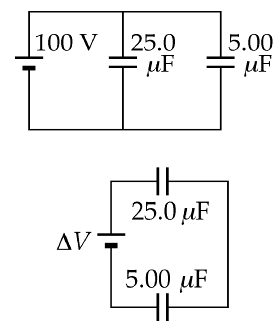


FIG. P26.28

(a) $C_p = C_1 + C_2 = 25.0\ \mu\text{F} + 5.00\ \mu\text{F} = 30.0\ \mu\text{F}$

$$U = \frac{1}{2}(30.0 \times 10^{-6})(100)^2 = \boxed{0.150\text{ J}}$$

(b) $C_s = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{25.0\ \mu\text{F}} + \frac{1}{5.00\ \mu\text{F}}\right)^{-1} = 4.17\ \mu\text{F}$

$$U = \frac{1}{2} C(\Delta V)^2$$

$$\Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2(0.150)}{4.17 \times 10^{-6}}} = \boxed{268\text{ V}}$$

P26.29 $W = U = \int F dx$

so $F = \frac{dU}{dx} = \frac{d}{dx}\left(\frac{Q^2}{2C}\right) = \frac{d}{dx}\left(\frac{Q^2 x}{2\epsilon_0 A}\right) = \boxed{\frac{Q^2}{2\epsilon_0 A}}$

P26.30 With switch closed, distance $d' = 0.500d$ and capacitance $C' = \frac{\epsilon_0 A}{d'} = \frac{2\epsilon_0 A}{d} = 2C$.

(a) $Q = C'(\Delta V) = 2C(\Delta V) = 2(2.00 \times 10^{-6}\text{ F})(100\text{ V}) = \boxed{400\ \mu\text{C}}$

(b) The force stretching out one spring is

$$F = \frac{Q^2}{2\epsilon_0 A} = \frac{4C^2(\Delta V)^2}{2\epsilon_0 A} = \frac{2C^2(\Delta V)^2}{(\epsilon_0 A/d)d} = \frac{2C(\Delta V)^2}{d}$$

One spring stretches by distance $x = \frac{d}{4}$, so

$$k = \frac{F}{x} = \frac{2C(\Delta V)^2}{d}\left(\frac{4}{d}\right) = \frac{8C(\Delta V)^2}{d^2} = \frac{8(2.00 \times 10^{-6}\text{ F})(100\text{ V})^2}{(8.00 \times 10^{-3}\text{ m})^2} = \boxed{2.50\text{ kN/m}}$$

P26.31 (a) $Q = C\Delta V = \left(150\times10^{-12}\text{ F}\right)\left(10\times10^{3}\text{ V}\right) = \boxed{1.50\times10^{-6}\text{ C}}$

(b) $U = \frac{1}{2}C\left(\Delta V\right)^2$

$$\Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2\left(250\times10^{-6}\text{ J}\right)}{150\times10^{-12}\text{ F}}} = \boxed{1.83\times10^{3}\text{ V}}$$

P26.32 (a) $U = \frac{1}{2}C\left(\Delta V\right)^2 + \frac{1}{2}C\left(\Delta V\right)^2 = \boxed{C\left(\Delta V\right)^2}$

(b) The altered capacitor has capacitance $C' = \frac{C}{2}$. The total charge is the same as before:

$$C(\Delta V) + C(\Delta V) = C(\Delta V') + \frac{C}{2}(\Delta V') \qquad \boxed{\Delta V' = \frac{4\Delta V}{3}}$$

(c) $U' = \frac{1}{2}C\left(\frac{4\Delta V}{3}\right)^2 + \frac{1}{2}\frac{1}{2}C\left(\frac{4\Delta V}{3}\right)^2 = \boxed{4C\frac{\left(\Delta V\right)^2}{3}}$

(d) The extra energy comes from work put into the system by the agent pulling the capacitor plates apart.

P26.33 $U = \frac{1}{2}C\left(\Delta V\right)^2$ where $C = 4\pi\in_0 R = \frac{R}{k_e}$ and $\Delta V = \frac{k_eQ}{R} - 0 = \frac{k_eQ}{R}$

$$U = \frac{1}{2}\left(\frac{R}{k_e}\right)\left(\frac{k_eQ}{R}\right)^2 = \boxed{\frac{k_eQ^2}{2R}}$$

P26.34 (a) The total energy is $U = U_1 + U_2 = \frac{1}{2}\frac{q_1^2}{C_1} + \frac{1}{2}\frac{q_2^2}{C_2} = \frac{1}{2}\frac{q_1^2}{4\pi\in_0 R_1} + \frac{1}{2}\frac{\left(Q-q_1\right)^2}{4\pi\in_0 R^2}$.

For a minimum we set $\frac{dU}{dq_1} = 0$:

$$\frac{1}{2}\frac{2q_1}{4\pi\in_0 R_1} + \frac{1}{2}\frac{2(Q-q_1)}{4\pi\in_0 R_2}\left(-1\right) = 0$$

$$R_2q_1 = R_1Q - R_1q_1 \qquad \boxed{q_1 = \frac{R_1Q}{R_1+R_2}}$$

Then $q_2 = Q - q_1 = \boxed{\frac{R_2Q}{R_1+R_2} = q_2}$.

(b) $V_1 = \frac{k_eq_1}{R_1} = \frac{k_eR_1Q}{R_1\left(R_1+R_2\right)} = \frac{k_eQ}{R_1+R_2}$

$$V_2 = \frac{k_eq_2}{R_2} = \frac{k_eR_2Q}{R_2\left(R_1+R_2\right)} = \frac{k_eQ}{R_1+R_2}$$

and $V_1 - V_2 = 0$

P26.35 The energy transferred is $T_{ET} = \frac{1}{2}Q\Delta V = \frac{1}{2}(50.0\text{ C})(1.00\times10^8\text{ V}) = 2.50\times10^9\text{ J}$

and 1% of this (or $\Delta E_{int} = 2.50\times10^7$ J) is absorbed by the tree. If m is the amount of water boiled away,

then $\Delta E_{int} = m(4\,186\text{ J/kg}\cdot°\text{C})(100°\text{C} - 30.0°\text{C}) + m(2.26\times10^6\text{ J/kg}) = 2.50\times10^7\text{ J}$

giving $m = \boxed{9.79\text{ kg}}$

Section 26.5 **Capacitors with Dielectrics**

P26.36 $Q_{max} = C\Delta V_{max}$,

but $\Delta V_{max} = E_{max}d$

Also, $C = \frac{\kappa \in_0 A}{d}$

Thus, $Q_{max} = \frac{\kappa \in_0 A}{d}(E_{max}d) = \kappa \in_0 AE_{max}$

(a) With air between the plates, $\kappa = 1.00$

and $E_{max} = 3.00\times10^6$ V/m

Therefore,

$$Q_{max} = \kappa \in_0 AE_{max} = (8.85\times10^{-12}\text{ F/m})(5.00\times10^{-4}\text{ m}^2)(3.00\times10^6\text{ V/m}) = \boxed{13.3\text{ nC}}$$

(b) With polystyrene between the plates, $\kappa = 2.56$ and $E_{max} = 24.0\times10^6$ V/m.

$$Q_{max} = \kappa \in_0 AE_{max} = 2.56(8.85\times10^{-12}\text{ F/m})(5.00\times10^{-4}\text{ m}^2)(24.0\times10^6\text{ V/m}) = \boxed{272\text{ nC}}$$

P26.37 (a) $C = \frac{\kappa \in_0 A}{d} = \frac{2.10(8.85\times10^{-12}\text{ F/m})(1.75\times10^{-4}\text{ m}^2)}{4.00\times10^{-5}\text{ m}} = 8.13\times10^{-11}\text{ F} = \boxed{81.3\text{ pF}}$

(b) $\Delta V_{max} = E_{max}d = (60.0\times10^6\text{ V/m})(4.00\times10^{-5}\text{ m}) = \boxed{2.40\text{ kV}}$

P26.38 Consider two sheets of aluminum foil, each 40 cm by 100 cm, with one sheet of plastic between them. Suppose the plastic has $\kappa \approx 3$, $E_{max} \sim 10^7$ V/m, and thickness 1 mil $= \frac{2.54\text{ cm}}{1\,000}$. Then,

$$C = \frac{\kappa \in_0 A}{d} \sim \frac{3(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.4\text{ m}^2)}{2.54\times10^{-5}\text{ m}} \sim \boxed{10^{-6}\text{ F}}$$

$$\Delta V_{max} = E_{max}d \sim (10^7\text{ V/m})(2.54\times10^{-5}\text{ m}) \sim \boxed{10^2\text{ V}}$$

P26.39 $C = \frac{\kappa \in_0 A}{d}$

or $95.0\times10^{-9} = \frac{3.70(8.85\times10^{-12})(0.070\,0)\ell}{0.025\,0\times10^{-3}}$

$\ell = \boxed{1.04\text{ m}}$

P26.40 Originally, $$C = \frac{\in_0 A}{d} = \frac{Q}{(\Delta V)_i}$$

(a) The charge is the same before and after immersion, with value $Q = \dfrac{\in_0 A(\Delta V)_i}{d}$.

$$Q = \frac{\left(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(25.0 \times 10^{-4} \text{ m}^2\right)(250 \text{ V})}{\left(1.50 \times 10^{-2} \text{ m}\right)} = \boxed{369 \text{ pC}}$$

(b) Finally,

$$C_f = \frac{\kappa \in_0 A}{d} = \frac{Q}{(\Delta V)_f} \qquad C_f = \frac{80.0\left(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(25.0 \times 10^{-4} \text{ m}^2\right)}{\left(1.50 \times 10^{-2} \text{ m}\right)} = \boxed{118 \text{ pF}}$$

$$(\Delta V)_f = \frac{Qd}{\kappa \in_0 A} = \frac{\in_0 A(\Delta V)_i d}{\kappa \in_0 Ad} = \frac{(\Delta V)_i}{\kappa} = \frac{250 \text{ V}}{80.0} = \boxed{3.12 \text{ V}}$$

(c) Originally, $$U_i = \frac{1}{2}C(\Delta V)_i^2 = \frac{\in_0 A(\Delta V)_i^2}{2d}$$

Finally, $$U_f = \frac{1}{2}C_f(\Delta V)_f^2 = \frac{\kappa \in_0 A(\Delta V)_i^2}{2d\kappa^2} = \frac{\in_0 A(\Delta V)_i^2}{2d\kappa}$$

So, $$\Delta U = U_f - U_i = \frac{-\in_0 A(\Delta V)_i^2(\kappa - 1)}{2d\kappa}$$

$$\Delta U = -\frac{\left(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(25.0 \times 10^{-4} \text{ m}^2\right)(250 \text{ V})^2(79.0)}{2\left(1.50 \times 10^{-2} \text{ m}\right)(80.0)} = \boxed{-45.5 \text{ nJ}}$$

P26.41 The given combination of capacitors is equivalent to the circuit diagram shown to the right.

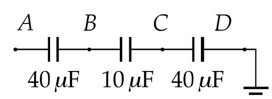


FIG. P26.41

Put charge Q on point A. Then,

$$Q = (40.0\ \mu\text{F})\Delta V_{AB} = (10.0\ \mu\text{F})\Delta V_{BC} = (40.0\ \mu\text{F})\Delta V_{CD}$$

So, $\Delta V_{BC} = 4\Delta V_{AB} = 4\Delta V_{CD}$, and the center capacitor will break down first, at $\Delta V_{BC} = 15.0$ V. When this occurs,

$$\Delta V_{AB} = \Delta V_{CD} = \frac{1}{4}(\Delta V_{BC}) = 3.75 \text{ V}$$

and $V_{AD} = V_{AB} + V_{BC} + V_{CD} = 3.75 \text{ V} + 15.0 \text{ V} + 3.75 \text{ V} = \boxed{22.5 \text{ V}}$.

Section 26.6 **Electric Dipole in an Electric Field**

P26.42 (a) The displacement from negative to positive charge is

$$2a = \left(-1.20\hat{\mathbf{i}} + 1.10\hat{\mathbf{j}}\right)\text{ mm} - \left(1.40\hat{\mathbf{i}} - 1.30\hat{\mathbf{j}}\right)\text{ mm} = \left(-2.60\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right)\times 10^{-3}\text{ m}$$

The electric dipole moment is

$$\vec{\mathbf{p}} = 2\vec{\mathbf{a}}q = \left(3.50\times 10^{-9}\text{ C}\right)\left(-2.60\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right)\times 10^{-3}\text{ m} = \boxed{\left(-9.10\hat{\mathbf{i}} + 8.40\hat{\mathbf{j}}\right)\times 10^{-12}\text{ C}\cdot\text{m}}$$

(b) $$\vec{\boldsymbol{\tau}} = \vec{\mathbf{p}}\times\vec{\mathbf{E}} = \left[\left(-9.10\hat{\mathbf{i}} + 8.40\hat{\mathbf{j}}\right)\times 10^{-12}\text{ C}\cdot\text{m}\right]\times\left[\left(7.80\hat{\mathbf{i}} - 4.90\hat{\mathbf{j}}\right)\times 10^{3}\text{ N/C}\right]$$

$$\vec{\boldsymbol{\tau}} = \left(+44.6\hat{\mathbf{k}} - 65.5\hat{\mathbf{k}}\right)\times 10^{-9}\text{ N}\cdot\text{m} = \boxed{-2.09\times 10^{-8}\text{ N}\cdot\text{m}\hat{\mathbf{k}}}$$

(c) $$U = -\vec{\mathbf{p}}\cdot\vec{\mathbf{E}} = -\left[\left(-9.10\hat{\mathbf{i}} + 8.40\hat{\mathbf{j}}\right)\times 10^{-12}\text{ C}\cdot\text{m}\right]\cdot\left[\left(7.80\hat{\mathbf{i}} - 4.90\hat{\mathbf{j}}\right)\times 10^{3}\text{ N/C}\right]$$

$$U = \left(71.0 + 41.2\right)\times 10^{-9}\text{ J} = \boxed{112\text{ nJ}}$$

(d) $$\left|\vec{\mathbf{p}}\right| = \sqrt{(9.10)^2 + (8.40)^2}\times 10^{-12}\text{ C}\cdot\text{m} = 12.4\times 10^{-12}\text{ C}\cdot\text{m}$$

$$\left|\vec{\mathbf{E}}\right| = \sqrt{(7.80)^2 + (4.90)^2}\times 10^{3}\text{ N/C} = 9.21\times 10^{3}\text{ N/C}$$

$$U_{\text{max}} = \left|\vec{\mathbf{p}}\right|\left|\vec{\mathbf{E}}\right| = 114\text{ nJ},\qquad U_{\text{min}} = -114\text{ nJ}$$

$$U_{\text{max}} - U_{\text{min}} = \boxed{228\text{ nJ}}$$

P26.43 (a) Let x represent the coordinate of the negative charge. Then $x + 2a\cos\theta$ is the coordinate of the positive charge. The force on the negative charge is $\vec{\mathbf{F}}_{-} = -qE(x)\hat{\mathbf{i}}$. The force on the positive charge is

$$\vec{\mathbf{F}}_{+} = +qE(x + 2a\cos\theta)\hat{\mathbf{i}} \approx qE(x)\hat{\mathbf{i}} + q\frac{dE}{dx}(2a\cos\theta)\hat{\mathbf{i}}.$$

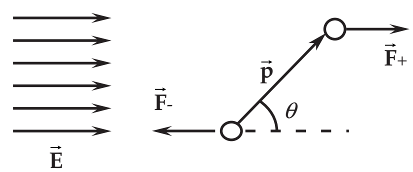


FIG. P26.43(a)

The force on the dipole is altogether $\vec{\mathbf{F}} = \vec{\mathbf{F}}_{-} + \vec{\mathbf{F}}_{+} = q\frac{dE}{dx}(2a\cos\theta)\hat{\mathbf{i}} = \boxed{p\frac{dE}{dx}\cos\theta\,\hat{\mathbf{i}}}$.

(b) The balloon creates field along the x-axis of $\frac{k_e q}{x^2}\hat{\mathbf{i}}$.

Thus, $\frac{dE}{dx} = \frac{(-2)k_e q}{x^3}$.

At $x = 16.0$ cm, $$\frac{dE}{dx} = \frac{(-2)\left(8.99\times 10^{9}\right)\left(2.00\times 10^{-6}\right)}{(0.160)^3} = \boxed{-8.78\text{ MN/C}\cdot\text{m}}$$

$$\vec{\mathbf{F}} = \left(6.30\times 10^{-9}\text{ C}\cdot\text{m}\right)\left(-8.78\times 10^{6}\text{ N/C}\cdot\text{m}\right)\cos 0°\hat{\mathbf{i}} = \boxed{-55.3\hat{\mathbf{i}}\text{ mN}}$$

Section 26.7 An Atomic Description of Dielectrics

P26.44 (a) Consider a gaussian surface in the form of a cylindrical pillbox with ends of area $A' << A$ parallel to the sheet. The side wall of the cylinder passes no flux of electric field since this surface is everywhere parallel to the field. Gauss's law becomes

$$EA' + EA' = \frac{Q}{\in A}A', \text{ so } \boxed{E = \frac{Q}{2\in A}} \text{ directed away from the positive sheet.}$$

(b) In the space between the sheets, each creates field $\frac{Q}{2\in A}$ away from the positive and toward the negative sheet. Together, they create a field of

$$\boxed{E = \frac{Q}{\in A}}$$

(c) Assume that the field is in the positive x-direction. Then, the potential of the positive plate relative to the negative plate is

$$\Delta V = -\int_{-\text{plate}}^{+\text{plate}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_{-\text{plate}}^{+\text{plate}} \frac{Q}{\in A}\hat{\mathbf{i}} \cdot \left(-\hat{\mathbf{i}}\,dx\right) = \boxed{+\frac{Qd}{\in A}}$$

(d) Capacitance is defined by: $C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\in A} = \boxed{\frac{\in A}{d} = \frac{\kappa \in_0 A}{d}}$.

P26.45 $E_{\max}$ occurs at the inner conductor's surface.

$E_{\max} = \frac{2k_e\lambda}{a}$ from an equation derived about this situation in Chapter 24.

$\Delta V = 2k_e\lambda \ln\left(\frac{b}{a}\right)$ from Example 26.1.

$$E_{\max} = \frac{\Delta V}{a\ln(b/a)}$$

$$\Delta V_{\max} = E_{\max}a\ln\left(\frac{b}{a}\right) = \left(18.0\times10^{6}\text{ V/m}\right)\left(0.800\times10^{-3}\text{ m}\right)\ln\left(\frac{3.00}{0.800}\right) = \boxed{19.0\text{ kV}}$$

Additional Problems

P26.46 Imagine the center plate is split along its midplane and pulled apart. We have two capacitors in parallel, supporting the same ΔV and carrying total charge Q. The upper has capacitance $C_1 = \frac{\in_0 A}{d}$ and the lower $C_2 = \frac{\in_0 A}{2d}$. Charge flows from ground onto each of the outside plates so that $Q_1 + Q_2 = Q \qquad \Delta V_1 = \Delta V_2 = \Delta V$

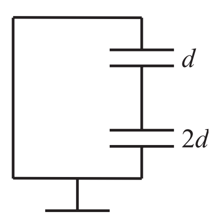


FIG. P26.46

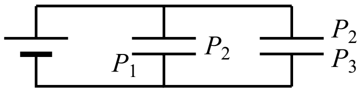
Then $\frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_1 d}{\in_0 A} = \frac{Q_2 2d}{\in_0 A} \qquad Q_1 = 2Q_2 \qquad 2Q_2 + Q_2 = Q$

(a) $Q_2 = \frac{Q}{3}$. $\boxed{\text{On the lower plate the charge is } -\frac{Q}{3}.}$

$Q_1 = \frac{2Q}{3}$. $\boxed{\text{On the upper plate the charge is } -\frac{2Q}{3}.}$

(b) $\Delta V = \frac{Q_1}{C_1} = \boxed{\frac{2Qd}{3\in_0 A}}$

P26.47 (a) Each face of P_2 carries charge, so the three-plate system is equivalent to



Each capacitor by itself has capacitance

$$C = \frac{\kappa \in_0 A}{d} = \frac{1\left(8.85\times10^{-12}\ \text{C}^2\right)7.5\times10^{-4}\ \text{m}^2}{\text{N}\cdot\text{m}^2\quad 1.19\times10^{-3}\ \text{m}} = 5.58\ \text{pF}$$

Then equivalent capacitance $= 5.58 + 5.58 = \boxed{11.2\ \text{pF}}$.

(b) $Q = C\Delta V + C\Delta V = 11.2\times10^{-12}\ \text{F}(12\ \text{V}) = \boxed{134\ \text{pC}}$

(c) Now P_3 has charge on two surfaces and in effect three capacitors are in parallel:

$$C = 3(5.58\ \text{pF}) = \boxed{16.7\ \text{pF}}$$

(d) Only one face of P_4 carries charge: $Q = C\Delta V = 5.58\times10^{-12}\ \text{F}(12\ \text{V}) = \boxed{66.9\ \text{pC}}$

P26.48 From the Example about a cylindrical capacitor,

$$V_b - V_a = -2k_e\lambda\ln\frac{b}{a}$$

$$V_b - 345\ \text{kV} = -2\left(8.99\times10^9\ \text{Nm}^2/\text{C}^2\right)\left(1.40\times10^{-6}\ \text{C/m}\right)\ln\frac{12\ \text{m}}{0.024\ \text{m}}$$

$$= -2(8.99)\left(1.4\times10^3\ \text{J/C}\right)\ln 500 = -1.564\,3\times10^5\ \text{V}$$

$$V_b = 3.45\times10^5\ \text{V} - 1.56\times10^5\ \text{V} = \boxed{1.89\times10^5\ \text{V}}$$

P26.49 (a) We use the equation $U = Q^2/2C$ to find the potential energy of the capacitor. As we will see, the potential difference ΔV changes as the dielectric is withdrawn. The initial and final energies are $U_i = \frac{1}{2}\left(\frac{Q^2}{C_i}\right)$ and $U_f = \frac{1}{2}\left(\frac{Q^2}{C_f}\right)$.

But the initial capacitance (with the dielectric) is $C_i = \kappa C_f$. Therefore, $U_f = \frac{1}{2}\kappa\left(\frac{Q^2}{C_i}\right)$.

Since the work done by the external force in removing the dielectric equals the change in potential energy, we have $W = U_f - U_i = \frac{1}{2}\kappa\left(\frac{Q^2}{C_i}\right) - \frac{1}{2}\left(\frac{Q^2}{C_i}\right) = \frac{1}{2}\left(\frac{Q^2}{C_i}\right)(\kappa - 1)$.

To express this relation in terms of potential difference ΔV_i, we substitute $Q = C_i\left(\Delta V_i\right)$, and evaluate: $W = \frac{1}{2}C_i\left(\Delta V_i\right)^2(\kappa-1) = \frac{1}{2}\left(2.00\times10^{-9}\ \text{F}\right)(100\ \text{V})^2(5.00 - 1.00) = \boxed{4.00\times10^{-5}\ \text{J}}$.

The positive result confirms that the final energy of the capacitor is greater than the initial energy. The extra energy comes from the work done *on* the system by the external force that pulled out the dielectric.

(b) The final potential difference across the capacitor is $\Delta V_f = \frac{Q}{C_f}$.

Substituting $C_f = \frac{C_i}{\kappa}$ and $Q = C_i\left(\Delta V_i\right)$ gives $\Delta V_f = \kappa\Delta V_i = 5.00(100\ \text{V}) = \boxed{500\ \text{V}}$.

Even though the capacitor is isolated and its charge remains constant, the potential difference across the plates does increase in this case.

***P26.50** (a)

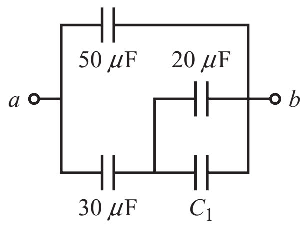


FIG. P26.50a

(b) $\frac{1}{1/30+1/(20+C_1)}=20\mu\text{F}$ gives $\frac{1}{30}+\frac{1}{20+C_1}=\frac{1}{20}$ so $\frac{1}{20+C_1}=\frac{1}{60}$

and $C_1 = \boxed{40.0\ \mu\text{F}}$

(c)

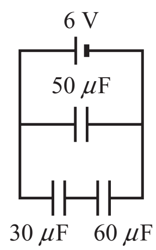


FIG. P26.50c

For the 50 μF, $\Delta V = \boxed{6.00\text{ V}}$ and $Q = C\Delta V = \boxed{300\ \mu\text{C}}$.

For the 30 μF and 60 μF, the equivalent capacitance is 20 μF and $Q = C\Delta V =$ 20 μF 6 V = $\boxed{120\ \mu\text{C}}$.

For 30 μF, $\Delta V = Q/C = 120\ \mu\text{C}/30\ \mu\text{F} = \boxed{4.00\text{ V}}$.

For 20 μF and 40 μF, $\Delta V = 120\ \mu\text{C}/60\ \mu\text{F} = \boxed{2.00\text{ V}}$.

For 20 μF, $Q = C\Delta V = 20\ \mu\text{F}\ 2\text{ V} = \boxed{40.0\ \mu\text{C}}$.

For 40 μF, $Q = C\Delta V = 40\ \mu\text{F}\ 2\text{ V} = \boxed{80.0\ \mu\text{C}}$.

P26.51 $\kappa = 3.00,\ E_{max} = 2.00\times10^{8}\ \text{V/m} = \frac{\Delta V_{max}}{d}$

For $C = \frac{\kappa\epsilon_0 A}{d} = 0.250\times10^{-6}\ \text{F}$

$$A = \frac{Cd}{\kappa\epsilon_0} = \frac{C\Delta V_{max}}{\kappa\epsilon_0 E_{max}} = \frac{\left(0.250\times10^{-6}\right)(4\,000)}{3.00\left(8.85\times10^{-12}\right)\left(2.00\times10^{8}\right)} = \boxed{0.188\ \text{m}^2}$$

***P26.52** (a) The partially filled capacitor constitutes two capacitors in series, with separate capacitances

$$\frac{1\in_0 A}{d(1-f)} \quad \text{and} \quad \frac{6.5\in_0 A}{fd} \quad \text{and equivalent capacitance}$$

$$\frac{1}{\frac{d(1-f)}{\in_0 A}+\frac{fd}{6.5\in_0 A}}=\frac{6.5\in_0 A}{6.5d-6.5df+fd}=\frac{\in_0 A}{d}\frac{6.5}{6.5-5.5f}=\boxed{25.0\ \mu\text{F}(1-0.846f)^{-1}}$$

(b) For $f = 0$, the capacitor is empty so we can expect capacitance $\boxed{25.0\ \mu\text{F}}$, and the expression in part (a) $\boxed{\text{agrees}}$ with this.

(c) For $f = 1$ we expect $6.5(25.0\ \mu\text{F}) = 162\ \mu\text{F}$. The expression in (a) becomes $25.0\ \mu\text{F}(1-0.846)^{-1} = \boxed{162\ \mu\text{F, in agreement}}$.

(d) The charge on the lower plate creates an electric field in the liquid given by

$$E=\frac{Q}{A\kappa\in_0}=\frac{300\ \mu\text{C}}{A\ 6.5\in_0}$$

The charge on the upper plate creates an electric field in the vacuum according to

$$E=\frac{Q}{A\in_0}=\frac{300\ \mu\text{C}}{A\in_0}$$

The change in strength of the field at the upper surface of the liquid is described by

$$\frac{300\ \mu\text{C}}{A\ 6.5\in_0}+\frac{Q_{induced}}{A\in_0}=\frac{300\ \mu\text{C}}{A\in_0}$$

which gives $Q_{induced} = \boxed{254\ \mu\text{C, independent of } f}$. The induced charge will be opposite in sign to the charge on the top capacitor plate and $\boxed{\text{the same in sign as the charge on the lower plate}}$.

P26.53 (a) Put charge Q on the sphere of radius a and $-Q$ on the other sphere. Relative to $V = 0$ at infinity,

the potential at the surface of a is $\quad V_a=\frac{k_eQ}{a}-\frac{k_eQ}{d}$

and the potential of b is $\quad V_b=\frac{-k_eQ}{b}+\frac{k_eQ}{d}$.

The difference in potential is $\quad V_a-V_b=\frac{k_eQ}{a}+\frac{k_eQ}{b}-\frac{k_eQ}{d}-\frac{k_eQ}{d}$

and $\quad C=\frac{Q}{V_a-V_b}=\boxed{\left(\frac{4\pi\in_0}{(1/a)+(1/b)-(2/d)}\right)}$

(b) As $d\to\infty$, $\frac{1}{d}$ becomes negligible compared to $\frac{1}{a}$. Then,

$$C=\frac{4\pi\in_0}{1/a+1/b} \text{ and } \frac{1}{C}=\boxed{\frac{1}{4\pi\in_0 a}+\frac{1}{4\pi\in_0 b}}$$

as for two spheres in series.

P26.54 The initial charge on the larger capacitor is

$$Q = C\Delta V = 10\ \mu\text{F}(15\ \text{V}) = 150\ \mu\text{C}$$

An additional charge q is pushed through the 50-V battery, giving the smaller capacitor charge q and the larger charge $150\ \mu\text{C} + q$.

Then $$50\ \text{V} = \frac{q}{5\ \mu\text{F}} + \frac{150\ \mu\text{C} + q}{10\ \mu\text{F}}$$

$$500\ \mu\text{C} = 2q + 150\ \mu\text{C} + q$$

$$q = 117\ \mu\text{C}$$

So across the $5\text{-}\mu\text{F}$ capacitor $$\Delta V = \frac{q}{C} = \frac{117\ \mu\text{C}}{5\ \mu\text{F}} = \boxed{23.3\ \text{V}}$$

Across the $10\text{-}\mu\text{F}$ capacitor $$\Delta V = \frac{150\ \mu\text{C} + 117\ \mu\text{C}}{10\ \mu\text{F}} = \boxed{26.7\ \text{V}}$$

P26.55 Gasoline: $$(126\,000\ \text{Btu/gal})(1\,054\ \text{J/Btu})\left(\frac{1.00\ \text{gal}}{3.786 \times 10^{-3}\ \text{m}^3}\right)\left(\frac{1.00\ \text{m}^3}{670\ \text{kg}}\right) = 5.24 \times 10^7\ \text{J/kg}$$

Battery: $$\frac{(12.0\ \text{J/C})(100\ \text{C/s})(3\,600\ \text{s})}{16.0\ \text{kg}} = 2.70 \times 10^5\ \text{J/kg}$$

Capacitor: $$\frac{\frac{1}{2}(0.100\ \text{F})(12.0\ \text{V})^2}{0.100\ \text{kg}} = 72.0\ \text{J/kg}$$

Gasoline has 194 times the specific energy content of the battery and 727 000 times that of the capacitor.

***P26.56** (a) The portion of the device containing the dielectric has plate area ℓx and capacitance $C_1 = \dfrac{\kappa \in_0 \ell x}{d}$. The unfilled part has area $\ell(\ell - x)$ and capacitance $C_2 = \dfrac{\in_0 \ell(\ell - x)}{d}$. The total capacitance is $C_1 + C_2 = \boxed{\dfrac{\in_0}{d}\left[\ell^2 + \ell x(\kappa - 1)\right]}$.

(b) The stored energy is $U = \dfrac{1}{2}\dfrac{Q^2}{C} = \boxed{\dfrac{Q^2 d}{2\in_0\left(\ell^2 + \ell x(\kappa - 1)\right)}}$.

(c) $\vec{\mathbf{F}} = -\left(\dfrac{dU}{dx}\right)\hat{\mathbf{i}} = \boxed{\dfrac{Q^2 d\,\ell(\kappa - 1)}{2\in_0\left(\ell^2 + \ell x(\kappa - 1)\right)^2}\hat{\mathbf{i}}}$. When $x = 0$, the original value of the force is $\dfrac{Q^2 d(\kappa - 1)}{2\in_0 \ell^3}\hat{\mathbf{i}}$. As the dielectric slides in, the charges on the plates redistribute themselves. The force decreases to its final value $\dfrac{Q^2 d(\kappa - 1)}{2\in_0 \ell^3 \kappa^2}\hat{\mathbf{i}}$.

continued on next page

(d) At $x = \frac{\ell}{2}$, $\vec{\mathbf{F}} = \frac{2Q^2 d(\kappa - 1)}{\in_0 \ell^3 (\kappa + 1)^2}\hat{\mathbf{i}}$.

For the constant charge on the capacitor and the initial voltage we have the relationship

$$Q = C_0 \Delta V = \frac{\in_0 \ell^2 \Delta V}{d}$$

Then the force is $\vec{\mathbf{F}} = \frac{2\in_0 \ell (\Delta V)^2 (\kappa - 1)}{d(\kappa + 1)^2}\hat{\mathbf{i}}$

$$\vec{\mathbf{F}} = \frac{2(8.85 \times 10^{-12}\ \text{C}^2)0.05\ \text{m}(2\ 000\ \text{Nm})^2(4.5-1)}{\text{Nm}^2(0.002\ \text{m})\text{C}^2(4.5+1)^2}\hat{\mathbf{i}} = \boxed{205\mu\text{N}\ \hat{\mathbf{i}}}$$

***P26.57** The portion of the capacitor nearly filled by metal has

capacitance $\frac{\kappa \in_0 (\ell x)}{d} \to \infty$

and stored energy $\frac{Q^2}{2C} \to 0$

The unfilled portion has

capacitance $\frac{\in_0 \ell(\ell - x)}{d}$

The charge on this portion is $Q = \frac{(\ell - x)Q_0}{\ell}$

(a) The stored energy is

$$U = \frac{Q^2}{2C} = \frac{[(\ell - x)Q_0/\ell]^2}{2\in_0 \ell(\ell - x)/d} = \boxed{\frac{Q_0^2(\ell - x)d}{2\in_0 \ell^3}}$$

(b) $F = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{Q_0^2(\ell - x)d}{2\in_0 \ell^3}\right) = +\frac{Q_0^2 d}{2\in_0 \ell^3}$

$$\vec{\mathbf{F}} = \boxed{\frac{Q_0^2 d}{2\in_0 \ell^3}\ \text{to the right}}\ \text{(into the capacitor)}$$

(c) Stress $= \frac{F}{\ell d} = \boxed{\frac{Q_0^2}{2\in_0 \ell^4}}$

(d) $u = \frac{1}{2}\in_0 E^2 = \frac{1}{2}\in_0\left(\frac{\sigma}{\in_0}\right)^2 = \frac{1}{2}\in_0\left(\frac{Q_0}{\in_0 \ell^2}\right)^2 = \boxed{\frac{Q_0^2}{2\in_0 \ell^4}}$. The answers to parts (c) and (d) are precisely the same.

***P26.58** One capacitor cannot be used by itself—it would burn out. She can use two capacitors in parallel, connected in series to another two capacitors in parallel. One capacitor will be left over. The equivalent capacitance is $\frac{1}{(200\ \mu\text{F})^{-1} + (200\ \mu\text{F})^{-1}} = 100\ \mu\text{F}$. When 90 V is connected across the combination, only $\boxed{45\ \text{V}}$ appears across each individual capacitor.

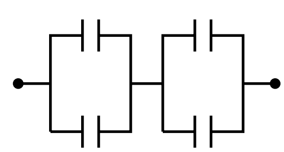


FIG. P26.58

P26.59 Call the unknown capacitance C_u

$$Q = C_u\left(\Delta V_i\right) = \left(C_u + C\right)\left(\Delta V_f\right)$$

$$\boxed{C_u = \frac{C\left(\Delta V_f\right)}{\left(\Delta V_i\right) - \left(\Delta V_f\right)}} = \frac{(10.0\ \mu\text{F})(30.0\ \text{V})}{(100\ \text{V} - 30.0\ \text{V})} = \boxed{4.29\ \mu\text{F}}$$

***P26.60** Consider a strip of width dx and length W at position x from the front left corner. The capacitance of the lower portion of this strip is $\frac{\kappa_1 \in_0 W\ dx}{t x/L}$. The capacitance of the upper portion is $\frac{\kappa_2 \in_0 W\ dx}{t(1 - x/L)}$. The series combination of the two elements has capacitance

$$\frac{1}{\dfrac{tx}{\kappa_1 \in_0 WL\ dx} + \dfrac{t(L-x)}{\kappa_2 \in_0 WL\ dx}} = \frac{\kappa_1\kappa_2 \in_0 WL\ dx}{\kappa_2 tx + \kappa_1 tL - \kappa_1 tx}$$

The whole capacitance is a combination of elements in parallel:

$$C = \int_0^L \frac{\kappa_1\kappa_2 \in_0 WL\ dx}{\left(\kappa_2 - \kappa_1\right)tx + \kappa_1 tL} = \frac{1}{\left(\kappa_2 - \kappa_1\right)t}\int_0^L \frac{\kappa_1\kappa_2 \in_0 WL\left(\kappa_2 - \kappa_1\right)tdx}{\left(\kappa_2 - \kappa_1\right)tx + \kappa_1 tL}$$

$$= \frac{\kappa_1\kappa_2 \in_0 WL}{\left(\kappa_2 - \kappa_1\right)t}\ln\left[\left(\kappa_2 - \kappa_1\right)tx + \kappa_1 tL\right]_0^L = \frac{\kappa_1\kappa_2 \in_0 WL}{\left(\kappa_2 - \kappa_1\right)t}\ln\left[\frac{\left(\kappa_2 - \kappa_1\right)tL + \kappa_1 tL}{0 + \kappa_1 tL}\right]$$

$$= \frac{\kappa_1\kappa_2 \in_0 WL}{\left(\kappa_2 - \kappa_1\right)t}\ln\left[\frac{\kappa_2}{\kappa_1}\right] = \frac{\kappa_1\kappa_2 \in_0 WL}{(-1)\left(\kappa_2 - \kappa_1\right)t}\ln\left[\left(\frac{\kappa_2}{\kappa_1}\right)^{-1}\right] = \boxed{\frac{\kappa_1\kappa_2 \in_0 WL}{\left(\kappa_1 - \kappa_2\right)t}\ln\left[\frac{\kappa_1}{\kappa_2}\right]}$$

(b) The capacitor physically has the same capacitance if it is turned upside down, so the answer should be the same with κ_1 and κ_2 interchanged. We have proved that it has this property in the solution to part (a).

(c) Let $\kappa_1 = \kappa_2\,(1 + x)$. Then $C = \frac{\kappa_2(1+x)\kappa_2 \in_0 WL}{\kappa_2 xt}\ln[1+x]$.

As x approaches zero we have $C = \frac{\kappa(1+0)\in_0 WL}{xt}x = \frac{\kappa \in_0 WL}{t}$ as was to be shown.

P26.61 (a) $C_0 = \frac{\in_0 A}{d} = \frac{Q_0}{\Delta V_0}$

When the dielectric is inserted at constant voltage,

$$C = \kappa C_0 = \frac{Q}{\Delta V_0}$$

$$U_0 = \frac{C_0\left(\Delta V_0\right)^2}{2}$$

$$U = \frac{C\left(\Delta V_0\right)^2}{2} = \frac{\kappa C_0\left(\Delta V_0^2\right)}{2}$$

and $\frac{U}{U_0} = \kappa$

The extra energy comes from (part of the) electrical work done by the battery in separating the extra charge.

(b) $Q_0 = C_0\Delta V_0$

and $Q = C\Delta V_0 = \kappa C_0\Delta V_0$

so the charge increases according to $\boxed{\frac{Q}{Q_0} = \kappa}$.

P26.62 Assume a potential difference across a and b, and notice that the potential difference across the $8.00\ \mu\text{F}$ capacitor must be zero by symmetry. Then the equivalent capacitance can be determined from the following circuit:

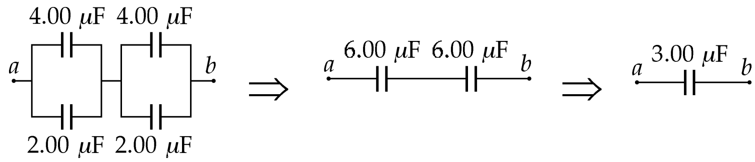


FIG. P26.62

$$C_{ab} = \boxed{3.00\ \mu\text{F}}$$

P26.63 Initially (capacitors charged in parallel),

$$q_1 = C_1(\Delta V) = (6.00\ \mu\text{F})(250\ \text{V}) = 1\,500\ \mu\text{C}$$

$$q_2 = C_2(\Delta V) = (2.00\ \mu\text{F})(250\ \text{V}) = 500\ \mu\text{C}$$

After reconnection (positive plate to negative plate),

$$q'_{\text{total}} = q_1 - q_2 = 1\,000\ \mu\text{C} \quad \text{and} \quad \Delta V' = \frac{q'_{\text{total}}}{C_{\text{total}}} = \frac{1\,000\ \mu\text{C}}{8.00\ \mu\text{F}} = 125\ \text{V}$$

Therefore,

$$q'_1 = C_1(\Delta V') = (6.00\ \mu\text{F})(125\ \text{V}) = \boxed{750\ \mu\text{C}}$$

$$q'_2 = C_2(\Delta V') = (2.00\ \mu\text{F})(125\ \text{V}) = \boxed{250\ \mu\text{C}}$$

***P26.64** Let charge λ per length be on one wire and $-\lambda$ be on the other. The electric field due to the charge on the positive wire is perpendicular to the wire, radial, and of magnitude

$$E_+ = \frac{\lambda}{2\pi \in_0 r}$$

The potential difference between the surfaces of the wires due to the presence of this charge is

$$\Delta V_1 = -\int_{-\text{wire}}^{+\text{wire}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = -\frac{\lambda}{2\pi \in_0} \int_{D-r}^{r} \frac{dr}{r} = \frac{\lambda}{2\pi \in_0} \ln\left(\frac{D-r}{r}\right)$$

The presence of the linear charge density $-\lambda$ on the negative wire makes an identical contribution to the potential difference between the wires. Therefore, the total potential difference is

$$\Delta V = 2(\Delta V_1) = \frac{\lambda}{\pi \in_0} \ln\left(\frac{D-r}{r}\right)$$

With D much larger than r we have nearly $\Delta V = \frac{\lambda}{\pi \in_0} \ln\left(\frac{D}{r}\right)$

and the capacitance of this system of two wires, each of length ℓ, is

$$C = \frac{Q}{\Delta V} = \frac{\lambda \ell}{\Delta V} = \frac{\lambda \ell}{(\lambda/\pi \in_0)\ln[D/r]} = \frac{\pi \in_0 \ell}{\ln[D/r]}$$

The capacitance per unit length is $\boxed{\dfrac{C}{\ell} = \dfrac{\pi \in_0}{\ln[D/r]}}$

P26.65 By symmetry, the potential difference across $3C$ is zero, so the circuit reduces to

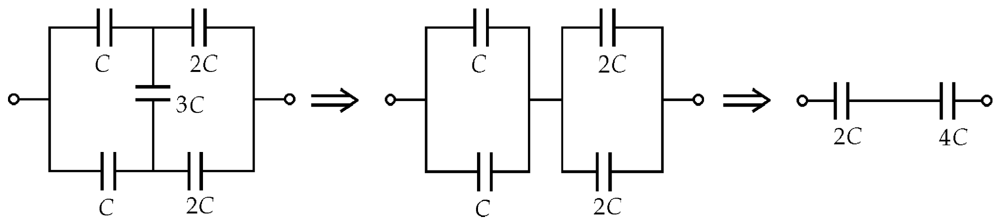


FIG. P26.65

$$C_{eq} = \left(\frac{1}{2C} + \frac{1}{4C}\right)^{-1} = \frac{8}{6}C = \boxed{\frac{4}{3}C}$$

P26.66 The condition that we are testing is that the capacitance increases by less than 10%, or,

$$\frac{C'}{C} < 1.10$$

Substituting the expressions for C and C' from Example 26.1, we have

$$\frac{C'}{C} = \frac{\dfrac{\ell}{2k_e \ln(b/1.10a)}}{\dfrac{\ell}{2k_e \ln(b/a)}} = \frac{\ln(b/a)}{\ln(b/1.10a)} < 1.10$$

This becomes

$$\ln\left(\frac{b}{a}\right) < 1.10\ln\left(\frac{b}{1.10a}\right) = 1.10\ln\left(\frac{b}{a}\right) + 1.10\ln\left(\frac{1}{1.10}\right) = 1.10\ln\left(\frac{b}{a}\right) - 1.10\ln(1.10)$$

We can rewrite this as,

$$-0.10\ln\left(\frac{b}{a}\right) < -1.10\ln(1.10)$$

$$\ln\left(\frac{b}{a}\right) > 11.0\ln(1.10) = \ln(1.10)^{11.0}$$

where we have reversed the direction of the inequality because we multiplied the whole expression by –1 to remove the negative signs. Comparing the arguments of the logarithms on both sides of the inequality, we see that

$$\frac{b}{a} > (1.10)^{11.0} = 2.85$$

Thus, if $b > 2.85a$, the increase in capacitance is less than 10% and it is more effective to increase ℓ.

ANSWERS TO EVEN PROBLEMS

P26.2 (a) $1.00\ \mu\text{F}$ (b) 100 V

P26.4 11.1 nF; 26.6 C

P26.6 $\dfrac{(2N-1)\in_0(\pi-\theta)R^2}{d}$

P26.8 $\dfrac{mgd\tan\theta}{q}$

P26.10 Yes; its total energy is sufficient to make the trip; 1.00×10^6 m/s.

P26.12 (a) $17.0\ \mu\text{F}$ (b) 9.00 V (c) $45.0\ \mu\text{C}$ and $108\ \mu\text{C}$

P26.14 (a) $2C$ (b) $Q_1 > Q_3 > Q_2$ (c) $\Delta V_1 > \Delta V_2 = \Delta V_3$ (d) Q_1 and Q_3 increase, Q_2 decreases

P26.16 $\dfrac{C_p}{2}+\sqrt{\dfrac{C_p^2}{4}-C_pC_s}$ and $\dfrac{C_p}{2}-\sqrt{\dfrac{C_p^2}{4}-C_pC_s}$

P26.18 (a) $398\ \mu\text{F}$ in series (b) $2.20\ \mu\text{F}$ in parallel

P26.20 $19.8\ \mu\text{C}$

P26.22 $\left(\sqrt{3}-1\right)\dfrac{C_0}{2}$

P26.24 $83.6\ \mu\text{C}$

P26.26 4.47 kV

P26.28 (a) See the solution. Stored energy = 0.150 J (b) See the solution. Potential difference = 268 V

P26.30 (a) $400\ \mu\text{C}$ (b) 2.50 kN/m

P26.32 (a) $C(\Delta V)^2$ (b) $4\Delta V/3$ (c) $4C(\Delta V)^2/3$ (d) Positive work is done by the agent pulling the plates apart.

P26.34 (a) $q_1=\dfrac{R_1Q}{R_1+R_2}$ and $q_2=\dfrac{R_2Q}{R_1+R_2}$ (b) See the solution.

P26.36 (a) 13.3 nC (b) 272 nC

P26.38 $\sim10^{-6}$ F and $\sim10^{2}$ V for two 40 cm by 100 cm sheets of aluminum foil sandwiching a thin sheet of plastic.

P26.40 (a) 369 pC (b) 118 pF, 3.12 V (c) –45.5 nJ

P26.42 (a) $\left(-9.10\hat{\mathbf{i}}+8.40\hat{\mathbf{j}}\right)$ pC·m (b) -20.9 nN·m$\hat{\mathbf{k}}$ (c) 112 nJ (d) 228 nJ

P26.44 See the solution.

P26.46 (a) $-2Q/3$ on upper plate, $-Q/3$ on lower plate (b) $2Qd/3\in_0A$

P26.48 189 kV

P26.50 (a) See the solution. (b) 40.0 μF (c) 6.00 V across 50 μF with charge 300 μF; 4.00 V across 30 μF with charge 120 μF; 2.00 V across 20 μF with charge 40 μF; 2.00 V across 40 μF with charge 80 μF

P26.52 (a) 25.0 μF $(1 - 0.846 f)^{-1}$ (b) 25.0 μF; the general expression agrees. (c) 162 μF; the general expression agrees. (d) It has the same sign as the lower capacitor plate and its magnitude is 254 μC, independent of f.

P26.54 23.3 V; 26.7 V

P26.56 (a) $\dfrac{\epsilon_0\left[\ell^2 + \ell x(\kappa - 1)\right]}{d}$ (b) $\dfrac{Q^2 d}{2\epsilon_0\left[\ell^2 + \ell x(\kappa - 1)\right]}$ (c) $\dfrac{Q^2 d\ell(\kappa - 1)}{2\epsilon_0\left[\ell^2 + \ell x(\kappa - 1)\right]^2}$ to the right
(d) 205 μN right

P26.58 One capacitor cannot be used by itself—it would burn out. She can use two capacitors in parallel, connected in series to another two capacitors in parallel. One capacitor will be left over. Each of the four capacitors will be exposed to a maximum voltage of 45 V.

P26.60 (a) $\dfrac{\kappa_1\kappa_2 \epsilon_0 WL}{(\kappa_1 - \kappa_2)t}\ln\dfrac{\kappa_1}{\kappa_2}$ (b) The capacitance is the same if κ_1 and κ_2 are interchanged, as it should be.

P26.62 3.00 μF

P26.64 See the solution.

P26.66 See the solution.

27

Current and Resistance

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q27.1 Voltage is a measure of potential difference, not of current. "Surge" implies a flow—and only charge, in coulombs, can flow through a system. It would also be correct to say that the victim carried a certain current, in amperes.

Q27.2 Geometry and resistivity. In turn, the resistivity of the material depends on the temperature.

***Q27.3** (i) We require $\rho L/A_A = 3\rho L/A_B$. Then $A_A/A_B = 1/3$, answer (f).
(ii) $\pi r_A^2/\pi r_B^2 = 1/3$ gives $r_A/r_B = 1/\sqrt{3}$, answer (e).

***Q27.4** Originally, $R = \dfrac{\rho \ell}{A}$. Finally, $R_f = \dfrac{\rho(\ell/3)}{3A} = \dfrac{\rho \ell}{9A} = \dfrac{R}{9}$. Answer (b).

Q27.5 The conductor does not follow Ohm's law, and must have a resistivity that is current-dependent, or more likely temperature-dependent.

Q27.6 The amplitude of atomic vibrations increases with temperature. Atoms can then scatter electrons more efficiently.

Q27.7 (i) The current density increases, so the drift speed must increase. Answer (a).
(ii) Answer (a).

Q27.8 The resistance of copper *increases* with temperature, while the resistance of silicon *decreases* with increasing temperature. The conduction electrons are scattered more by vibrating atoms when copper heats up. Silicon's charge carrier density increases as temperature increases and more atomic electrons are promoted to become conduction electrons.

***Q27.9** In a normal metal, suppose that we could proceed to a limit of zero resistance by lengthening the average time between collisions. The classical model of conduction then suggests that a constant applied voltage would cause constant acceleration of the free electrons. The drift speed and the current would increase steadily in time.

It is not the situation envisioned in the question, but we can actually switch to zero resistance by substituting a superconducting wire for the normal metal. In this case, the drift velocity of electrons is established by vibrations of atoms in the crystal lattice; the maximum current is limited; and it becomes impossible to establish a potential difference across the superconductor.

Q27.10 Because there are so many electrons in a conductor (approximately 10^{28} electrons/m^3) the average velocity of charges is very slow. When you connect a wire to a potential difference, you establish an electric field everywhere in the wire nearly instantaneously, to make electrons start drifting everywhere all at once.

***Q27.11** Action (a) makes the current three times larger.
(b) causes no change in current.
(c) corresponds to a current $\sqrt{3}$ times larger.
(d) R is 1/4 as large, so current is 4 times larger.
(e) R is 2 times larger, so current is half as large.
(f) R increases by a small percentage as current has a small decrease.
(g) Current decreases by a large factor.
The ranking is then d > a > c > b > f > e > g.

***Q27.12** $R_A = \dfrac{\rho L_A}{\pi(d_A/2)^2} = \dfrac{\rho 2L_B}{\pi(2d_B/2)^2} = \dfrac{1}{2}\dfrac{\rho L_B}{\pi(d_B/2)^2} = \dfrac{R_B}{2}$

$\mathcal{P}_A = I_A\Delta V = (\Delta V)^2/R_A = 2(\Delta V)^2/R_B = 2\mathcal{P}_B$ Answer (e).

***Q27.13** $R_A = \dfrac{\rho_A L}{A} = \dfrac{2\rho_B L}{A} = 2R_B$

$\mathcal{P}_A = I_A\Delta V = (\Delta V)^2/R_A = (\Delta V)^2/2R_B = \mathcal{P}_B/2$ Answer (f).

***Q27.14** (i) Bulb (a) must have higher resistance so that it will carry less current and have lower power.
(ii) Bulb (b) carries more current.

***Q27.15** One ampere–hour is (1 C/s)(3 600 s) = 3 600 coulombs. The ampere–hour rating is the quantity of charge that the battery can lift though its nominal potential difference. Answer (d).

Q27.16 Choose the voltage of the power supply you will use to drive the heater. Next calculate the required resistance R as $\dfrac{\Delta V^2}{\mathcal{P}}$. Knowing the resistivity ρ of the material, choose a combination of wire length and cross-sectional area to make $\left(\dfrac{\ell}{A}\right) = \left(\dfrac{R}{\rho}\right)$. You will have to pay for less material if you make both ℓ and A smaller, but if you go too far the wire will have too little surface area to radiate away the energy; then the resistor will melt.

SOLUTIONS TO PROBLEMS

Section 27.1 **Electric Current**

P27.1 $I = \dfrac{\Delta Q}{\Delta t}$ $\quad \Delta Q = I\Delta t = \left(30.0\times10^{-6}\text{ A}\right)\left(40.0\text{ s}\right) = 1.20\times10^{-3}\text{ C}$

$$N = \frac{Q}{e} = \frac{1.20\times10^{-3}\text{ C}}{1.60\times10^{-19}\text{ C/electron}} = \boxed{7.50\times10^{15}\text{ electrons}}$$

P27.2 The molar mass of silver $= 107.9 \text{ g/mole}$ and the volume V is

$$V = (\text{area})(\text{thickness}) = \left(700\times10^{-4}\ \text{m}^2\right)\left(0.133\times10^{-3}\ \text{m}\right) = 9.31\times10^{-6}\ \text{m}^3$$

The mass of silver deposited is $m_{\text{Ag}} = \rho V = \left(10.5\times10^{3}\ \text{kg/m}^3\right)\left(9.31\times10^{-6}\,\text{m}^3\right) = 9.78\times10^{-2}\ \text{kg}$.

And the number of silver atoms deposited is

$$N = \left(9.78\times10^{-2}\ \text{kg}\right)\left(\frac{6.02\times10^{23}\ \text{atoms}}{107.9\ \text{g}}\right)\left(\frac{1\,000\ \text{g}}{1\ \text{kg}}\right) = 5.45\times10^{23}\ \text{atoms}$$

$$I = \frac{\Delta V}{R} = \frac{12.0\ \text{V}}{1.80\ \Omega} = 6.67\ \text{A} = 6.67\ \text{C/s}$$

$$\Delta t = \frac{\Delta Q}{I} = \frac{Ne}{I} = \frac{\left(5.45\times10^{23}\right)\left(1.60\times10^{-19}\ \text{C}\right)}{6.67\ \text{C/s}} = 1.31\times10^{4}\ \text{s} = \boxed{3.64\ \text{h}}$$

P27.3 $Q(t) = \int_0^t I\,dt = I_0\tau\left(1 - e^{-t/\tau}\right)$

(a) $Q(\tau) = I_0\tau\left(1 - e^{-1}\right) = \boxed{(0.632)I_0\tau}$

(b) $Q(10\tau) = I_0\tau\left(1 - e^{-10}\right) = \boxed{(0.999\,95)I_0\tau}$

(c) $Q(\infty) = I_0\tau\left(1 - e^{-\infty}\right) = \boxed{I_0\tau}$

P27.4 The period of revolution for the sphere is $T = \dfrac{2\pi}{\omega}$, and the average current represented by this revolving charge is $I = \dfrac{q}{T} = \boxed{\dfrac{q\omega}{2\pi}}$.

P27.5 $q = 4t^3 + 5t + 6$

$A = \left(2.00\ \text{cm}^2\right)\left(\dfrac{1.00\ \text{m}}{100\ \text{cm}}\right)^2 = 2.00\times10^{-4}\ \text{m}^2$

(a) $I(1.00\ \text{s}) = \left.\dfrac{dq}{dt}\right|_{t=1.00\ \text{s}} = \left.\left(12t^2 + 5\right)\right|_{t=1.00\ \text{s}} = \boxed{17.0\ \text{A}}$

(b) $J = \dfrac{I}{A} = \dfrac{17.0\ \text{A}}{2.00\times10^{-4}\ \text{m}^2} = \boxed{85.0\ \text{kA/m}^2}$

P27.6 $I = \dfrac{dq}{dt}$ $\quad q = \int dq = \int I\,dt = \int_0^{1/240\ \text{s}} (100\ \text{A})\sin\left(\dfrac{120\pi t}{\text{s}}\right)dt$

$$q = \frac{-100\ \text{C}}{120\pi}\left[\cos\left(\frac{\pi}{2}\right) - \cos 0\right] = \frac{+100\ \text{C}}{120\pi} = \boxed{0.265\ \text{C}}$$

P27.7 (a) $J = \dfrac{I}{A} = \dfrac{8.00\times10^{-6}\ \text{A}}{\pi\left(1.00\times10^{-3}\ \text{m}\right)^2} = \boxed{2.55\ \text{A/m}^2}$

(b) From $J = nev_d$, we have $n = \dfrac{J}{ev_d} = \dfrac{2.55\ \text{A/m}^2}{\left(1.60\times10^{-19}\ \text{C}\right)\left(3.00\times10^{8}\ \text{m/s}\right)} = \boxed{5.31\times10^{10}\ \text{m}^{-3}}$.

(c) From $I = \dfrac{\Delta Q}{\Delta t}$, we have $\Delta t = \dfrac{\Delta Q}{I} = \dfrac{N_{\text{A}}e}{I} = \dfrac{\left(6.02\times10^{23}\right)\left(1.60\times10^{-19}\ \text{C}\right)}{8.00\times10^{-6}\ \text{A}} = \boxed{1.20\times10^{10}\ \text{s}}$.

(This is about 382 years!)

***P27.8** (a) $J = \frac{I}{A} = \frac{5.00\text{ A}}{\pi\left(4.00\times10^{-3}\text{ m}\right)^2} = \boxed{99.5\text{ kA/m}^2}$

(b) Current is the same and current density is smaller. Then $\boxed{I = 5.00\text{ A}}$,

$$J_2 = \frac{1}{4}J_1 = \frac{1}{4}9.95\times10^4\text{ A/m}^2 = \boxed{2.49\times10^4\text{ A/m}^2}$$

$A_2 = 4A_1$ or $\pi r_2^2 = 4\pi r_1^2$ so $r_2 = 2r_1 = \boxed{0.800\text{ cm}}$

P27.9 (a) The speed of each deuteron is given by $K = \frac{1}{2}mv^2$

$\left(2.00\times10^6\right)\left(1.60\times10^{-19}\text{ J}\right) = \frac{1}{2}\left(2\times1.67\times10^{-27}\text{ kg}\right)v^2$ and $v = 1.38\times10^7\text{ m/s}$

The time between deuterons passing a stationary point is t in $I = \frac{q}{t}$

$10.0\times10^{-6}\text{ C/s} = 1.60\times10^{-19}\text{ C}/t$ or $t = 1.60\times10^{-14}\text{ s}$

So the distance between them is $vt = \left(1.38\times10^7\text{ m/s}\right)\left(1.60\times10^{-14}\text{ s}\right) = \boxed{2.21\times10^{-7}\text{ m}}$.

(b) One nucleus will put its nearest neighbor at potential

$$V = \frac{k_e q}{r} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)}{2.21\times10^{-7}\text{ m}} = 6.49\times10^{-3}\text{ V}$$

This is very small compared to the 2 MV accelerating potential, so repulsion within the beam is a small effect.

P27.10 We use $I = nqAv_d$ n is the number of charge carriers per unit volume, and is identical to the number of atoms per unit volume. We assume a contribution of 1 free electron per atom in the relationship above. For aluminum, which has a molar mass of 27, we know that Avogadro's number of atoms, N_A, has a mass of 27.0 g. Thus, the mass per atom is

$$\frac{27.0\text{ g}}{N_A} = \frac{27.0\text{ g}}{6.02\times10^{23}} = 4.49\times10^{-23}\text{ g/atom}$$

Thus, $n = \frac{\text{density of aluminum}}{\text{mass per atom}} = \frac{2.70\text{ g/cm}^3}{4.49\times10^{-23}\text{ g/atom}}$

$$n = 6.02\times10^{22}\text{ atoms/cm}^3 = 6.02\times10^{28}\text{ atoms/m}^3$$

Therefore, $v_d = \frac{I}{nqA} = \frac{5.00\text{ A}}{\left(6.02\times10^{28}\text{ m}^{-3}\right)\left(1.60\times10^{-19}\text{ C}\right)\left(4.00\times10^{-6}\text{ m}^2\right)} = 1.30\times10^{-4}\text{ m/s}$

or, $v_d = \boxed{0.130\text{ mm/s}}$

Section 27.2 **Resistance**

P27.11 $\Delta V = IR$

and $R = \frac{\rho\ell}{A}$: $A = \left(0.600\text{ mm}\right)^2\left(\frac{1.00\text{ m}}{1\,000\text{ mm}}\right)^2 = 6.00\times10^{-7}\text{ m}^2$

$\Delta V = \frac{I\rho\ell}{A}$: $I = \frac{\Delta VA}{\rho\ell} = \frac{\left(0.900\text{ V}\right)\left(6.00\times10^{-7}\text{ m}^2\right)}{\left(5.60\times10^{-8}\ \Omega\cdot\text{m}\right)\left(1.50\text{ m}\right)}$

$I = \boxed{6.43\text{ A}}$

P27.12 $I = \frac{\Delta V}{R} = \frac{120\text{ V}}{240\ \Omega} = 0.500\text{ A} = \boxed{500\text{ mA}}$

P27.13 (a) Given $M = \rho_d V = \rho_d A\ell$ where $\rho_d \equiv$ mass density,

we obtain: $A = \frac{M}{\rho_d \ell}$ Taking $\rho_r \equiv$ resistivity, $R = \frac{\rho_r \ell}{A} = \frac{\rho_r \ell}{M/\rho_d \ell} = \frac{\rho_r \rho_d \ell^2}{M}$

Thus, $\ell = \sqrt{\frac{MR}{\rho_r \rho_d}} = \sqrt{\frac{(1.00\times10^{-3})(0.500)}{(1.70\times10^{-8})(8.92\times10^{3})}}$ $\ell = \boxed{1.82\text{ m}}$

(b) $V = \frac{M}{\rho_d}$, or $\pi r^2 \ell = \frac{M}{\rho_d}$

Thus, $r = \sqrt{\frac{M}{\pi \rho_d \ell}} = \sqrt{\frac{1.00\times10^{-3}}{\pi(8.92\times10^{3})(1.82)}}$ $r = 1.40\times10^{-4}\text{ m}$

The diameter is twice this distance: diameter $= \boxed{280\ \mu\text{m}}$

P27.14 (a) Suppose the rubber is 10 cm long and 1 mm in diameter.

$$R = \frac{\rho\ell}{A} = \frac{4\rho\ell}{\pi d^2} \sim \frac{4(10^{13}\ \Omega\cdot\text{m})(10^{-1}\text{ m})}{\pi(10^{-3}\text{ m})^2} = \boxed{\sim 10^{18}\ \Omega}$$

(b) $R = \frac{4\rho\ell}{\pi d^2} \sim \frac{4(1.7\times10^{-8}\ \Omega\cdot\text{m})(10^{-3}\text{ m})}{\pi(2\times10^{-2}\text{ m})^2}\boxed{\sim 10^{-7}\ \Omega}$

(c) $I = \frac{\Delta V}{R} \sim \frac{10^2\text{ V}}{10^{18}\ \Omega}\boxed{\sim 10^{-16}\text{ A}}$

$I \sim \frac{10^2\text{ V}}{10^{-7}\ \Omega}\boxed{\sim 10^{9}\text{ A}}$

P27.15 $J = \sigma E$ so $\sigma = \frac{J}{E} = \frac{6.00\times10^{-13}\text{ A/m}^2}{100\text{ V/m}} = \boxed{6.00\times10^{-15}\ (\Omega\cdot\text{m})^{-1}}$

Section 27.3 **A Model for Electrical Conduction**

***P27.16** (a) The density of charge carriers n is set by the material and is $\boxed{\text{unaffected}}$.

(b) The current density is proportional to current according to $|J| = \frac{I}{A}$ so it $\boxed{\text{doubles}}$.

(c) For larger current density in $J = nev_d$ the drift speed v_d $\boxed{\text{doubles}}$.

(d) The time between collisions $\tau = \frac{m\sigma}{nq^2}$ is $\boxed{\text{unchanged}}$ as long as σ does not change due to a temperature change in the conductor.

P27.17 $\rho = \dfrac{m}{nq^2\tau}$ We take the density of conduction electrons from an Example in the chapter text.

so $$\tau = \frac{m}{\rho n q^2} = \frac{(9.11\times10^{-31})}{(1.70\times10^{-8})(8.46\times10^{28})(1.60\times10^{-19})^2} = 2.47\times10^{-14}\text{ s}$$

$$v_d = \frac{qE}{m}\tau$$

gives $$7.84\times10^{-4} = \frac{(1.60\times10^{-19})E(2.47\times10^{-14})}{9.11\times10^{-31}}$$

Therefore, $E = \boxed{0.180\text{ V/m}}$

Section 27.4 **Resistance and Temperature**

P27.18 $R = R_0[1+\alpha(\Delta T)]$ gives $140\ \Omega = (19.0\ \Omega)\left[1+(4.50\times10^{-3}/°\text{C})\Delta T\right]$

Solving, $\Delta T = 1.42\times10^{3}°\text{C} = T - 20.0°\text{C}$

And the final temperature is $\boxed{T = 1.44\times10^{3}°\text{C}}$

P27.19 (a) $\rho = \rho_0[1+\alpha(T-T_0)] = (2.82\times10^{-8}\ \Omega\cdot\text{m})[1+3.90\times10^{-3}(30.0°)] = \boxed{3.15\times10^{-8}\ \Omega\cdot\text{m}}$

(b) $J = \dfrac{E}{\rho} = \dfrac{0.200\text{ V/m}}{3.15\times10^{-8}\ \Omega\cdot\text{m}} = \boxed{6.35\times10^{6}\text{ A/m}^2}$

(c) $I = JA = J\left(\dfrac{\pi d^2}{4}\right) = (6.35\times10^{6}\text{ A/m}^2)\left[\dfrac{\pi(1.00\times10^{-4}\text{ m})^2}{4}\right] = \boxed{49.9\text{ mA}}$

(d) $n = \dfrac{6.02\times10^{23}\text{ electrons}}{[26.98\text{ g}/(2.70\times10^{6}\text{ g/m}^3)]} = 6.02\times10^{28}\text{ electrons/m}^3$

$$v_d = \frac{J}{ne} = \frac{(6.35\times10^{6}\text{ A/m}^2)}{(6.02\times10^{28}\text{ electrons/m}^3)(1.60\times10^{-19}\text{ C})} = \boxed{659\ \mu\text{m/s}}$$

(e) $\Delta V = E\ell = (0.200\text{ V/m})(2.00\text{ m}) = \boxed{0.400\text{ V}}$

***P27.20** We require $10\ \Omega = \dfrac{3.5\times10^{-5}\,\Omega\cdot\text{m}\ \ell_1}{\pi(1.5\times10^{-3}\text{m})^2} + \dfrac{1.5\times10^{-6}\,\Omega\cdot\text{m}\ \ell_2}{\pi(1.5\times10^{-3}\text{m})^2}$ and for

any ΔT $\quad 10\ \Omega = \dfrac{3.5\times10^{-5}\,\Omega\cdot\text{m}\ \ell_1}{\pi(1.5\times10^{-3}\text{m})^2}\left(1-0.5\times10^{-3}\dfrac{\Delta T}{°\text{C}}\right) + \dfrac{1.5\times10^{-6}\,\Omega\cdot\text{m}\ \ell_2}{\pi(1.5\times10^{-3}\text{m})^2}\left(1+0.4\times10^{-3}\dfrac{\Delta T}{°\text{C}}\right)$

simplifying gives $10 = 4.951\,5\ \ell_1 + 0.212\,21\ \ell_2$

and $0 = -2.475\,7\times10^{-3}\ \ell_1 + 8.488\,3\times10^{-5}\ \ell_2$

These conditions are just sufficient to determine ℓ_1 and ℓ_2. $\boxed{\text{The design goal can be met.}}$

We have $\ell_2 = 29.167\ \ell_1$ so $10 = 4.951\,5\ \ell_1 + 0.212\,21\ (29.167\ \ell_1)$

and $\ell_1 = 10/11.141 = \boxed{0.898\text{ m} = \ell_1 \quad \ell_2 = 26.2\text{ m}}$

P27.21 $R = R_0\left[1+\alpha T\right]$

$R - R_0 = R_0\alpha\Delta T$

$\dfrac{R-R_0}{R_0} = \alpha\Delta T = \left(5.00\times10^{-3}\right)25.0 = \boxed{0.125}$

P27.22 For aluminum, $\alpha_E = 3.90\times10^{-3}\,°\text{C}^{-1}$ (Table 27.2)

$\alpha = 24.0\times10^{-6}\,°\text{C}^{-1}$ (Table 19.1)

$$R = \frac{\rho\ell}{A} = \frac{\rho_0\left(1+\alpha_E\Delta T\right)\ell\left(1+\alpha\Delta T\right)}{A\left(1+\alpha\Delta T\right)^2} = R_0\frac{\left(1+\alpha_E\Delta T\right)}{\left(1+\alpha\Delta T\right)} = (1.234\ \Omega)\left(\frac{1.39}{1.002\ 4}\right) = \boxed{1.71\ \Omega}$$

Section 27.5 **Superconductors**

Problem 50 in Chapter 43 can be assigned with this section.

Section 27.6 **Electrical Power**

P27.23 $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{600\ \text{W}}{120\ \text{V}} = \boxed{5.00\ \text{A}}$

and $R = \dfrac{\Delta V}{I} = \dfrac{120\ \text{V}}{5.00\ \text{A}} = \boxed{24.0\ \Omega}$

P27.24 $\mathcal{P} = I\Delta V = 500\times10^{-6}\ \text{A}\left(15\times10^{3}\ \text{V}\right) = \boxed{7.50\ \text{W}}$

***P27.25** The energy that must be added to the water is

$Q = mc\Delta T = (109\ \text{kg})(4\ 186\ \text{J/kg°C})(29.0°\text{C}) = 1.32\times10^{7}\ \text{J}$

Thus, the power supplied by the heater is

$\mathcal{P} = \dfrac{W}{\Delta t} = \dfrac{Q}{\Delta t} = \dfrac{1.32\times10^{7}\ \text{J}}{25\times60\ \text{s}} = 8\ 820\ \text{W}$

and the resistance is $R = \dfrac{(\Delta V)^2}{\mathcal{P}} = \dfrac{(220\ \text{V})^2}{8\ 820\ \text{W}} = \boxed{5.49\ \Omega}$.

***P27.26** (a) $\textit{efficiency} = \dfrac{\textit{mechanical power output}}{\textit{total power input}} = 0.900 = \dfrac{2.50\ \text{hp}(746\ \text{W/1 hp})}{(120\ \text{V})\,I}$

$I = \dfrac{1\ 860\ \text{J/s}}{0.9(120\ \text{V})} = \dfrac{2\ 070\ \text{J/s}}{120\ \text{J/C}} = \boxed{17.3\ \text{A}}$

(b) $\textit{energy input} = \mathcal{P}_{\text{input}}\,\Delta t = (2\ 070\ \text{J/s})\ 3\ (3\ 600\ \text{s}) = \boxed{2.24\times10^{7}\ \text{J}}$

(c) $\textit{cost} = 2.24\times 107\ \text{J}\left(\dfrac{\$\ 0.16}{1\ \text{kWh}}\right)\left(\dfrac{\text{k}}{10^3}\dfrac{\text{J}}{\text{W s}}\dfrac{\text{h}}{3\ 600\ \text{s}}\right) = \boxed{\$\ 0.995}$

P27.27 $\frac{\mathcal{P}}{\mathcal{P}_0} = \frac{(\Delta V)^2/R}{(\Delta V_0)^2/R} = \left(\frac{\Delta V}{\Delta V_0}\right)^2 = \left(\frac{140}{120}\right)^2 = 1.361$

$$\Delta\% = \left(\frac{\mathcal{P}-\mathcal{P}_0}{\mathcal{P}_0}\right)(100\%) = \left(\frac{\mathcal{P}}{\mathcal{P}_0}-1\right)(100\%) = (1.361-1)100\% = \boxed{36.1\%}$$

P27.28 The battery takes in energy by electric transmission

$$\mathcal{P}\Delta t = (\Delta V)I(\Delta t) = 2.3\ \text{J/C}\left(13.5\times10^{-3}\ \text{C/s}\right)4.2\ \text{h}\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right) = 469\ \text{J}$$

It puts out energy by electric transmission

$$(\Delta V)I(\Delta t) = 1.6\ \text{J/C}\left(18\times10^{-3}\ \text{C/s}\right)2.4\ \text{h}\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right) = 249\ \text{J}$$

(a) efficiency $= \frac{\text{useful output}}{\text{total input}} = \frac{249\ \text{J}}{469\ \text{J}} = \boxed{0.530}$

(b) The only place for the missing energy to go is into internal energy:
$469\ \text{J} = 249\ \text{J} + \Delta E_{\text{int}}$

$\Delta E_{\text{int}} = \boxed{221\ \text{J}}$

(c) We imagine toasting the battery over a fire with 221 J of heat input:

$Q = mc\Delta T$

$\Delta T = \frac{Q}{mc} = \frac{221\ \text{J}}{0.015\ \text{kg}}\frac{\text{kg}^\circ\text{C}}{975\ \text{J}} = \boxed{15.1^\circ\text{C}}$

P27.29 $\mathcal{P} = I(\Delta V) = \frac{(\Delta V)^2}{R} = 500\ \text{W} \qquad R = \frac{(110\ \text{V})^2}{(500\ \text{W})} = 24.2\ \Omega$

(a) $R = \frac{\rho}{A}\ell \qquad \text{so} \qquad \ell = \frac{RA}{\rho} = \frac{(24.2\ \Omega)\pi\left(2.50\times10^{-4}\ \text{m}\right)^2}{1.50\times10^{-6}\ \Omega\cdot\text{m}} = \boxed{3.17\ \text{m}}$

(b) $R = R_0[1+\alpha\Delta T] = 24.2\ \Omega\left[1+\left(0.400\times10^{-3}\right)(1\,180)\right] = 35.6\ \Omega$

$\mathcal{P} = \frac{(\Delta V)^2}{R} = \frac{(110)^2}{35.6} = \boxed{340\ \text{W}}$

P27.30 $R = \frac{\rho\ell}{A} = \frac{\left(1.50\times10^{-6}\ \Omega\cdot\text{m}\right)25.0\ \text{m}}{\pi\left(0.200\times10^{-3}\ \text{m}\right)^2} = 298\ \Omega$

$\Delta V = IR = (0.500\ \text{A})(298\ \Omega) = 149\ \text{V}$

(a) $E = \frac{\Delta V}{\ell} = \frac{149\ \text{V}}{25.0\ \text{m}} = \boxed{5.97\ \text{V/m}}$

(b) $\mathcal{P} = (\Delta V)I = (149\ \text{V})(0.500\ \text{A}) = \boxed{74.6\ \text{W}}$

(c) $R = R_0\left[1+\alpha(T-T_0)\right] = 298\ \Omega\left[1+\left(0.400\times10^{-3}/^\circ\text{C}\right)320^\circ\text{C}\right] = 337\ \Omega$

$I = \frac{\Delta V}{R} = \frac{(149\ \text{V})}{(337\ \Omega)} = 0.443\ \text{A}$

$\mathcal{P} = (\Delta V)I = (149\ \text{V})(0.443\ \text{A}) = \boxed{66.1\ \text{W}}$

P27.31 (a) $\Delta U = q(\Delta V) = It(\Delta V) = (55.0\text{ A}\cdot\text{h})(12.0\text{ V})\left(\frac{1\text{ C}}{1\text{ A}\cdot\text{s}}\right)\left(\frac{1\text{ J}}{1\text{ V}\cdot\text{C}}\right)\left(\frac{1\text{ W}\cdot\text{s}}{1\text{ J}}\right)$

$$= 660\text{ W}\cdot\text{h} = \boxed{0.660\text{ kWh}}$$

(b) $\text{Cost } = 0.660\text{ kWh}\left(\frac{\$0.060\,0}{1\text{ kWh}}\right) = \boxed{3.96¢}$

***P27.32** (a) The resistance of 1 m of 12-gauge copper wire is

$$R = \frac{\rho\ell}{A} = \frac{\rho\ell}{\pi(d/2)^2} = \frac{4\rho\ell}{\pi d^2} = \frac{4(1.7\times10^{-8}\ \Omega\cdot\text{m})1\text{ m}}{\pi(0.205\,3\times10^{-2}\text{ m})^2} = 5.14\times10^{-3}\ \Omega$$

The rate of internal energy production is $\mathcal{P} = I\Delta V = I^2R = (20\text{ A})^2\,5.14\times10^{-3}\ \Omega = \boxed{2.05\text{ W}}$.

(b) $\mathcal{P}_{\text{Al}} = I^2R = \frac{I^2 4\rho_{\text{Al}}\ell}{\pi d^2}$

$$\frac{\mathcal{P}_{\text{Al}}}{\mathcal{P}_{\text{Cu}}} = \frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}} \qquad \mathcal{P}_{\text{Al}} = \frac{2.82\times10^{-8}\ \Omega\cdot\text{m}}{1.7\times10^{-8}\ \Omega\cdot\text{m}}2.05\text{ W} = \boxed{3.41\text{ W}}$$

Aluminum of the same diameter will get hotter than copper. It would not be as safe. If it is surrounded by thermal insulation, it could get much hotter than a copper wire.

P27.33 The energy taken in by electric transmission for the fluorescent lamp is

$$\mathcal{P}\Delta t = 11\text{ J/s}(100\text{ h})\left(\frac{3\,600\text{ s}}{1\text{ h}}\right) = 3.96\times10^6\text{ J}$$

$$\text{cost} = 3.96\times10^6\text{ J}\left(\frac{\$0.08}{\text{kWh}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3\,600\text{ s}}\right) = \$0.088$$

For the incandescent bulb,

$$\mathcal{P}\Delta t = 40\text{ W}(100\text{ h})\left(\frac{3\,600\text{ s}}{1\text{ h}}\right) = 1.44\times10^7\text{ J}$$

$$\text{cost} = 1.44\times10^7\text{ J}\left(\frac{\$0.08}{3.6\times10^6\text{ J}}\right) = \$0.32$$

$$\text{saving} = \$0.32 - \$0.088 = \boxed{\$0.232}$$

P27.34 The total clock power is

$$(270\times10^6\text{ clocks})\left(2.50\ \frac{\text{J/s}}{\text{clock}}\right)\left(\frac{3\,600\text{ s}}{1\text{ h}}\right) = 2.43\times10^{12}\text{ J/h}$$

From $e = \frac{W_{\text{out}}}{Q_{\text{in}}}$, the power input to the generating plants must be:

$$\frac{Q_{\text{in}}}{\Delta t} = \frac{W_{\text{out}}/\Delta t}{e} = \frac{2.43\times10^{12}\text{ J/h}}{0.250} = 9.72\times10^{12}\text{ J/h}$$

and the rate of coal consumption is

$$\text{Rate} = (9.72\times10^{12}\text{ J/h})\left(\frac{1.00\text{ kg coal}}{33.0\times10^6\text{ J}}\right) = 2.95\times10^5\text{ kg coal/h} = \boxed{295\text{ metric ton/h}}$$

P27.35 $\mathcal{P} = I(\Delta V) = (1.70\text{ A})(110\text{ V}) = 187\text{ W}$

Energy used in a 24-hour day $= (0.187\text{ kW})(24.0\text{ h}) = 4.49\text{ kWh}$.

Therefore daily $\text{cost} = 4.49\text{ kWh}\left(\dfrac{\$0.060\,0}{\text{kWh}}\right) = \$0.269 = \boxed{26.9¢}$.

P27.36 $\mathcal{P} = I\Delta V = (2.00\text{ A})(120\text{ V}) = 240\text{ W}$

$$\Delta E_{\text{int}} = (0.500\text{ kg})(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(77.0^\circ\text{C}) = 161\text{ kJ}$$

$$\Delta t = \frac{\Delta E_{\text{int}}}{\mathcal{P}} = \frac{1.61\times10^5\text{ J}}{240\text{ W}} = \boxed{672\text{ s}}$$

P27.37 At operating temperature,

(a) $\mathcal{P} = I\Delta V = (1.53\text{ A})(120\text{ V}) = \boxed{184\text{ W}}$

(b) Use the change in resistance to find the final operating temperature of the toaster.

$$R = R_0(1+\alpha\Delta T) \qquad \frac{120}{1.53} = \frac{120}{1.80}\left[1+(0.400\times10^{-3})\Delta T\right]$$

$$\Delta T = 441^\circ\text{C} \qquad T = 20.0^\circ\text{C} + 441^\circ\text{C} = \boxed{461^\circ\text{C}}$$

P27.38 You pay the electric company for energy transferred in the amount $E = \mathcal{P}\Delta t$.

(a) $\mathcal{P}\Delta t = 40\text{ W}(2\text{ weeks})\left(\dfrac{7\text{ d}}{1\text{ week}}\right)\left(\dfrac{86\,400\text{ s}}{1\text{ d}}\right)\left(\dfrac{1\text{ J}}{1\text{ W}\cdot\text{s}}\right) = 48.4\text{ MJ}$

$$\mathcal{P}\Delta t = 40\text{ W}(2\text{ weeks})\left(\frac{7\text{ d}}{1\text{ week}}\right)\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{\text{k}}{1\,000}\right) = 13.4\text{ kWh}$$

$$\mathcal{P}\Delta t = 40\text{ W}(2\text{ weeks})\left(\frac{7\text{ d}}{1\text{ week}}\right)\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{0.12\ \$}{\text{kWh}}\right) = \boxed{\$1.61}$$

(b) $\mathcal{P}\Delta t = 970\text{ W}(3\text{ min})\left(\dfrac{1\text{ h}}{60\text{ min}}\right)\left(\dfrac{\text{k}}{1\,000}\right)\left(\dfrac{0.12\ \$}{\text{kWh}}\right) = \boxed{\$0.005\,82} = 0.582¢$

(c) $\mathcal{P}\Delta t = 5\,200\text{ W}(40\text{ min})\left(\dfrac{1\text{ h}}{60\text{ min}}\right)\left(\dfrac{\text{k}}{1\,000}\right)\left(\dfrac{0.12\ \$}{\text{kWh}}\right) = \boxed{\$0.416}$

P27.39 Consider a 400-W blow dryer used for ten minutes daily for a year. The energy transferred to the dryer is

$$\mathcal{P}\Delta t = (400\text{ J/s})(600\text{ s/d})(365\text{ d}) \approx 9\times10^7\text{ J}\left(\frac{1\text{ kWh}}{3.6\times10^6\text{ J}}\right) \approx 20\text{ kWh}$$

We suppose that electrically transmitted energy costs on the order of ten cents per kilowatt-hour. Then the cost of using the dryer for a year is on the order of

$$\text{Cost} \approx (20\text{ kWh})(\$0.10/\text{kWh}) = \$2\ \boxed{\sim\$1}$$

Additional Problems

***P27.40** (a) $I = \dfrac{\Delta V}{R}$ so $\mathcal{P} = I\Delta V = \dfrac{(\Delta V)^2}{R}$

$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{25.0\text{ W}} = \boxed{576\ \Omega} \quad \text{and} \quad R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ V})^2}{100\text{ W}} = \boxed{144\ \Omega}$$

(b) $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{25.0\text{ W}}{120\text{ V}} = 0.208\text{ A} = \dfrac{Q}{\Delta t} = \dfrac{1.00\text{ C}}{\Delta t}$

$$\Delta t = \frac{1.00\text{ C}}{0.208\text{ A}} = \boxed{4.80\text{ s}}$$

The charge itself is the same. It comes out at a location that is at lower potential.

(c) $\mathcal{P} = 25.0\text{ W} = \dfrac{\Delta U}{\Delta t} = \dfrac{1.00\text{ J}}{\Delta t}$ $\qquad \Delta t = \dfrac{1.00\text{ J}}{25.0\text{ W}} = \boxed{0.040\,0\text{ s}}$

The energy itself is the same. It enters the bulb by electrical transmission and leaves by heat and electromagnetic radiation.

(d) $\Delta U = \mathcal{P}\Delta t = (25.0\text{ J/s})(86\,400\text{ s/d})(30.0\text{ d}) = 64.8\times10^6\text{ J}$

The electric company sells $\boxed{\text{energy}}$.

$$\text{Cost} = 64.8\times10^6\text{ J}\left(\frac{\$0.070\,0}{\text{kWh}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3\,600\text{ s}}\right) = \boxed{\$1.26}$$

$$\text{Cost per joule} = \frac{\$0.070\,0}{\text{kWh}}\left(\frac{\text{kWh}}{3.60\times10^6\text{ J}}\right) = \boxed{\$1.94\times10^{-8}/\text{J}}$$

***P27.41** The heater should put out constant power

$$\mathcal{P} = \frac{Q}{\Delta t} = \frac{mc(T_f - T_i)}{\Delta t} = \frac{(0.250\text{ kg})(4\,186\text{ J})(100°\text{C} - 20°\text{C})}{\text{kg}\cdot°\text{C}(4\text{ min})}\left(\frac{1\text{ min}}{60\text{ s}}\right) = 349\text{ J/s}$$

Then its resistance should be described by

$$\mathcal{P} = (\Delta V)I = \frac{(\Delta V)(\Delta V)}{R} \qquad R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\text{ J/C})^2}{349\text{ J/s}} = 41.3\ \Omega$$

Its resistivity at 100°C is given by

$$\rho = \rho_0\left[1 + \alpha(T - T_0)\right] = \left(1.50\times10^{-6}\ \Omega\cdot\text{m}\right)\left[1 + 0.4\times10^{-3}(80)\right] = 1.55\times10^{-6}\ \Omega\cdot\text{m}$$

Then for a wire of circular cross section

$$R = \rho\frac{\ell}{A} = \rho\frac{\ell}{\pi r^2} = \rho\frac{\ell 4}{\pi d^2}$$

$$41.3\ \Omega = \left(1.55\times10^{-6}\ \Omega\cdot\text{m}\right)\frac{4\ell}{\pi d^2}$$

$$\frac{\ell}{d^2} = 2.09\times10^{+7}/\text{m} \qquad \text{or} \qquad d^2 = \left(4.77\times10^{-8}\text{ m}\right)\ell$$

One possible choice is $\ell = 0.900\text{ m}$ and $d = 2.07\times10^{-4}$ m. If ℓ and d are made too small, the surface area will be inadequate to transfer heat into the water fast enough to prevent overheating of the filament. To make the volume less than 0.5 cm^3, we want ℓ and d less than those described by $\dfrac{\pi d^2}{4}\ell = 0.5\times10^{-6}\text{ m}^3$. Substituting $d^2 = \left(4.77\times10^{-8}\text{ m}\right)\ell$ gives $\dfrac{\pi}{4}\left(4.77\times10^{-8}\text{ m}\right)\ell^2 = 0.5\times10^{-6}\text{ m}^3$, $\ell = 3.65\text{ m}$ and $d = 4.18\times10^{-4}$ m. Thus our answer is: Any diameter d and length ℓ related by $d^2 = \left(4.77\times10^{-8}\text{ m}\right)\ell$ would have the right resistance. One possibility is length 0.900 m and diameter 0.207 mm, but such a small wire might overheat rapidly if it were not surrounded by water. The volume can be less than 0.5 cm^3.

P27.42 The original stored energy is $U_i = \frac{1}{2}Q\Delta V_i = \frac{1}{2}\frac{Q^2}{C}$.

(a) When the switch is closed, charge Q distributes itself over the plates of C and $3C$ in parallel, presenting equivalent capacitance $4C$. Then the final potential difference is

$\boxed{\Delta V_f = \frac{Q}{4C}}$ for both.

(b) The smaller capacitor then carries charge $C\Delta V_f = \frac{Q}{4C}C = \boxed{\frac{Q}{4}}$. The larger capacitor carries charge $3C\frac{Q}{4C} = \boxed{\frac{3Q}{4}}$.

(c) The smaller capacitor stores final energy $\frac{1}{2}C(\Delta V_f)^2 = \frac{1}{2}C\left(\frac{Q}{4C}\right)^2 = \boxed{\frac{Q^2}{32C}}$. The larger capacitor possesses energy $\frac{1}{2}3C\left(\frac{Q}{4C}\right)^2 = \boxed{\frac{3Q^2}{32C}}$.

(d) The total final energy is $\frac{Q^2}{32C} + \frac{3Q^2}{32C} = \frac{Q^2}{8C}$. The loss of potential energy is the energy appearing as internal energy in the resistor: $\frac{Q^2}{2C} = \frac{Q^2}{8C} + \Delta E_{\text{int}}$ $\quad \Delta E_{\text{int}} = \boxed{\frac{3Q^2}{8C}}$

P27.43 We begin with the differential equation $\quad \alpha = \frac{1}{\rho}\frac{d\rho}{dT}$

(a) Separating variables, $\quad \int_{\rho_0}^{\rho}\frac{d\rho}{\rho} = \int_{T_0}^{T}\alpha dT$

$\ln\left(\frac{\rho}{\rho_0}\right) = \alpha(T - T_0)$ and $\boxed{\rho = \rho_0 e^{\alpha(T-T_0)}}$.

(b) From the series expansion $e^x \approx 1 + x$, $(x << 1)$, we have $\boxed{\rho \approx \rho_0\left[1 + \alpha(T - T_0)\right]}$.

P27.44 We find the drift velocity from $\quad I = nqv_dA = nqv_d\pi r^2$

$$v_d = \frac{I}{nq\pi r^2} = \frac{1\,000 \text{ A}}{8.46\times10^{28} \text{ m}^{-3}(1.60\times10^{-19} \text{ C})\pi(10^{-2} \text{ m})^2} = 2.35\times10^{-4} \text{ m/s}$$

$$v = \frac{x}{t} \quad t = \frac{x}{v} = \frac{200\times10^3 \text{ m}}{2.35\times10^{-4} \text{ m/s}} = 8.50\times10^8 \text{ s} = \boxed{27.0 \text{ yr}}$$

***P27.45** From $\rho = \frac{RA}{\ell} = \frac{(\Delta V)}{I}\frac{A}{\ell}$ we compute

ℓ (m)	R (Ω)	ρ ($\Omega\cdot$m)
0.540	10.4	1.41×10^{-6}
1.028	21.1	1.50×10^{-6}
1.543	31.8	1.50×10^{-6}

$\bar{\rho} = \boxed{1.47\times10^{-6}\ \Omega\cdot\text{m}}$. With its uncertainty range from 1.41 to 1.50, this average value agrees with the tabulated value of $1.50\times10^{-6}\ \Omega\cdot\text{m}$ in Table 27.2.

P27.46 2 wires $\rightarrow \ell = 100$ m

$$R = \frac{0.108\ \Omega}{300\text{ m}}(100\text{ m}) = 0.036\ 0\ \Omega$$

(a) $(\Delta V)_{\text{home}} = (\Delta V)_{\text{line}} - IR = 120 - (110)(0.036\ 0) = \boxed{116\text{ V}}$

(b) $\mathcal{P} = I(\Delta V) = (110\text{ A})(116\text{ V}) = \boxed{12.8\text{ kW}}$

(c) $\mathcal{P}_{\text{wires}} = I^2R = (110\text{ A})^2(0.036\ 0\ \Omega) = \boxed{436\text{ W}}$

***P27.47** (a) $\vec{\mathbf{E}} = -\dfrac{dV}{dx}\hat{\mathbf{i}} = -\dfrac{(0-4.00)\text{ V}}{(0.500-0)\text{ m}} = \boxed{8.00\hat{\mathbf{i}}\text{ V/m}}$

(b) $R = \dfrac{\rho\ell}{A} = \dfrac{(4.00\times10^{-8}\ \Omega\cdot\text{m})(0.500\text{ m})}{\pi(1.00\times10^{-4}\text{ m})^2} = \boxed{0.637\ \Omega}$

(c) $I = \dfrac{\Delta V}{R} = \dfrac{4.00\text{ V}}{0.637\ \Omega} = \boxed{6.28\text{ A}}$

(d) $J = \dfrac{I}{A} = \dfrac{6.28\text{ A}}{\pi(1.00\times10^{-4}\text{ m})^2} = 2.00\times10^8\text{ A/m}^2 = \boxed{200\text{ MA/m}^2}$ The field and the current are both in the x direction.

(e) $\rho J = (4.00\times10^{-8}\ \Omega\cdot\text{m})(2.00\times10^8\text{ A/m}^2) = 8.00\text{ V/m} = E$

***P27.48** (a) $\vec{\mathbf{E}} = -\dfrac{dV(x)}{dx}\hat{\mathbf{i}} = \boxed{\dfrac{V}{L}\hat{\mathbf{i}}}$

(b) $R = \dfrac{\rho\ell}{A} = \boxed{\dfrac{4\rho L}{\pi d^2}}$

(c) $I = \dfrac{\Delta V}{R} = \boxed{\dfrac{V\pi d^2}{4\rho L}}$

(d) $J = \dfrac{I}{A} = \boxed{\dfrac{V}{\rho L}}$ The field and the current are both in the x direction.

(e) $\rho J = \dfrac{V}{L} = \boxed{E}$

P27.49 (a) $\mathcal{P} = I\Delta V$

so $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{8.00 \times 10^3 \text{ W}}{12.0 \text{ V}} = \boxed{667 \text{ A}}$

(b) $\Delta t = \dfrac{\Delta U}{\mathcal{P}} = \dfrac{2.00 \times 10^7 \text{ J}}{8.00 \times 10^3 \text{ W}} = 2.50 \times 10^3 \text{ s}$

and $\Delta x = v\Delta t = (20.0 \text{ m/s})(2.50 \times 10^3 \text{ s}) = \boxed{50.0 \text{ km}}$

***P27.50** (a) We begin with $R = \dfrac{\rho\ell}{A} = \dfrac{\rho_0[1+\alpha(T-T_0)]\ell_0[1+\alpha'(T-T_0)]}{A_0[1+2\alpha'(T-T_0)]}$,

which reduces to $R = \boxed{\dfrac{R_0[1+\alpha(T-T_0)][1+\alpha'(T-T_0)]}{[1+2\alpha'(T-T_0)]}}$

(b) For copper: $\rho_0 = 1.70 \times 10^{-8} \ \Omega\cdot\text{m}$, $\alpha = 3.90 \times 10^{-3}\,{}^\circ\text{C}^{-1}$, and

$\alpha' = 17.0 \times 10^{-6}\,{}^\circ\text{C}^{-1}$

$$R_0 = \frac{\rho_0\ell_0}{A_0} = \frac{(1.70 \times 10^{-8})(2.00)}{\pi(0.100 \times 10^{-3})^2} = 1.08 \ \Omega$$

The simple formula for R gives:

$$R = (1.08 \ \Omega)\left[1 + (3.90 \times 10^{-3}\,{}^\circ\text{C}^{-1})(100^\circ\text{C} - 20.0^\circ\text{C})\right] = \boxed{1.420 \ \Omega}$$

while the more complicated formula gives:

$$R = \frac{(1.08 \ \Omega)\left[1 + (3.90 \times 10^{-3}\,{}^\circ\text{C}^{-1})(80.0^\circ\text{C})\right]\left[1 + (17.0 \times 10^{-6}\,{}^\circ\text{C}^{-1})(80.0^\circ\text{C})\right]}{\left[1 + 2(17.0 \times 10^{-6}\,{}^\circ\text{C}^{-1})(80.0^\circ\text{C})\right]}$$

$$= \boxed{1.418 \ \Omega}$$

The results agree to three digits. The variation of resistance with temperature is typically a much larger effect than thermal expansion in size.

P27.51 Let α be the temperature coefficient at 20.0°C, and α' be the temperature coefficient at 0°C. Then $\rho = \rho_0\left[1+\alpha\left(T-20.0°\text{C}\right)\right]$, and $\rho = \rho'\left[1+\alpha'\left(T-0°\text{C}\right)\right]$ must both give the correct resistivity at any temperature *T*. That is, we must have:

$$\rho_0\left[1+\alpha(T-20.0°\text{C})\right] = \rho'\left[1+\alpha'(T-0°\text{C})\right] \qquad (1)$$

Setting $T = 0$ in equation (1) yields: $\rho' = \rho_0\left[1-\alpha\left(20.0°\text{C}\right)\right]$,

and setting $T = 20.0°\text{C}$ in equation (1) gives: $\rho_0 = \rho'\left[1+\alpha'(20.0°\text{C})\right]$

Put ρ' from the first of these results into the second to obtain:

$$\rho_0 = \rho_0\left[1-\alpha(20.0°\text{C})\right]\left[1+\alpha'(20.0°\text{C})\right]$$

Therefore $1+\alpha'\left(20.0°\text{C}\right) = \dfrac{1}{1-\alpha\left(20.0°\text{C}\right)}$

which simplifies to $$\alpha' = \frac{\alpha}{\left[1-\alpha(20.0°\text{C})\right]}$$

From this, the temperature coefficient, based on a reference temperature of 0°C, may be computed for any material. For example, using this, Table 27.2 becomes at 0°C :

Material	Temp Coefficients at 0°C
Silver	$4.1\times10^{-3}/°\text{C}$
Copper	$4.2\times10^{-3}/°\text{C}$
Gold	$3.6\times10^{-3}/°\text{C}$
Aluminum	$4.2\times10^{-3}/°\text{C}$
Tungsten	$4.9\times10^{-3}/°\text{C}$
Iron	$5.6\times10^{-3}/°\text{C}$
Platinum	$4.25\times10^{-3}/°\text{C}$
Lead	$4.2\times10^{-3}/°\text{C}$
Nichrome	$0.4\times10^{-3}/°\text{C}$
Carbon	$-0.5\times10^{-3}/°\text{C}$
Germanium	$-24\times10^{-3}/°\text{C}$
Silicon	$-30\times10^{-3}/°\text{C}$

P27.52 (a) A thin cylindrical shell of radius *r*, thickness *dr*, and length *L* contributes resistance

$$dR = \frac{\rho d\ell}{A} = \frac{\rho dr}{(2\pi r)L} = \left(\frac{\rho}{2\pi L}\right)\frac{dr}{r}$$

The resistance of the whole annulus is the series summation of the contributions of the thin shells:

$$R = \frac{\rho}{2\pi L}\int_{r_a}^{r_b}\frac{dr}{r} = \boxed{\frac{\rho}{2\pi L}\ln\left(\frac{r_b}{r_a}\right)}$$

(b) In this equation $\frac{\Delta V}{I} = \frac{\rho}{2\pi L}\ln\left(\frac{r_b}{r_a}\right)$

we solve for $\boxed{\rho = \frac{2\pi L\Delta V}{I\ln\left(r_b/r_a\right)}}$

***P27.53** The original resistance is $R_i = \rho L_i/A_i$.

The new length is $L = L_i + \delta L = L_i(1 + \delta)$.

Constancy of volume implies $AL = A_iL_i$ so $A = \frac{A_iL_i}{L} = \frac{A_iL_i}{L_i(1+\delta)} = \frac{A_i}{(1+\delta)}$

The new resistance is $R = \frac{\rho L}{A} = \frac{\rho L_i(1+\delta)}{A_i/(1+\delta)} = R_i(1+\delta)^2 = R_i(1+2\delta+\delta^2)$.

The result is exact if the assumptions are precisely true. Our derivation contains no approximation steps where delta is assumed to be small.

P27.54 Each speaker receives 60.0 W of power. Using $\mathcal{P} = I^2R$, we then have

$$I = \sqrt{\frac{\mathcal{P}}{R}} = \sqrt{\frac{60.0\text{ W}}{4.00\ \Omega}} = 3.87\text{ A}$$

The system is $\boxed{\text{not adequately protected}}$ since the $\boxed{\text{fuse should be set to melt at 3.87 A, or less}}$.

P27.55 (a) $\Delta V = -E\cdot\ell$ or $dV = -E\cdot dx$

$\Delta V = -IR = -E\cdot\ell$

$$I = \frac{dq}{dt} = \frac{E\cdot\ell}{R} = \frac{A}{\rho\ell}E\cdot\ell = \frac{A}{\rho}E = -\sigma A\frac{dV}{dx} = \boxed{\sigma A\left|\frac{dV}{dx}\right|}$$

(b) Current flows in the direction of decreasing voltage. Energy flows by heat in the direction of decreasing temperature.

P27.56 From the geometry of the longitudinal section of the resistor shown in the figure, we see that

$$\frac{(b-r)}{y} = \frac{(b-a)}{h}$$

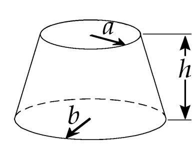


FIG. P27.56

From this, the radius at a distance *y* from the base is $r = (a-b)\frac{y}{h} + b$

For a disk-shaped element of volume $dR = \frac{\rho dy}{\pi r^2}$: $R = \frac{\rho}{\pi}\int_0^h \frac{dy}{\left[(a-b)(y/h)+b\right]^2}$

Using the integral formula $\int\frac{du}{(au+b)^2} = -\frac{1}{a(au+b)}$, $\boxed{R = \frac{\rho}{\pi}\frac{h}{ab}}$

P27.57 $R = \int \frac{\rho dx}{A} = \int \frac{\rho dx}{wy}$ where $y = y_1 + \frac{y_2 - y_1}{L} x$

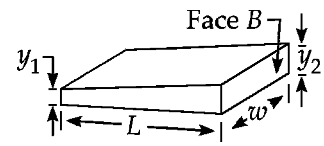


FIG. P27.57

$$R = \frac{\rho}{w} \int_0^L \frac{dx}{y_1 + [(y_2 - y_1)/L] x} = \frac{\rho L}{w(y_2 - y_1)} \ln\left[y_1 + \frac{y_2 - y_1}{L} x \right]\Bigg|_0^L$$

$$R = \boxed{\frac{\rho L}{w(y_2 - y_1)} \ln\left(\frac{y_2}{y_1}\right)}$$

***P27.58** A spherical layer within the shell, with radius r and thickness dr, has resistance

$$dR = \frac{\rho dr}{4\pi r^2}$$

The whole resistance is the absolute value of the quantity

$$R = \int_a^b dR = \int_{r_a}^{r_b} \frac{\rho dr}{4\pi r^2} = \frac{\rho}{4\pi} \frac{r^{-1}}{-1}\Bigg|_{r_a}^{r_b} = -\frac{\rho}{4\pi}\left(-\frac{1}{r_a} + \frac{1}{r_b} \right) = \frac{\rho}{4\pi}\left(\frac{1}{r_a} - \frac{1}{r_b} \right)$$

***P27.59** Coat the surfaces of entry and exit with material of much higher conductivity than the bulk material of the object. The electric potential will be essentially uniform over each of these electrodes. Current will be distributed over the whole area where each electrode is in contact with the resistive object.

P27.60 (a) The resistance of the dielectric block is $R = \frac{\rho \ell}{A} = \frac{d}{\sigma A}$.

The capacitance of the capacitor is $C = \frac{\kappa \in_0 A}{d}$.

Then $RC = \frac{d}{\sigma A} \frac{\kappa \in_0 A}{d} = \frac{\kappa \in_0}{\sigma}$ is a characteristic of the material only.

(b) $$R = \frac{\kappa \in_0}{\sigma C} = \frac{\rho \kappa \in_0}{C} = \frac{75 \times 10^{16}\ \Omega \cdot \text{m}(3.78) 8.85 \times 10^{-12}\ \text{C}^2}{14 \times 10^{-9}\ \text{F} \qquad \text{N} \cdot \text{m}^2} = \boxed{1.79 \times 10^{15}\ \Omega}$$

P27.61 (a) Think of the device as two capacitors in parallel. The one on the left has $\kappa_1 = 1$, $A_1 = \left(\frac{\ell}{2} + x \right) \ell$. The equivalent capacitance is

$$\frac{\kappa_1 \in_0 A_1}{d} + \frac{\kappa_2 \in_0 A_2}{d} = \frac{\in_0 \ell}{d}\left(\frac{\ell}{2} + x \right) + \frac{\kappa \in_0 \ell}{d}\left(\frac{\ell}{2} - x \right) = \boxed{\frac{\in_0 \ell}{2d}(\ell + 2x + \kappa \ell - 2\kappa x)}$$

(b) The charge on the capacitor is $Q = C\Delta V$

$$Q = \frac{\in_0 \ell \Delta V}{2d}(\ell + 2x + \kappa \ell - 2\kappa x)$$

The current is

$$I = \frac{dQ}{dt} = \frac{dQ}{dx}\frac{dx}{dt} = \frac{\in_0 \ell \Delta V}{2d}(0 + 2 + 0 - 2\kappa) v = -\frac{\in_0 \ell \Delta V v}{d}(\kappa - 1)$$

The negative value indicates that the current drains charge from the capacitor. Positive current is $\boxed{\text{clockwise } \frac{\in_0 \ell \Delta V v}{d}(\kappa - 1)}$.

P27.62 $I = I_0\left[\exp\left(\frac{e\Delta V}{k_B T}\right) - 1\right]$ and $R = \frac{\Delta V}{I}$

with $I_0 = 1.00 \times 10^{-9}$ A, $e = 1.60 \times 10^{-19}$ C, and $k_B = 1.38 \times 10^{-23}$ J/K

The following includes a partial table of calculated values and a graph for each of the specified temperatures.

(i) For $T = 280$ K:

ΔV (V)	I (A)	R (Ω)
0.400	0.015 6	25.6
0.440	0.081 8	5.38
0.480	0.429	1.12
0.520	2.25	0.232
0.560	11.8	0.047 6
0.600	61.6	0.009 7

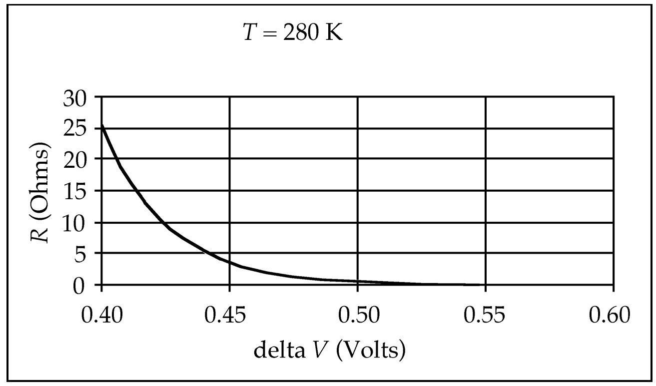


FIG. P27.62(i)

(ii) For $T = 300$ K:

ΔV (V)	I (A)	R (Ω)
0.400	0.005	77.3
0.440	0.024	18.1
0.480	0.114	4.22
0.520	0.534	0.973
0.560	2.51	0.223
0.600	11.8	0.051

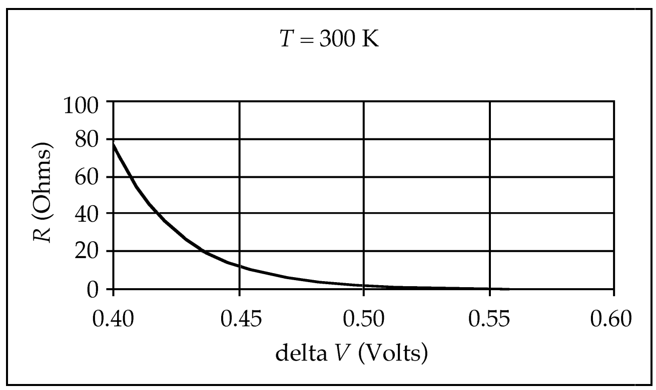


FIG. P27.62(ii)

(iii) For $T = 320$ K:

ΔV (V)	I (A)	R (Ω)
0.400	0.002 0	203
0.440	0.008 4	52.5
0.480	0.035 7	13.4
0.520	0.152	3.42
0.560	0.648	0.864
0.600	2.76	0.217

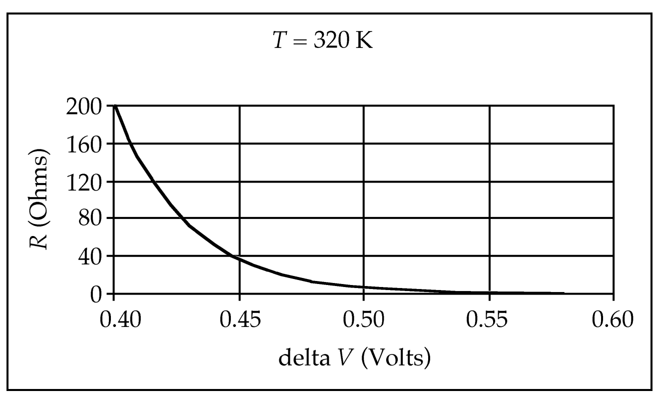


FIG. P27.62(iii)

P27.63 The volume of the gram of gold is given by $\rho = \dfrac{m}{V}$

$$V = \frac{m}{\rho} = \frac{10^{-3}\ \text{kg}}{19.3\times 10^{3}\ \text{kg/m}^{3}} = 5.18\times 10^{-8}\ \text{m}^{3} = A\left(2.40\times 10^{3}\ \text{m}\right)$$

$$A = 2.16\times 10^{-11}\ \text{m}^{2}$$

$$R = \frac{\rho \ell}{A} = \frac{2.44\times 10^{-8}\ \Omega\cdot\text{m}\left(2.4\times 10^{3}\ \text{m}\right)}{2.16\times 10^{-11}\ \text{m}^{2}} = \boxed{2.71\times 10^{6}\ \Omega}$$

P27.64 The resistance of one wire is $\left(\dfrac{0.500\ \Omega}{\text{mi}}\right)(100\ \text{mi}) = 50.0\ \Omega.$

The whole wire is at nominal 700 kV away from ground potential, but the potential difference between its two ends is

$$IR = (1\,000\ \text{A})(50.0\ \Omega) = 50.0\ \text{kV}$$

Then it radiates as heat power $\mathcal{P} = (\Delta V)I = \left(50.0\times 10^{3}\ \text{V}\right)(1\,000\ \text{A}) = \boxed{50.0\ \text{MW}}$.

P27.65 $R = R_0\left[1+\alpha(T-T_0)\right]$ so $T = T_0 + \dfrac{1}{\alpha}\left[\dfrac{R}{R_0}-1\right] = T_0 + \dfrac{1}{\alpha}\left[\dfrac{I_0}{I}-1\right]$

In this case, $I = \dfrac{I_0}{10}$, so $T = T_0 + \dfrac{1}{\alpha}(9) = 20° + \dfrac{9}{0.004\,50/°\text{C}} = \boxed{2\,020°\text{C}}$

ANSWERS TO EVEN PROBLEMS

P27.2 3.64 h

P27.4 $q\omega/2\pi$

P27.6 0.265 C

P27.8 (a) 99.5 kA/m^2 (b) Current is the same, current density is smaller. 5.00 A, 24.9 kA/m^2, 0.800 cm

P27.10 0.130 mm/s

P27.12 500 mA

P27.14 (a) ~10^{18} Ω (b) ~10^{-7} Ω (c) ~100 aA, ~1 GA

P27.16 (a) no change (b) doubles (c) doubles (d) no change

P27.18 1.44×10^{3} °C

P27.20 She can meet the design goal by choosing $\ell_1 = 0.898$ m and $\ell_2 = 26.2$ m.

P27.22 1.71 Ω

P27.24 7.50 W

P27.26 (a) 17.3 A (b) 22.4 MJ (c) \$0.995

P27.28 (a) 0.530 (b) 221 J (c) 15.1°C

P27.30 (a) 5.97 V/m (b) 74.6 W (c) 66.1 W

P27.32 (a) 2.05 W (b) 3.41 W. It would not be as safe. If surrounded by thermal insulation, it would get much hotter than a copper wire.

P27.34 295 metric ton/h

P27.36 672 s

P27.38 (a) \$1.61 (b) \$0.005 82 (c) \$0.416

P27.40 (a) 576 Ω and 144 Ω (b) 4.80 s. The charge itself is the same. It is at a location that is lower in potential. (c) 0.040 0 s. The energy itself is the same. It enters the bulb by electric transmission and leaves by heat and electromagnetic radiation. (d) \$1.26, energy, 1.94×10^{-8} \$/J

P27.42 (a) $Q/4C$ (b) $Q/4$ and $3Q/4$ (c) $Q^2/32C$ and $3Q^2/32C$ (d) $3Q^2/8C$

P27.44 8.50×10^8 s = 27.0 yr

P27.46 (a) 116 V (b) 12.8 kW (c) 436 W

P27.48 (a) $E = V/L$ in the x direction (b) $R = 4\rho L/\pi d^2$ (c) $I = V\pi d^2/4\rho L$ (d) $J = V/\rho L$ (e) See the solution.

P27.50 (a) See the solution. (b) 1.418 Ω nearly agrees with 1.420 Ω.

P27.52 (a) $R = \dfrac{\rho}{2\pi L}\ln\dfrac{r_b}{r_a}$ (b) $\rho = \dfrac{2\pi L\Delta V}{I\ln(r_b/r_a)}$

P27.54 No. The fuses should pass no more than 3.87 A.

P27.56 See the solution.

P27.58 See the solution.

P27.60 (b) 1.79 PΩ

P27.62 See the solution.

P27.64 50.0 MW

28

Direct Current Circuits

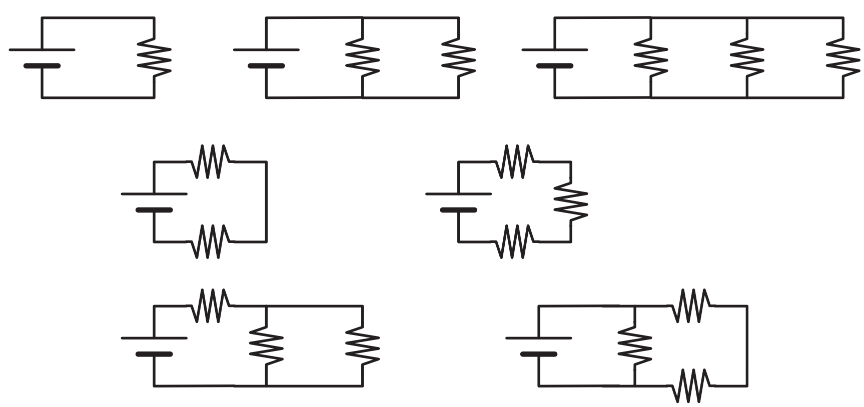
CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q28.1 No. If there is one battery in a circuit, the current inside it will be from its negative terminal to its positive terminal. Whenever a battery is delivering energy to a circuit, it will carry current in this direction. On the other hand, when another source of emf is charging the battery in question, it will have a current pushed through it from its positive terminal to its negative terminal.

***Q28.2** The terminal potential difference is $\mathcal{E} - Ir$ where I is the current in the battery in the direction from its negative to its positive pole. So the answer to (i) is (d) and the answer to (ii) is (b). The current might be zero or an outside agent might push current backward through the battery from positive to negative terminal.

***Q28.3**



***Q28.4** Answers (b) and (d), as described by Kirchhoff's junction rule.

***Q28.5** Answer (a).

Q28.6 The whole wire is very nearly at one uniform potential. There is essentially zero *difference* in potential between the bird's feet. Then negligible current goes through the bird. The resistance through the bird's body between its feet is much larger than the resistance through the wire between the same two points.

***Q28.7** Answer (b). Each headlight's terminals are connected to the positive and negative terminals of the battery. Each headlight can operate if the other is burned out.

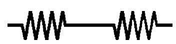
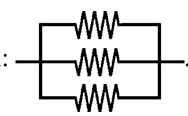
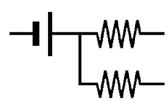
Q28.8 Answer their question with a challenge. If the student is just looking at a diagram, provide the materials to build the circuit. If you are looking at a circuit where the second bulb really is fainter, get the student to unscrew them both and interchange them. But check that the student's understanding of potential has not been impaired: if you patch past the first bulb to short it out, the second gets brighter.

***Q28.9** Answer (a). When the breaker trips to off, current does not go through the device.

***Q28.10** (i) For both batteries to be delivering electric energy, currents are in the direction g to a to b, h to d to c, and so e to f. Points f, g, and h are all at zero potential. Points b, c, and e are at the same higher voltage, d still higher, and a highest of all. The ranking is a > d > b = c = e > f = g = h. (ii) The current in ef must be the sum of the other two currents. The ranking is e = f > g = a = b > h = d = c.

***Q28.11** Closing the switch removes lamp C from the circuit, decreasing the resistance seen by the battery, and so increasing the current in the battery. (i) Answer (a). (ii) Answer (d). (iii) Answer (a). (iv) Answer (a). (v) Answer (d). (vi) Answer (a).

***Q28.12** Closing the switch lights lamp C. The action increases the battery current so it decreases the terminal voltage of the battery. (i) Answer (b). (ii) Answer (a). (iii) Answer (a). (iv) Answer (b). (v) Answer (a). (vi) Answer (a).

Q28.13 Two runs in series:  . Three runs in parallel: . Junction of one lift and two runs:  .

Gustav Robert Kirchhoff, Professor of Physics at Heidelberg and Berlin, was master of the obvious. A junction rule: The number of skiers coming into any junction must be equal to the number of skiers leaving. A loop rule: the total change in altitude must be zero for any skier completing a closed path.

Q28.14 The bulb will light up for a while immediately after the switch is closed. As the capacitor charges, the bulb gets progressively dimmer. When the capacitor is fully charged the current in the circuit is zero and the bulb does not glow at all. If the value of RC is small, this whole process might occupy a very short time interval.

Q28.15 The hospital maintenance worker is right. A hospital room is full of electrical grounds, including the bed frame. If your grandmother touched the faulty knob and the bed frame at the same time, she could receive quite a jolt, as there would be a potential difference of 120 V across her. If the 120 V is DC, the shock could send her into ventricular fibrillation, and the hospital staff could use the defibrillator you read about in Chapter 26. If the 120 V is AC, which is most likely, the current could produce external and internal burns along the path of conduction. Likely no one got a shock from the radio back at home because her bedroom contained no electrical grounds—no conductors connected to zero volts. Just like the bird in Question 28.6, granny could touch the "hot" knob without getting a shock so long as there was no path to ground to supply a potential difference across her. A new appliance in the bedroom or a flood could make the radio lethal. Repair it or discard it. Enjoy the news from Lake Wobegon on the new plastic radio.

Q28.16 Both 120-V and 240-V lines can deliver injurious or lethal shocks, but there is a somewhat better safety factor with the lower voltage. To say it a different way, the insulation on a 120-V line can be thinner. On the other hand, a 240-V device carries less current to operate a device with the same power, so the conductor itself can be thinner. Finally, as we will see in Chapter 33, the last step-down transformer can also be somewhat smaller if it has to go down only to 240 volts from the high voltage of the main power line.

SOLUTIONS TO PROBLEMS

Section 28.1 **Electromotive Force**

P28.1 (a) $\mathcal{P} = \frac{(\Delta V)^2}{R}$

becomes $20.0\text{ W} = \frac{(11.6\text{ V})^2}{R}$

so $R = \boxed{6.73\ \Omega}$

(b) $\Delta V = IR$

so $11.6\text{ V} = I(6.73\ \Omega)$

and $I = 1.72\text{ A}$

$\mathcal{E} = IR + Ir$

so $15.0\text{ V} = 11.6\text{ V} + (1.72\text{ A})r$

$r = \boxed{1.97\ \Omega}$

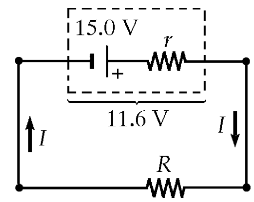


FIG. P28.1

P28.2 The total resistance is $R = \frac{3.00\text{ V}}{0.600\text{ A}} = 5.00\ \Omega$.

(a) $R_{\text{lamp}} = R - r_{\text{batteries}} = 5.00\ \Omega - 0.408\ \Omega = \boxed{4.59\ \Omega}$

(b) $\frac{\mathcal{P}_{\text{batteries}}}{\mathcal{P}_{\text{total}}} = \frac{(0.408\ \Omega)I^2}{(5.00\ \Omega)I^2} = 0.0816 = \boxed{8.16\%}$

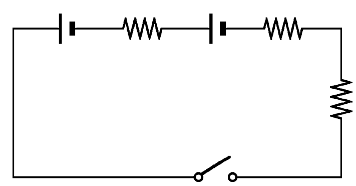


FIG. P28.2

P28.3 (a) Here $\mathcal{E} = I(R+r)$, so $I = \frac{\mathcal{E}}{R+r} = \frac{12.6\text{ V}}{(5.00\ \Omega + 0.0800\ \Omega)} = 2.48\text{ A}$.

Then, $\Delta V = IR = (2.48\text{ A})(5.00\ \Omega) = \boxed{12.4\text{ V}}$.

(b) Let I_1 and I_2 be the currents flowing through the battery and the headlights, respectively.

Then, $I_1 = I_2 + 35.0\text{ A}$, and $\mathcal{E} - I_1 r - I_2 r = 0$

so $\mathcal{E} = (I_2 + 35.0\text{ A})(0.0800\ \Omega) + I_2(5.00\ \Omega) = 12.6\text{ V}$

giving $I_2 = 1.93\text{ A}$

Thus, $\Delta V_2 = (1.93\text{ A})(5.00\ \Omega) = \boxed{9.65\text{ V}}$

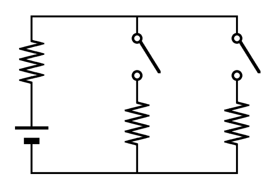


FIG. P28.3

***P28.4** (a) At maximum power transfer, $r = R$. Equal powers are delivered to r and R. The efficiency is $\boxed{50.0\%}$.

(b) For maximum fractional energy transfer to R, we want zero energy absorbed by r, so we want $r = \boxed{0}$.

(c) High efficiency. The electric company's economic interest is to minimize internal energy production in its power lines, so that it can sell a large fraction of the energy output of its generators to the customers.

(d) High power transfer. Energy by electric transmission is so cheap compared to the sound system that she does not spend extra money to buy an efficient amplifier.

Section 28.2 **Resistors in Series and Parallel**

P28.5 (a) $R_p = \dfrac{1}{(1/7.00\ \Omega)+(1/10.0\ \Omega)} = 4.12\ \Omega$

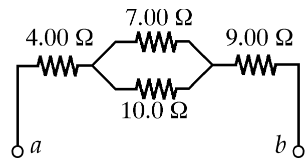


FIG. P28.5

$R_s = R_1 + R_2 + R_3 = 4.00 + 4.12 + 9.00 = \boxed{17.1\ \Omega}$

(b) $\Delta V = IR$

$34.0\ \text{V} = I(17.1\ \Omega)$

$I = \boxed{1.99\ \text{A}}$ for $4.00\ \Omega$, $9.00\ \Omega$ resistors

Applying $\Delta V = IR$, $(1.99\ \text{A})(4.12\ \Omega) = 8.18\ \text{V}$

$8.18\ \text{V} = I(7.00\ \Omega)$

so $I = \boxed{1.17\ \text{A}}$ for $7.00\ \Omega$ resistor

$8.18\ \text{V} = I(10.0\ \Omega)$

so $I = \boxed{0.818\ \text{A}}$ for $10.0\ \Omega$ resistor

***P28.6** (a) The conductors in the cord have resistance. There is a potential difference across each when current is flowing. The potential difference applied to the light bulb is less than 120 V, so it will carry less current than it is designed to, and will operate at lower power than 75 W.

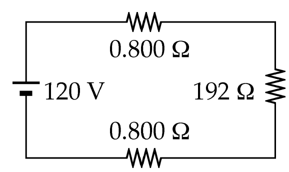


FIG. P28.6

(b) If the temperature of the bulb does not change much between the design operating point and the actual operating point, we can take the resistance of the filament as constant.
For the bulb in use as intended,

$$I = \frac{\mathcal{P}}{\Delta V} = \frac{75.0\ \text{W}}{120\ \text{V}} = 0.625\ \text{A}$$

and $$R = \frac{\Delta V}{I} = \frac{120\ \text{V}}{0.625\ \text{A}} = 192\ \Omega$$

Now, presuming the bulb resistance is unchanged,

$$I = \frac{120\ \text{V}}{193.6\ \Omega} = 0.620\ \text{A}$$

Across the bulb is $\Delta V = IR = 192\ \Omega(0.620\ \text{A}) = 119\ \text{V}$

so its power is $\mathcal{P} = I\Delta V = 0.620\ \text{A}(119\ \text{V}) = \boxed{73.8\ \text{W}}$

P28.7 If we turn the given diagram on its side, we find that it is the same as figure (a). The $20.0\ \Omega$ and $5.00\ \Omega$ resistors are in series, so the first reduction is shown in (b). In addition, since the $10.0\ \Omega$, $5.00\ \Omega$, and $25.0\ \Omega$ resistors are then in parallel, we can solve for their equivalent resistance as:

$$R_{\text{eq}} = \frac{1}{\left(\dfrac{1}{10.0\ \Omega} + \dfrac{1}{5.00\ \Omega} + \dfrac{1}{25.0\ \Omega}\right)} = 2.94\ \Omega$$

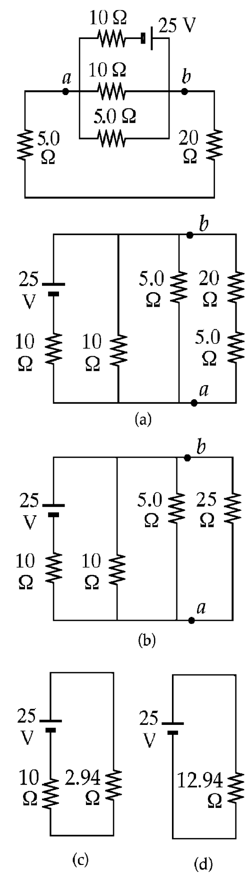


FIG. P28.7

This is shown in figure (c), which in turn reduces to the circuit shown in figure (d).

Next, we work backwards through the diagrams applying $I = \dfrac{\Delta V}{R}$ and $\Delta V = IR$ alternately to every resistor, real and equivalent. The $12.94\ \Omega$ resistor is connected across 25.0 V, so the current through the battery in every diagram is

$$I = \frac{\Delta V}{R} = \frac{25.0\ \text{V}}{12.94\ \Omega} = 1.93\ \text{A}$$

In figure (c), this 1.93 A goes through the $2.94\ \Omega$ equivalent resistor to give a potential difference of:

$$\Delta V = IR = (1.93\ \text{A})(2.94\ \Omega) = 5.68\ \text{V}$$

From figure (b), we see that this potential difference is the same across ΔV_{ab}, the 10 Ω resistor, and the 5.00 Ω resistor.

Thus we have first found the answer to part (b), which is $\Delta V_{ab} = \boxed{5.68\ \text{V}}$.

(a) Since the current through the $20.0\ \Omega$ resistor is also the current through the $25.0\ \Omega$ line *ab*,

$$I = \frac{\Delta V_{ab}}{R_{ab}} = \frac{5.68\ \text{V}}{25.0\ \Omega} = 0.227\ \text{A} = \boxed{227\ \text{mA}}$$

P28.8 We assume that the metal wand makes low-resistance contact with the person's hand and that the resistance through the person's body is negligible compared to the resistance R_{shoes} of the shoe soles. The equivalent resistance seen by the power supply is $1.00\ \text{M}\Omega + R_{\text{shoes}}$. The current through both resistors is $\dfrac{50.0\ \text{V}}{1.00\ \text{M}\Omega + R_{\text{shoes}}}$. The voltmeter displays

$$\Delta V = I(1.00\ \text{M}\Omega) = \frac{50.0\ \text{V}(1.00\ \text{M}\Omega)}{1.00\ \text{M}\Omega + R_{\text{shoes}}} = \Delta V$$

(a) We solve to obtain $\quad 50.0\ \text{V}(1.00\ \text{M}\Omega) = \Delta V(1.00\ \text{M}\Omega) + \Delta V\left(R_{\text{shoes}}\right)$

$$R_{\text{shoes}} = \frac{1.00\ \text{M}\Omega(50.0 - \Delta V)}{\Delta V}$$

(b) With $R_{\text{shoes}} \to 0$, the current through the person's body is

$$\frac{50.0\ \text{V}}{1.00\ \text{M}\Omega} = 50.0\ \mu\text{A} \qquad \boxed{\text{The current will never exceed } 50\ \mu\text{A.}}$$

P28.9 (a) Since all the current in the circuit must pass through the series 100 Ω resistor, $\mathcal{P} = I^2R$

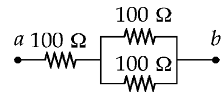


FIG. P28.9

$$\mathcal{P}_{\max} = RI_{\max}^2$$

so $$I_{\max} = \sqrt{\frac{\mathcal{P}}{R}} = \sqrt{\frac{25.0\text{ W}}{100\ \Omega}} = 0.500\text{ A}$$

$$R_{eq} = 100\ \Omega + \left(\frac{1}{100} + \frac{1}{100}\right)^{-1}\ \Omega = 150\ \Omega$$

$$\Delta V_{\max} = R_{\text{eq}} I_{\max} = \boxed{75.0\text{ V}}$$

(b) $$\mathcal{P}_1 = I\Delta V = (0.500\text{ A})(75.0\text{ V}) = \boxed{37.5\text{ W}}\text{ total power}$$

$$\mathcal{P}_1 = \boxed{25.0\text{ W}}$$

$$\mathcal{P}_2 = \mathcal{P}_3 = RI^2(100\ \Omega)(0.250\text{ A})^2 = \boxed{6.25\text{ W}}$$

P28.10 Using 2.00-Ω, 3.00-Ω, and 4.00-Ω resistors, there are 7 series, 4 parallel, and 6 mixed combinations:

Series		Parallel	Mixed
2.00 Ω	6.00 Ω	0.923 Ω	1.56 Ω
3.00 Ω	7.00 Ω	1.20 Ω	2.00 Ω
4.00 Ω	9.00 Ω	1.33 Ω	2.22 Ω
5.00 Ω		1.71 Ω	3.71 Ω
			4.33 Ω
			5.20 Ω

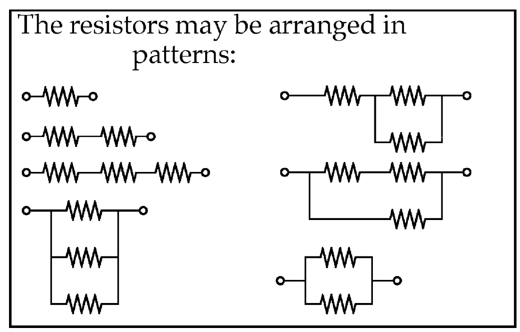


FIG. P28.10

P28.11 *When S is open*, R_1, R_2, R_3 are in series with the battery. Thus:

$$R_1 + R_2 + R_3 = \frac{6\text{ V}}{10^{-3}\text{ A}} = 6\text{ k}\Omega \qquad (1)$$

When S is closed in position 1, the parallel combination of the two R_2's is in series with R_1, R_3, and the battery. Thus:

$$R_1 + \frac{1}{2}R_2 + R_3 = \frac{6\text{ V}}{1.2\times 10^{-3}\text{ A}} = 5\text{ k}\Omega \qquad (2)$$

When S is closed in position 2, R_1 and R_2 are in series with the battery. R_3 is shorted. Thus:

$$R_1 + R_2 = \frac{6\text{ V}}{2\times 10^{-3}\text{ A}} = 3\text{ k}\Omega \qquad (3)$$

From (1) and (3): $R_3 = 3\text{ k}\Omega$.

Subtract (2) from (1): $R_2 = 2\text{ k}\Omega$.

From (3): $R_1 = 1\text{ k}\Omega$.

Answers: $\boxed{R_1 = 1.00\text{ k}\Omega,\ R_2 = 2.00\text{ k}\Omega,\ R_3 = 3.00\text{ k}\Omega}$.

P28.12 Denoting the two resistors as x and y,

$$x+y=690, \quad \text{and} \quad \frac{1}{150}=\frac{1}{x}+\frac{1}{y}$$

$$\frac{1}{150}=\frac{1}{x}+\frac{1}{690-x}=\frac{(690-x)+x}{x(690-x)}$$

$$x^2-690x+103\,500=0$$

$$x=\frac{690\pm\sqrt{(690)^2-414\,000}}{2}$$

$$x=\boxed{470\ \Omega} \qquad y=\boxed{220\ \Omega}$$

***P28.13** The resistance between a and b $\boxed{\text{decreases}}$. Closing the switch opens a new path with resistance of only 20 Ω. The original resistance is $R+\dfrac{1}{\dfrac{1}{90+10}+\dfrac{1}{10+90}}=R+50\ \Omega$.

The final resistance is $R+\dfrac{1}{\dfrac{1}{90}+\dfrac{1}{10}}+\dfrac{1}{\dfrac{1}{10}+\dfrac{1}{90}}=R+18\ \Omega$.

We require $R+50\ \Omega=2(R+18\ \Omega)$ so $R=\boxed{14.0\ \Omega}$

***P28.14** (a) The resistors 2, 3, and 4 can be combined to a single $2R$ resistor. This is in series with resistor 1, with resistance R, so the equivalent resistance of the whole circuit is $3R$. In series, potential difference is shared in proportion to the resistance, so resistor 1 gets $\frac{1}{3}$ of the battery voltage and the 2-3-4 parallel combination gets $\frac{2}{3}$ of the battery voltage. This is the potential difference across resistor 4, but resistors 2 and 3 must share this voltage. In this branch $\frac{1}{3}$ goes to 2 and $\frac{2}{3}$ to 3. The ranking by potential difference is $\boxed{\Delta V_4>\Delta V_3>\Delta V_1>\Delta V_2}$.

(b) Based on the reasoning above the potential differences are $\boxed{\Delta V_1=\dfrac{\mathcal{E}}{3},\ \Delta V_2=\dfrac{2\mathcal{E}}{9},\ \Delta V_3=\dfrac{4\mathcal{E}}{9},\ \Delta V_4=\dfrac{2\mathcal{E}}{3}}$.

(c) All the current goes through resistor 1, so it gets the most. The current then splits at the parallel combination. Resistor 4 gets more than half, because the resistance in that branch is less than in the other branch. Resistors 2 and 3 have equal currents because they are in series. The ranking by current is $\boxed{I_1>I_4>I_2=I_3}$.

(d) Resistor 1 has a current of I. Because the resistance of 2 and 3 in series is twice that of resistor 4, twice as much current goes through 4 as through 2 and 3. The currents through the resistors are $\boxed{I_1=I,\ I_2=I_3=\dfrac{I}{3},\ I_4=\dfrac{2I}{3}}$.

(e) Increasing resistor 3 increases the equivalent resistance of the entire circuit. The current in the battery, which is the current through resistor 1, decreases. This decreases the potential difference across resistor 1, increasing the potential difference across the parallel combination. With a larger potential difference the current through resistor 4 is increased. With

continued on next page

more current through 4, and less in the circuit to start with, the current through resistors 2 and 3 must decrease. To summarize, $\boxed{I_4 \text{ increases and } I_1,\ I_2, \text{ and } I_3 \text{ decrease}}$.

(f) If resistor 3 has an infinite resistance it blocks any current from passing through that branch, and the circuit effectively is just resistor 1 and resistor 4 in series with the battery. The circuit now has an equivalent resistance of $4R$. The current in the circuit drops to $\frac{3}{4}$ of the original current because the resistance has increased by $\frac{4}{3}$. All this current passes through resistors 1 and 4, and none passes through 2 or 3. Therefore $\boxed{I_1 = \frac{3I}{4},\ I_2 = I_3 = 0,\ I_4 = \frac{3I}{4}}$.

P28.15 $R_p = \left(\frac{1}{3.00} + \frac{1}{1.00}\right)^{-1} = 0.750\ \Omega$

$R_s = (2.00 + 0.750 + 4.00)\ \Omega = 6.75\ \Omega$

$I_{\text{battery}} = \frac{\Delta V}{R_s} = \frac{18.0\ \text{V}}{6.75\ \Omega} = 2.67\ \text{A}$

$\mathcal{P}_2 = I^2R$: $\quad \mathcal{P}_2 = (2.67\ \text{A})^2(2.00\ \Omega)$

$\mathcal{P}_2 = \boxed{14.2\ \text{W}}$ in $2.00\ \Omega$

$\mathcal{P}_4 = (2.67\ \text{A})^2(4.00\ \text{A}) = \boxed{28.4\ \text{W}}$ in $4.00\ \Omega$

$\Delta V_2 = (2.67\ \text{A})(2.00\ \Omega) = 5.33\ \text{V},$

$\Delta V_4 = (2.67\ \text{A})(4.00\ \Omega) = 10.67\ \text{V}$

$\Delta V_p = 18.0\ \text{V} - \Delta V_2 - \Delta V_4 = 2.00\ \text{V}\left(= \Delta V_3 = \Delta V_1\right)$

$\mathcal{P}_3 = \frac{(\Delta V_3)^2}{R_3} = \frac{(2.00\ \text{V})^2}{3.00\ \Omega} = \boxed{1.33\ \text{W}}$ in $3.00\ \Omega$

$\mathcal{P}_1 = \frac{(\Delta V_1)}{R_1} = \frac{(2.00\ \text{V})^2}{1.00\ \Omega} = \boxed{4.00\ \text{W}}$ in $1.00\ \Omega$

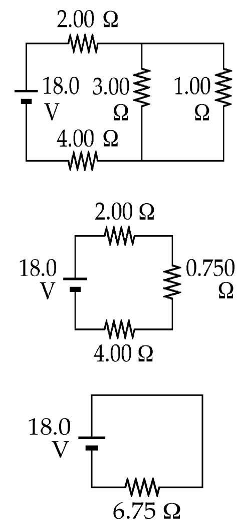


FIG. P28.15

Section 28.3 Kirchhoff's Rules

P28.16 $+15.0 - (7.00)I_1 - (2.00)(5.00) = 0$

$5.00 = 7.00\ I_1$ so $\boxed{I_1 = 0.714\ \text{A}}$

$I_3 = I_1 + I_2 = 2.00\ \text{A}$

$0.714 + I_2 = 2.00$ so $\boxed{I_2 = 1.29\ \text{A}}$

$+\mathcal{E} - 2.00(1.29) - 5.00(2.00) = 0$ $\quad \boxed{\mathcal{E} = 12.6\ \text{V}}$

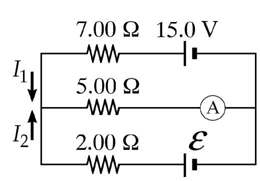


FIG. P28.16

P28.17 We name currents I_1, I_2, and I_3 as shown.

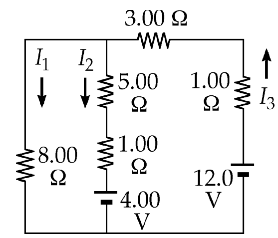


FIG. P28.17

From Kirchhoff's current rule, $I_3 = I_1 + I_2$.

Applying Kirchhoff's voltage rule to the loop containing I_2 and I_3,

$12.0\text{ V} - (4.00)I_3 - (6.00)I_2 - 4.00\text{ V} = 0$

$8.00 = (4.00)I_3 + (6.00)I_2$

Applying Kirchhoff's voltage rule to the loop containing I_1 and I_2,

$$-(6.00)I_2 - 4.00\text{ V} + (8.00)I_1 = 0 \qquad (8.00)I_1 = 4.00 + (6.00)I_2$$

Solving the above linear system, we proceed to the pair of simultaneous equations:

$$\begin{cases} 8 = 4I_1 + 4I_2 + 6I_2 \\ 8I_1 = 4 + 6I_2 \end{cases} \quad \text{or} \quad \begin{cases} 8 = 4I_1 + 10I_2 \\ I_2 = 1.33I_1 - 0.667 \end{cases}$$

and to the single equation $8 = 4I_1 + 13.3I_1 - 6.67$

$$I_1 = \frac{14.7\text{ V}}{17.3\ \Omega} = 0.846\text{ A} \qquad \text{Then} \qquad I_2 = 1.33(0.846\text{ A}) - 0.667$$

and $\quad I_3 = I_1 + I_2 \qquad$ give $\quad \boxed{I_1 = 846\text{ mA},\ I_2 = 462\text{ mA},\ I_3 = 1.31\text{ A}}$

All currents are in the directions indicated by the arrows in the circuit diagram.

P28.18 The solution figure is shown to the right.

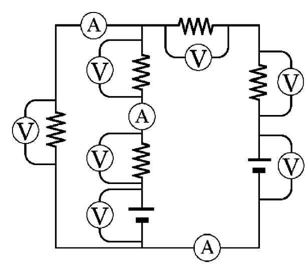


FIG. P28.18

***P28.19** We use the results of Problem 28.17.

(a) By the 4.00-V battery: $\Delta U = (\Delta V)I\Delta t = (4.00\text{ V})(-0.462\text{ A})120\text{ s} = \boxed{-222\text{ J}}$

By the 12.0-V battery: $(12.0\text{ V})(1.31\text{ A})120\text{ s} = \boxed{1.88\text{ kJ}}$

(b) By the 8.00-Ω resistor: $I^2R\Delta t = (0.846\text{ A})^2(8.00\ \Omega)120\text{ s} = \boxed{687\text{ J}}$

By the 5.00-Ω resistor: $(0.462\text{ A})^2(5.00\ \Omega)120\text{ s} = \boxed{128\text{ J}}$

By the 1.00-Ω resistor: $(0.462\text{ A})^2(1.00\ \Omega)120\text{ s} = \boxed{25.6\text{ J}}$

By the 3.00-Ω resistor: $(1.31\text{ A})^2(3.00\ \Omega)120\text{ s} = \boxed{616\text{ J}}$

By the 1.00-Ω resistor: $(1.31\text{ A})^2(1.00\ \Omega)120\text{ s} = \boxed{205\text{ J}}$

(c) $-222\text{ J} + 1.88\text{ kJ} = \boxed{1.66\text{ kJ}}$ from chemical to electrically transmitted. Like a child counting his lunch money twice, we can count the same energy again, $687\text{ J} + 128\text{ J} + 25.6\text{ J} + 616\text{ J} + 205\text{ J} = 1.66\text{ kJ}$, as it is transformed from electrically transmitted to internal. The net energy transformation is $\boxed{\text{from chemical to internal}}$.

***P28.20** (a) The first equation represents Kirchhoff's loop theorem. We choose to think of it as describing a clockwise trip around the left-hand loop in a circuit; see Figure (a). For the right-hand loop see Figure (b). The junctions must be between the 5.8 V and the $370\ \Omega$ and between the $370\ \Omega$ and the $150\ \Omega$. Then we have Figure (c). This is consistent with the third equation,

$$I_1 + I_3 - I_2 = 0$$
$$I_2 = I_1 + I_3$$

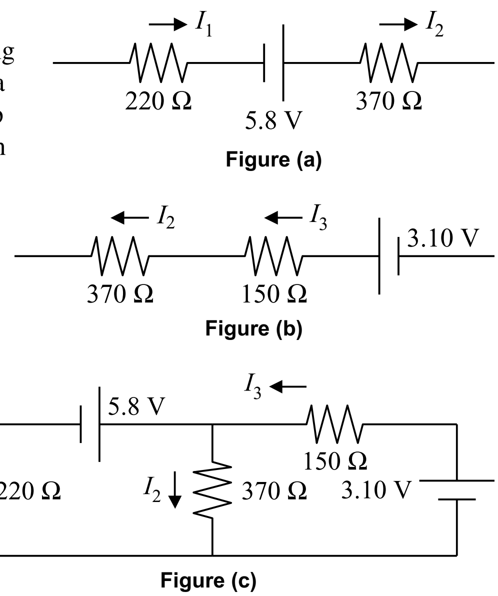


FIG. P28.20

(b) We substitute:

$$-220I_1 + 5.8 - 370I_1 - 370I_3 = 0$$
$$+370I_1 + 370I_3 + 150I_3 - 3.1 = 0$$

Next

$$I_3 = \frac{5.8 - 590I_1}{370}$$
$$370I_1 + \frac{520}{370}(5.8 - 590I_1) - 3.1 = 0$$
$$370I_1 + 8.15 - 829I_1 - 3.1 = 0$$
$$I_1 = \frac{5.05\text{ V}}{459\ \Omega} = \boxed{\text{11.0 mA in the 220-}\Omega\text{ resistor and out of the positive pole of the 5.8-V battery}}$$
$$I_3 = \frac{5.8 - 590(0.011\,0)}{370} = -1.87\text{ mA}$$

$\boxed{\text{The current is 1.87 mA in the 150-}\Omega\text{ resistor and out of the negative pole of the 3.1-V battery.}}$

$$I_2 = 11.0 - 1.87 = \boxed{9.13\text{ mA in the 370-}\Omega\text{ resistor}}$$

***P28.21** Let I_6 represent the current in the ammeter and the top 6-Ω resistor. The bottom 6-Ω resistor has the same potential difference across it, so it carries an equal current. For the top loop we have

$6\text{ V} - 10\ \Omega\, I_{10} - 6\ \Omega\, I_6 = 0$

For the bottom loop, $4.5 - 5\, I_5 - 6\, I_6 = 0$.

For the junctions on the left side, taken together, $+ I_{10} + I_5 - I_6 - I_6 = 0$.

We eliminate $I_{10} = 0.6 - 0.6\, I_6$ and $I_5 = 0.9 - 1.2\, I_6$ by substitution:

$0.6 - 0.6\, I_6 + 0.9 - 1.2\, I_6 - 2\, I_6 = 0 \quad I_6 = 1.5/3.8 = \boxed{0.395\text{ A}}$

The loop theorem for the little loop containing the voltmeter gives

$+ 6\text{ V} - \Delta V - 4.5\text{ V} = 0 \qquad \Delta V = \boxed{1.50\text{ V}}$

P28.22 Label the currents in the branches as shown in the first figure. Reduce the circuit by combining the two parallel resistors as shown in the second figure.

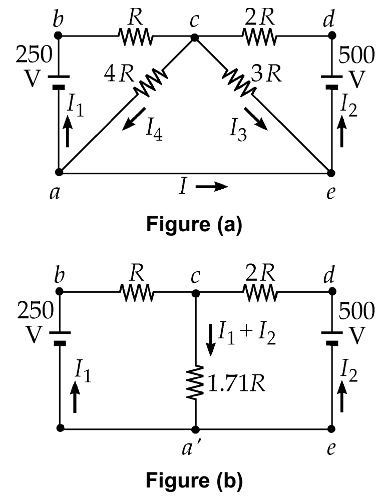


FIG. P28.22

Apply Kirchhoff's loop rule to both loops in Figure (b) to obtain:

$$(2.71R)I_1 + (1.71R)I_2 = 250$$

and $$(1.71R)I_1 + (3.71R)I_2 = 500$$

With $R = 1\,000\ \Omega$, simultaneous solution of these equations yields:

$$I_1 = 10.0 \text{ mA}$$

and $$I_2 = 130.0 \text{ mA}$$

From Figure (b), $$V_c - V_a = (I_1 + I_2)(1.71R) = 240 \text{ V}$$

Thus, from Figure (a), $$I_4 = \frac{V_c - V_a}{4R} = \frac{240 \text{ V}}{4\,000\ \Omega} = 60.0 \text{ mA}$$

Finally, applying Kirchhoff's point rule at point a in Figure (a) gives:

$$I = I_4 - I_1 = 60.0 \text{ mA} - 10.0 \text{ mA} = +50.0 \text{ mA}$$

or $$I = \boxed{50.0 \text{ mA from point } a \text{ to point } e}$$

P28.23 Name the currents as shown in the figure to the right. Then $w + x + z = y$. Loop equations are

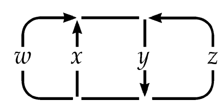


FIG. P28.23

$$-200w - 40.0 + 80.0x = 0$$
$$-80.0x + 40.0 + 360 - 20.0y = 0$$
$$+360 - 20.0y - 70.0z + 80.0 = 0$$

Eliminate y by substitution. $$\begin{cases} x = 2.50w + 0.500 \\ 400 - 100x - 20.0w - 20.0z = 0 \\ 440 - 20.0w - 20.0x - 90.0z = 0 \end{cases}$$

Eliminate x. $$\begin{cases} 350 - 270w - 20.0z = 0 \\ 430 - 70.0w - 90.0z = 0 \end{cases}$$

Eliminate $z = 17.5 - 13.5w$ to obtain $$430 - 70.0w - 1\,575 + 1\,215w = 0$$

$$w = \frac{70.0}{70.0} = \boxed{1.00 \text{ A upward in } 200\ \Omega}$$

Now $$z = \boxed{4.00 \text{ A upward in } 70.0\ \Omega}$$

$$x = \boxed{3.00 \text{ A upward in } 80.0\ \Omega}$$

$$y = \boxed{8.00 \text{ A downward in } 20.0\ \Omega}$$

and for the 200 Ω, $$\Delta V = IR = (1.00 \text{ A})(200\ \Omega) = \boxed{200 \text{ V}}$$

P28.24 Using Kirchhoff's rules,

$$12.0-(0.010\,0)I_1-(0.060\,0)I_3=0$$
$$10.0+(1.00)I_2-(0.060\,0)I_3=0$$

and $I_1=I_2+I_3$

$$12.0-(0.010\,0)I_2-(0.070\,0)I_3=0$$
$$10.0+(1.00)I_2-(0.060\,0)I_3=0$$

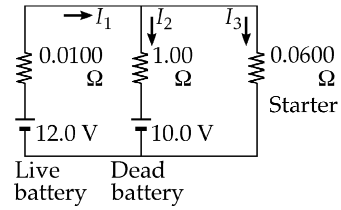


FIG. P28.24

Solving simultaneously,

$I_2=$ **0.283 A downward** in the dead battery

and $I_3=$ **171 A downward** in the starter

The currents are forward in the live battery and in the starter, relative to normal starting operation. The current is backward in the dead battery, tending to charge it up.

P28.25 We name the currents I_1, I_2, and I_3 as shown.

(a) $I_1=I_2+I_3$

Counterclockwise around the top loop,

$$12.0\text{ V}-(2.00\ \Omega)I_3-(4.00\ \Omega)I_1=0$$

Traversing the bottom loop,

$$8.00\text{ V}-(6.00\ \Omega)I_2+(2.00\ \Omega)I_3=0$$

$$I_1=3.00-\frac{1}{2}I_3,\quad I_2=\frac{4}{3}+\frac{1}{3}I_3,\quad\text{and}\quad \boxed{I_3=909\text{ mA}}$$

(b) $V_a-(0.909\text{ A})(2.00\ \Omega)=V_b$

$$V_b-V_a=\boxed{-1.82\text{ V}}$$

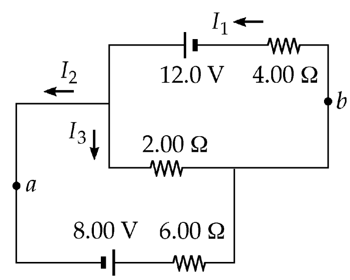


FIG. P28.25

P28.26

$$\Delta V_{ab}=(1.00)I_1+(1.00)(I_1-I_2)$$
$$\Delta V_{ab}=(1.00)I_1+(1.00)I_2+(5.00)(I-I_1+I_2)$$
$$\Delta V_{ab}=(3.00)(I-I_1)+(5.00)(I-I_1+I_2)$$

Let $I=1.00$ A, $I_1=x$, and $I_2=y$.

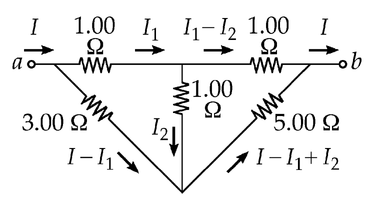


FIG. P28.26

Then, the three equations become:

$$\Delta V_{ab}=2.00x-y,\ \text{or}\ y=2.00x-\Delta V_{ab}$$

$$\Delta V_{ab}=-4.00x+6.00y+5.00$$

and $\Delta V_{ab}=8.00-8.00x+5.00y$

Substituting the first into the last two gives:

$$7.00\Delta V_{ab}=8.00x+5.00\quad\text{and}\quad 6.00\Delta V_{ab}=2.00x+8.00$$

Solving these simultaneously yields $\Delta V_{ab}=\frac{27}{17}$ V.

Then, $R_{ab}=\frac{\Delta V_{ab}}{I}=\frac{\frac{27}{17}\text{ V}}{1.00\text{ A}}$ or $\boxed{R_{ab}=\frac{27}{17}\ \Omega}$

Section 28.4 ***RC* Circuits**

P28.27 (a) $RC = \left(1.00\times10^{6}\ \Omega\right)\left(5.00\times10^{-6}\ \text{F}\right) = \boxed{5.00\ \text{s}}$

(b) $Q = C\mathcal{E} = \left(5.00\times10^{-6}\ \text{C}\right)(30.0\ \text{V}) = \boxed{150\ \mu\text{C}}$

(c) $I(t) = \dfrac{\mathcal{E}}{R}e^{-t/RC} = \left(\dfrac{30.0}{1.00\times10^{6}}\right)\exp\left[\dfrac{-10.0}{\left(1.00\times10^{6}\right)\left(5.00\times10^{-6}\right)}\right]$

$= \boxed{4.06\ \mu\text{A}}$

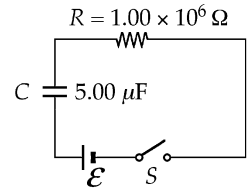


FIG. P28.27

P28.28 The potential difference across the capacitor $\Delta V(t) = \Delta V_{\max}\left(1 - e^{-t/RC}\right)$

Using 1 Farad $= 1\ \text{s}/\Omega$, $4.00\ \text{V} = (10.0\ \text{V})\left[1 - e^{-(3.00\ \text{s})/\left[R\left(10.0\times10^{-6}\ \text{s}/\Omega\right)\right]}\right]$

Therefore, $0.400 = 1.00 - e^{-\left(3.00\times10^{5}\ \Omega\right)/R}$

Or $e^{-\left(3.00\times10^{5}\ \Omega\right)/R} = 0.600$

Taking the natural logarithm of both sides, $-\dfrac{3.00\times10^{5}\ \Omega}{R} = \ln(0.600)$

and $R = -\dfrac{3.00\times10^{5}\ \Omega}{\ln(0.600)} = +5.87\times10^{5}\ \Omega = \boxed{587\ \text{k}\Omega}$

P28.29 (a) $I(t) = -I_0 e^{-t/RC}$

$$I_0 = \frac{Q}{RC} = \frac{5.10\times10^{-6}\ \text{C}}{(1\,300\ \Omega)\left(2.00\times10^{-9}\ \text{F}\right)} = 1.96\ \text{A}$$

$$I(t) = -(1.96\ \text{A})\exp\left[\frac{-9.00\times10^{-6}\ \text{s}}{(1\,300\ \Omega)\left(2.00\times10^{-9}\ \text{F}\right)}\right] = \boxed{-61.6\ \text{mA}}$$

(b) $q(t) = Qe^{-t/RC} = (5.10\ \mu\text{C})\exp\left[\dfrac{-8.00\times10^{-6}\ \text{s}}{(1\ 300\ \Omega)\left(2.00\times10^{-9}\ \text{F}\right)}\right] = \boxed{0.235\ \mu\text{C}}$

(c) The magnitude of the maximum current is $I_0 = \boxed{1.96\ \text{A}}$.

P28.30 We are to calculate

$$\int_0^{\infty} e^{-2t/RC}dt = -\frac{RC}{2}\int_0^{\infty} e^{-2t/RC}\left(-\frac{2dt}{RC}\right) = -\frac{RC}{2}e^{-2t/RC}\Big|_0^{\infty} = -\frac{RC}{2}\left[e^{-\infty} - e^{0}\right] = -\frac{RC}{2}[0-1] = \boxed{+\frac{RC}{2}}$$

P28.31 (a) Call the potential at the left junction V_L and at the right V_R. After a "long" time, the capacitor is fully charged.

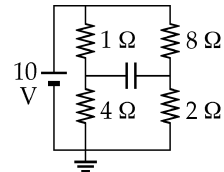


FIG. P28.31(a)

$V_L = 8.00$ V because of voltage divider:

$$I_L = \frac{10.0 \text{ V}}{5.00 \ \Omega} = 2.00 \text{ A}$$

$$V_L = 10.0 \text{ V} - (2.00 \text{ A})(1.00 \ \Omega) = 8.00 \text{ V}$$

Likewise, $$V_R = \left(\frac{2.00 \ \Omega}{2.00 \ \Omega + 8.00 \ \Omega}\right)(10.0 \text{ V}) = 2.00 \text{ V}$$

or $$I_R = \frac{10.0 \text{ V}}{10.0 \ \Omega} = 1.00 \text{ A}$$

$$V_R = (10.0 \text{ V}) - (8.00 \ \Omega)(1.00 \text{ A}) = 2.00 \text{ V}$$

Therefore, $$\Delta V = V_L - V_R = 8.00 - 2.00 = \boxed{6.00 \text{ V}}$$

(b) Redraw the circuit $$R = \frac{1}{(1/9.00 \ \Omega) + (1/6.00 \ \Omega)} = 3.60 \ \Omega$$

$$RC = 3.60 \times 10^{-6} \text{ s}$$

and $$e^{-t/RC} = \frac{1}{10}$$

so $$t = RC \ln 10 = \boxed{8.29 \ \mu\text{s}}$$

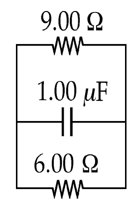


FIG. P28.31(b)

P28.32 (a) $$\tau = RC = \left(1.50 \times 10^5 \ \Omega\right)\left(10.0 \times 10^{-6} \text{ F}\right) = \boxed{1.50 \text{ s}}$$

(b) $$\tau = \left(1.00 \times 10^5 \ \Omega\right)\left(10.0 \times 10^{-6} \text{ F}\right) = \boxed{1.00 \text{ s}}$$

(c) The battery carries current $$\frac{10.0 \text{ V}}{50.0 \times 10^3 \ \Omega} = 200 \ \mu\text{A}$$

The 100 kΩ carries current of magnitude $$I = I_0 e^{-t/RC} = \left(\frac{10.0 \text{ V}}{100 \times 10^3 \ \Omega}\right) e^{-t/1.00 \text{ s}}$$

So the switch carries downward current $$\boxed{200 \ \mu\text{A} + (100 \ \mu\text{A}) e^{-t/1.00 \text{ s}}}$$

Section 28.5 **Electrical Meters**

P28.33 $\Delta V = I_g r_g = \left(I - I_g\right) R_p$, or $R_p = \dfrac{I_g r_g}{\left(I - I_g\right)} = \dfrac{I_g (60.0 \ \Omega)}{\left(I - I_g\right)}$

Therefore, to have $I = 0.100 \text{ A} = 100 \text{ mA}$ when $I_g = 0.500 \text{ mA}$:

$$R_p = \frac{(0.500 \text{ mA})(60.0 \ \Omega)}{99.5 \text{ mA}} = \boxed{0.302 \ \Omega}$$

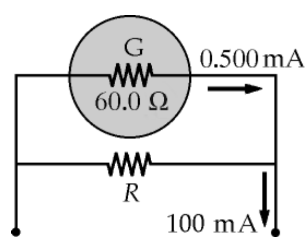


FIG. P28.33

P28.34 *Ammeter:* $I_g r = (0.500\text{ A} - I_g)(0.220\ \Omega)$

or $I_g(r + 0.220\ \Omega) = 0.110\text{ V}$ (1)

Voltmeter: $2.00\text{ V} = I_g(r + 2\,500\ \Omega)$ (2)

Solve (1) and (2) simultaneously to find:

$$I_g = \boxed{0.756\text{ mA}} \quad \text{and} \quad r = \boxed{145\ \Omega}$$

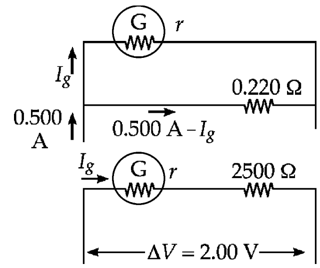


FIG. P28.34

P28.35 Series Resistor → Voltmeter

$\Delta V = IR$: $25.0 = 1.50 \times 10^{-3}(R_s + 75.0)$

Solving, $\boxed{R_s = 16.6\text{ k}\Omega}$

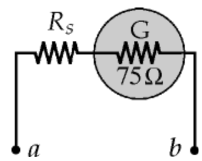


FIG. P28.35

***P28.36** (a) In Figure (a), the emf sees an equivalent resistance of 200.00 Ω.

$$I = \frac{6.000\,0\text{ V}}{200.00\ \Omega} = \boxed{0.030\,000\text{ A}}$$

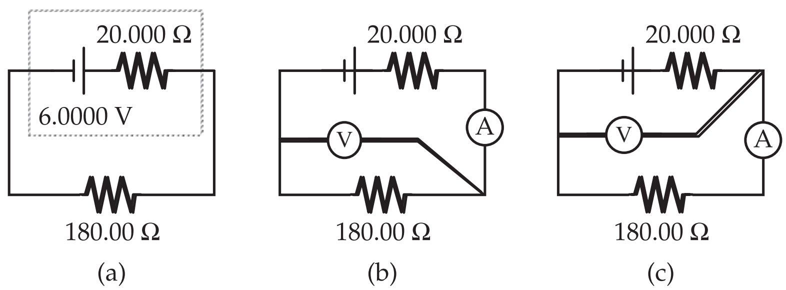


FIG. P28.36

The terminal potential difference is $\Delta V = IR = (0.030\,000\text{ A})(180.00\ \Omega) = \boxed{5.400\,0\text{ V}}$

(b) In Figure (b), $R_{eq} = \left(\dfrac{1}{180.00\ \Omega} + \dfrac{1}{20\,000\ \Omega}\right)^{-1} = 178.39\ \Omega$

The equivalent resistance across the emf is $178.39\ \Omega + 0.500\,00\ \Omega + 20.000\ \Omega = 198.89\ \Omega$

The ammeter reads $I = \dfrac{\mathcal{E}}{R} = \dfrac{6.000\,0\text{ V}}{198.89\ \Omega} = \boxed{0.030\,167\text{ A}}$

and the voltmeter reads $\Delta V = IR = (0.030\,167\text{ A})(178.39\ \Omega) = \boxed{5.381\,6\text{ V}}$

(c) In Figure (c), $\left(\dfrac{1}{180.50\ \Omega} + \dfrac{1}{20\,000\ \Omega}\right)^{-1} = 178.89\ \Omega$

Therefore, the emf sends current through $R_{tot} = 178.89\ \Omega + 20.000\ \Omega = 198.89\ \Omega$

The current through the battery is but not all of this goes through the ammeter. $I = \dfrac{6.000\,0\text{ V}}{198.89\ \Omega} = 0.030\,168\text{ A}$

The voltmeter reads $\Delta V = IR = (0.030\,168\text{ A})(178.89\ \Omega) = \boxed{5.396\,6\text{ V}}$

The ammeter measures current $I = \dfrac{\Delta V}{R} = \dfrac{5.396\,6\text{ V}}{180.50\ \Omega} = \boxed{0.029\,898\text{ A}}$.

continued on next page

(d) Both circuits are good enough for some measurements. The connection in Figure (c) gives data leading to value of resistance that is too high by only about 0.3%. Its value is more accurate than the value, 0.9% too low, implied by the data from the circuit in part (b).

Section 28.6 **Household Wiring and Electrical Safety**

P28.37 (a) $\mathcal{P} = I\Delta V$: So for the Heater, $I = \dfrac{\mathcal{P}}{\Delta V} = \dfrac{1\,500\text{ W}}{120\text{ V}} = \boxed{12.5\text{ A}}$

For the Toaster, $I = \dfrac{750\text{ W}}{120\text{ V}} = \boxed{6.25\text{ A}}$

And for the Grill, $I = \dfrac{1\,000\text{ W}}{120\text{ V}} = \boxed{8.33\text{ A}}$

(b) $12.5 + 6.25 + 8.33 = \boxed{27.1\text{ A}}$

The current draw is greater than 25.0 amps, so a circuit with this circuit breaker would not be sufficient.

P28.38 (a) Suppose that the insulation between either of your fingers and the conductor adjacent is a chunk of rubber with contact area 4 mm^2 and thickness 1 mm. Its resistance is

$$R = \frac{\rho\ell}{A} \approx \frac{\left(10^{13}\ \Omega\cdot\text{m}\right)\left(10^{-3}\text{ m}\right)}{4\times10^{-6}\text{ m}^2} \approx 2\times10^{15}\ \Omega$$

The current will be driven by 120 V through total resistance (series)

$$2\times10^{15}\ \Omega + 10^{4}\ \Omega + 2\times10^{15}\ \Omega \approx 5\times10^{15}\ \Omega$$

It is: $I = \dfrac{\Delta V}{R} \sim \dfrac{120\text{ V}}{5\times10^{15}\ \Omega}\boxed{\sim10^{-14}\text{ A}}$.

(b) The resistors form a voltage divider, with the center of your hand at potential $\dfrac{V_h}{2}$, where V_h is the potential of the "hot" wire. The potential difference between your finger and thumb is $\Delta V = IR \sim \left(10^{-14}\text{ A}\right)\left(10^{4}\ \Omega\right) \sim 10^{-10}\text{ V}$. So the points where the rubber meets your fingers are at potentials of

$$\boxed{\sim\frac{V_h}{2} + 10^{-10}\text{ V}} \quad \text{and} \quad \boxed{\sim\frac{V_h}{2} - 10^{-10}\text{ V}}$$

Additional Problems

***P28.39** Several seconds is many time constants, so the capacitor is fully charged and (d) the current in its branch is $\boxed{\text{zero}}$.

Center loop: $+8\text{ V} + 3\ \Omega\, I_2 - 5\ \Omega\, I_1 = 0$

Right loop: $+4\text{ V} - 3\ \Omega\, I_2 - 5\ \Omega\, I_3 = 0$

Top junction: $+I_1 + I_2 - I_3 = 0$

Now we will eliminate $I_1 = 1.6 + 0.6I_2$ and $I_3 = 0.8 - 0.6I_2$

by substitution: $1.6 + 0.6I_2 + I_2 - 0.8 + 0.6I_2 = 0$ Then $I_2 = -0.8/2.2 = -0.3636$

So (b) $\boxed{\text{the current in 3 }\Omega\text{ is 0.364 A down}}$.

Now (a) $I_3 = 0.8 - 0.6(-0.364) = \boxed{1.02\text{ A down in 4 V and in 5 }\Omega}$.

(c) $I_1 = 1.6 + 0.6(-0.364) = \boxed{1.38\text{ A up in the 8 V battery}}$

(e) For the left loop $+3\text{ V} - Q/6\ \mu\text{F} + 8\text{ V} = 0$ so $Q = 6\ \mu\text{F}\ 11\text{ V} = \boxed{66.0\ \mu\text{C}}$

***P28.40** The current in the battery is $\dfrac{15\text{ V}}{10\ \Omega+\dfrac{1}{1/5\ \Omega+1/8\ \Omega}}=1.15\text{ A}.$

The voltage across 5 Ω is 15 V – 10 Ω 1.15 A = 3.53 V.

(a) The current in it is 3.53 V/5 Ω = 0.706 A.

(b) $\mathcal{P}=3.53\text{ V }0.706\text{ A}=\boxed{2.49\text{ W}}$

(c) Only the circuit in Figure P28.40c requires the use of Kirchhoff's rules for solution. In the other circuits the 5-Ω and 8-Ω resistors are still in parallel with each other.

(d) The power is lowest in Figure P28.40c. The circuits in Figures P28.40b and P28.40d have in effect 30-V batteries driving the current.

P28.41 The set of four batteries boosts the electric potential of each bit of charge that goes through them by $4\times1.50\text{ V}=6.00\text{ V}$. The chemical energy they store is

$$\Delta U=q\Delta V=(240\text{ C})(6.00\text{ J/C})=1\,440\text{ J}$$

The radio draws current $I=\dfrac{\Delta V}{R}=\dfrac{6.00\text{ V}}{200\ \Omega}=0.030\,0\text{ A}$

So, its power is $\mathcal{P}=(\Delta V)I=(6.00\text{ V})(0.030\,0\text{ A})=0.180\text{ W}=0.180\text{ J/s}$

Then for the time the energy lasts,
we have $\mathcal{P}=\dfrac{E}{\Delta t}$: $\Delta t=\dfrac{E}{\mathcal{P}}=\dfrac{1\,440\text{ J}}{0.180\text{ J/s}}=8.00\times10^3\text{ s}$

We could also compute this from $I=\dfrac{Q}{\Delta t}$: $\Delta t=\dfrac{Q}{I}=\dfrac{240\text{ C}}{0.030\,0\text{ A}}=8.00\times10^3\text{ s}=\boxed{2.22\text{ h}}$

***P28.42** $I=\dfrac{\mathcal{E}}{R+r}$, so $\mathcal{P}=I^2R=\dfrac{\mathcal{E}^2R}{(R+r)^2}$ or $(R+r)^2=\left(\dfrac{\mathcal{E}^2}{\mathcal{P}}\right)R$

Let $x\equiv\dfrac{\mathcal{E}^2}{\mathcal{P}}$, then $(R+r)^2=xR$ or $R^2+(2r-x)R-r^2=0$

With $r=1.20\ \Omega$, this becomes $R^2+(2.40-x)R-1.44=0$

which has solutions of $R=\dfrac{-(2.40-x)\pm\sqrt{(2.40-x)^2-5.76}}{2}$

(a) With $\mathcal{E}=9.20\text{ V}$ and $\mathcal{P}=12.8\text{ W}$, $x=6.61$:

$$R=\frac{+4.21\pm\sqrt{(4.21)^2-5.76}}{2}=\boxed{3.84\ \Omega}\text{ or }\boxed{0.375\ \Omega}.$$ Either external resistance extracts the same power from the battery.

(b) For $\mathcal{E}=9.20\text{ V}$ and $\mathcal{P}=21.2\text{ W}$, $x\equiv\dfrac{\mathcal{E}^2}{\mathcal{P}}=3.99$

$$R=\frac{+1.59\pm\sqrt{(1.59)^2-5.76}}{2}=\frac{1.59\pm\sqrt{-3.22}}{2}$$

The equation for the load resistance yields a complex number, so $\boxed{\text{there is no resistance}}$ that will extract 21.2 W from this battery. The maximum power output occurs when $R=r=1.20\ \Omega$, and that maximum is $\mathcal{P}_{max}=\dfrac{\mathcal{E}^2}{4r}=17.6\text{ W}$.

P28.43 Using Kirchhoff's loop rule for the closed loop, $+12.0-2.00I-4.00I=0$, so $I=2.00\text{ A}$

$$V_b-V_a=+4.00\text{ V}-(2.00\text{ A})(4.00\ \Omega)-(0)(10.0\ \Omega)=-4.00\text{ V}$$

Thus, $|\Delta V_{ab}|=\boxed{4.00\text{ V}}$ and $\boxed{\text{point } a \text{ is at the higher potential}}$.

P28.44 (a) $R_{\text{eq}}=3R$ $\quad I=\dfrac{\mathcal{E}}{3R}$ $\quad \mathcal{P}_{\text{series}}=\mathcal{E}I=\boxed{\dfrac{\mathcal{E}^2}{3R}}$

(b) $R_{\text{eq}}=\dfrac{1}{(1/R)+(1/R)+(1/R)}=\dfrac{R}{3}$ $\quad I=\dfrac{3\mathcal{E}}{R}$ $\quad \mathcal{P}_{\text{parallel}}=\mathcal{E}I=\boxed{\dfrac{3\mathcal{E}^2}{R}}$

(c) Nine times more power is converted in the $\boxed{\text{parallel}}$ connection.

***P28.45** The charging current is given by $14.7\text{ V}-13.2\text{ V}-I\,0.85\ \Omega=0$ $\quad I=1.76\text{ A}$

The energy delivered by the 14.7 V supply is $\Delta VIt=14.7\text{ V }1.76\text{ A }1.80\text{ h }(3\,600\text{ s/h})=168\,000\text{ J}$

The energy stored in the battery is $13.2\text{ V }1.76\text{ A }1.8\,(3\,600\text{ s})=151\,000\text{ J}$

The same energy is released by the emf of the battery: $13.2\text{ V }(I)\,7.3\,(3\,600\text{ s})=151\,000\text{ J}$

so the discharge current is $I=0.435\text{ A}$

The load resistor is given by $13.2\text{ V}-(0.435\text{ A})\,R-(0.435\text{ A})\,0.85\ \Omega=0$

$R=(12.8\text{ V})/0.435\text{ A}=29.5\ \Omega$

The energy delivered to the load is $\Delta VIt=I^2R\,t=(0.435\text{ A})^2\,29.5\ \Omega\,(7.3)\,3\,600\text{ s}=147\,000\text{ J}$

The efficiency is $147\,000\text{ J}/168\,000\text{ J}=\boxed{0.873}$

P28.46 (a) $\mathcal{E}-I\left(\sum R\right)-(\mathcal{E}_1+\mathcal{E}_2)=0$

$$40.0\text{ V}-(4.00\text{ A})\left[(2.00+0.300+0.300+R)\Omega\right]-(6.00+6.00)\text{ V}=0;\quad \text{so}\quad R=\boxed{4.40\ \Omega}$$

(b) Inside the supply, $\mathcal{P}=I^2R=(4.00\text{ A})^2(2.00\ \Omega)=\boxed{32.0\text{ W}}$

Inside both batteries together, $\mathcal{P}=I^2R=(4.00\text{ A})^2(0.600\ \Omega)=\boxed{9.60\text{ W}}$

For the limiting resistor, $\mathcal{P}=(4.00\text{ A})^2(4.40\ \Omega)=\boxed{70.4\text{ W}}$

(c) $\mathcal{P}=I(\mathcal{E}_1+\mathcal{E}_2)=(4.00\text{ A})\left[(6.00+6.00)\text{ V}\right]=\boxed{48.0\text{ W}}$

P28.47 Let the two resistances be x and y.

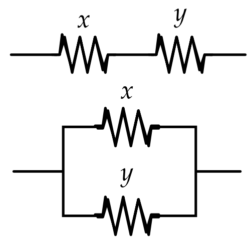


FIG. P28.47

Then, $R_s = x + y = \frac{\mathcal{P}_s}{I^2} = \frac{225\text{ W}}{(5.00\text{ A})^2} = 9.00\ \Omega \qquad y = 9.00\ \Omega - x$

and $R_p = \frac{xy}{x+y} = \frac{\mathcal{P}_p}{I^2} = \frac{50.0\text{ W}}{(5.00\text{ A})^2} = 2.00\ \Omega$

so $\frac{x(9.00\ \Omega - x)}{x + (9.00\ \Omega - x)} = 2.00\ \Omega \qquad x^2 - 9.00x + 18.0 = 0$

Factoring the second equation, $(x - 6.00)(x - 3.00) = 0$

so $x = 6.00\ \Omega$ or $x = 3.00\ \Omega$

Then, $y = 9.00\ \Omega - x$ gives $y = 3.00\ \Omega$ or $y = 6.00\ \Omega$

There is only one physical answer: The two resistances are $\boxed{6.00\ \Omega}$ and $\boxed{3.00\ \Omega}$.

P28.48 Let the two resistances be x and y.

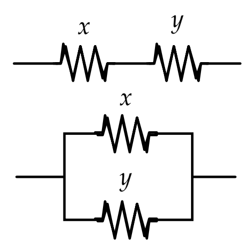


FIG. P28.48

Then, $R_s = x + y = \frac{\mathcal{P}_s}{I^2}$ and $R_p = \frac{xy}{x+y} = \frac{\mathcal{P}_p}{I^2}$.

From the first equation, $y = \frac{\mathcal{P}_s}{I^2} - x$, and the second

becomes $\frac{x\left(\mathcal{P}_s/I^2 - x\right)}{x + \left(\mathcal{P}_s/I^2 - x\right)} = \frac{\mathcal{P}_p}{I^2}$ or $x^2 - \left(\frac{\mathcal{P}_s}{I^2}\right)x + \frac{\mathcal{P}_s\mathcal{P}_p}{I^4} = 0$.

Using the quadratic formula, $x = \frac{\mathcal{P}_s \pm \sqrt{\mathcal{P}_s^2 - 4\mathcal{P}_s\mathcal{P}_p}}{2I^2}$.

Then, $y = \frac{\mathcal{P}_s}{I^2} - x$ gives $y = \frac{\mathcal{P}_s \mp \sqrt{\mathcal{P}_s^2 - 4\mathcal{P}_s\mathcal{P}_p}}{2I^2}$.

The two resistances are $\boxed{\frac{\mathcal{P}_s + \sqrt{\mathcal{P}_s^2 - 4\mathcal{P}_s\mathcal{P}_p}}{2I^2}}$ and $\boxed{\frac{\mathcal{P}_s - \sqrt{\mathcal{P}_s^2 - 4\mathcal{P}_s\mathcal{P}_p}}{2I^2}}$.

P28.49 (a) $\Delta V_1 = \Delta V_2 \qquad I_1R_1 = I_2R_2$

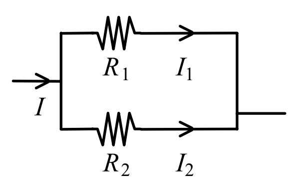


FIG. P28.49(a)

$$I = I_1 + I_2 = I_1 + \frac{I_1R_1}{R_2} = I_1\frac{R_2 + R_1}{R_2}$$

$$\boxed{I_1 = \frac{IR_2}{R_1 + R_2}} \qquad I_2 = \frac{I_1R_1}{R_2} = \boxed{\frac{IR_1}{R_1 + R_2} = I_2}$$

(b) The power delivered to the pair is $\mathcal{P} = I_1^2R_1 + I_2^2R_2 = I_1^2R_1 + \left(I - I_1\right)^2 R_2$. For minimum power we want to find I_1 such that $\frac{d\mathcal{P}}{dI_1} = 0$.

$$\frac{d\mathcal{P}}{dI_1} = 2I_1R_1 + 2\left(I - I_1\right)(-1)R_2 = 0 \qquad I_1R_1 - IR_2 + I_1R_2 = 0$$

$I_1 = \frac{IR_2}{R_1 + R_2}$ This is the same condition as that found in part (a).

***P28.50** (a) When the capacitor is fully charged, no current exists in its branch. The current in the left resistors is 5 V/83 Ω = 0.060 2 A. The current in the right resistors is 5 V/(2 Ω + R). Relative to the negative side of the battery, the left capacitor plate is at voltage 80 Ω (0.060 2 A) = 4.82 V. The right plate is at R (5 V)/(2 Ω + R).
The voltage across the capacitor is 4.82 V – R (5 V)/(2 Ω + R).
The charge on the capacitor is

$$Q = 3\ \mu\text{F}\ [4.82\ \text{V} - R\ (5\ \text{V})/(2\ \Omega + R)] = \boxed{(28.9\ \Omega - 0.542\ R)\ \mu\text{C}/(2\ \Omega + R)}$$

(b) With $R = 10\ \Omega$, $Q = (28.9 - 5.42)\ \mu\text{C}/(2 + 10) = \boxed{1.96\ \mu\text{C}}$

(c) $\boxed{\text{Yes.}}$ $Q = 0$ when $28.9\ \Omega - 0.542\ R = 0$ $R = \boxed{53.3\ \Omega}$

(d) The maximum charge occurs for $R = 0$. It is 28.9/2 = $\boxed{14.5\ \mu\text{C}}$.

(e) $\boxed{\text{Yes. Taking } R = \infty \text{ corresponds to disconnecting a wire to remove the branch containing } R.}$

In this case $|Q| = 0.542\ R/R = \boxed{0.542\ \mu\text{C}}$.

P28.51 Let R_m = measured value, R = actual value,

I_R = current through the resistor R

I = current measured by the ammeter

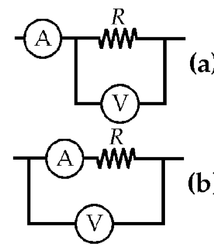


FIG. P28.51

(a) When using circuit (a), $I_R R = \Delta V = 20\ 000(I - I_R)$ or

$$R = 20\ 000\left[\frac{I}{I_R} - 1\right]$$

But since $I = \dfrac{\Delta V}{R_m}$ and $I_R = \dfrac{\Delta V}{R}$, we have $\dfrac{I}{I_R} = \dfrac{R}{R_m}$

and $R = 20\ 000\dfrac{(R - R_m)}{R_m}$ (1)

When $R > R_m$, we require $\dfrac{(R - R_m)}{R} \le 0.050\ 0$

Therefore, $R_m \ge R(1 - 0.050\ 0)$ and from (1) we find $\boxed{R \le 1\ 050\ \Omega}$

(b) When using circuit (b), $I_R R = \Delta V - I_R(0.5\ \Omega)$

But since $I_R = \dfrac{\Delta V}{R_m}$, $R_m = (0.500 + R)$ (2)

When $R_m > R$, we require $\dfrac{(R_m - R)}{R} \le 0.050\ 0$

From (2) we find $\boxed{R \ge 10.0\ \Omega}$

P28.52 The battery supplies energy at a changing rate $\frac{dE}{dt} = \mathcal{P} = \mathcal{E}I = \mathcal{E}\left(\frac{\mathcal{E}}{R}e^{-1/RC}\right)$

Then the total energy put out by the battery is $\int dE = \int_{t=0}^{\infty} \frac{\mathcal{E}^2}{R} \exp\left(-\frac{t}{RC}\right) dt$

$$\int dE = \frac{\mathcal{E}^2}{R}(-RC)\int_0^{\infty} \exp\left(-\frac{t}{RC}\right)\left(-\frac{dt}{RC}\right) = -\mathcal{E}^2 C \exp\left(-\frac{t}{RC}\right)\Big|_0^{\infty} = -\mathcal{E}^2 C[0-1] = \mathcal{E}^2 C$$

The power delivered to the resistor is $\frac{dE}{dt} = \mathcal{P} = \Delta V_R I = I^2 R = R\frac{\mathcal{E}^2}{R^2}\exp\left(-\frac{2t}{RC}\right)$

So the total internal energy appearing in the resistor is $\int dE = \int_0^{\infty} \frac{\mathcal{E}^2}{R} \exp\left(-\frac{2t}{RC}\right) dt$

$$\int dE = \frac{\mathcal{E}^2}{R}\left(-\frac{RC}{2}\right)\int_0^{\infty} \exp\left(-\frac{2t}{RC}\right)\left(-\frac{2dt}{RC}\right) = -\frac{\mathcal{E}^2 C}{2} \exp\left(-\frac{2t}{RC}\right)\Big|_0^{\infty} = -\frac{\mathcal{E}^2 C}{2}[0-1] = \frac{\mathcal{E}^2 C}{2}$$

The energy finally stored in the capacitor is $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}C\mathcal{E}^2$. Thus, energy of the circuit is conserved $\mathcal{E}^2 C = \frac{1}{2}\mathcal{E}^2 C + \frac{1}{2}\mathcal{E}^2 C$ and resistor and capacitor share equally in the energy from the battery.

P28.53 (a) $q = C\Delta V\left(1 - e^{-t/RC}\right)$

$$q = \left(1.00\times10^{-6}\text{ F}\right)(10.0\text{ V})\left[1 - e^{-10.0/\left[\left(2.00\times10^{6}\right)\left(1.00\times10^{-6}\right)\right]}\right] = \boxed{9.93\ \mu\text{C}}$$

(b) $I = \frac{dq}{dt} = \left(\frac{\Delta V}{R}\right)e^{-t/RC}$

$$I = \left(\frac{10.0\text{ V}}{2.00\times10^{6}\ \Omega}\right)e^{-5.00} = 3.37\times10^{-8}\text{ A} = \boxed{33.7\text{ nA}}$$

(c) $\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}\frac{q^2}{C}\right) = \left(\frac{q}{C}\right)\frac{dq}{dt} = \left(\frac{q}{C}\right)I$

$$\frac{dU}{dt} = \left(\frac{9.93\times10^{-6}\text{ C}}{1.00\times10^{-6}\text{ C/V}}\right)\left(3.37\times10^{-8}\text{ A}\right) = 3.34\times10^{-7}\text{ W} = \boxed{334\text{ nW}}$$

(d) $\mathcal{P}_{\text{battery}} = I\mathcal{E} = \left(3.37\times10^{-8}\text{ A}\right)(10.0\text{ V}) = 3.37\times10^{-7}\text{ W} = \boxed{337\text{ nW}}$

The battery power could also be computed as the sum of the instantaneous powers delivered to the resistor and to the capacitor:

$$I^2R + dU/dt = (33.7\times10^{-9}\text{ A})^2\, 2\times10^{6}\ \Omega + 334\text{ nW} = 337\text{ nW}$$

***P28.54** (a) We find the resistance intrinsic to the vacuum cleaner:

$$\mathcal{P} = I\Delta V = \frac{(\Delta V)^2}{R}$$

$$R = \frac{(\Delta V)^2}{\mathcal{P}} = \frac{(120\ \text{V})^2}{535\ \text{W}} = 26.9\ \Omega$$

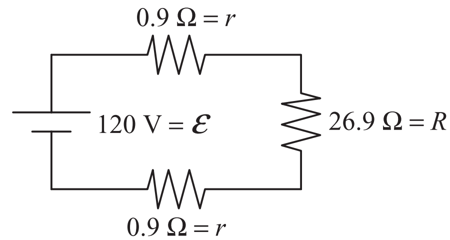


FIG. P28.54

with the inexpensive cord, the equivalent resistance is $0.9\ \Omega + 26.9\ \Omega + 0.9\ \Omega = 28.7\ \Omega$ so the current throughout the circuit is

$$I = \frac{\mathcal{E}}{R_{\text{Tot}}} = \frac{120\ \text{V}}{28.7\ \Omega} = 4.18\ \text{A}$$

and the cleaner power is

$$\mathcal{P}_{\text{cleaner}} = I(\Delta V)_{\text{cleaner}} = I^2 R = (4.18\ \text{A})^2 (26.9\ \Omega) = \boxed{470\ \text{W}}$$

In symbols, $R_{\text{Tot}} = R + 2r$, $I = \dfrac{\mathcal{E}}{R + 2r}$ and $\mathcal{P}_{\text{cleaner}} = I^2 R = \dfrac{\mathcal{E}^2 R}{(R+2r)^2}$

(b) $$R + 2r = \left(\frac{\mathcal{E}^2 R}{\mathcal{P}_{\text{cleaner}}}\right)^{1/2} = 120\ \text{V}\left(\frac{26.9\ \Omega}{525\ \text{W}}\right)^{1/2} = 27.2\ \Omega$$

$$r = \frac{27.2\ \Omega - 26.9\ \Omega}{2} = 0.128\ \Omega = \frac{\rho\ell}{A} = \frac{\rho\ell 4}{\pi d^2}$$

$$d = \left(\frac{4\rho\ell}{\pi r}\right)^{1/2} = \left(\frac{4\left(1.7\times 10^{-8}\ \Omega\cdot\text{m}\right)(15\ \text{m})}{\pi(0.128\ \Omega)}\right)^{1/2} = \boxed{1.60\ \text{mm or more}}$$

(c) Unless the extension cord is a superconductor, it is impossible to attain cleaner power 535 W. To move from 525 W to 532 W will require a lot more copper, as we show here:

$$r = \frac{\mathcal{E}}{2}\left(\frac{R}{\mathcal{P}_{\text{cleaner}}}\right)^{1/2} - \frac{R}{2} = \frac{120\ \text{V}}{2}\left(\frac{26.9\ \Omega}{532\ \text{W}}\right)^{1/2} - \frac{26.9\ \Omega}{2} = 0.037\,9\ \Omega$$

$$d = \left(\frac{4\rho\ell}{\pi r}\right)^{1/2} = \left(\frac{4\left(1.7\times 10^{-8}\ \Omega\cdot\text{m}\right)(15\ \text{m})}{\pi\left(0.037\,9\ \Omega\right)}\right)^{1/2} = \boxed{2.93\ \text{mm or more}}$$

P28.55 (a) First determine the resistance of each light bulb: $\mathcal{P}=\frac{(\Delta V)^2}{R}$

$$R=\frac{(\Delta V)^2}{\mathcal{P}}=\frac{(120\text{ V})^2}{60.0\text{ W}}=240\ \Omega$$

We obtain the equivalent resistance R_{eq} of the network of light bulbs by identifying series and parallel equivalent resistances:

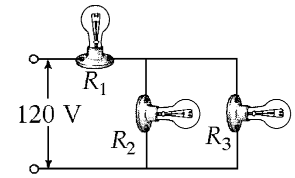


FIG. P28.55

$$R_{eq}=R_1+\frac{1}{(1/R_2)+(1/R_3)}=240\ \Omega+120\ \Omega=360\ \Omega$$

The total power dissipated in the 360 Ω is $\mathcal{P}=\frac{(\Delta V)^2}{R_{eq}}=\frac{(120\text{ V})^2}{360\ \Omega}=\boxed{40.0\text{ W}}$

(b) The current through the network is given by $\mathcal{P}=I^2R_{eq}$: $I=\sqrt{\frac{\mathcal{P}}{R_{eq}}}=\sqrt{\frac{40.0\text{ W}}{360\ \Omega}}=\frac{1}{3}\text{ A}$

The potential difference across R_1 is $\Delta V_1=IR_1=\left(\frac{1}{3}\text{ A}\right)(240\ \Omega)=\boxed{80.0\text{ V}}$

The potential difference ΔV_{23} across the parallel combination of R_2 and R_3 is

$$\Delta V_{23}=IR_{23}=\left(\frac{1}{3}\text{ A}\right)\left(\frac{1}{(1/240\ \Omega)+(1/240\ \Omega)}\right)=\boxed{40.0\text{ V}}$$

P28.56 (a) With the switch closed, current exists in a simple series circuit as shown. The capacitors carry no current. For R_2 we have

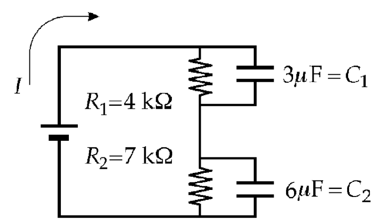


FIG. P28.56(a)

$$\mathcal{P}=I^2R_2 \qquad I=\sqrt{\frac{\mathcal{P}}{R_2}}=\sqrt{\frac{2.40\text{ V}\cdot\text{A}}{7\,000\text{ V/A}}}=18.5\text{ mA}$$

The potential difference across R_1 and C_1 is

$$\Delta V=IR_1=(1.85\times10^{-2}\text{ A})(4\,000\text{ V/A})=74.1\text{ V}$$

The charge on C_1

$$Q=C_1\Delta V=(3.00\times10^{-6}\text{ C/V})(74.1\text{ V})=\boxed{222\ \mu\text{C}}$$

The potential difference across R_2 and C_2 is

$$\Delta V=IR_2=(1.85\times10^{-2}\text{ A})(7\,000\ \Omega)=130\text{ V}$$

The charge on C_2

$$Q=C_2\Delta V=(6.00\times10^{-6}\text{ C/V})(130\text{ V})=778\ \mu\text{C}$$

The battery emf is

$$IR_{eq}=I(R_1+R_2)=1.85\times10^{-2}\text{ A}(4\,000+7\,000)\text{ V/A}=204\text{ V}$$

(b) In equilibrium after the switch has been opened, no current exists. The potential difference across each resistor is zero. The full 204 V appears across both capacitors. The new charge on C_2 is

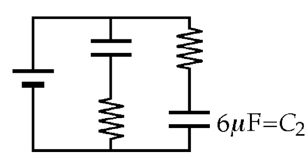


FIG. P28.56(b)

$$Q=C_2\Delta V=(6.00\times10^{-6}\text{ C/V})(204\text{ V})=1\,222\ \mu\text{C}$$

for a change of $1\,222\ \mu\text{C}-778\ \mu\text{C}=\boxed{444\ \mu\text{C}}$.

***P28.57** (a) The emf of the battery is $\boxed{9.30\text{ V}}$. Its internal resistance is given by

$9.30\text{ V} - 3.70\text{ A}\,r = 0 \qquad r = \boxed{2.51\ \Omega}$

(b) Total emf = 20(9.30 V) = $\boxed{186\text{ V}}$. The maximum current is given by

$20(9.30\text{ V}) - 20(2.51\ \Omega)\,I - 0\,I = 0 \qquad I = \boxed{3.70\text{ A}}$

(c) For the circuit $20(9.30\text{ V}) - 20(2.51\ \Omega)\,I - 120\ \Omega\,I = 0 \quad I = 186\text{ V}/170\ \Omega = \boxed{1.09\text{ A}}$

(d) $\mathcal{P} = I^2R = (1.09\text{ A})^2\,120\ \Omega = \boxed{143\text{ W}}$. This is a potentially deadly situation.

(e) The potential difference across his body is 120 Ω (0.005 00 A) = 0.600 V.

This must be the terminal potential difference of the bank of batteries:

$186\text{ V} - I_{tot}\,20(2.51\ \Omega) = 0.6\text{ V} \qquad I_{tot} = 185.4\text{ V}/50.3\ \Omega = 3.688\text{ A}$

For the copper wire we then have $0.6\text{ V} = (3.688\text{ A} - 0.005\text{ A})\,R \quad R = \boxed{0.163\ \Omega}$

(f) For the experimenter's body, $\mathcal{P} = I\Delta V = 0.005\text{ A}\ 0.6\text{ V} = \boxed{3.00\text{ mW}}$.

(g) For the wire $\mathcal{P} = I\Delta V = 3.683\text{ A}\ 0.6\text{ V} = \boxed{2.21\text{ W}}$.

(h) The power output of the emf depends on the resistance connected to it. A question about "the rest of the power" is not meaningful when it compares circuits with different currents. The net emf produces more current in the circuit where the copper wire is used. The net emf delivers more power when the copper wire is used, 687 W rather than 203 W without the wire. Nearly all of this power results in extra internal energy in the internal resistance of the batteries, which rapidly rise to a high temperature. The circuit with the copper wire is unsafe because the batteries overheat. The circuit without the copper wire is unsafe because it delivers an electric shock to the experimenter.

***P28.58** The battery current is

$(150+45+14+4)\text{ mA} = 213\text{ mA}$

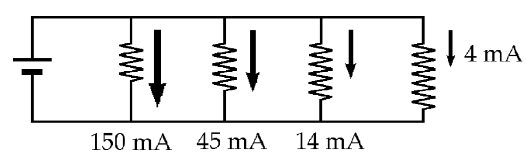
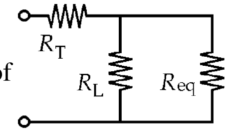


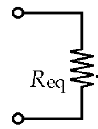
FIG. P28.58

(a) The resistor with highest resistance is that carrying 4 mA. Doubling its resistance will reduce the current it carries to 2 mA. Then the total current is $(150+45+14+2)\text{ mA} = 211\text{ mA}$, nearly the same as before. The ratio is $\frac{211}{213} = \boxed{0.991}$.

(b) The resistor with least resistance carries 150 mA. Doubling its resistance changes this current to 75 mA and changes the total to $(75+45+14+4)\text{ mA} = 138\text{ mA}$. The ratio is $\frac{138}{213} = \boxed{0.648}$, representing a much larger reduction (35.2% instead of 0.9%).

(c) This problem is precisely analogous. As a battery maintained a potential difference in parts (a) and (b), a furnace maintains a temperature difference here. Energy flow by heat is analogous to current and takes place through thermal resistances in parallel. Each resistance can have its "*R*-value" increased by adding insulation. Doubling the thermal resistance of the attic door will produce only a negligible (0.9%) saving in fuel. The ceiling originally has the smallest thermal resistance. Doubling the thermal resistance of $\boxed{\text{the ceiling}}$ will produce a much larger saving.

P28.59 From the hint, the equivalent resistance of 

is just 

.

That is, $$R_T + \frac{1}{1/R_L + 1/R_{eq}} = R_{eq}$$

$$R_T + \frac{R_L R_{eq}}{R_L + R_{eq}} = R_{eq}$$

$$R_T R_L + R_T R_{eq} + R_L R_{eq} = R_L R_{eq} + R_{eq}^2$$

$$R_{eq}^2 - R_T R_{eq} - R_T R_L = 0$$

$$R_{eq} = \frac{R_T \pm \sqrt{R_T^2 - 4(1)(-R_T R_L)}}{2(1)}$$

Only the + sign is physical:

$$R_{eq} = \frac{1}{2}\left(\sqrt{4R_T R_L + R_T^2} + R_T\right)$$

For example, if $R_T = 1\ \Omega$

And $R_L = 20\ \Omega,\ R_{eq} = 5\ \Omega$

P28.60 (a) First let us flatten the circuit on a 2-D plane as shown; then reorganize it to a format easier to read. Notice that the two resistors shown in the top horizontal branch carry the same current as the resistors in the horizontal branch second from the top. The center junctions in these two branches are at the same potential. The vertical resistor between these two junctions has no potential difference across it and carries no current. This middle resistor can be removed without affecting the circuit. The remaining resistors over the three parallel branches have equivalent resistance

$$R_{eq} = \left(\frac{1}{20} + \frac{1}{20} + \frac{1}{10}\right)^{-1} = \boxed{5.00\ \Omega}$$

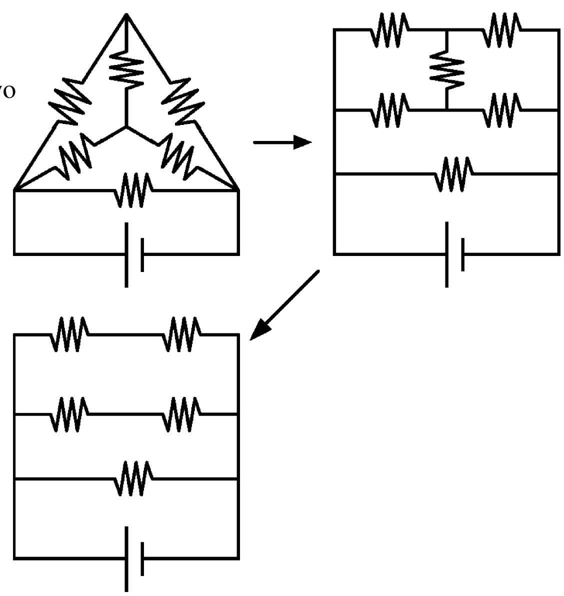


FIG. P28.60(a)

(b) So the current through the battery is

$$\frac{\Delta V}{R_{eq}} = \frac{12.0\ \text{V}}{5.00\ \Omega} = \boxed{2.40\ \text{A}}$$

P28.61 (a) After steady-state conditions have been reached, there is no DC current through the capacitor.

Thus, for R_3: $\boxed{I_{R_3} = 0 \text{ (steady-state)}}$

For the other two resistors, the steady-state current is simply determined by the 9.00-V emf across the 12-kΩ and 15-kΩ resistors in series:

For R_1 and R_2: $$I_{(R_1+R_2)} = \frac{\mathcal{E}}{R_1+R_2} = \frac{9.00 \text{ V}}{(12.0 \text{ k}\Omega + 15.0 \text{ k}\Omega)} = \boxed{333\ \mu\text{A (steady-state)}}$$

(b) After the transient currents have ceased, the potential difference across C is the same as the potential difference across $R_2 (= IR_2)$ because there is no voltage drop across R_3. Therefore, the charge Q on C is

$$Q = C(\Delta V)_{R_2} = C(IR_2) = (10.0\ \mu\text{F})(333\ \mu\text{A})(15.0 \text{ k}\Omega)$$
$$= \boxed{50.0\ \mu\text{C}}$$

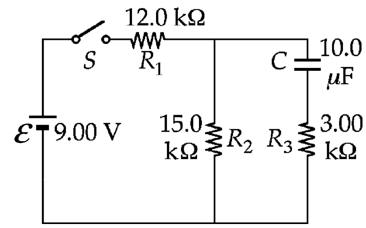


FIG. P28.61(b)

(c) When the switch is opened, the branch containing R_1 is no longer part of the circuit. The capacitor discharges through $(R_2 + R_3)$ with a time constant of $(R_2 + R_3)C = (15.0 \text{ k}\Omega + 3.00 \text{ k}\Omega)(10.0\ \mu\text{F}) = 0.180$ s. The initial current I_i in this discharge circuit is determined by the initial potential difference across the capacitor applied to $(R_2 + R_3)$ in series:

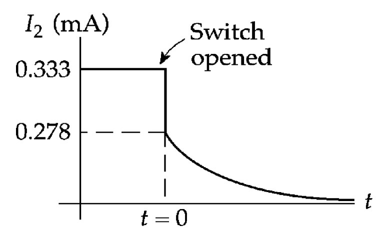


FIG. P28.61(c)

$$I_i = \frac{(\Delta V)_C}{(R_2+R_3)} = \frac{IR_2}{(R_2+R_3)} = \frac{(333\ \mu\text{A})(15.0 \text{ k}\Omega)}{(15.0 \text{ k}\Omega + 3.00 \text{ k}\Omega)} = 278\ \mu\text{A}$$

Thus, when the switch is opened, the current through R_2 changes instantaneously from $333\ \mu\text{A}$ (downward) to $278\ \mu\text{A}$ (downward) as shown in the graph. Thereafter, it decays according to

$$I_{R_2} = I_i e^{-t/(R_2+R_3)C} = \boxed{(278\ \mu\text{A})e^{-t/(0.180 \text{ s})} \text{ (for } t > 0)}$$

(d) The charge q on the capacitor decays from Q_i to $\frac{Q_i}{5}$ according to

$$q = Q_i e^{-t/(R_2+R_3)C}$$
$$\frac{Q_i}{5} = Q_i e^{(-t/0.180 \text{ s})}$$
$$5 = e^{t/0.180 \text{ s}}$$
$$\ln 5 = \frac{t}{180 \text{ ms}}$$
$$t = (0.180 \text{ s})(\ln 5) = \boxed{290 \text{ ms}}$$

P28.62 $\Delta V = \mathcal{E} e^{-t/RC}$

so $\ln\left(\frac{\mathcal{E}}{\Delta V}\right) = \left(\frac{1}{RC}\right)t$

A plot of $\ln\left(\frac{\mathcal{E}}{\Delta V}\right)$ versus t should be a straight line with slope equal to $\frac{1}{RC}$.

Using the given data values:

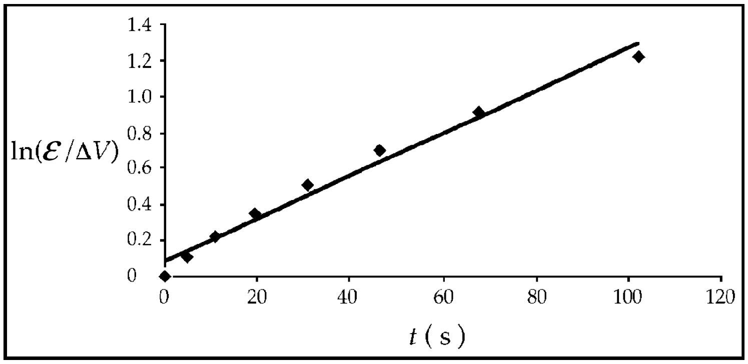


FIG. P28.62

(a) A least-square fit to this data yields the graph above.

$\sum x_i = 282,\qquad \sum x_i^2 = 1.86\times10^4,$

$\sum x_i y_i = 244,\qquad \sum y_i = 4.03,\qquad N = 8$

$$\text{Slope} = \frac{N\left(\sum x_i y_i\right) - \left(\sum x_i\right)\left(\sum y_i\right)}{N\left(\sum x_i^2\right) - \left(\sum x_i\right)^2} = 0.0118$$

$$\text{Intercept} = \frac{\left(\sum x_i^2\right)\left(\sum y_i\right) - \left(\sum x_i\right)\left(\sum x_i y_i\right)}{N\left(\sum x_i^2\right) - \left(\sum x_i\right)^2} = 0.088\,2$$

$t(s)$	$\Delta V(V)$	$\ln(\mathcal{E}/\Delta V)$
0	6.19	0
4.87	5.55	0.109
11.1	4.93	0.228
19.4	4.34	0.355
30.8	3.72	0.509
46.6	3.09	0.695
67.3	2.47	0.919
102.2	1.83	1.219

The equation of the best fit line is: $\boxed{\ln\left(\frac{\mathcal{E}}{\Delta V}\right) = (0.0118)t + 0.088\,2}$

(b) Thus, the time constant is $\tau = RC = \frac{1}{\text{slope}} = \frac{1}{0.0118} = \boxed{84.7\text{ s}}$

and the capacitance is $C = \frac{\tau}{R} = \frac{84.7\text{ s}}{10.0\times10^6\ \Omega} = \boxed{8.47\ \mu\text{F}}$

P28.63 (a) For the first measurement, the equivalent circuit is as shown in Figure 1.

$R_{ab} = R_1 = R_y + R_y = 2R_y$

so $R_y = \frac{1}{2}R_1$ (1)

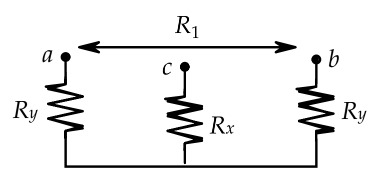


Figure 1

For the second measurement, the equivalent circuit is shown in Figure 2.

Thus, $R_{ac} = R_2 = \frac{1}{2}R_y + R_x$ (2)

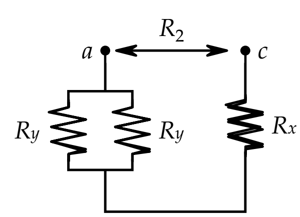


Figure 2

Substitute (1) into (2) to obtain:

$R_2 = \frac{1}{2}\left(\frac{1}{2}R_1\right) + R_x$, or $\boxed{R_x = R_2 - \frac{1}{4}R_1}$

FIG. P28.63

(b) If $R_1 = 13.0\ \Omega$ and $R_2 = 6.00\ \Omega$, then $\boxed{R_x = 2.75\ \Omega}$.

$\boxed{\text{The antenna is inadequately grounded}}$ since this exceeds the limit of $2.00\ \Omega$

P28.64 Start at the point when the voltage has just reached $\frac{2}{3}\Delta V$ and the switch has just closed. The voltage is $\frac{2}{3}\Delta V$ and is decaying towards 0 V with a time constant R_2C

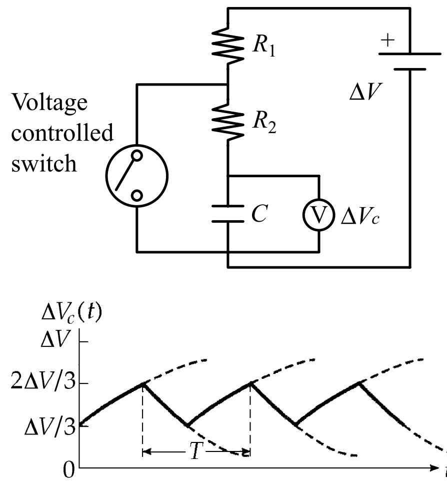


FIG. P28.64

$$\Delta V_C(t) = \left[\frac{2}{3}\Delta V\right]e^{-t/R_2C}$$

We want to know when $\Delta V_C(t)$ will reach $\frac{1}{3}\Delta V$.

Therefore, $\frac{1}{3}\Delta V = \left[\frac{2}{3}\Delta V\right]e^{-t/R_2C}$

or $e^{-t/R_2C} = \frac{1}{2}$

or $t_1 = R_2C\ln 2$

After the switch opens, the voltage is $\frac{1}{3}\Delta V$, increasing toward ΔV with time constant $(R_1+R_2)C$:

$$\Delta V_C(t) = \Delta V - \left[\frac{2}{3}\Delta V\right]e^{-t/(R_1+R_2)C}$$

When $\Delta V_C(t) = \frac{2}{3}\Delta V$

$\frac{2}{3}\Delta V = \Delta V - \frac{2}{3}\Delta V e^{-t/(R_1+R_2)C}$ or $e^{-t/(R_1+R_2)C} = \frac{1}{2}$

So $t_2 = (R_1+R_2)C\ln 2$ and $T = t_1 + t_2 = \boxed{(R_1+2R_2)C\ln 2}$

P28.65 A certain quantity of energy $\Delta E_{\text{int}} = \mathcal{P}(\text{time})$ is required to raise the temperature of the water to 100°C. For the power delivered to the heaters we have $\mathcal{P} = I\Delta V = \frac{(\Delta V)^2}{R}$ where (ΔV) is a constant. Thus, comparing coils 1 and 2, we have for the energy $\frac{(\Delta V)^2\Delta t}{R_1} = \frac{(\Delta V)^2 2\Delta t}{R_2}$. Then $R_2 = 2R_1$.

(a) When connected in parallel, the coils present equivalent resistance

$$R_p = \frac{1}{1/R_1 + 1/R_2} = \frac{1}{1/R_1 + 1/2R_1} = \frac{2R_1}{3}. \text{ Now } \frac{(\Delta V)^2\Delta t}{R_1} = \frac{(\Delta V)^2\Delta t_p}{2R_1/3} \qquad \Delta t_p = \boxed{\frac{2\Delta t}{3}}$$

(b) For the series connection, $R_s = R_1 + R_2 = R_1 + 2R_1 = 3R_1$ and $\frac{(\Delta V)^2\Delta t}{R_1} = \frac{(\Delta V)^2\Delta t_s}{3R_1}$

$\Delta t_s = \boxed{3\Delta t}$

P28.66 (a) We model the person's body and street shoes as shown. For the discharge to reach 100 V,

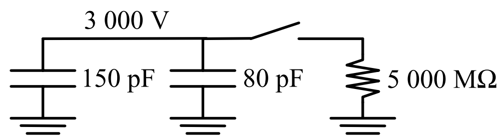


FIG. P28.66(a)

$$q(t) = Qe^{-t/RC} = C\Delta V(t) = C\Delta V_0 e^{-t/RC}$$

$$\frac{\Delta V}{\Delta V_0} = e^{-t/RC} \qquad \frac{\Delta V_0}{\Delta V} = e^{+t/RC}$$

$$\frac{t}{RC} = \ln\left(\frac{\Delta V_0}{\Delta V}\right)$$

$$t = RC\ln\left(\frac{\Delta V_0}{\Delta V}\right) = 5\,000\times 10^{6}\ \Omega\left(230\times 10^{-12}\ \text{F}\right)\ln\left(\frac{3\,000}{100}\right) = \boxed{3.91\ \text{s}}$$

(b) $t = \left(1\times 10^{6}\ \text{V/A}\right)\left(230\times 10^{-12}\ \text{C/V}\right)\ln 30 = \boxed{782\ \mu\text{s}}$

ANSWERS TO EVEN PROBLEMS

P28.2 (a) 4.59 Ω (b) 8.16%

P28.4 (a) 50.0% (b) $r = 0$ (c) High efficiency. The electric company's economic interest is to minimize internal energy production in its power lines, so that it can sell a large fraction of the energy output of its generators to the customers. (d) High power transfer. Energy by electric transmission is so cheap compared to the sound system that she does not spend extra money to buy an efficient amplifier.

P28.6 (a) The 120-V potential difference is applied across the series combination of the two conductors in the extension cord and the light bulb. The potential difference across the light bulb is less than 120 V and its power is less than 75 W. (b) We assume the bulb has constant resistance—that is, that its temperature does not change much from the design operating point. See the solution. 73.8 W

P28.8 (a) See the solution. (b) no

P28.10 See the solution.

P28.12 470 Ω and 220 Ω

P28.14 (a) $\Delta V_4 > \Delta V_3 > \Delta V_1 > \Delta V_2$ (b) $\Delta V_1 = \mathcal{E}/3$, $\Delta V_2 = 2\mathcal{E}/9$, $\Delta V_3 = 4\mathcal{E}/9$, $\Delta V_4 = 2\mathcal{E}/3$ (c) $I_1 > I_4 > I_2 = I_3$ (d) $I_1 = I$, $I_2 = I_3 = I/3$, $I_4 = 2I/3$ (e) Increasing the value of resistor 3 increases the equivalent resistance of the entire circuit. The current in the battery, which is also the current in resistor 1, therefore decreases. Then the potential difference across resistor 1 decreases and the potential difference across the parallel combination increases. Driven by a larger potential difference, the current in resistor 4 increases. This effect makes the current in resistors 2 and 3 decrease. In summary, I_4 increases while I_1, I_2, and I_3 decrease. (f) $I_1 = 3I/4$, $I_2 = I_3 = 0$, $I_4 = 3I/4$

P28.16 $I_1 = 714$ mA $I_2 = 1.29$ A $\mathcal{E} = 12.6$ V

P28.18 See the solution.

P28.20 (a) See the solution. (b) The current in the 220-Ω resistor and the 5.80-V battery is 11.0 mA out of the positive battery pole. The current in the 370-Ω resistor is 9.13 mA. The current in the 150-Ω resistor and the 3.10-V battery is 1.87 mA out of the negative battery pole.

P28.22 50.0 mA from *a* to *e*

P28.24 starter 171 A downward in the diagram; battery 0.283 A downward

P28.26 See the solution.

P28.28 587 kΩ

P28.30 See the solution.

P28.32 (a) 1.50 s (b) 1.00 s (c) $200\ \mu\text{A} + (100\ \mu\text{A})e^{-t/1.00\text{ s}}$

P28.34 145 Ω, 0.756 mA

P28.36 (a) 30.000 mA, 5.400 0 V (b) 30.167 mA, 5.381 6 V (c) 29.898 mA, 5.396 6 V (d) Both circuits are good enough for some measurements. The circuit in part (c) gives data leading to a value of resistance that is too high by only about 0.3%. Its value is more accurate than the value, 0.9% too low, implied by the data from the circuit in part (b).

P28.38 (a) $\sim 10^{-14}$ A (b) $V_h/2 + \sim 10^{-10}$ V and $V_h/2 - \sim 10^{-10}$ V, where V_h is the potential of the live wire, $\sim 10^2$ V

P28.40 (a) 0.706 A (b) 2.49 W (c) Only the circuit in Figure P28.40c requires the use of Kirchhoff's rules for solution. In the other circuits the 5-Ω and 8-Ω resistors are still in parallel with each other. (d) The power is lowest in Figure P28.40c. The circuits in Figures P28.40b and P28.40d have in effect 30-V batteries driving the current.

P28.42 (a) either 3.84 Ω or 0.375 Ω (b) No load resistor can extract more than 17.6 W from this battery.

P28.44 (a) $\mathcal{E}^2/3R$ (b) $3\mathcal{E}^2/R$ (c) in the parallel connection

P28.46 (a) 4.40 Ω (b) 32.0 W, 9.60 W, 70.4 W (c) 48.0 W

P28.48 $\dfrac{\mathcal{P}_s + \sqrt{\mathcal{P}_s^2 - 4\mathcal{P}_s\mathcal{P}_p}}{2I^2}$ and $\dfrac{\mathcal{P}_s - \sqrt{\mathcal{P}_s^2 - 4\mathcal{P}_s\mathcal{P}_p}}{2I^2}$

P28.50 (a) $15.0\ \mu\text{C}\dfrac{160\ \Omega - 3R}{166\ \Omega + 83R}$ (b) 1.96 μC (c) Yes; 53.3 Ω (d) 14.5 μC for $R = 0$ (e) Yes; it corresponds to disconnecting the wire; 0.542 μC

P28.52 See the solution.

P28.54 (a) 470 W (b) 1.60 mm or more (c) 2.93 mm or more

P28.56 (a) 222 μC (b) increase by 444 μC

P28.58 (a) 0.991 (b) 0.648 (c) The energy flows are precisely analogous to the currents in parts (a) and (b). The ceiling has the smallest *R*-value of the thermal resistors in parallel, so increasing its thermal resistance will produce the biggest reduction in the total energy flow.

P28.60 (a) 5.00 Ω (b) 2.40 A

P28.62 (a) $\ln(\mathcal{E}/\Delta V) = 0.011\,8\ t + 0.088\,2$ (b) 84.7 s, 8.47 μF

P28.64 $(R_1 + 2R_2)C\ln 2$

P28.66 (a) 3.91 s (b) 0.782 ms

29

Magnetic Fields

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q29.1** (a) Yes, as described by $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$.
(b) No, as described by $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$
(c) Yes. (d) Yes. (e) No. The wire is uncharged.
(f) Yes. (g) Yes. (h) Yes.

***Q29.2** (i) (b). (ii) (a). Electron A has a smaller radius of curvature, as described by $qvB = mv^2/r$.

***Q29.3** (i) (c) (ii) (c) (iii) (c) (iv) (b) (v) (d) (vi) (b) (vii) (b) (viii) (b)

***Q29.4** We consider the quantity $|\, qvB \sin\theta \,|$, in units of e (m/s)(T). For (a) it is $1 \times 10^6\ 10^{-3}\ 1 = 10^3$. For (b) it is $1 \times 10^6\ 10^{-3}\ 0 = 0$. For (c) $2 \times 10^6\ 10^{-3}\ 1 = 2\,000$. For (d) $1 \times 10^6\ 2 \times 10^{-3}\ 1 = 2\,000$. For (e) $1 \times 10^6\ 10^{-3}\ 1 = 10^3$. For (f) $1 \times 10^6\ 10^{-3}\ 0.707 = 707$. The ranking is then $c = d > a = e > f > b$.

***Q29.5** $\hat{\mathbf{i}} \times (-\hat{\mathbf{k}}) = \hat{\mathbf{j}}$. Answer (c).

***Q29.6** Answer (c). It is not necessarily zero. If the magnetic field is parallel or antiparallel to the velocity of the charged particle, then the particle will experience no magnetic force.

Q29.7 If they are projected in the same direction into the same magnetic field, the charges are of opposite sign.

Q29.8 Send the particle through the uniform field and look at its path. If the path of the particle is parabolic, then the field must be electric, as the electric field exerts a constant force on a charged particle, independent of its velocity. If you shoot a proton through an electric field, it will feel a constant force in the same direction as the electric field—it's similar to throwing a ball through a gravitational field.
If the path of the particle is helical or circular, then the field is magnetic.
If the path of the particle is straight, then observe the speed of the particle. If the particle accelerates, then the field is electric, as a constant force on a proton with or against its motion will make its speed change. If the speed remains constant, then the field is magnetic.

***Q29.9** Answer (d). The electrons will feel a constant electric force and a magnetic force that will change in direction and in magnitude as their speed changes.

Q29.10 Yes. If the magnetic field is perpendicular to the plane of the loop, then it exerts no torque on the loop.

Q29.11 If you can hook a spring balance to the particle and measure the force on it in a known electric field, then $q = \frac{F}{E}$ will tell you its charge. You cannot hook a spring balance to an electron. Measuring the acceleration of small particles by observing their deflection in known electric and magnetic fields can tell you the charge-to-mass ratio, but not separately the charge or mass. Both an acceleration produced by an electric field and an acceleration caused by a magnetic field depend on the properties of the particle only by being proportional to the ratio $\frac{q}{m}$.

Q29.12 If the current loop feels a torque, it must be caused by a magnetic field. If the current loop feels no torque, try a different orientation—the torque is zero if the field is along the axis of the loop.

Q29.13 The Earth's magnetic field exerts force on a charged incoming cosmic ray, tending to make it spiral around a magnetic field line. If the particle energy is low enough, the spiral will be tight enough that the particle will first hit some matter as it follows a field line down into the atmosphere or to the surface at a high geographic latitude.

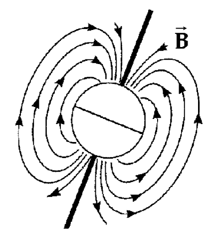


FIG. Q29.13

Q29.14 No. Changing the velocity of a particle requires an accelerating force. The magnetic force is proportional to the speed of the particle. If the particle is not moving, there can be no magnetic force on it.

SOLUTIONS TO PROBLEMS

Section 29.1 Magnetic Fields and Forces

P29.1 (a) up

(b) out of the page, since the charge is negative.

(c) no deflection

(d) into the page

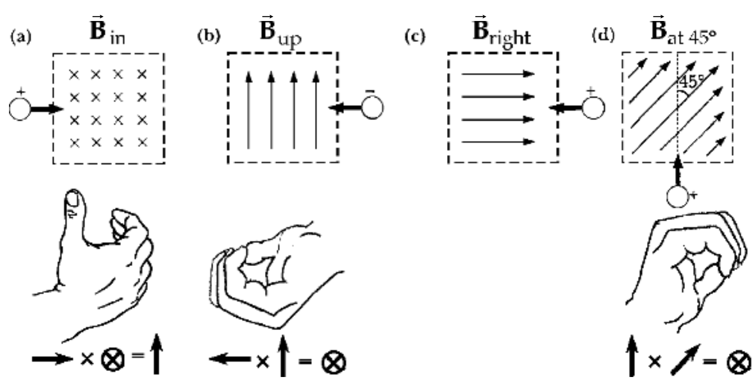


FIG. P29.1

P29.2 At the equator, the Earth's magnetic field is horizontally north. Because an electron has negative charge, $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ is opposite in direction to $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. Figures are drawn looking down.

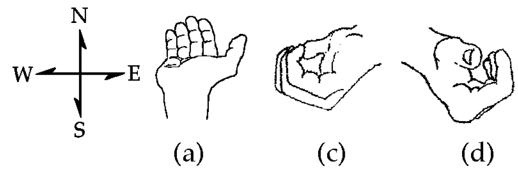


FIG. P29.2

(a) Down × North = East, so the force is directed $\boxed{\text{West}}$.

(b) North × North $= \sin 0° = 0$: $\boxed{\text{Zero deflection}}$.

(c) West × North = Down, so the force is directed $\boxed{\text{Up}}$.

(d) Southeast × North = Up, so the force is $\boxed{\text{Down}}$.

P29.3 $F = ma = \left(1.67 \times 10^{-27} \text{ kg}\right)\left(2.00 \times 10^{13} \text{ m/s}^2\right) = 3.34 \times 10^{-14} \text{ N} = qvB \sin 90°$

$$B = \frac{F}{qv} = \frac{3.34 \times 10^{-14} \text{ N}}{\left(1.60 \times 10^{-19} \text{ C}\right)\left(1.00 \times 10^{7} \text{ m/s}\right)} = \boxed{2.09 \times 10^{-2} \text{ T}}$$

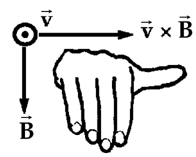


FIG. P29.3

The right-hand rule shows that B must be in the $-y$ direction to yield a force in the $+x$ direction when v is in the z direction.

P29.4 (a) $F_B = qvB \sin\theta = \left(1.60 \times 10^{-19} \text{ C}\right)\left(3.00 \times 10^{6} \text{ m/s}\right)\left(3.00 \times 10^{-1} \text{ T}\right) \sin 37.0°$

$F_B = \boxed{8.67 \times 10^{-14} \text{ N}}$

(b) $a = \dfrac{F}{m} = \dfrac{8.67 \times 10^{-14} \text{ N}}{1.67 \times 10^{-27} \text{ kg}} = \boxed{5.19 \times 10^{13} \text{ m/s}^2}$

P29.5 $F_B = qvB \sin\theta$ so $8.20 \times 10^{-13} \text{ N} = \left(1.60 \times 10^{-19} \text{ C}\right)\left(4.00 \times 10^{6} \text{ m/s}\right)\left(1.70 \text{ T}\right) \sin\theta$

$\sin\theta = 0.754$ and $\theta = \sin^{-1}(0.754) = \boxed{48.9° \text{ or } 131°}$

P29.6 First find the speed of the electron.

$$\Delta K = \frac{1}{2}mv^2 = e\Delta V = \Delta U: \qquad v = \sqrt{\frac{2e\Delta V}{m}} = \sqrt{\frac{2\left(1.60 \times 10^{-19} \text{ C}\right)\left(2\,400 \text{ J/C}\right)}{\left(9.11 \times 10^{-31} \text{ kg}\right)}} = 2.90 \times 10^{7} \text{ m/s}$$

(a) $F_{B,\,\max} = qvB = \left(1.60 \times 10^{-19} \text{ C}\right)\left(2.90 \times 10^{7} \text{ m/s}\right)\left(1.70 \text{ T}\right) = \boxed{7.90 \times 10^{-12} \text{ N}}$

(b) $F_{B,\,\min} = \boxed{0}$ occurs when $\vec{\mathbf{v}}$ is either parallel to or anti-parallel to $\vec{\mathbf{B}}$.

P29.7 $\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$

$$\vec{\mathbf{v}} \times \vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ +2 & -4 & +1 \\ +1 & +2 & -3 \end{vmatrix} = (12 - 2)\hat{\mathbf{i}} + (1 + 6)\hat{\mathbf{j}} + (4 + 4)\hat{\mathbf{k}} = 10\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$$

$$\left|\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right| = \sqrt{10^2 + 7^2 + 8^2} = 14.6 \text{ T} \cdot \text{m/s}$$

$$\left|\vec{\mathbf{F}}_B\right| = q\left|\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right| = \left(1.60 \times 10^{-19} \text{ C}\right)\left(14.6 \text{ T} \cdot \text{m/s}\right) = \boxed{2.34 \times 10^{-18} \text{ N}}$$

P29.8 $q\vec{\mathbf{E}} = \left(-1.60\times10^{-19}\,\text{C}\right)(20.0\ \text{N/C})\hat{\mathbf{k}} = \left(-3.20\times10^{-18}\,\text{N}\right)\hat{\mathbf{k}}$

$$\sum\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}}\times\vec{\mathbf{B}} = m\vec{\mathbf{a}}$$

$$\left(-3.20\times10^{-18}\,\text{N}\right)\hat{\mathbf{k}} - 1.60\times10^{-19}\,\text{C}\left(1.20\times10^{4}\ \text{m/s}\,\hat{\mathbf{i}}\right)\times\vec{\mathbf{B}} = \left(9.11\times10^{-31}\right)\left(2.00\times10^{12}\ \text{m/s}^2\right)\hat{\mathbf{k}}$$

$$-\left(3.20\times10^{-18}\,\text{N}\right)\hat{\mathbf{k}} - \left(1.92\times10^{-15}\,\text{C}\cdot\text{m/s}\right)\hat{\mathbf{i}}\times\vec{\mathbf{B}} = \left(1.82\times10^{-18}\,\text{N}\right)\hat{\mathbf{k}}$$

$$\left(1.92\times10^{-15}\,\text{C}\cdot\text{m/s}\right)\hat{\mathbf{i}}\times\vec{\mathbf{B}} = -\left(5.02\times10^{-18}\,\text{N}\right)\hat{\mathbf{k}}$$

The magnetic field $\boxed{\text{may have any } x\text{-component}}$. $B_z = \boxed{0}$ and $B_y = \boxed{-2.62\ \text{mT}}$.

Section 29.2 **Motion of a Charged Particle in a Uniform Magnetic Field**

P29.9 (a) $B = 50.0\times10^{-6}\ \text{T}$; $v = 6.20\times10^{6}\ \text{m/s}$

Direction is given by the right-hand-rule: $\boxed{\text{southward}}$

$$F_B = qvB\sin\theta$$

$$F_B = \left(1.60\times10^{-19}\,\text{C}\right)\left(6.20\times10^{6}\ \text{m/s}\right)\left(50.0\times10^{-6}\,\text{T}\right)\sin 90.0°$$

$$= \boxed{4.96\times10^{-17}\,\text{N}}$$

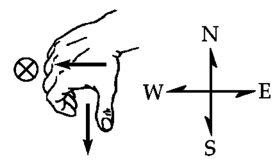


FIG. P29.9

(b) $F = \dfrac{mv^2}{r}$ so

$$r = \frac{mv^2}{F} = \frac{\left(1.67\times10^{-27}\,\text{kg}\right)\left(6.20\times10^{6}\ \text{m/s}\right)^2}{4.96\times10^{-17}\,\text{N}} = \boxed{1.29\ \text{km}}$$

***P29.10** (a) The horizontal velocity component of the electrons is given by $(1/2)mv_x^2 = |q|V$.

$$v_x = \sqrt{\frac{2|q|V}{m}} = \sqrt{\frac{2(1.6\times10^{-19}\,\text{C})\ 2\,500\ \text{J/C}}{9.11\times10^{-31}\,\text{kg}}} = 2.96\times10^{7}\ \text{m/s}$$

Its time of flight is $t = x/v_x = 0.35\ \text{m}/2.96\times10^7\ \text{m/s} = 1.18\times10^{-8}\ \text{s}$.

Its vertical deflection is $y = (1/2)\,gt^2 = (1/2)\,9.8\ \text{m/s}^2\,(1.18\times10^{-8}\ \text{s})^2 = \boxed{6.84\times10^{-16}\ \text{m down}}$, which is unobservably small.

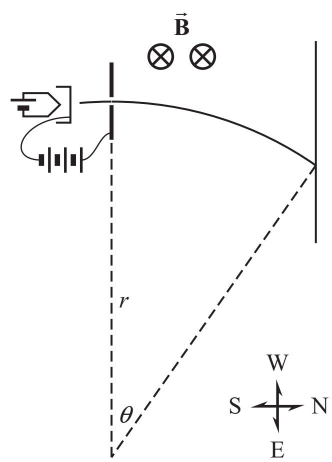


FIG. P29.10

(b) The magnetic force is in the direction –north × down = –west = east. The beam is deflected into a circular path with radius

$$r = \frac{mv}{|q|B} = \frac{9.11\times10^{-31}\,\text{kg}\ 2.96\times10^{7}\text{m/s}}{1.6\times10^{-19}\,\text{C}\ 20\times10^{-6}\text{N}\cdot\text{s/C}\cdot\text{m}} = 8.44\ \text{m}.$$

Their path to the screen subtends at the center of curvature an angle given by $\sin\theta = 0.35\ \text{m}/8.44\ \text{m} = 2.38°$. Their deflection is $8.44\ \text{m}(1-\cos 2.38°) = \boxed{7.26\ \text{mm east}}$. It does not move as a projectile, but its northward velocity component stays nearly constant, changing from 2.96×10^7 m/s cos 0° to 2.96×10^7 m/s cos 2.38°. That is, it is constant within 0.09%. It is a good approximation to think of it as moving on a parabola as it really moves on a circle.

P29.11 $q(\Delta V) = \frac{1}{2}mv^2$ or $v = \sqrt{\frac{2q(\Delta V)}{m}}$

Also, $qvB = \frac{mv^2}{r}$ so $r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2q(\Delta V)}{m}} = \sqrt{\frac{2m(\Delta V)}{qB^2}}$

Therefore,
$$r_p^2 = \frac{2m_p(\Delta V)}{eB^2}$$

$$r_d^2 = \frac{2m_d(\Delta V)}{q_d B^2} = \frac{2(2m_p)(\Delta V)}{eB^2} = 2\left(\frac{2m_p(\Delta V)}{eB^2}\right) = 2r_p^2$$

and
$$r_\alpha^2 = \frac{2m_\alpha(\Delta V)}{q_\alpha B^2} = \frac{2(4m_p)(\Delta V)}{(2e)B^2} = 2\left(\frac{2m_p(\Delta V)}{eB^2}\right) = 2r_p^2$$

The conclusion is: $\boxed{r_\alpha = r_d = \sqrt{2}r_p}$

P29.12 For each electron, $|q|vB\sin 90.0° = \frac{mv^2}{r}$ and $v = \frac{eBr}{m}$.

The electrons have no internal structure to absorb energy, so the collision must be perfectly elastic:

$$K = \frac{1}{2}mv_{1i}^2 + 0 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2$$

$$K = \frac{1}{2}m\left(\frac{e^2B^2R_1^2}{m^2}\right) + \frac{1}{2}m\left(\frac{e^2B^2R_2^2}{m^2}\right) = \frac{e^2B^2}{2m}\left(R_1^2 + R_2^2\right)$$

$$K = \frac{e\left(1.60\times10^{-19}\text{ C}\right)\left(0.044\,0\text{ N}\cdot\text{s/C}\cdot\text{m}\right)^2}{2\left(9.11\times10^{-31}\text{ kg}\right)}\left[(0.010\,0\text{ m})^2 + (0.024\,0\text{ m})^2\right] = \boxed{115\text{ keV}}$$

P29.13 (a) We begin with $qvB = \frac{mv^2}{R}$

or $qRB = mv$

But $L = mvR = qR^2B$

Therefore,
$$R = \sqrt{\frac{L}{qB}} = \sqrt{\frac{4.00\times10^{-25}\text{ J}\cdot\text{s}}{\left(1.60\times10^{-19}\text{ C}\right)\left(1.00\times10^{-3}\text{ T}\right)}} = 0.050\,0\text{ m} = \boxed{5.00\text{ cm}}$$

(b) Thus,
$$v = \frac{L}{mR} = \frac{4.00\times10^{-25}\text{ J}\cdot\text{s}}{\left(9.11\times10^{-31}\text{ kg}\right)(0.050\,0\text{ m})} = \boxed{8.78\times10^6\text{ m/s}}$$

P29.14 $\frac{1}{2}mv^2 = q(\Delta V)$ so $v = \sqrt{\frac{2q(\Delta V)}{m}}$

$r = \frac{mv}{qB}$ so $r = \frac{m\sqrt{2q(\Delta V)/m}}{qB}$

$r^2 = \frac{m}{q}\cdot\frac{2(\Delta V)}{B^2}$ and $(r')^2 = \frac{m'}{q'}\cdot\frac{2(\Delta V)}{B^2}$

$m = \frac{qB^2r^2}{2(\Delta V)}$ and $(m') = \frac{(q')B^2(r')^2}{2(\Delta V)}$ so $\frac{m'}{m} = \frac{q'}{q}\cdot\frac{(r')^2}{r^2} = \left(\frac{2e}{e}\right)\left(\frac{2R}{R}\right)^2 = \boxed{8}$

P29.15 $E = \frac{1}{2}mv^2 = e\Delta V$

and $evB\sin 90° = \frac{mv^2}{R}$

$$B = \frac{mv}{eR} = \frac{m}{eR}\sqrt{\frac{2e\Delta V}{m}} = \frac{1}{R}\sqrt{\frac{2m\Delta V}{e}}$$

$$B = \frac{1}{5.80\times10^{10}\text{ m}}\sqrt{\frac{2\left(1.67\times10^{-27}\text{ kg}\right)\left(10.0\times10^{6}\text{ V}\right)}{1.60\times10^{-19}\text{ C}}} = \boxed{7.88\times10^{-12}\text{ T}}$$

P29.16 (a) The boundary between a region of strong magnetic field and a region of zero field cannot be perfectly sharp, but we ignore the thickness of the transition zone. In the field the electron moves on an arc of a circle:

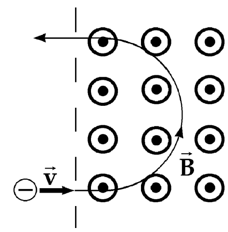


FIG. P29.16(a)

$\sum F = ma$:

$$|q|vB\sin 90° = \frac{mv^2}{r}$$

$$\frac{v}{r} = \omega = \frac{|q|B}{m} = \frac{\left(1.60\times10^{-19}\text{C}\right)\left(10^{-3}\text{ N}\cdot\text{s/C}\cdot\text{m}\right)}{\left(9.11\times10^{-31}\text{kg}\right)} = 1.76\times10^{8}\text{ rad/s}$$

The time for one half revolution is,

from $\Delta\theta = \omega\Delta t$

$$\Delta t = \frac{\Delta\theta}{\omega} = \frac{\pi\text{ rad}}{1.76\times10^{8}\text{ rad/s}} = \boxed{1.79\times10^{-8}\text{ s}}$$

(b) The maximum depth of penetration is the radius of the path.

Then $v = \omega r = \left(1.76\times10^{8}\text{s}^{-1}\right)(0.02\text{ m}) = 3.51\times10^{6}\text{ m/s}$

and

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(9.11\times10^{-31}\text{kg}\right)\left(3.51\times10^{6}\text{ m/s}\right)^2 = 5.62\times10^{-18}\text{ J} = \frac{5.62\times10^{-18}\text{ J}\cdot\text{e}}{1.60\times10^{-19}\text{ C}}$$

$$= \boxed{35.1\text{ eV}}$$

Section 29.3 **Applications Involving Charged Particles Moving in a Magnetic Field**

P29.17 $F_B = F_e$

so $qvB = qE$

where $v = \sqrt{\frac{2K}{m}}$ and K is kinetic energy of the electron.

$$E = vB = \sqrt{\frac{2K}{m}}B = \sqrt{\frac{2(750)\left(1.60\times10^{-19}\right)}{9.11\times10^{-31}}}(0.015\,0) = \boxed{244\text{ kV/m}}$$

P29.18 $K = \frac{1}{2}mv^2 = q(\Delta V)$ so $v = \sqrt{\frac{2q(\Delta V)}{m}}$

$$\left|\vec{\mathbf{F}}_B\right| = \left|q\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right| = \frac{mv^2}{r} \qquad r = \frac{mv}{qB} = \frac{m}{q}\sqrt{\frac{2q(\Delta V)/m}{B}} = \frac{1}{B}\sqrt{\frac{2m(\Delta V)}{q}}$$

(a) $r_{238} = \sqrt{\frac{2\left(238 \times 1.66 \times 10^{-27}\right)2\,000}{1.60 \times 10^{-19}}}\left(\frac{1}{1.20}\right) = 8.28 \times 10^{-2}\text{ m} = \boxed{8.28\text{ cm}}$

(b) $r_{235} = \boxed{8.23\text{ cm}}$

$$\frac{r_{238}}{r_{235}} = \sqrt{\frac{m_{238}}{m_{235}}} = \sqrt{\frac{238.05}{235.04}} = 1.006\,4$$

The ratios of the orbit radius for different ions are independent of ΔV and B.

P29.19 In the velocity selector: $v = \frac{E}{B} = \frac{2\,500\text{ V/m}}{0.035\,0\text{ T}} = 7.14 \times 10^4\text{ m/s}$

In the deflection chamber: $r = \frac{mv}{qB} = \frac{\left(2.18 \times 10^{-26}\text{ kg}\right)\left(7.14 \times 10^4\text{ m/s}\right)}{\left(1.60 \times 10^{-19}\text{ C}\right)\left(0.035\,0\text{ T}\right)} = \boxed{0.278\text{ m}}$

P29.20 Note that the "cyclotron frequency" is an angular speed. The motion of the proton is described by

$\sum F = ma$:

$$|q|vB\sin 90° = \frac{mv^2}{r}$$

$$|q|B = m\frac{v}{r} = m\omega$$

(a) $\omega = \frac{|q|B}{m} = \frac{\left(1.60 \times 10^{-19}\text{C}\right)\left(0.8\text{ N}\cdot\text{s/C}\cdot\text{m}\right)}{\left(1.67 \times 10^{-27}\text{kg}\right)}\left(\frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right) = \boxed{7.66 \times 10^7\text{ rad/s}}$

(b) $v = \omega r = \left(7.66 \times 10^7\text{ rad/s}\right)(0.350\text{ m})\left(\frac{1}{1\text{ rad}}\right) = \boxed{2.68 \times 10^7\text{ m/s}}$

(c) $K = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.67 \times 10^{-27}\text{kg}\right)\left(2.68 \times 10^7\text{ m/s}\right)^2\left(\frac{1\text{ eV}}{1.6 \times 10^{-19}\text{J}}\right) = \boxed{3.76 \times 10^6\text{ eV}}$

(d) The proton gains 600 eV twice during each revolution, so the number of revolutions is

$$\frac{3.76 \times 10^6\text{ eV}}{2(600\text{ eV})} = \boxed{3.13 \times 10^3\text{ revolutions}}$$

(e) $\theta = \omega t \qquad t = \frac{\theta}{\omega} = \frac{3.13 \times 10^3\text{ rev}}{7.66 \times 10^7\text{ rad/s}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = \boxed{2.57 \times 10^{-4}\text{ s}}$

P29.21 (a) $F_B = qvB = \frac{mv^2}{R}$

$$\omega = \frac{v}{R} = \frac{qBR}{mR} = \frac{qB}{m} = \frac{\left(1.60 \times 10^{-19}\text{C}\right)(0.450\text{ T})}{1.67 \times 10^{-27}\text{kg}} = \boxed{4.31 \times 10^7\text{ rad/s}}$$

(b) $v = \frac{qBR}{m} = \frac{\left(1.60 \times 10^{-19}\text{C}\right)(0.450\text{ T})(1.20\text{ m})}{1.67 \times 10^{-27}\text{kg}} = \boxed{5.17 \times 10^7\text{ m/s}}$

***P29.22** (a) The path radius is $r = mv/qB$, which we can put in terms of energy E by $(1/2)mv^2 = E$.

$v = (2E/m)^{1/2}$ so $r = (m/qB)(2E/m)^{1/2} = (2m)^{1/2}(qB)^{-1}E^{1/2}$

Then $dr/dt = = (2m)^{1/2}(qB)^{-1}(1/2)\,E^{-1/2}\,dE/dt = \dfrac{\sqrt{2m}}{qB2}\dfrac{\sqrt{2m}}{qBr}\dfrac{q^2B\Delta V}{\pi m} = \dfrac{1}{r}\dfrac{\Delta V}{\pi B}$

(b) The dashed red line should spiral around many times, with its turns relatively far apart on the inside and closer together on the outside.

(c) $$\frac{dr}{dt} = \frac{1}{r}\frac{\Delta V}{\pi B} = \frac{600\ \text{V}}{0.35\ \text{m}\ \pi\ 0.8\ \text{N s}}\frac{\text{C m N m}}{\text{V C}} = \boxed{682\ \text{m/s}}$$

(d) $$\Delta r = \frac{dr}{dt}T = \frac{1}{r}\frac{\Delta V}{\pi B}\frac{2\pi m}{qB} = \frac{2\Delta V m}{rqB^2} = \frac{2\times 600\ \text{V}\ 1.67\times 10^{-27}\text{kg}\ \text{C}^2\ \text{m}^2\ \text{N m}}{0.35\ \text{m}\ 1.6\times 10^{-19}\text{C}\ 0.8^2\ \text{N}^2\ \text{s}^2\ \text{V C}} = \boxed{55.9\ \mu\text{m}}$$

P29.23 $\theta = \tan^{-1}\left(\dfrac{25.0}{10.0}\right) = 68.2°$ and $R = \dfrac{1.00\ \text{cm}}{\sin 68.2°} = 1.08\ \text{cm}$

Ignoring relativistic correction, the kinetic energy of the electrons is

$\dfrac{1}{2}mv^2 = q\Delta V$ so $v = \sqrt{\dfrac{2q\Delta V}{m}} = 1.33\times 10^8\ \text{m/s}$

From Newton's second law, $\dfrac{mv^2}{R} = qvB$, we find the magnetic field

$$B = \frac{mv}{|q|R} = \frac{(9.11\times 10^{-31}\,\text{kg})(1.33\times 10^8\ \text{m/s})}{(1.60\times 10^{-19}\,\text{C})(1.08\times 10^{-2}\,\text{m})} = \boxed{70.1\ \text{mT}}$$

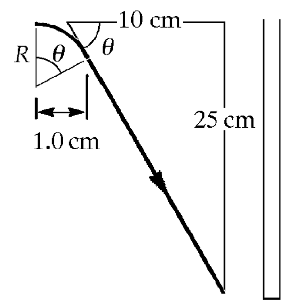


FIG. P29.23

***P29.24** (a) Yes: The constituent of the beam is present in all kinds of atoms.

(b) Yes: Everything in the beam has a single charge-to-mass ratio.

(c) In a charged macroscopic object most of the atoms are uncharged. A molecule never has all of its atoms ionized. Any atom other than hydrogen contains neutrons and so has more mass per charge if it is ionized than hydrogen does. The greatest charge-to-mass ratio Thomson could expect was then for ionized hydrogen, 1.6×10^{-19} C/1.67×10^{-27} kg, smaller than the value e/m he measured, 1.6×10^{-19} C/9.11×10^{-31} kg, by 1 836 times. The particles in his beam could not be whole atoms, but rather must be much smaller in mass.

(d) With kinetic energy 100 eV, an electron has speed given by $(1/2)mv^2 = 100$ eV

$v = \sqrt{\dfrac{200\cdot 1.6\times 10^{-19}\,\text{C}\ 1\ \text{J/C}}{9.11\times 10^{-31}\,\text{kg}}} = 5.93\times 10^6\,\text{m/s}.$ The time to travel 40 cm is

0.4 m/(5.93×10^6 m/s) = 6.75×10^{-8} s. If it is fired horizontally it will fall vertically by $(1/2)gt^2 = (1/2)(9.8\ \text{m/s}^2)(6.75\times 10^{-8}\ \text{s})^2 = 2.23\times 10^{-14}$ m, an immeasurably small amount. An electron with higher energy falls by a smaller amount.

Section 29.4 **Magnetic Force Acting on a Current-Carrying Conductor**

P29.25 $F_B = ILB\sin\theta$ with $F_B = F_g = mg$

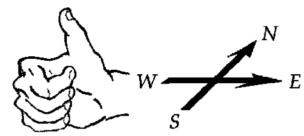


FIG. P29.25

$mg = ILB\sin\theta$ so $\frac{m}{L}g = IB\sin\theta$

$I = 2.00\text{ A}$ and $\frac{m}{L} = (0.500\text{ g/cm})\left(\frac{100\text{ cm/m}}{1\,000\text{ g/kg}}\right) = 5.00\times10^{-2}\text{ kg/m}$

Thus $(5.00\times10^{-2})(9.80) = (2.00)B\sin 90.0°$

$B = \boxed{0.245\text{ Tesla}}$ with the direction given by right-hand rule: $\boxed{\text{eastward}}$.

P29.26 $\vec{\mathbf{F}}_B = I\vec{\ell}\times\vec{\mathbf{B}} = (2.40\text{ A})(0.750\text{ m})\hat{\mathbf{i}}\times(1.60\text{ T})\hat{\mathbf{k}} = \boxed{\left(-2.88\hat{\mathbf{j}}\right)\text{ N}}$

P29.27 (a) $F_B = ILB\sin\theta = (5.00\text{ A})(2.80\text{ m})(0.390\text{ T})\sin 60.0° = \boxed{4.73\text{ N}}$

(b) $F_B = (5.00\text{ A})(2.80\text{ m})(0.390\text{ T})\sin 90.0° = \boxed{5.46\text{ N}}$

(c) $F_B = (5.00\text{ A})(2.80\text{ m})(0.390\text{ T})\sin 120° = \boxed{4.73\text{ N}}$

***P29.28** The magnetic force should counterbalance the gravitational force on each section of wire:

$I\ell B\sin 90° = mg$ $I = \frac{m}{\ell}\frac{g}{B} = 2.4\times10^{-3}\frac{\text{kg}}{\text{m}}\frac{9.8\text{ m/s}^2\text{C m}}{28\times10^{-6}\text{N s}} = \boxed{840\text{ A}}$

The current should be $\boxed{\text{east}}$ so that the magnetic force will be east × north = up.

P29.29 The rod feels force $\vec{\mathbf{F}}_B = I\left(\vec{\mathbf{d}}\times\vec{\mathbf{B}}\right) = Id\left(\hat{\mathbf{k}}\right)\times B\left(-\hat{\mathbf{j}}\right) = IdB\left(\hat{\mathbf{i}}\right)$.

The work-energy theorem is $\left(K_{\text{trasn}} + K_{\text{rot}}\right)_i + \Delta E = \left(K_{\text{trans}} + K_{\text{rot}}\right)_f$

$0 + 0 + F_s\cos\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$IdBL\cos 0° = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2$ and $IdBL = \frac{3}{4}mv^2$

$v = \sqrt{\frac{4IdBL}{3m}} = \sqrt{\frac{4(48.0\text{ A})(0.120\text{ m})(0.240\text{ T})(0.450\text{ m})}{3(0.720\text{ kg})}} = \boxed{1.07\text{ m/s}}$

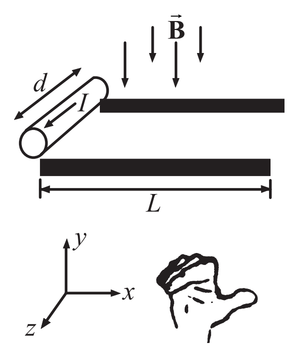


FIG. P29.29

P29.30 The rod feels force $\vec{\mathbf{F}}_B = I\left(\vec{\mathbf{d}}\times\vec{\mathbf{B}}\right) = Id\left(\hat{\mathbf{k}}\right)\times B\left(-\hat{\mathbf{j}}\right) = IdB\left(\hat{\mathbf{i}}\right)$.

The work-energy theorem is $\left(K_{\text{trans}} + K_{\text{rot}}\right)_i + \Delta E = \left(K_{\text{trans}} + K_{\text{rot}}\right)_f$

$$0 + 0 + Fs\cos\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$IdBL\cos 0° = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 \text{ and } v = \boxed{\sqrt{\frac{4IdBL}{3m}}}$$

P29.31 The magnetic force on each bit of ring is $Id\vec{\mathbf{s}}\times\vec{\mathbf{B}} = IdsB$ radially inward and upward, at angle θ above the radial line. The radially inward components tend to squeeze the ring but all cancel out as forces. The upward components $IdsB\sin\theta$ all add to $\boxed{I2\pi rB\sin\theta \text{ up}}$.

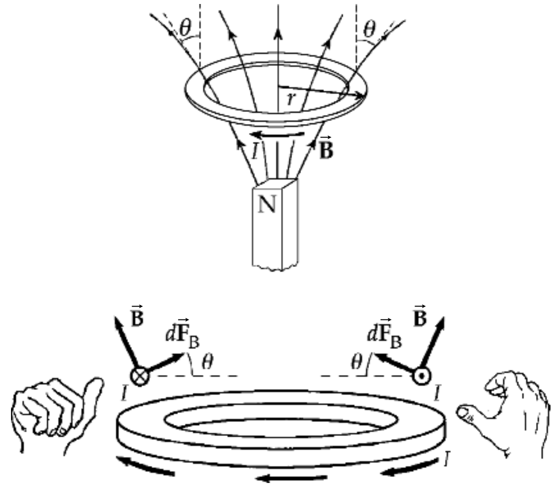


FIG. P29.31

***P29.32** (a) For each segment, $I = 5.00$ A and $\vec{\mathbf{B}} = 0.020\,0$ N/A·m $\hat{\mathbf{j}}$.

Segment	$\vec{\ell}$	$\vec{\mathbf{F}}_B = I\left(\vec{\ell}\times\vec{\mathbf{B}}\right)$
ab	-0.400 m $\hat{\mathbf{j}}$	$\boxed{0}$
bc	0.400 m $\hat{\mathbf{k}}$	$\boxed{(40.0 \text{ mN})\left(-\hat{\mathbf{i}}\right)}$
cd	-0.400 m $\hat{\mathbf{i}} + 0.400$ m $\hat{\mathbf{j}}$	$\boxed{(40.0 \text{ mN})\left(-\hat{\mathbf{k}}\right)}$
da	0.400 m $\hat{\mathbf{i}} - 0.400$ m $\hat{\mathbf{k}}$	$\boxed{(40.0 \text{ mN})\left(\hat{\mathbf{k}}+\hat{\mathbf{i}}\right)}$

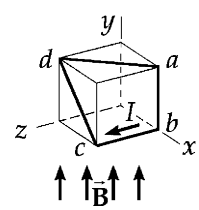


FIG. P29.32

(b) The forces on the four segments must add to zero, so the force on the fourth segment must be the negative of the resultant of the forces on the other three.

P29.33 Take the x-axis east, the y-axis up, and the z-axis south. The field is

$$\vec{\mathbf{B}} = (52.0\ \mu\text{T})\cos 60.0°\left(-\hat{\mathbf{k}}\right) + (52.0\ \mu\text{T})\sin 60.0°\left(-\hat{\mathbf{j}}\right)$$

The current then has equivalent length: $\vec{\mathbf{L}}' = 1.40\ \text{m}\left(-\hat{\mathbf{k}}\right) + 0.850\ \text{m}\left(\hat{\mathbf{j}}\right)$

$$\vec{\mathbf{F}}_B = I\vec{\mathbf{L}}' \times \vec{\mathbf{B}} = (0.035\,0\ \text{A})\left(0.850\hat{\mathbf{j}} - 1.40\hat{\mathbf{k}}\right)\ \text{m} \times \left(-45.0\hat{\mathbf{j}} - 26.0\hat{\mathbf{k}}\right)10^{-6}\,\text{T}$$

$$\vec{\mathbf{F}}_B = 3.50 \times 10^{-8}\ \text{N}\left(-22.1\hat{\mathbf{i}} - 63.0\hat{\mathbf{i}}\right) = 2.98 \times 10^{-6}\ \text{N}\left(-\hat{\mathbf{i}}\right) = \boxed{2.98\ \mu\text{N west}}$$

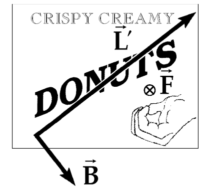


FIG. P29.33

Section 29.5 **Torque on a Current Loop in a Uniform Magnetic Field**

P29.34 (a) $2\pi r = 2.00\ \text{m}$ so $r = 0.318\ \text{m}$

$$\mu = IA = \left(17.0 \times 10^{-3}\ \text{A}\right)\left[\pi(0.318)^2\ \text{m}^2\right] = \boxed{5.41\ \text{mA}\cdot\text{m}^2}$$

(b) $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$ so $\tau = \left(5.41 \times 10^{-3}\ \text{A}\cdot\text{m}^2\right)(0.800\ \text{T}) = \boxed{4.33\ \text{mN}\cdot\text{m}}$

P29.35 $\tau = NBAI\sin\phi$

$\tau = 100(0.800\ \text{T})\left(0.400 \times 0.300\ \text{m}^2\right)(1.20\ \text{A})\sin 60°$

$\tau = \boxed{9.98\ \text{N}\cdot\text{m}}$

Note that ϕ is the angle between the magnetic moment and the $\vec{\mathbf{B}}$ field. The loop will rotate so as to align the magnetic moment with the $\vec{\mathbf{B}}$ field. Looking down along the y-axis, the loop will rotate in a $\boxed{\text{clockwise}}$ direction.

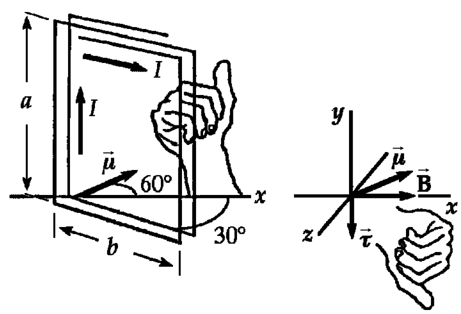


FIG. P29.35

P29.36 Choose $U = 0$ when the dipole moment is at $\theta = 90.0°$ to the field. The field exerts torque of magnitude $\mu B\sin\theta$ on the dipole, tending to turn the dipole moment in the direction of decreasing θ. According to the mechanical equations $\Delta U = -\int dW$ and $dW = \tau\, d\theta$, the potential energy of the dipole-field system is given by

$$U - 0 = \int_{90.0°}^{\theta} \mu B\sin\theta\, d\theta = \mu B(-\cos\theta)\Big|_{90.0°}^{\theta} = -\mu B\cos\theta + 0 \qquad \text{or} \qquad \boxed{U = -\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}}.$$

P29.37 (a) The field exerts torque on the needle tending to align it with the field, so the minimum energy orientation of the needle is:

$\boxed{\text{pointing north at 48.0° below the horizontal}}$

where its energy is $U_{\min} = -\mu B\cos 0° = -\left(9.70 \times 10^{-3}\ \text{A}\cdot\text{m}^2\right)\left(55.0 \times 10^{-6}\ \text{T}\right) = -5.34 \times 10^{-7}\ \text{J}$.

It has maximum energy when pointing in the opposite direction,

$\boxed{\text{south at 48.0° above the horizontal}}$

where its energy is $U_{\max} = -\mu B\cos 180° = +\left(9.70 \times 10^{-3}\ \text{A}\cdot\text{m}^2\right)\left(55.0 \times 10^{-6}\,\text{T}\right) = +5.34 \times 10^{-7}\,\text{J}$.

(b) $U_{\min} + W = U_{\max}$: $\quad W = U_{\max} - U_{\min} = +5.34 \times 10^{-7}\ \text{J} - \left(-5.34 \times 10^{-7}\ \text{J}\right) = \boxed{1.07\ \mu\text{J}}$

P29.38 (a) $\vec{\tau} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$, so $\tau = \left|\vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}\right| = \mu B \sin\theta = NIAB \sin\theta$

$$\tau_{\max} = NIAB \sin 90.0° = 1(5.00\text{ A})\left[\pi(0.050\,0\text{ m})^2\right](3.00\times10^{-3}\text{ T}) = \boxed{118\ \mu\text{N}\cdot\text{m}}$$

(b) $U = -\vec{\mu}\cdot\vec{\mathbf{B}}$, so $-\mu B \le U \le +\mu B$

Since $\mu B = (NIA)B = 1(5.00\text{ A})\left[\pi(0.050\,0\text{ m})^2\right](3.00\times10^{-3}\text{ T}) = 118\ \mu\text{J}$,

the range of the potential energy is: $\boxed{-118\ \mu\text{J} \le U \le +118\ \mu\text{J}}$.

***P29.39** For a single-turn circle, $r = 1.5\text{ m}/2\pi$. The magnetic moment is

$\mu = NIA = 1\ (30\times10^{-3}\text{ A})\ \pi(1.5\text{ m}/2\pi)^2 = 5.37\times10^{-3}\text{ A}\cdot\text{m}^2$

For a single-turn square, $\ell = 1.5\text{ m}/4$. The magnetic moment is

$\mu = NIA = 1\ (30\times10^{-3}\text{ A})\ (1.5\text{ m}/4)^2 = 4.22\times10^{-3}\text{ A}\cdot\text{m}^2$

For a single-turn triangle, $\ell = 1.5\text{ m}/3 = 0.5\text{ m}$. The magnetic moment is

$\mu = NIA = 1\ (30\times10^{-3}\text{ A})\ (1/2)\ 0.5\text{ m}\ (0.5^2 - 0.25^2)^{1/2}\text{ m} = 3.25\times10^{-3}\text{ A}\cdot\text{m}^2$

For a flat compact circular coil with N turns, $r = 1.5\text{ m}/2\pi N$. The magnetic moment is

$\mu = NIA = N\ (30\times10^{-3}\text{ A})\ \pi(1.5\text{ m}/2\pi N)^2 = 5.37\times10^{-3}\text{ A}\cdot\text{m}^2/N$

> The magnetic moment cannot go to infinity. Its maximum value is 5.37 mA·m² for a single-turn circle. Smaller by 21% and by 40% are the magnetic moments for the single-turn square and triangle. Circular coils with several turns have magnetic moments inversely proportional to the number of turns, approaching zero as the number of turns goes to infinity.

P29.40 (a) $\left|\vec{\boldsymbol{\tau}}\right| = \left|\vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}\right| = NIAB\sin\theta$

$$\tau_{\max} = 80(10^{-2}\text{ A})(0.025\text{ m}\cdot0.04\text{ m})(0.8\text{ N/A}\cdot\text{m})\sin 90° = \boxed{6.40\times10^{-4}\text{ N}\cdot\text{m}}$$

(b) $\mathcal{P}_{\max} = \tau_{\max}\omega = 6.40\times10^{-4}\text{ N}\cdot\text{m}(3\,600\text{ rev/min})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{0.241\text{ W}}$

(c) In one half revolution the work is

$$W = U_{\max} - U_{\min} = -\mu B\cos180° - (-\mu B\cos0°) = 2\mu B$$
$$= 2NIAB = 2(6.40\times10^{-4}\text{ N}\cdot\text{m}) = 1.28\times10^{-3}\text{ J}$$

In one full revolution, $W = 2(1.28\times10^{-3}\text{ J}) = \boxed{2.56\times10^{-3}\text{ J}}$.

(d) $\mathcal{P}_{\text{avg}} = \frac{W}{\Delta t} = \frac{2.56\times10^{-3}\text{ J}}{(1/60)\text{ s}} = \boxed{0.154\text{ W}}$

The peak power in (b) is greater by the factor $\frac{\pi}{2}$.

Section 29.6 **The Hall Effect**

P29.41 $B=\dfrac{nqt(\Delta V_H)}{I}=\dfrac{\left(8.46\times10^{28}\ \text{m}^{-3}\right)\left(1.60\times10^{-19}\ \text{C}\right)\left(5.00\times10^{-3}\ \text{m}\right)\left(5.10\times10^{-12}\ \text{V}\right)}{8.00\ \text{A}}$

$B=4.31\times10^{-5}\ \text{T}=\boxed{43.1\ \mu\text{T}}$

P29.42 (a) $\Delta V_H=\dfrac{IB}{nqt}$ so $\dfrac{nqt}{I}=\dfrac{B}{\Delta V_H}=\dfrac{0.080\,0\ \text{T}}{0.700\times10^{-6}\ \text{V}}=1.14\times10^{5}\ \text{T/V}$

Then, the unknown field is $B=\left(\dfrac{nqt}{I}\right)(\Delta V_H)$

$B=\left(1.14\times10^{5}\ \text{T/V}\right)\left(0.330\times10^{-6}\ \text{V}\right)=0.037\,7\ \text{T}=\boxed{37.7\ \text{mT}}$

(b) $\dfrac{nqt}{I}=1.14\times10^{5}\ \text{T/V}$ so $n=\left(1.14\times10^{5}\ \text{T/V}\right)\dfrac{I}{qt}$

$n=\left(1.14\times10^{5}\ \text{T/V}\right)\dfrac{0.120\ \text{A}}{\left(1.60\times10^{-19}\ \text{C}\right)\left(2.00\times10^{-3}\ \text{m}\right)}=\boxed{4.29\times10^{25}\ \text{m}^{-3}}$

Additional Problems

***P29.43** (a) Define vector $\vec{\mathbf{h}}$ to have the downward direction of the current, and vector $\vec{\mathbf{L}}$ to be along the pipe into the page as shown. The $\boxed{\text{electric current experiences a magnetic force}}$.

$I\left(\vec{\mathbf{h}}\times\vec{\mathbf{B}}\right)$ in the direction of $\vec{\mathbf{L}}$

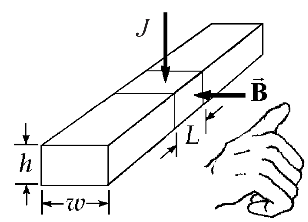


FIG. P29.43

(b) The sodium, consisting of ions and electrons, flows along the pipe transporting no net charge. But inside the section of length *L*, electrons drift upward to constitute downward electric current $J\times(\text{area})=J\,Lw$.

The current then feels a magnetic force $I\left|\vec{\mathbf{h}}\times\vec{\mathbf{B}}\right|=JLwhB\sin 90°$.

This force along the pipe axis will make the fluid move, exerting pressure

$\dfrac{F}{\text{area}}=\dfrac{JLwhB}{hw}=\boxed{JLB}$

(c) Charge moves within the fluid inside the length *L*, but charge does not accumulate: the fluid is not charged after it leaves the pump. It is not current-carrying and it is not magnetized.

P29.44 (a) At the moment shown in Figure 29.10, the particle must be moving upward in order for the magnetic force on it to be  into the page, toward the center of this turn of its spiral path. Throughout its motion it circulates clockwise.

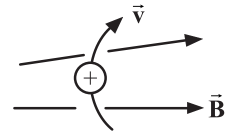


FIG. P29.44(a)

(b) After the particle has passed the middle of the bottle and moves into the region of increasing magnetic field, the magnetic force on it has a component to the left (as well as a radially inward component) as shown. This force in the $-x$ direction slows and reverses the particle's motion along the axis.

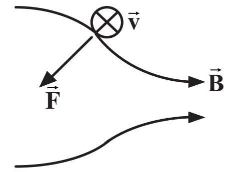


FIG. P29.44(b)

(c) The magnetic force is perpendicular to the velocity and does no work on the particle. The particle keeps constant kinetic energy. As its axial velocity component decreases, its tangential velocity component increases.

(d) The orbiting particle constitutes a loop of current in the yz plane and therefore a magnetic dipole moment $IA = \frac{q}{T}A$ in the $-x$ direction. It is like a little bar magnet with its N pole on the left.

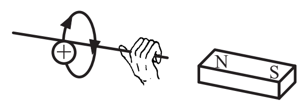


FIG. P29.44(d)

(e) Problem 31 showed that a nonuniform magnetic field exerts a net force on a magnetic dipole. When the dipole is aligned opposite to the external field, the force pushes it out of the region of stronger field. Here it is to the left, a force of repulsion of one magnetic south pole on another south pole.

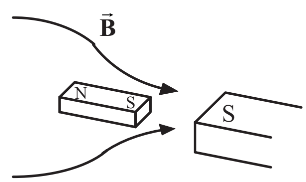


FIG. P29.44(e)

***P29.45** The particle moves in an arc of a circle with radius

$$r = \frac{mv}{qB} = \frac{1.67\times10^{-27}\ \text{kg}\ \ 3\times10^{7}\ \text{m/s}\ \ \text{C m}}{1.6\times10^{-19}\ \text{C}\ \ 25\times10^{-6}\ \text{N s}} = \boxed{12.5\ \text{km}}$$

. It will not hit the Earth, but perform a hairpin turn and go back parallel to its original direction.

P29.46 $\sum F_y = 0$: $\quad +n - mg = 0$

$\sum F_x = 0$: $\quad -\mu_k n + IBd\sin 90.0° = 0$

$$B = \frac{\mu_k mg}{Id} = \frac{0.100(0.200\ \text{kg})(9.80\ \text{ms}^2)}{(10.0\ \text{A})(0.500\ \text{m})} = \boxed{39.2\ \text{mT}}$$

P29.47 The magnetic force on each proton, $\vec{\mathbf{F}}_B = q\vec{\mathbf{v}}\times\vec{\mathbf{B}} = qvB\sin 90°$ downward perpendicular to velocity, causes centripetal acceleration, guiding it into a circular path of radius r, with

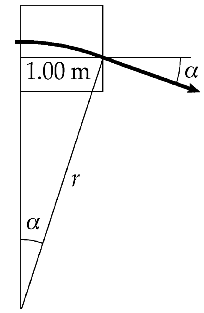


FIG. P29.47

$$qvB = \frac{mv^2}{r}$$

and $$r = \frac{mv}{qB}$$

We compute this radius by first finding the proton's speed:

$$K = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(5.00\times10^{6}\,\text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}{1.67\times10^{-27}\,\text{kg}}} = 3.10\times10^{7}\ \text{m/s}$$

Now, $$r = \frac{mv}{qB} = \frac{\left(1.67\times10^{-27}\ \text{kg}\right)\left(3.10\times10^{7}\ \text{m/s}\right)}{\left(1.60\times10^{-19}\ \text{C}\right)\left(0.050\,0\ \text{N}\cdot\text{s/C}\cdot\text{m}\right)} = 6.46\ \text{m}$$

(b) We can most conveniently do part (b) first. From the figure observe that

$$\sin\alpha = \frac{1.00\ \text{m}}{r} = \frac{1\ \text{m}}{6.46\ \text{m}}$$

$$\boxed{\alpha = 8.90°}$$

(a) The magnitude of the proton momentum stays constant, and its final y component is

$$-\left(1.67\times10^{-27}\ \text{kg}\right)\left(3.10\times10^{7}\ \text{m/s}\right)\sin 8.90° = \boxed{-8.00\times10^{-21}\ \text{kg}\cdot\text{m/s}}$$

***P29.48** (a) If $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$, $\quad \vec{\mathbf{F}}_B = q\vec{\mathbf{v}}\times\vec{\mathbf{B}} = e\left(v_i\hat{\mathbf{i}}\right)\times\left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right) = 0 + ev_iB_y\hat{\mathbf{k}} - ev_iB_z\hat{\mathbf{j}}$.

Since the force actually experienced is $\vec{\mathbf{F}}_B = F_i\hat{\mathbf{j}}$, observe that

$\boxed{B_x \text{ could have any value}}$, $\boxed{B_y = 0}$, and $\boxed{B_z = -\frac{F_i}{ev_i}}$.

(b) If $\vec{\mathbf{v}} = -v_i\hat{\mathbf{i}}$, then $\quad \vec{\mathbf{F}}_B = q\vec{\mathbf{v}}\times\vec{\mathbf{B}} = e\left(-v_i\hat{\mathbf{i}}\right)\times\left(B_x\hat{\mathbf{i}} + 0\hat{\mathbf{j}} - \frac{F_i}{ev_i}\hat{\mathbf{k}}\right) = \boxed{-F_i\hat{\mathbf{j}}}$

(c) If $q = -e$ and $\vec{\mathbf{v}} = -v_i\hat{\mathbf{i}}$, then $\quad \vec{\mathbf{F}}_B = q\vec{\mathbf{v}}\times\vec{\mathbf{B}} = -e\left(-v_i\hat{\mathbf{i}}\right)\times\left(B_x\hat{\mathbf{i}} + 0\hat{\mathbf{j}} - \frac{F_i}{ev_i}\hat{\mathbf{k}}\right) = \boxed{+F_i\hat{\mathbf{j}}}$

Reversing either the velocity or the sign of the charge reverses the force.

P29.49 (a) The net force is the Lorentz force given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}}\times\vec{\mathbf{B}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)$$

$$\vec{\mathbf{F}} = \left(3.20\times10^{-19}\right)\left[\left(4\hat{\mathbf{i}} - 1\hat{\mathbf{j}} - 2\hat{\mathbf{k}}\right) + \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 1\hat{\mathbf{k}}\right)\times\left(2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 1\hat{\mathbf{k}}\right)\right]\ \text{N}$$

Carrying out the indicated operations, we find:

$$\vec{\mathbf{F}} = \boxed{\left(3.52\hat{\mathbf{i}} - 1.60\hat{\mathbf{j}}\right)\times10^{-18}\ \text{N}}.$$

(b) $$\theta = \cos^{-1}\left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{3.52}{\sqrt{(3.52)^2 + (1.60)^2}}\right) = \boxed{24.4°}$$

***P29.50** (a) The field should be in the $+z$ direction, perpendicular to the final as well as to the initial velocity, and with $\hat{\mathbf{i}}\times\hat{\mathbf{k}}=-\hat{\mathbf{j}}$ as the direction of the initial force.

(b) $r=\dfrac{mv}{qB}=\dfrac{\left(1.67\times10^{-27}\text{ kg}\right)\left(20\times10^{6}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.3\text{ N}\cdot\text{s/C}\cdot\text{m}\right)}=\boxed{0.696\text{ m}}$

(c) The path is a quarter circle, of length $(\pi/2)0.696\text{ m}=\boxed{1.09\text{ m}}$

(d) $\Delta t=1.09\text{ m}/20\times10^{6}\text{ m/s}=\boxed{54.7\text{ ns}}$

P29.51 A key to solving this problem is that reducing the normal force will reduce the friction force: $F_B=BIL$ or $B=\dfrac{F_B}{IL}$.

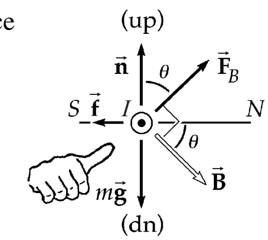


FIG. P29.51

When the wire is just able to move, $\sum F_y=n+F_B\cos\theta-mg=0$

so $n=mg-F_B\cos\theta$

and $f=\mu\left(mg-F_B\cos\theta\right)$

Also, $\sum F_x=F_B\sin\theta-f=0$

so $F_B\sin\theta=f$: $F_B\sin\theta=\mu\left(mg-F_B\cos\theta\right)$ and $F_B=\dfrac{\mu mg}{\sin\theta+\mu\cos\theta}$

We minimize B by minimizing F_B: $\dfrac{dF_B}{d\theta}=(\mu mg)\dfrac{\cos\theta-\mu\sin\theta}{\left(\sin\theta+\mu\cos\theta\right)^2}=0\Rightarrow\mu\sin\theta=\cos\theta$

Thus, $\theta=\tan^{-1}\left(\dfrac{1}{\mu}\right)=\tan^{-1}(5.00)=78.7°$ for the smallest field, and

$$B=\frac{F_B}{IL}=\left(\frac{\mu g}{I}\right)\frac{(m/L)}{\sin\theta+\mu\cos\theta}$$

$$B_{\text{min}}=\left[\frac{(0.200)\left(9.80\text{ m/s}^2\right)}{1.50\text{ A}}\right]\frac{0.100\text{ kg/m}}{\sin 78.7°+(0.200)\cos 78.7°}=0.128\text{ T}$$

$$\boxed{B_{\text{min}}=0.128\text{ T pointing north at an angle of }78.7°\text{ below the horizontal}}$$

P29.52 Let v_i represent the original speed of the alpha particle. Let v_α and v_p represent the particles' speeds after the collision. We have conservation of momentum $4m_pv_i=4m_pv_\alpha+m_pv_p$ and the relative velocity equation $v_i-0=v_p-v_\alpha$. Eliminating v_i,

$$4v_p-4v_\alpha=4v_\alpha+v_p \qquad 3v_p=8v_\alpha \qquad v_\alpha=\frac{3}{8}v_p$$

For the proton's motion in the magnetic field,

$$\sum F=ma \qquad ev_pB\sin 90°=\frac{m_pv_p^2}{R} \qquad \frac{eBR}{m_p}=v_p$$

For the alpha particle,

$$2ev_\alpha B\sin 90°=\frac{4m_pv_\alpha^2}{r_\alpha} \qquad r_\alpha=\frac{2m_pv_\alpha}{eB} \qquad r_\alpha=\frac{2m_p}{eB}\frac{3}{8}v_p=\frac{2m_p}{eB}\frac{3}{8}\frac{eBR}{m_p}=\boxed{\frac{3}{4}R}$$

P29.53 Let Δx_1 be the elongation due to the weight of the wire and let Δx_2 be the additional elongation of the springs when the magnetic field is turned on. Then $F_{magnetic} = 2k\Delta x_2$ where k is the force constant of the spring and can be determined from $k = \dfrac{mg}{2\Delta x_1}$. (The factor 2 is included in the two previous equations since there are 2 springs in parallel.) Combining these two equations, we find

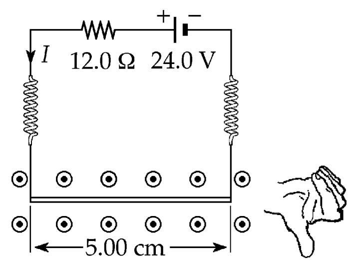


FIG. P29.53

$$F_{magnetic} = 2\left(\frac{mg}{2\Delta x_1}\right)\Delta x_2 = \frac{mg\Delta x_2}{\Delta x_1}; \quad \text{but } \left|\vec{\mathbf{F}}_B\right| = I\left|\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right| = ILB$$

Therefore, where $I = \dfrac{24.0\text{ V}}{12.0\ \Omega} = 2.00\text{ A}$, $B = \dfrac{mg\Delta x_2}{IL\Delta x_1} = \dfrac{(0.100)(9.80)\left(3.00\times 10^{-3}\right)}{(2.00)(0.050\,0)\left(5.00\times 10^{-3}\right)} = \boxed{0.588\text{ T}}$.

P29.54 Suppose the input power is

$120\text{ W} = (120\text{ V})I$: $\boxed{I \sim 1\text{ A} = 10^0\text{ A}}$

Suppose $\omega = 2\,000\text{ rev/min}\left(\dfrac{1\text{ min}}{60\text{ s}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right) \sim 200\text{ rad/s}$

and the output power is $20\text{ W} = \tau\omega = \tau(200\text{ rad/s})$ $\boxed{\tau \sim 10^{-1}\text{ N}\cdot\text{m}}$

Suppose the area is about $(3\text{ cm})\times(4\text{ cm})$, or $\boxed{A \sim 10^{-3}\text{ m}^2}$

Suppose that the field is $\boxed{B \sim 10^{-1}\text{ T}}$

Then, the number of turns in the coil may be found from $\tau \cong NIAB$:

$$0.1\text{ N}\cdot\text{m} \sim N(1\text{ C/s})\left(10^{-3}\text{ m}^2\right)\left(10^{-1}\text{ N}\cdot\text{s/C}\cdot\text{m}\right)$$

giving $\boxed{N \sim 10^3}$

P29.55 The sphere is in translational equilibrium; thus

$$f_s - Mg\sin\theta = 0 \qquad (1)$$

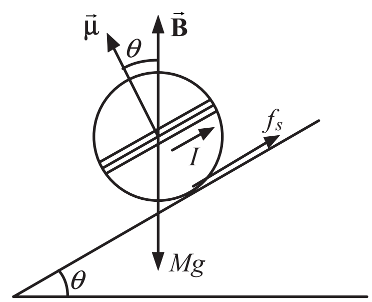


FIG. P29.55

The sphere is in rotational equilibrium. If torques are taken about the center of the sphere, the magnetic field produces a clockwise torque of magnitude $\mu B\sin\theta$, and the frictional force a counterclockwise torque of magnitude $f_s R$, where R is the radius of the sphere. Thus:

$$f_s R - \mu B\sin\theta = 0 \qquad (2)$$

From (1): $f_s = Mg\sin\theta$. Substituting this in (2) and canceling out $\sin\theta$, one obtains

$$\mu B = MgR. \qquad (3)$$

Now $\mu = NI\pi R^2$. Thus (3) gives $I = \dfrac{Mg}{\pi NBR} = \dfrac{(0.08\text{ kg})\left(9.80\text{ m/s}^2\right)}{\pi(5)(0.350\text{ T})(0.2\text{ m})} = \boxed{0.713\text{ A}}$.

The current must be counterclockwise as seen from above.

P29.56 Call the length of the rod L and the tension in each wire alone $\frac{T}{2}$. Then, at equilibrium:

$$\sum F_x = T\sin\theta - ILB\sin 90.0° = 0 \quad \text{or} \quad T\sin\theta = ILB$$

$$\sum F_y = T\cos\theta - mg = 0, \quad \text{or} \quad T\cos\theta = mg$$

$$\tan\theta = \frac{ILB}{mg} = \frac{IB}{(m/L)g} \quad \text{or} \quad B = \frac{(m/L)g}{I}\tan\theta = \boxed{\frac{\lambda g}{I}\tan\theta}$$

P29.57 $\sum F = ma$ or $qvB\sin 90.0° = \frac{mv^2}{r}$

$\therefore$ the angular frequency for each ion is $\frac{v}{r} = \omega = \frac{qB}{m} = 2\pi f$ and

$$\Delta\omega = \omega_{12} - \omega_{14} = qB\left(\frac{1}{m_{12}} - \frac{1}{m_{14}}\right) = \frac{(1.60\times10^{-19}\text{ C})(2.40\text{ T})}{(1.66\times10^{-27}\text{ kg/u})}\left(\frac{1}{12.0\text{ u}} - \frac{1}{14.0\text{ u}}\right)$$

$$\Delta\omega = 2.75\times10^{6}\text{ s}^{-1} = \boxed{2.75\text{ Mrad/s}}$$

P29.58 Let v_x and $v_\perp$ be the components of the velocity of the positron parallel to and perpendicular to the direction of the magnetic field.

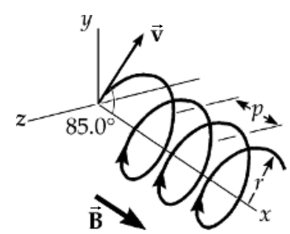


FIG. P29.58

(a) The pitch of trajectory is the distance moved along x by the positron during each period, T (determined by the cyclotron frequency):

$$p = v_x T = (v\cos 85.0°)\left(\frac{2\pi m}{Bq}\right)$$

$$p = \frac{(5.00\times10^{6})(\cos 85.0°)(2\pi)(9.11\times10^{-31})}{0.150(1.60\times10^{-19})} = \boxed{1.04\times10^{-4}\text{ m}}$$

(b) The equation about circular motion in a magnetic field still applies to the radius of the spiral: $r = \frac{mv_\perp}{Bq} = \frac{mv\sin 85.0°}{Bq}$

$$r = \frac{(9.11\times10^{-31})(5.00\times10^{6})(\sin 85.0°)}{(0.150)(1.60\times10^{-19})} = \boxed{1.89\times10^{-4}\text{ m}}$$

P29.59 $|\tau| = IAB$ where the effective current due to the orbiting electrons is $I = \frac{\Delta q}{\Delta t} = \frac{q}{T}$

and the period of the motion is $T = \frac{2\pi R}{v}$

The electron's speed in its orbit is found by requiring $\frac{k_e q^2}{R^2} = \frac{mv^2}{R}$ or $v = q\sqrt{\frac{k_e}{mR}}$

Substituting this expression for v into the equation for T, we find $T = 2\pi\sqrt{\frac{mR^3}{q^2 k_e}}$

$$T = 2\pi\sqrt{\frac{(9.11\times10^{-31})(5.29\times10^{-11})^3}{(1.60\times10^{-19})^2(8.99\times10^{9})}} = 1.52\times10^{-16}\text{ s}$$

Therefore, $|\tau| = \left(\frac{q}{T}\right)AB = \frac{1.60\times10^{-19}}{1.52\times10^{-16}}\pi(5.29\times10^{-11})^2(0.400) = \boxed{3.70\times10^{-24}\text{ N}\cdot\text{m}}$.

P29.60 (a) $K = \frac{1}{2}mv^2 = 6.00 \text{ MeV} = \left(6.00\times10^6 \text{ eV}\right)\left(1.60\times10^{-19} \text{ J/eV}\right)$

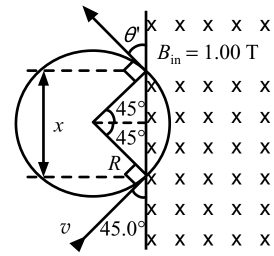


FIG. P29.60

$K = 9.60\times10^{-13}$ J

$$v = \sqrt{\frac{2\left(9.60\times10^{-13} \text{ J}\right)}{1.67\times10^{-27} \text{ kg}}} = 3.39\times10^7 \text{ m/s}$$

$$F_B = qvB = \frac{mv^2}{R} \text{ so}$$

$$R = \frac{mv}{qB} = \frac{\left(1.67\times10^{-27} \text{kg}\right)\left(3.39\times10^7 \text{ m/s}\right)}{\left(1.60\times10^{-19} \text{C}\right)(1.00 \text{ T})} = 0.354 \text{ m}$$

Then, from the diagram, $x = 2R\sin 45.0° = 2(0.354 \text{ m})\sin 45.0° = \boxed{0.501 \text{ m}}$.

(b) From the diagram, observe that $\theta' = \boxed{45.0°}$.

P29.61 (a) The magnetic force acting on ions in the blood stream will deflect positive charges toward point *A* and negative charges toward point *B*. This separation of charges produces an electric field directed from *A* toward *B*. At equilibrium, the electric force caused by this field must balance the magnetic force, so

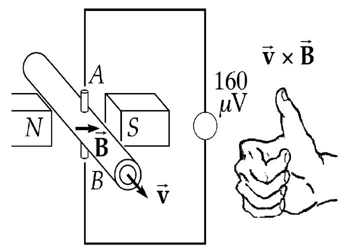


FIG. P29.61

$$qvB = qE = q\left(\frac{\Delta V}{d}\right)$$

or $$v = \frac{\Delta V}{Bd} = \frac{\left(160\times10^{-6} \text{V}\right)}{(0.040\,0 \text{ T})\left(3.00\times10^{-3} \text{m}\right)} = \boxed{1.33 \text{ m/s}}$$

(b) $\boxed{\text{No}}$. Negative ions moving in the direction of v would be deflected toward point *B*, giving *A* a higher potential than *B*. Positive ions moving in the direction of v would be deflected toward *A*, again giving *A* a higher potential than *B*. Therefore, the sign of the potential difference does not depend on whether the ions in the blood are positively or negatively charged.

P29.62 (a) See graph to the right. The Hall voltage is directly proportional to the magnetic field. A least-square fit to the data gives the equation of the best fitting line as:

$$\boxed{\Delta V_H = \left(1.00\times10^{-4} \text{ V/T}\right)B}$$

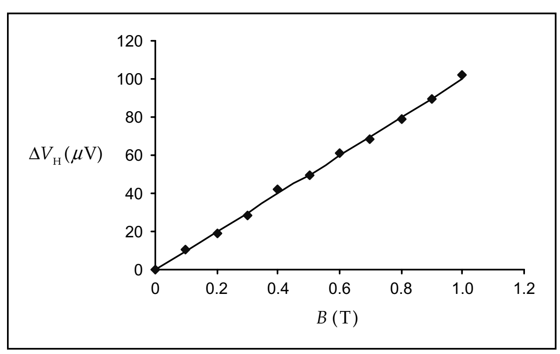


FIG. P29.62

(b) Comparing the equation of the line which fits the data best to

$$\Delta V_H = \left(\frac{1}{nqt}\right)B$$

continued on next page

observe that: $\dfrac{I}{nqt} = 1.00\times10^{-4}\ \text{V/T}$, or $t = \dfrac{I}{nq\left(1.00\times10^{-4}\ \text{V/T}\right)}$.

Then, if $I = 0.200$ A, $q = 1.60\times10^{-19}$ C, and $n = 1.00\times10^{26}\,\text{m}^{-3}$, the thickness of the sample is

$$t = \frac{0.200\ \text{A}}{\left(1.00\times10^{26}\text{m}^{-3}\right)\left(1.60\times10^{-19}\text{C}\right)\left(1.00\times10^{-4}\ \text{V/T}\right)} = 1.25\times10^{-4}\,\text{m} = \boxed{0.125\ \text{mm}}$$

P29.63 When in the field, the particles follow a circular path according to $qvB = \dfrac{mv^2}{r}$, so the radius of the path is $r = \dfrac{mv}{qB}$.

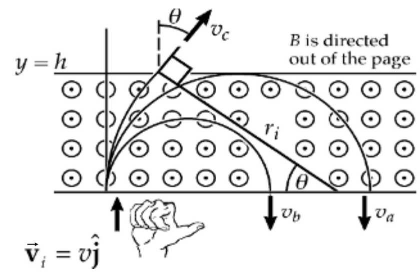


FIG. P29.63

(a) When $r = h = \dfrac{mv}{qB}$, that is, when $\boxed{v = \dfrac{qBh}{m}}$, the particle will cross the band of field. It will move in a full semicircle of radius h, leaving the field at $(2h,\ 0,\ 0)$ with velocity $\boxed{\vec{\mathbf{v}}_f = -v\hat{\mathbf{j}}}$.

(b) When $v < \dfrac{qBh}{m}$, the particle will move in a smaller $\boxed{\text{semicircle}}$ of radius $r = \dfrac{mv}{qB} < h$. It will leave the field at $(2r,\ 0,\ 0)$ with velocity $\boxed{\vec{\mathbf{v}}_f = -v\hat{\mathbf{j}}}$.

(c) When $v > \dfrac{qBh}{m}$, the particle moves in a $\boxed{\text{circular arc}}$ of radius $r = \dfrac{mv}{qB} > h$, centered at $(r,\ 0,\ 0)$. The arc subtends an angle given by $\theta = \sin^{-1}\left(\dfrac{h}{r}\right)$. It will leave the field at the point with coordinates $\left[r(1-\cos\theta),\ h,\ 0\right]$ with velocity $\boxed{\vec{\mathbf{v}}_f = v\sin\theta\,\hat{\mathbf{i}} + v\cos\theta\,\hat{\mathbf{j}}}$.

P29.64 (a) $I = \dfrac{ev}{2\pi r}$ $\qquad \mu = IA = \left(\dfrac{ev}{2\pi r}\right)\pi r^2 = \boxed{9.27\times10^{-24}\ \text{A}\cdot\text{m}^2}$

The Bohr model predicts the correct magnetic moment. However, the "planetary model" is seriously deficient in other regards.

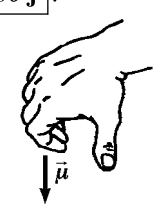


FIG. P29.64

(b) Because the electron is (–), its [conventional] current is clockwise, as seen from above, and μ points $\boxed{\text{downward}}$.

***P29.65** (a) The torque on the dipole $\vec{\tau} = \vec{\mu}\times\vec{\mathbf{B}}$ has magnitude $\mu B \sin\theta \approx \mu B\theta$, proportional to the angular displacement if the angle is small. It is a restoring torque, tending to turn the dipole toward its equilibrium orientation. Then the statement that its motion is simple harmonic is true for small angular displacements.

(b) $\tau = I\alpha$ becomes $\quad -\mu B\theta = I\, d^2\theta/dt^2 \qquad d^2\theta/dt^2 = -(\mu B/I)\theta = -\omega^2\theta$

where $\omega = (\mu B/I)^{1/2}$ is the angular frequency and $f = \omega/2\pi = \dfrac{1}{2\pi}\sqrt{\dfrac{\mu B}{I}}$ is the frequency in hertz.

(c) The equilibrium orientation of the needle shows the direction of the field. In a stronger field, the frequency is higher. The frequency is easy to measure precisely over a wide range of values. The equation in part (c) gives
$0.680\ \text{Hz} = (1/2\pi)\,(\mu/I)^{1/2}\,(39.2\ \mu\text{T})^{1/2}$ and $4.90\ \text{Hz} = (1/2\pi)\,(\mu/I)^{1/2}\,(B_2)^{1/2}$. We square and divide to find $B_2/39.2\ \mu\text{T} = (4.90\ \text{Hz}/0.680\ \text{Hz})^2$ so $B_2 = 51.9(39.2\ \mu\text{T}) = \boxed{2.04\ \text{mT}}$.

ANSWERS TO EVEN PROBLEMS

P29.2 (a) west (b) no deflection (c) up (d) down

P29.4 (a) 86.7 fN (b) 51.9 Tm/s^2

P29.6 (a) 7.90 pN (b) 0

P29.8 $B_y = -2.62$ mT. $B_z = 0$. B_x may have any value.

P29.10 (a) 6.84×10^{-16} m down (b) 7.24 mm east. The beam moves on an arc of a circle rather than on a parabola, but its northward velocity component stays constant within 0.09%, so it is a good approximation to treat it as constant.

P29.12 115 keV

P29.14 $\frac{m'}{m} = 8$

P29.16 (a) 17.9 ns (b) 35.1 eV

P29.18 (a) 8.28 cm (b) 8.23 cm; ratio is independent of both ΔV and B

P29.20 (a) 7.66×10^7 rad/s (b) 26.8 Mm/s (c) 3.76 MeV (d) 3.13×10^3 rev (e) 257 μs

P29.22 (b) The dashed red line should spiral around many times, with its turns relatively far apart on the inside and closer together on the outside. (c) 682 m/s (d) 55.9 μm

P29.24 (a) Yes: The constituent of the beam is present in all kinds of atoms. (b) Yes: Everything in the beam has a single charge-to-mass ratio. (c) Thomson pointed out that ionized hydrogen had the largest charge-to-mass ratio previously known, and that the particles in his beam had a charge-to-mass ratio about 2 000 times larger. The particles in his beam could not be whole atoms, but rather must be much smaller in mass. (d) No. The particles move with speed on the order of ten million meters per second, to fall by an immeasurably small amount over a distance of less than a meter.

P29.26 $\left(-2.88\hat{\mathbf{j}}\right)$ N

P29.28 840 A east

P29.30 $\left(\frac{4IdBL}{3m}\right)^{1/2}$

P29.32 (a) $\vec{\mathbf{F}}_{ab} = 0$; $\vec{\mathbf{F}}_{bc} = 40.0\text{ mN}\left(-\hat{\mathbf{i}}\right)$; $\vec{\mathbf{F}}_{cd} = 40.0\text{ mN}\left(-\hat{\mathbf{k}}\right)$; $\vec{\mathbf{F}}_{da} = (40.0\text{ mN})\left(\hat{\mathbf{i}} + \hat{\mathbf{k}}\right)$ (b) The forces on the four segments must add to zero, so the force on the fourth segment must be the negative of the resultant of the forces on the other three.

P29.34 (a) $5.41\text{ mA}\cdot\text{m}^2$ (b) $4.33\text{ mN}\cdot\text{m}$

P29.36 See the solution.

P29.38 (a) 118 μN · m (b) $-118\ \mu\text{J} \le U \le 118\ \mu\text{J}$

P29.40 (a) 640 μN · m (b) 241 mW (c) 2.56 mJ (d) 154 mW

P29.42 (a) 37.7 mT (b) $4.29\times 10^{25}/\text{m}^3$

P29.44 See the solution.

P29.46 39.2 mT

P29.48 (a) B_x is indeterminate. $B_y = 0$. $B_z = \dfrac{-F_i}{ev_i}$ (b) $-F_i\hat{\mathbf{j}}$ (c) $+F_i\hat{\mathbf{j}}$

P29.50 (a) the $+z$ direction (b) 0.696 m (c) 1.09 m (d) 54.7 ns

P29.52 $\dfrac{3R}{4}$

P29.54 $B \sim 10^{-1}$ T; $\tau \sim 10^{-1}$ N·m; $I \sim 1$ A; $A \sim 10^{-3}$ m^2; $N \sim 10^3$

P29.56 $\dfrac{\lambda g \tan\theta}{I}$

P29.58 (a) 0.104 mm; (b) 0.189 mm

P29.60 (a) 0.501 m (b) 45.0°

P29.62 (a) See the solution. Empirically, $\Delta V_{\text{H}} = (100\ \mu\text{V/T})B$ (b) 0.125 mm

P28.64 (a) 9.27×10^{-24} A · m^2 (b) down

30

Sources of the Magnetic Field

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q30.1** Answers (b) and (c).

***Q30.2** (i) Magnetic field lines line in horizontal planes and go around the wire clockwise as seen from above. East of the wire the field points horizontally south. Answer (b). (ii) The same. Answer (b).

***Q30.3** (i) Answer (f). (ii) Answer (e).

Q30.4 The magnetic field created by wire 1 at the position of wire 2 is into the paper. Hence, the magnetic force on wire 2 is in direction down × into the paper = to the right, away from wire 1. Now wire 2 creates a magnetic field into the page at the location of wire 1, so wire 1 feels force up × into the paper = left, away from wire 2.

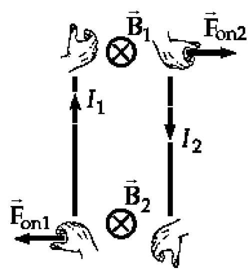


FIG. Q30.4

***Q30.5** Newton's third law describes the relationship. Answer (c).

***Q30.6** (a) No (b) Yes, if all are alike in sign. (c) Yes, if all carry current in the same direction. (d) no

Q30.7 Ampère's law is valid for all closed paths surrounding a conductor, but not always convenient. There are many paths along which the integral is cumbersome to calculate, although not impossible. Consider a circular path around but *not* coaxial with a long, straight current-carrying wire.

Q30.8 The Biot-Savart law considers the contribution of each element of current in a conductor to determine the magnetic field, while for Ampère's law, one need only know the current passing through a given surface. Given situations of high degrees of symmetry, Ampère's law is more convenient to use, even though both laws are equally valid in all situations.

Q30.9 Apply Ampère's law to the circular path labeled 1 in the picture. Since there is no current inside this path, the magnetic field inside the tube must be zero. On the other hand, the current through path 2 is the current carried by the conductor. Therefore the magnetic field outside the tube is nonzero.

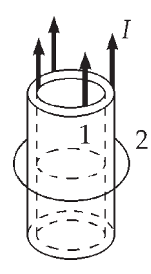


FIG. Q30.9

***Q30.10** (i) Answer (b). (ii) Answer (d), according to $B = \frac{\mu_0 NI}{\ell}$. (iii) Answer (b). (iv) Answer (c).

***Q30.11** Answer (a). The adjacent wires carry currents in the same direction.

***Q30.12** Answer (c). The magnetic flux is $\Phi_B = BA\cos\theta$. Therefore the flux is maximum when the field is perpendicular to the area of the loop of wire. The flux is zero when there is no component of magnetic field perpendicular to the loop—that is, when the plane of the loop contains the x axis.

***Q30.13** Zero in each case. The fields have no component perpendicular to the area.

***Q30.14** (a) Positive charge for attraction. (b) Larger. The contributions away from + and toward – are in the same direction at the midpoint. (c) Downward (d) Smaller. Clockwise around the left-hand wire and clockwise around the right-hand wire are in opposite directions at the midpoint.

***Q30.15** (a) In units of μ_0(ampere/cm), the field of the straight wire, from $\mu_0 I/2\pi r$, is $3/(2\pi 2) = 0.75/\pi$. As a multiple of the same quantity, $N\mu_0 I/2r$ gives for (b) $10 \times 0.3/2 \times 2 = 0.75$.

(c) $N\mu_0 I/\ell$ gives $1000 \times 0.3/200 = 1.5$ times μ_0(ampere/cm), which is also $1.5 \times 4\pi \times 10^{-7}/0.01$ T = 0.19 mT.

(d) The field is zero at the center.

(e) 1 mT is larger than 0.19 mT, so it is largest of all.
The ranking is then e > c > b > a > d.

***Q30.16** Yes. Either pole of the magnet creates field that turns atoms inside the iron to align their magnetic moments with the external field. Then the nonuniform field exerts a net force on each atom toward the direction in which the field is getting stronger.

A magnet on a refrigerator door goes through the same steps to exert a strong normal force on the door. Then the magnet is supported by a frictional force.

Q30.17 Magnetic domain alignment within the magnet and then within the first piece of iron creates an external magnetic field. The field of the first piece of iron in turn can align domains in another iron sample. A nonuniform magnetic field exerts a net force of attraction on magnetic dipoles aligned with the field.

Q30.18 The shock misaligns the domains. Heating will also decrease magnetism.

Q30.19 The north magnetic pole is off the coast of Antarctica, near the south geographic pole. Straight up.

***Q30.20** (a) The third magnet from the top repels the second one with a force equal to the weight of the top two. The yellow magnet repels the blue one with a force equal to the weight of the blue one.

(b) The rods (or a pencil) prevents motion to the side and prevents the magnets from rotating under their mutual torques. Its constraint changes unstable equilibrium into stable.

(c) Most likely, the disks are magnetized perpendicular to their flat faces, making one face a north pole and the other a south pole. One disk has its north pole on the top side and the adjacent magnets have their north poles on their bottom sides.

(d) If the blue magnet were inverted, it and the yellow one would stick firmly together. The pair would still produce an external field and would float together above the red magnets.

SOLUTIONS TO PROBLEMS

Section 30.1 **The Biot-Savart Law**

P30.1 $B = \dfrac{\mu_0 I}{2R} = \dfrac{\mu_0 q (v/2\pi R)}{2R} = \boxed{12.5 \text{ T}}$

P30.2 $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{\left(4\pi \times 10^{-7}\right)(1.00 \text{ A})}{2\pi (1.00 \text{ m})} = \boxed{2.00 \times 10^{-7} \text{ T}}$

P30.3 (a) $B = \dfrac{4\mu_0 I}{4\pi a}\left(\cos\dfrac{\pi}{4} - \cos\dfrac{3\pi}{4}\right)$ where $a = \dfrac{\ell}{2}$

is the distance from any side to the center.

$$B = \frac{4.00 \times 10^{-6}}{0.200}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) = 2\sqrt{2} \times 10^{-5} \text{ T} = \boxed{28.3\ \mu\text{T into the paper}}$$

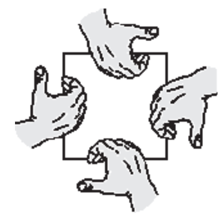


FIG. P30.3

(b) For a single circular turn with $4\ell = 2\pi R$,

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 \pi I}{4\ell} = \frac{\left(4\pi^2 \times 10^{-7}\right)(10.0)}{4(0.400)} = \boxed{24.7\ \mu\text{T into the paper}}$$

P30.4 We can think of the total magnetic field as the superposition of the field due to the long straight wire (having magnitude $\dfrac{\mu_0 I}{2\pi R}$ and directed into the page) and the field due to the circular loop (having magnitude $\dfrac{\mu_0 I}{2R}$ and directed into the page). The resultant magnetic field is:

$$\boxed{\vec{\mathbf{B}} = \left(1 + \frac{1}{\pi}\right)\frac{\mu_0 I}{2R} \text{ (directed into the page)}}.$$

P30.5 For leg 1, $d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = 0$, so there is no contribution to the field from this segment. For leg 2, the wire is only semi-infinite; thus,

$$B = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi x}\right) = \boxed{\frac{\mu_0 I}{4\pi x} \text{ into the paper}}$$

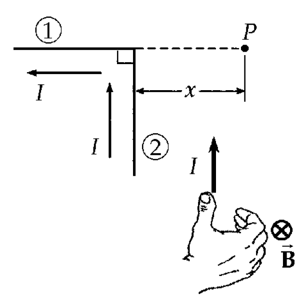


FIG. P30.5

P30.6 Along the axis of a circular loop of radius R,

$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

or $$\frac{B}{B_0} = \left[\frac{1}{(x/R)^2 + 1}\right]^{3/2}$$

where $$B_0 \equiv \frac{\mu_0 I}{2R}$$

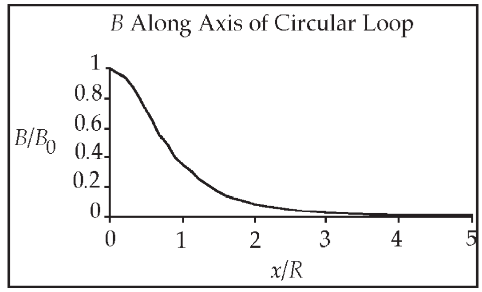
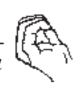


FIG. P30.6

x/R	B/B_0
0.00	1.00
1.00	0.354
2.00	0.0894
3.00	0.0316
4.00	0.0143
5.00	0.00754

P30.7 Wire 1 creates at the origin magnetic field

$\vec{\mathbf{B}}_1 = \frac{\mu_0 I}{2\pi r}$ right hand rule $= \frac{\mu_0 I_1}{2\pi a}$  $= \frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}}$

(a) If the total field at the origin is $\frac{2\mu_0 I_1}{2\pi a}\hat{\mathbf{j}} = \frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}} + \vec{\mathbf{B}}_2$ then the second wire must create field

according to $\vec{\mathbf{B}}_2 = \frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}} = \frac{\mu_0 I_2}{2\pi(2a)}$ .

Then $I_2 =$ $\boxed{2I_1 \text{ out of the paper}}$.

(b) The other possibility is $\vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \frac{2\mu_0 I_1}{2\pi a}\left(-\hat{\mathbf{j}}\right) = \frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}} + \vec{\mathbf{B}}_2$.

Then $\vec{\mathbf{B}}_2 = \frac{3\mu_0 I_1}{2\pi a}\left(-\hat{\mathbf{j}}\right) = \frac{\mu_0 I_2}{2\pi(2a)}$  $I_2 =$ $\boxed{6I_1 \text{ into the paper}}$

P30.8 Every element of current creates magnetic field in the same direction, into the page, at the center of the arc. The upper straight portion creates one-half of the field that an infinitely long straight wire would create. The curved portion creates one quarter of the field that a circular loop produces at its center. The lower straight segment also creates field $\frac{1}{2}\frac{\mu_0 I}{2\pi r}$.

The total field is

$$\vec{\mathbf{B}} = \left(\frac{1}{2}\frac{\mu_0 I}{2\pi r} + \frac{1}{4}\frac{\mu_0 I}{2r} + \frac{1}{2}\frac{\mu_0 I}{2\pi r}\right) \text{ into the page} = \boxed{\frac{\mu_0 I}{2r}\left(\frac{1}{\pi} + \frac{1}{4}\right) \text{ into the plane of the paper}}$$

$$= \left(\frac{0.284\,15\mu_0 I}{r}\right) \text{ into the page}$$

P30.9 (a) Above the pair of wires, the field out of the page of the 50 A current will be stronger than the $\left(-\hat{\mathbf{k}}\right)$ field of the 30 A current, so they cannot add to zero. Between the wires, both produce fields into the page. They can only add to zero below the wires, at coordinate $y=-|y|$. Here the total field is

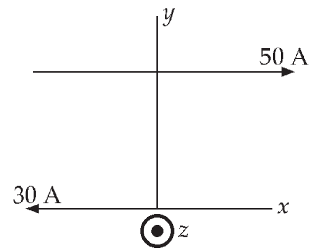


FIG. P30.9

$\vec{\mathbf{B}}=\frac{\mu_0 I}{2\pi r}$  $+\frac{\mu_0 I}{2\pi r}$  :

$$0=\frac{\mu_0}{2\pi}\left[\frac{50\text{ A}}{(|y|+0.28\text{ m})}\left(-\hat{\mathbf{k}}\right)+\frac{30\text{ A}}{|y|}\left(\hat{\mathbf{k}}\right)\right]$$

$$50|y|=30\left(|y|+0.28\text{ m}\right)$$

$$50(-y)=30(0.28\text{ m}-y)$$

$$-20y=30(0.28\text{ m})\quad\boxed{\text{at } y=-0.420\text{ m}}$$

(b) At $y=0.1$ m the total field is $\vec{\mathbf{B}}=\frac{\mu_0 I}{2\pi r}$  $+\frac{\mu_0 I}{2\pi r}$  :

$$\vec{\mathbf{B}}=\frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{2\pi}\left(\frac{50\text{ A}}{(0.28-0.10)\text{ m}}\left(-\hat{\mathbf{k}}\right)+\frac{30\text{ A}}{0.10\text{ m}}\left(-\hat{\mathbf{k}}\right)\right)=1.16\times10^{-4}\text{ T}\left(-\hat{\mathbf{k}}\right)$$

The force on the particle is

$$\vec{\mathbf{F}}=q\vec{\mathbf{v}}\times\vec{\mathbf{B}}=\left(-2\times10^{-6}\text{ C}\right)\left(150\times10^{6}\text{ m/s}\right)\left(\hat{\mathbf{i}}\right)\times\left(1.16\times10^{-4}\text{ N}\cdot\text{s/C}\cdot\text{m}\right)\left(-\hat{\mathbf{k}}\right)$$

$$=\boxed{3.47\times10^{-2}\text{ N}\left(-\hat{\mathbf{j}}\right)}$$

(c) We require $\vec{\mathbf{F}}_e=3.47\times10^{-2}\text{ N}\left(+\hat{\mathbf{j}}\right)=q\vec{\mathbf{E}}=\left(-2\times10^{-6}\text{ C}\right)\vec{\mathbf{E}}$

So $\vec{\mathbf{E}}=\boxed{-1.73\times10^{4}\,\hat{\mathbf{j}}\text{ N/C}}$

***P30.10** We use the Biot-Savart law. For bits of wire along the straight-line sections, $d\vec{\mathbf{s}}$ is at 0° or 180° to $\hat{\mathbf{r}}$, so $d\vec{\mathbf{s}}\times\hat{\mathbf{r}}=0$. Thus, only the curved section of wire contributes to $\vec{\mathbf{B}}$ at *P*. Hence, $d\vec{\mathbf{s}}$ is tangent to the arc and $\hat{\mathbf{r}}$ is radially inward; so $d\vec{\mathbf{s}}\times\hat{\mathbf{r}}=|ds|1\sin 90°\otimes=|ds|\otimes$. All points along the curve are the same distance $r=0.600$ m from the field point, so

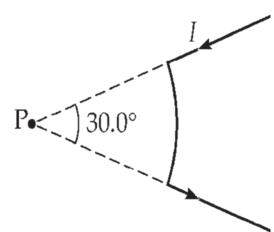


FIG. P30.10

$$B=\int_{\text{all current}}\left|d\vec{\mathbf{B}}\right|=\int\frac{\mu_0}{4\pi}\frac{I\left|d\vec{\mathbf{s}}\times\hat{\mathbf{r}}\right|}{r^2}=\frac{\mu_0}{4\pi}\frac{I}{r^2}\int|ds|=\frac{\mu_0}{4\pi}\frac{I}{r^2}s$$

where *s* is the arc length of the curved wire,

$$s=r\theta=(0.600\text{ m})(30.0°)\left(\frac{2\pi}{360°}\right)=0.314\text{ m}$$

Then, $B=\left(10^{-7}\text{ T}\cdot\text{m/A}\right)\frac{(3.00\text{ A})}{(0.600\text{ m})^2}(0.314\text{ m})$ $\qquad B=\boxed{262\text{ nT into the page}}$

***P30.11** Label the wires 1, 2, and 3 as shown in Figure (a) and let the magnetic field created by the currents in these wires be $\vec{\mathbf{B}}_1$, $\vec{\mathbf{B}}_2$, and $\vec{\mathbf{B}}_3$, respectively.

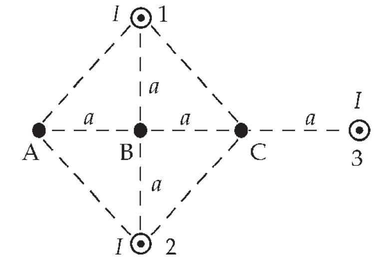


Figure (a)

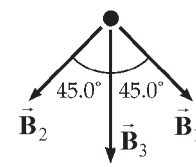


Figure (b)

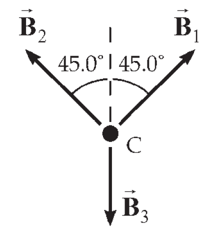


Figure (c)

FIG. P30.11

(a) At point A: $B_1 = B_2 = \dfrac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}$ and $B_3 = \dfrac{\mu_0 I}{2\pi(3a)}$.

The directions of these fields are shown in Figure (b). Observe that the horizontal components of $\vec{\mathbf{B}}_1$ and $\vec{\mathbf{B}}_2$ cancel while their vertical components both add onto $\vec{\mathbf{B}}_3$.

Therefore, the net field at point A is:

$$B_A = B_1\cos 45.0° + B_2\cos 45.0° + B_3$$

$$= \frac{\mu_0 I}{2\pi a}\left[\frac{2}{\sqrt{2}}\cos 45.0° + \frac{1}{3}\right]$$

$$B_A = \frac{\left(4\pi\times 10^{-7}\ \mathrm{T\cdot m/A}\right)(2.00\ \mathrm{A})}{2\pi\left(1.00\times 10^{-2}\ \mathrm{m}\right)}\left[\frac{2}{\sqrt{2}}\cos 45° + \frac{1}{3}\right]$$

$$B_A = \boxed{53.3\ \mu\mathrm{T\ toward\ the\ bottom\ of\ the\ page}}$$

(b) At point B: $\vec{\mathbf{B}}_1$ and $\vec{\mathbf{B}}_2$ cancel, leaving

$$B_B = B_3 = \frac{\mu_0 I}{2\pi(2a)}$$

$$B_B = \frac{\left(4\pi\times 10^{-7}\ \mathrm{T\cdot m/A}\right)(2.00\ \mathrm{A})}{2\pi(2)\left(1.00\times 10^{-2}\ \mathrm{m}\right)}$$

$$= \boxed{20.0\ \mu\mathrm{T\ toward\ the\ bottom\ of\ the\ page}}$$

(c) At point C: $B_1 = B_2 = \dfrac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}$ and $B_3 = \dfrac{\mu_0 I}{2\pi a}$ with the directions shown in Figure (c). Again, the horizontal components of $\vec{\mathbf{B}}_1$ and $\vec{\mathbf{B}}_2$ cancel. The vertical components both oppose $\vec{\mathbf{B}}_3$ giving

$$B_C = 2\left[\frac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}\cos 45.0°\right] - \frac{\mu_0 I}{2\pi a} = \frac{\mu_0 I}{2\pi a}\left[\frac{2}{\sqrt{2}}\cos 45.0° - 1\right] = \boxed{0}$$

***P30.12** (a) The upward lightning current creates field in counterclockwise horizontal circles.

$$\frac{\mu_0 I}{2\pi r}\ \text{righthand rule} = \frac{4\pi\times 10^{-7}\ \mathrm{T\ m}\ 20\times 10^{3}\ \mathrm{A}}{2\pi\ \mathrm{A}\ 50\ \mathrm{m}}\ \text{north} = 8.00\times 10^{-5}\ \mathrm{T\ north}$$

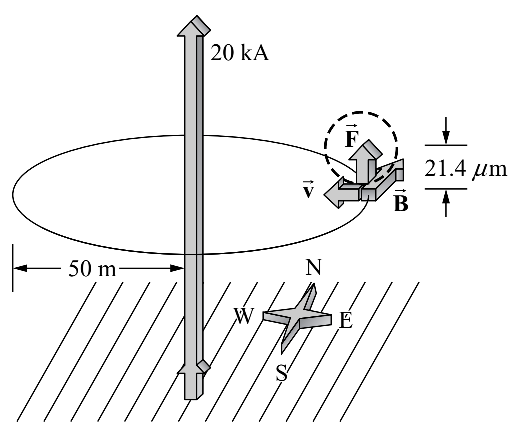


FIG. P30.12

continued on next page

The force on the electron is

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = -1.6 \times 10^{-19}\ \text{C (300 m/s west)} \times 8 \times 10^{-5}\ \text{(N s/C m) north}$$
$$= -3.84 \times 10^{-21}\ \text{N down}\ \sin 90° \ = \boxed{3.84 \times 10^{-21}\ \text{N up}}$$

(b) $r = \dfrac{mv}{qB} = \dfrac{9.11\times10^{-31}\ \text{kg 300 m/s}}{1.6\times10^{-19}\ \text{C } 8\times10^{-5}\ \text{N s/C m}} = \boxed{2.14\times10^{-5}\ \text{m}}$. This distance is negligible compared to 50 m, so the electron does move in a uniform field.

(c) $\omega = qB/m = 2\pi N/t$

$$N = \frac{qBt}{2\pi m} = \frac{1.6\times10^{-19}\ \text{C}\left(8\times10^{-5}\ \text{N s/C m}\right)60\times10^{-6}\ \text{s}}{2\pi\ 9.11\times10^{-31}\ \text{kg}} = \boxed{134\ \text{rev}}$$

***P30.13** (a) We use Equation 30.4 in the chapter text for the field created by a straight wire of limited length. The sines of the angles appearing in that equation are equal to the cosines of the complementary angles shown in our diagram. For the distance a from the wire to the field point we have $\tan 30° = \dfrac{a}{L/2}$, $a = 0.288\,7L$. One wire contributes to the field at P

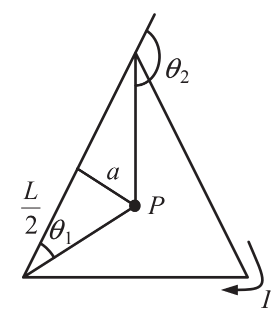


FIG. P30.13(a)

$$B = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2) = \frac{\mu_0 I}{4\pi(0.288\,7L)}(\cos 30° - \cos 150°)$$
$$= \frac{\mu_0 I(1.732)}{4\pi(0.288\,7L)} = \frac{1.50\mu_0 I}{\pi L}$$

Each side contributes the same amount of field in the same direction, which is perpendicularly into the paper in the picture. So the total field is

$$3\left(\frac{1.50\mu_0 I}{\pi L}\right) = \boxed{\frac{4.50\mu_0 I}{\pi L}}.$$

(b) As we showed in part (a), one whole side of the triangle creates field at the center $\dfrac{\mu_0 I(1.732)}{4\pi a}$. Now one-half of one nearby side of the triangle will be half as far away from point P_b and have a geometrically similar situation. Then it creates at P_b field $\dfrac{\mu_0 I(1.732)}{4\pi(a/2)} = \dfrac{2\mu_0 I(1.732)}{4\pi a}$. The two half-sides shown crosshatched in the picture create at P_b field $2\left(\dfrac{2\mu_0 I(1.732)}{4\pi a}\right) = \dfrac{4\mu_0 I(1.732)}{4\pi(0.288\,7L)} = \dfrac{6\mu_0 I}{\pi L}$. The rest of the triangle will contribute somewhat more field in the same direction, so we already have a proof that the field at P_b is $\boxed{\text{stronger}}$.

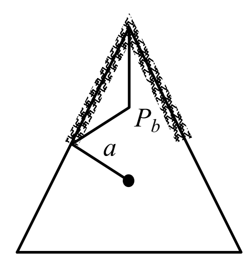


FIG. P30.13(b)

P30.14 Apply the equation from the chapter text for the field created by a straight wire of limited length, three times:

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi a}\left(\cos 0 - \frac{d}{\sqrt{d^2+a^2}}\right)\text{ toward you} + \frac{\mu_0 I}{4\pi d}\left(\frac{a}{\sqrt{d^2+a^2}} + \frac{a}{\sqrt{d^2+a^2}}\right)\text{ away from you}$$

$$+\frac{\mu_0 I}{4\pi a}\left(\frac{-d}{\sqrt{d^2+a^2}} - \cos 180°\right)\text{ toward you}$$

$$\vec{\mathbf{B}} = \boxed{\frac{\mu_0 I\left(a^2+d^2-d\sqrt{a^2+d^2}\right)}{2\pi ad\sqrt{a^2+d^2}}\text{ away from you}}$$

P30.15 Take the x-direction to the right and the y-direction up in the plane of the paper. Current 1 creates at P a field

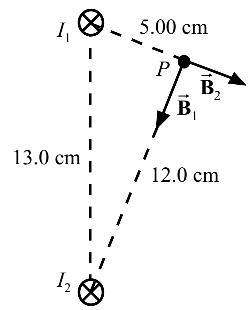


FIG. P30.15

$$B_1 = \frac{\mu_0 I}{2\pi a} = \frac{\left(2.00\times 10^{-7}\ \text{T}\cdot\text{m}\right)(3.00\ \text{A})}{\text{A}(0.050\ 0\ \text{m})}$$

$\vec{\mathbf{B}}_1 = 12.0\ \mu\text{T}$ downward and leftward, at angle 67.4° below the $-x$ axis.
Current 2 contributes

$$B_2 = \frac{\left(2.00\times 10^{-7}\ \text{T}\cdot\text{m}\right)(3.00\ \text{A})}{\text{A}(0.120\ \text{m})}\quad\text{clockwise perpendicular to 12.0 cm}$$

$\vec{\mathbf{B}}_2 = 5.00\ \mu\text{T}$ to the right and down, at angle $-22.6°$

Then, $\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = (12.0\ \mu\text{T})\left(-\hat{\mathbf{i}}\cos 67.4° - \hat{\mathbf{j}}\sin 67.4°\right) + (5.00\ \mu\text{T})\left(\hat{\mathbf{i}}\cos 22.6° - \hat{\mathbf{j}}\sin 22.6°\right)$

$$\vec{\mathbf{B}} = (-11.1\ \mu\text{T})\hat{\mathbf{j}} - (1.92\ \mu\text{T})\hat{\mathbf{j}} = \boxed{(-13.0\ \mu\text{T})\hat{\mathbf{j}}}$$

***P30.16** We apply $B = (\mu_0/2\pi)\ \mu/x^3$ to the center of a face, where $B = 40\ 000\ \mu$T and $x = 0.6$ mm, and also to the exterior weak-field point, where $B = 50\ \mu$T and x is the unknown.
Then $Bx^3 = Bx^3$ $\quad 40\ 000\ \mu\text{T}\ (0.6\ \text{mm})^3 = 50\ \mu\text{T}\ (d + 0.6\ \text{mm})^3$
$d = (40\ 000/50)^{1/3}\ 0.6\ \text{mm} - 0.6\ \text{mm} = \boxed{4.97\ \text{mm}}$
The strong field does not penetrate your painful joint.

Section 30.2 The Magnetic Force Between Two Parallel Conductors

P30.17 By symmetry, we note that the magnetic forces on the top and bottom segments of the rectangle cancel. The net force on the vertical segments of the rectangle is (using an equation from the chapter text)

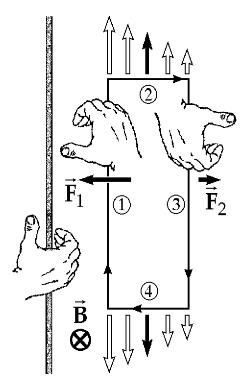


FIG. P30.17

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\hat{\mathbf{i}} = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{-a}{c(c+a)}\right)\hat{\mathbf{i}}$$

$$\vec{\mathbf{F}} = \frac{\left(4\pi\times 10^{-7}\ \text{N/A}^2\right)(5.00\ \text{A})(10.0\ \text{A})(0.450\ \text{m})}{2\pi}\left(\frac{-0.150\ \text{m}}{(0.100\ \text{m})(0.250\ \text{m})}\right)\hat{\mathbf{i}}$$

$$\vec{\mathbf{F}} = \left(-2.70\times 10^{-5}\,\hat{\mathbf{i}}\right)\ \text{N}$$

or $\quad\vec{\mathbf{F}} = \boxed{2.70\times 10^{-5}\ \text{N toward the left}}$

P30.18 Let both wires carry current in the x direction, the first at $y = 0$ and the second at $y = 10.0$ cm.

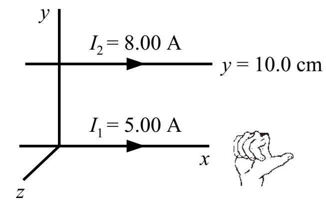


FIG. P30.18(a)

(a) $\vec{\mathbf{B}} = \dfrac{\mu_0 I}{2\pi r}\hat{\mathbf{k}} = \dfrac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(5.00\ \text{A})}{2\pi(0.100\ \text{m})}\hat{\mathbf{k}}$

$\vec{\mathbf{B}} = \boxed{1.00\times10^{-5}\ \text{T out of the page}}$

(b) $\vec{\mathbf{F}}_B = I_2\vec{\ell}\times\vec{\mathbf{B}} = (8.00\ \text{A})\left[(1.00\ \text{m})\hat{\mathbf{i}}\times(1.00\times10^{-5}\ \text{T})\hat{\mathbf{k}}\right] = (8.00\times10^{-5}\ \text{N})(-\hat{\mathbf{j}})$

$\boxed{\vec{\mathbf{F}}_B = 8.00\times10^{-5}\ \text{N toward the first wire}}$

(c) $\vec{\mathbf{B}} = \dfrac{\mu_0 I}{2\pi r}(-\hat{\mathbf{k}}) = \dfrac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(8.00\ \text{A})}{2\pi(0.100\ \text{m})}(-\hat{\mathbf{k}}) = (1.60\times10^{-5}\ \text{T})(-\hat{\mathbf{k}})$

$\vec{\mathbf{B}} = \boxed{1.60\times10^{-5}\ \text{T into the page}}$

(d) $\vec{\mathbf{F}}_B = I_1\vec{\ell}\times\vec{\mathbf{B}} = (5.00\ \text{A})\left[(1.00\ \text{m})\hat{\mathbf{i}}\times(1.60\times10^{-5}\ \text{T})(-\hat{\mathbf{k}})\right] = (8.00\times10^{-5}\ \text{N})(+\hat{\mathbf{j}})$

$\vec{\mathbf{F}}_B = \boxed{8.00\times10^{-5}\ \text{N towards the second wire}}$

P30.19 To attract, both currents must be to the right. The attraction is described by

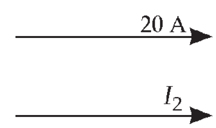


FIG. P30.19

$$F = I_2\ell B\sin 90° = I_2\ell\frac{\mu_0 I}{2\pi r}$$

So $I_2 = \dfrac{F}{\ell}\dfrac{2\pi r}{\mu_0 I_1} = (320\times10^{-6}\ \text{N/m})\left(\dfrac{2\pi(0.5\ \text{m})}{(4\pi\times10^{-7}\ \text{N}\cdot\text{s/C}\cdot\text{m})(20\ \text{A})}\right) = 40.0\ \text{A}$

Let y represent the distance of the zero-field point below the upper wire.

Then $\vec{\mathbf{B}} = \dfrac{\mu_0 I}{2\pi r}$  $+\dfrac{\mu_0 I}{2\pi r}$  $\quad 0 = \dfrac{\mu_0}{2\pi}\left(\dfrac{20\ \text{A}}{y}(\text{away}) + \dfrac{40\ \text{A}}{(0.5\ \text{m}-y)}(\text{toward})\right)$

$20(0.5\ \text{m}-y) = 40y \qquad 20(0.5\ \text{m}) = 60y$

$y = \boxed{0.167\ \text{m below the upper wire}}$

***P30.20** Carrying oppositely directed currents, wires 1 and 2 repel each other. If wire 3 were between them, it would have to repel either 1 or 2, so the force on that wire could not be zero. If wire 3 were to the right of wire 2, it would feel a larger force exerted by 2 than that exerted by 1, so the total force on 3 could not be zero. Therefore wire 3 must be to the left of both other wires as shown. It must carry downward current so that it can attract wire 2. We answer part (b) first.

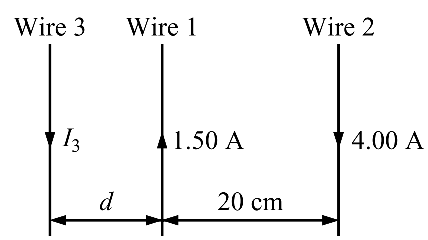


FIG. P30.20

(b) For the equilibrium of wire 3 we have

$F_{1\text{ on }3} = F_{2\text{ on }3} \qquad \dfrac{\mu_0(1.50\ \text{A})I_3}{2\pi d} = \dfrac{\mu_0(4\ \text{A})I_3}{2\pi(20\ \text{cm}+d)}$

$1.5(20\ \text{cm}+d) = 4d \qquad d = \dfrac{30\ \text{cm}}{2.5} = \boxed{12.0\ \text{cm to the left of wire 1}}$

continued on next page

(a) Thus the situation is possible in just one way.

(c) For the equilibrium of wire 1,

$$\frac{\mu_0 I_3 (1.5\text{ A})}{2\pi(12\text{ cm})} = \frac{\mu_0 (4\text{ A})(1.5\text{ A})}{2\pi(20\text{ cm})} \qquad I_3 = \frac{12}{20}4\text{ A} = \boxed{2.40\text{ A down}}$$

We know that wire 2 must be in equilibrium because the forces on it are equal in magnitude to the forces that it exerts on wires 1 and 3, which are equal because they both balance the equal-magnitude forces that 1 exerts on 3 and that 3 exerts on 1.

***P30.21** The separation between the wires is

$$a = 2(6.00\text{ cm})\sin 8.00° = 1.67\text{ cm}$$

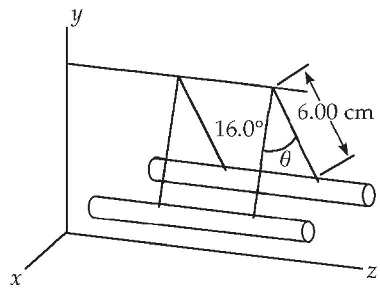


FIG. P30.21

(a) Because the wires repel, the currents are in $\boxed{\text{opposite directions}}$.

(b) Because the magnetic force acts horizontally,

$$\frac{F_B}{F_g} = \frac{\mu_0 I^2 \ell}{2\pi a m g} = \tan 8.00°$$

$$I^2 = \frac{mg2\pi a}{\ell \mu_0}\tan 8.00° \qquad \text{so} \qquad I = \boxed{67.8\text{ A}}$$

(c) Smaller. A smaller gravitational force would be pulling down on the wires and so tending to pull the wires together. Then a smaller magnetic force is required to keep the wires apart.

***P30.22** (a) If the conductors are wires, $F = I_2 \ell B \sin 90°$ and $B = \mu_0 I_1/2\pi r$ give $F = I_1 I_2 \mu_0 \ell/2\pi r$

$$r = \frac{I_1 I_2 \mu_0 \ell}{2\pi F} = \frac{(10\text{ A})^2 2\times10^{-7}\text{ T m } 0.5\text{ m}}{1\text{ N A}} = \boxed{10.0\ \mu\text{m}}$$

(b) For a force of ordinary size in a tabletop mechanics experiments to act on ordinary-to-large size currents, the distance between them must be quite small, but the situation is physically possible. If we tried to use wires with diameter 10 μm on a tabletop, they would feel the force only momentarily after we turn on the current, until they melt. We can use wide, thin sheets of copper, perhaps plated onto glass, and perhaps with water cooling, to have the forces act continuously. A practical electric motor must use coils of wire with many turns.

Section 30.3 **Ampère's Law**

P30.23 Each wire is distant from P by

$$(0.200\text{ m})\cos 45.0° = 0.141\text{ m}$$

Each wire produces a field at P of equal magnitude:

$$B_A = \frac{\mu_0 I}{2\pi a} = \frac{(2.00\times10^{-7}\text{ T}\cdot\text{m/A})(5.00\text{ A})}{(0.141\text{ m})} = 7.07\ \mu\text{T}$$

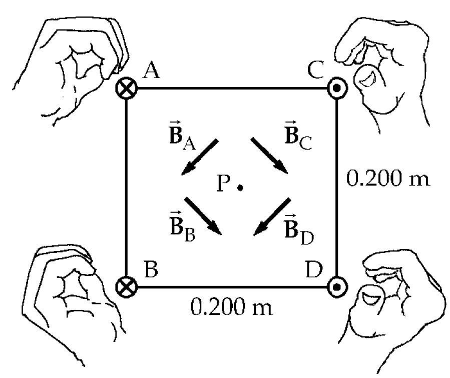


FIG. P30.23

Carrying currents into the page, A produces at P a field of $7.07\ \mu$T to the left and down at –135°, while B creates a field to the right and down at –45°. Carrying currents toward you, C produces a field downward and to the right at –45°, while D's contribution is downward and to the left. The total field is then

$$4(7.07\ \mu\text{T})\sin 45.0° = \boxed{20.0\ \mu\text{T}} \text{ toward the bottom of the page.}$$

P30.24 Let the current I be to the right. It creates a field $B = \frac{\mu_0 I}{2\pi d}$ at the proton's location. And we have a balance between the weight of the proton and the magnetic force $mg(-\hat{\mathbf{j}}) + qv(-\hat{\mathbf{i}}) \times \frac{\mu_0 I}{2\pi d}(\hat{\mathbf{k}}) = 0$ at a distance d from the wire

$$d = \frac{qv\mu_0 I}{2\pi mg} = \frac{(1.60 \times 10^{-19}\text{ C})(2.30 \times 10^4\text{ m/s})(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(1.20 \times 10^{-6}\text{ A})}{2\pi(1.67 \times 10^{-27}\text{ kg})(9.80\text{ m/s}^2)} = \boxed{5.40\text{ cm}}$$

P30.25 From Ampère's law, the magnetic field at point a is given by $B_a = \frac{\mu_0 I_a}{2\pi r_a}$, where I_a is the net current through the area of the circle of radius r_a. In this case, $I_a = 1.00$ A out of the page (the current in the inner conductor), so

$$B_a = \frac{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(1.00\text{ A})}{2\pi(1.00 \times 10^{-3}\text{ m})} = \boxed{200\ \mu\text{T toward top of page}}$$

Similarly at point b: $B_b = \frac{\mu_0 I_b}{2\pi r_b}$, where I_b is the net current through the area of the circle having radius r_b.

Taking out of the page as positive, $I_b = 1.00\text{ A} - 3.00\text{ A} = -2.00\text{ A}$, or $I_b = 2.00$ A into the page. Therefore,

$$B_b = \frac{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(2.00\text{ A})}{2\pi(3.00 \times 10^{-3}\text{ m})} = \boxed{133\ \mu\text{T toward bottom of page}}$$

P30.26 (a) In $B = \frac{\mu_0 I}{2\pi r}$, the field will be one-tenth as large at a ten-times larger distance: $\boxed{400\text{ cm}}$.

(b) $\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r_1}\hat{\mathbf{k}} + \frac{\mu_0 I}{2\pi r_2}(-\hat{\mathbf{k}})$ so

$$B = \frac{4\pi \times 10^{-7}\text{ T}\cdot\text{m}(2.00\text{ A})}{2\pi\text{ A}}\left(\frac{1}{0.398\,5\text{ m}} - \frac{1}{0.401\,5\text{ m}}\right) = \boxed{7.50\text{ nT}}$$

(c) Call r the distance from cord center to field point and $2d = 3.00$ mm the distance between conductors.

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r-d} - \frac{1}{r+d}\right) = \frac{\mu_0 I}{2\pi}\frac{2d}{r^2 - d^2}$$

$$7.50 \times 10^{-10}\text{ T} = (2.00 \times 10^{-7}\text{ T}\cdot\text{m/A})(2.00\text{ A})\frac{(3.00 \times 10^{-3}\text{ m})}{r^2 - 2.25 \times 10^{-6}\text{ m}^2} \quad \text{so} \quad r = \boxed{1.26\text{ m}}$$

The field of the two-conductor cord is weak to start with and falls off rapidly with distance.

(d) The cable creates $\boxed{\text{zero}}$ field at exterior points, since a loop in Ampère's law encloses zero total current. Shall we sell coaxial-cable power cords to people who worry about biological damage from weak magnetic fields?

P30.27 (a) One wire feels force due to the field of the other ninety-nine.

$$B=\frac{\mu_0 I_0 r}{2\pi R^2}=\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(99)(2.00\ \text{A})(0.200\times10^{-2}\ \text{m})}{2\pi(0.500\times10^{-2}\ \text{m})^2}$$

$$=3.17\times10^{-3}\ \text{T}$$

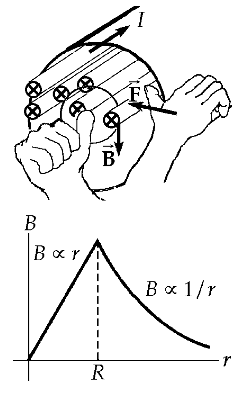


FIG. P30.27

This field points tangent to a circle of radius 0.200 cm and exerts force $\vec{\mathbf{F}}=I\vec{\ell}\times\vec{\mathbf{B}}$ toward the center of the bundle, on the single hundredth wire:

$$\frac{F}{\ell}=IB\sin\theta=(2.00\ \text{A})(3.17\times10^{-3}\text{T})\sin 90°=6.34\ \text{mN/m}$$

$$\frac{F_B}{\ell}=\boxed{6.34\times10^{-3}\ \text{N/m inward}}$$

(b) $B\propto r$, so B is greatest at the outside of the bundle. Since each wire carries the same current, F is $\boxed{\text{greatest at the outer surface}}$.

P30.28 (a) $B_{\text{inner}}=\frac{\mu_0 NI}{2\pi r}=\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(900)(14.0\times10^{3}\ \text{A})}{2\pi(0.700\ \text{m})}=\boxed{3.60\ \text{T}}$

(b) $B_{\text{outer}}=\frac{\mu_0 NI}{2\pi r}=\frac{(2\times10^{-7}\ \text{T}\cdot\text{m/A})(900)(14.0\times10^{3}\ \text{A})}{1.30\ \text{m}}=\boxed{1.94\ \text{T}}$

P30.29 We assume the current is vertically upward.

(a) Consider a circle of radius r slightly less than R. It encloses no current so from

$$\oint\vec{\mathbf{B}}\cdot d\vec{\mathbf{s}}=\mu_0 I_{\text{inside}}\qquad B(2\pi r)=0$$

we conclude that the magnetic field is $\boxed{\text{zero}}$.

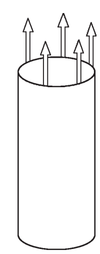


FIG. P30.29(a)

(b) Now let the r be barely larger than R. Ampère's law becomes $B(2\pi R)=\mu_0 I$,

so $\boxed{B=\frac{\mu_0 I}{2\pi R}}$

The field's direction is  $\boxed{\text{tangent to the wall of the cylinder in a counterclockwise sense}}$.

(c) Consider a strip of the wall of width dx and length ℓ. Its width is so small compared to $2\pi R$ that the field at its location would be essentially unchanged if the current in the strip were turned off.

The current it carries is $I_s=\frac{Idx}{2\pi R}$ up.

The force on it is

$$\vec{\mathbf{F}}=I_s\vec{\ell}\times\vec{\mathbf{B}}=\frac{Idx}{2\pi R}\left(\ell\frac{\mu_0 I}{2\pi R}\right)\widehat{\textbf{up}\times\textbf{into page}}=\frac{\mu_0 I^2\ell dx}{4\pi^2R^2}\widehat{\textbf{radially inward}}.$$

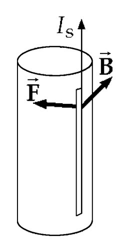


FIG. P30.29(c)

The pressure on the strip and everywhere on the cylinder is

$$P=\frac{F}{A}=\frac{\mu_0 I^2\ell dx}{4\pi^2R^2\ell dx}=\boxed{\frac{\mu_0 I^2}{(2\pi R)^2}\ \text{inward}}$$

The pinch effect makes an effective demonstration when an aluminum can crushes itself as it carries a large current along its length.

P30.30 From $\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I$, $\quad I = \dfrac{2\pi rB}{\mu_0} = \dfrac{2\pi(1.00\times10^{-3})(0.100)}{4\pi\times10^{-7}} = \boxed{500\text{ A}}$

P30.31 Use Ampère's law, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I$. For current density J, this becomes

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \int J \cdot dA.$$

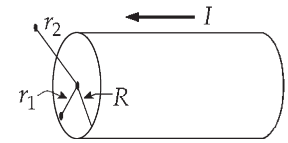


FIG. P30.31

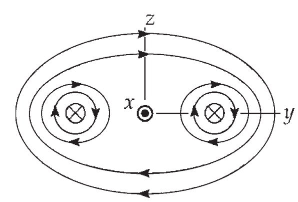
(a) For $r_1 < R$, this gives $\quad B2\pi r_1 = \mu_0 \int_0^{r_1} (br)(2\pi r dr)$ and

$$\boxed{B = \frac{\mu_0 b r_1^2}{3} \quad (\text{for } r_1 < R \text{ or inside the cylinder})}$$

(b) When $r_2 > R$, Ampère's law yields $\quad (2\pi r_2)B = \mu_0 \int_0^{R} (br)(2\pi r dr) = \dfrac{2\pi\mu_0 bR^3}{3}$,

or $\quad \boxed{B = \dfrac{\mu_0 bR^3}{3r_2} \quad (\text{for } r_2 > R \text{ or outside the cylinder})}$

***P30.32** (a) See Figure (a) to the right.

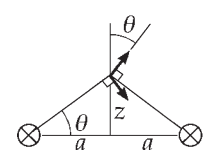


(Currents are into the paper) Figure (a)

(c) We choose to do part (c) before part (b). At a point on the z axis, the contribution from each wire has magnitude $B = \dfrac{\mu_0 I}{2\pi\sqrt{a^2+z^2}}$ and is perpendicular to the line from this point to the wire as shown in Figure (b). Combining fields, the vertical components cancel while the horizontal components add, yielding

$$B_y = 2\left(\frac{\mu_0 I}{2\pi\sqrt{a^2+z^2}}\sin\theta\right) = \frac{\mu_0 I}{\pi\sqrt{a^2+z^2}}\left(\frac{z}{\sqrt{a^2+z^2}}\right) = \frac{\mu_0 I z}{\pi(a^2+z^2)}$$

$$B_y = \frac{4\pi\times10^{-7}\text{ T m 8 A } z}{\pi\left[(0.03\text{ m})^2+z^2\right]} \quad \text{so} \quad \vec{\mathbf{B}} = \frac{32\times10^{-7} z\text{ T}\cdot\text{m}}{9\times10^{-4}\text{ m}^2+z^2}\hat{\mathbf{j}}$$



At a distance z above the plane of the conductors Figure (b)

FIG. P30.32

(b) Substituting $z = 0$ gives zero for the field. We can see this from cancellation of the separate fields in either diagram.
Taking the limit $z \to \infty$ gives $1/z \to 0$, as we should expect.

(d) The condition for a maximum is:

$$\frac{dB_y}{dz} = \frac{-\mu_0 I z(2z)}{\pi(a^2+z^2)^2} + \frac{\mu_0 I}{\pi(a^2+z^2)} = 0 \quad \text{or} \quad \frac{\mu_0 I}{\pi}\frac{(a^2-z^2)}{(a^2+z^2)^2} = 0$$

Thus, along the z axis, the field is a maximum at $\boxed{d = a = 3.00\text{ cm}}$.

(e) The value of the maximum field is $\vec{\mathbf{B}} = \dfrac{32\times10^{-7}0.03\text{ m T}\cdot\text{m}}{9\times10^{-4}\text{ m}^2+9\times10^{-4}\text{ m}^2}\hat{\mathbf{j}} = \boxed{53.3\ \hat{\mathbf{j}}\ \mu\text{T}}$.

***P30.33** $J_s = \frac{I}{\ell}$. Each filament of current creates a contribution to the total field that goes counterclockwise around that filament's location. Together, they create field straight up to the right of the sheet and straight down to the left of the sheet.

From Ampère's law applied to the suggested rectangle,

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I$$

$B \cdot 2\ell + 0 = \mu_0 J_s \ell$ Therefore the field is uniform in space, with the magnitude $\boxed{B = \frac{\mu_0 J_s}{2}}$.

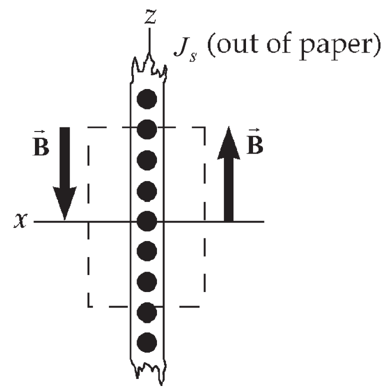


FIG. P30.33

Section 30.4 **The Magnetic Field of a Solenoid**

***P30.34** In the expression $B = N\mu_0 I/\ell$ for the field within a solenoid with radius much less than 20 cm, all we want to do is increase N.
(a) Make the wire as long and thin as possible without melting when it carries the 5-A current. Then the solenoid can have many turns.
(b) As small in radius as possible with your experiment fitting inside. Then with a smaller circumference the wire can form a solenoid with more turns.

P30.35 $B = \mu_0 \frac{N}{\ell} I$ so $I = \frac{B}{\mu_0 n} = \frac{(1.00 \times 10^{-4}\text{ T})0.400\text{ m}}{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})1\,000} = \boxed{31.8\text{ mA}}$

P30.36 Let the axis of the solenoid lie along the y–axis from $y = 0$ to $y = \ell$. We will determine the field at $y = a$. This point will be inside the solenoid if $0 < a < \ell$ and outside if $a < 0$ or $a > \ell$. We think of solenoid as formed of rings, each of thickness dy. Now I is the symbol for the current in each turn of wire and the number of turns per length is $\left(\frac{N}{\ell}\right)$. So the number of turns in the ring is $\left(\frac{N}{\ell}\right)dy$ and the current in the ring is $I_{\text{ring}} = I\left(\frac{N}{\ell}\right)dy$. Now we use the result derived in the chapter text for the field created by one ring:

$$B_{\text{ring}} = \frac{\mu_0 I_{\text{ring}} R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

where x is the name of the distance from the center of the ring, at location y, to the field point $x = a - y$. Each ring creates field in the same direction, along our y–axis, so the whole field of the solenoid is

$$B = \sum_{\text{all rings}} B_{\text{ring}} = \sum \frac{\mu_0 I_{\text{ring}} R^2}{2\left(x^2 + R^2\right)^{3/2}} = \int_0^\ell \frac{\mu_0 I (N/\ell) dy R^2}{2\left((a-y)^2 + R^2\right)^{3/2}} = \frac{\mu_0 I N R^2}{2\ell} \int_0^\ell \frac{dy}{2\left((a-y)^2 + R^2\right)^{3/2}}$$

To perform the integral we change variables to $u = a - y$.

$$B = \frac{\mu_0 I N R^2}{2\ell} \int_a^{a-\ell} \frac{-du}{\left(u^2 + R^2\right)^{3/2}}$$

continued on next page

and then use the table of integrals in the appendix:

(a) $B=\dfrac{\mu_0 INR^2}{2\ell}\dfrac{-u}{R^2\sqrt{u^2+R^2}}\Bigg|_a^{a-\ell}=\boxed{\dfrac{\mu_0 IN}{2\ell}\left[\dfrac{a}{\sqrt{a^2+R^2}}-\dfrac{a-\ell}{\sqrt{(a-\ell)^2+R^2}}\right]}$

(b) If ℓ is much larger than R and $a=0$,

we have $B\cong\dfrac{\mu_0 IN}{2\ell}\left[0-\dfrac{-\ell}{\sqrt{\ell^2}}\right]=\dfrac{\mu_0 IN}{2\ell}$.

This is just half the magnitude of the field deep within the solenoid. We would get the same result by substituting $a=\ell$ to describe the other end.

P30.37 The field produced by the solenoid in its interior is given by

$$\vec{\mathbf{B}}=\mu_0 nI\left(-\hat{\mathbf{i}}\right)=\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(\frac{30.0}{10^{-2}\ \mathrm{m}}\right)(15.0\ \mathrm{A})\left(-\hat{\mathbf{i}}\right)$$

$$\vec{\mathbf{B}}=-\left(5.65\times10^{-2}\ \mathrm{T}\right)\hat{\mathbf{i}}$$ 

The force exerted on side AB of the square current loop is

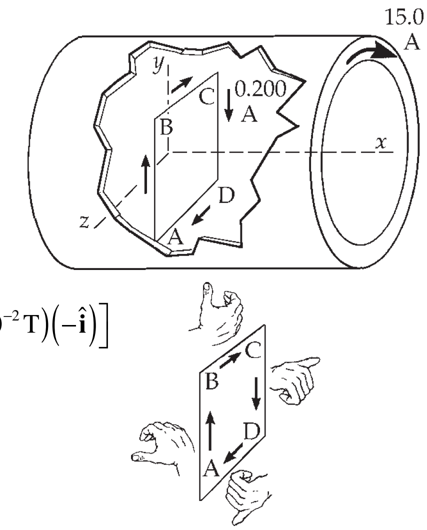


FIG. P30.37

$$\left(\vec{\mathbf{F}}_B\right)_{AB}=I\vec{\mathbf{L}}\times\vec{\mathbf{B}}=(0.200\ \mathrm{A})\left[\left(2.00\times10^{-2}\ \mathrm{m}\right)\hat{\mathbf{j}}\times\left(5.65\times10^{-2}\ \mathrm{T}\right)\left(-\hat{\mathbf{i}}\right)\right]$$

$$\left(\vec{\mathbf{F}}_B\right)_{AB}=\left(2.26\times10^{-4}\ \mathrm{N}\right)\hat{\mathbf{k}}$$ 

Similarly, each side of the square loop experiences a force, lying in the plane of the loop, of

$\boxed{226\ \mu\mathrm{N}\text{ directed away from the center}}$.

From the above result, it is seen that the net torque exerted on the square loop by the field of the solenoid should be zero. More formally, the magnetic dipole moment of the square loop is given by

$$\vec{\boldsymbol{\mu}}=I\vec{\mathbf{A}}=(0.200\ \mathrm{A})\left(2.00\times10^{-2}\ \mathrm{m}\right)^2\left(-\hat{\mathbf{i}}\right)=-80.0\ \mu\mathrm{A\cdot m^2}\hat{\mathbf{i}}$$ 

The torque exerted on the loop is then $\vec{\boldsymbol{\tau}}=\vec{\boldsymbol{\mu}}\times\vec{\mathbf{B}}=\left(-80.0\ \mu\mathrm{A\cdot m^2}\hat{\mathbf{i}}\right)\times\left(-5.65\times10^{-2}\ \mathrm{T}\hat{\mathbf{i}}\right)=\boxed{0}$

***P30.38** The number of turns is $N=\dfrac{75\ \mathrm{cm}}{0.1\ \mathrm{cm}}=750$. We assume that the solenoid is long enough to qualify as a long solenoid. Then the field within it (not close to the ends) is $B=\dfrac{N\mu_0 I}{\ell}$ so

$$I=\frac{B\ell}{N\mu_0}=\frac{\left(8\times10^{-3}\ \mathrm{T}\right)(0.75\ \mathrm{m\cdot A})}{750\left(4\pi\times10^{-7}\ \mathrm{T\cdot m}\right)}=6.37\ \mathrm{A}$$

The resistance of the wire is

$$R=\frac{\rho\ell_{\text{wire}}}{A}=\frac{\left(1.7\times10^{-8}\ \Omega\cdot\mathrm{m}\right)2\pi(0.05\ \mathrm{m})750}{\pi\left(0.05\times10^{-2}\ \mathrm{m}\right)^2}=5.10\ \Omega$$

The power delivered is

$$\mathcal{P}=I\Delta V=I^2R=(6.37\ \mathrm{A})^2(5.10\ \Omega)=\boxed{207\ \mathrm{W}}$$

The power required would be smaller if wire were wrapped in several layers.

Section 30.5 Gauss's Law in Magnetism

P30.39 (a) $\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = \left(5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right) \text{ T} \cdot \left(2.50 \times 10^{-2} \text{ m}\right)^2 \hat{\mathbf{i}}$

$$\Phi_B = 3.12 \times 10^{-3} \text{ T} \cdot \text{m}^2 = 3.12 \times 10^{-3} \text{ Wb} = \boxed{3.12 \text{ mWb}}$$

(b) $\left(\Phi_B\right)_{\text{total}} = \oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \boxed{0}$ for *any* closed surface (Gauss's law for magnetism)

P30.40 (a) $\left(\Phi_B\right)_{\text{flat}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = B\pi R^2 \cos(180 - \theta) = \boxed{-B\pi R^2 \cos\theta}$

(b) The net flux out of the closed surface is zero: $\left(\Phi_B\right)_{\text{flat}} + \left(\Phi_B\right)_{\text{curved}} = 0$.

$$\left(\Phi_B\right)_{\text{curved}} = \boxed{B\pi R^2 \cos\theta}$$

P30.41 (a) $\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA$ where A is the cross-sectional area of the solenoid.

$$\Phi_B = \left(\frac{\mu_0 N I}{\ell}\right)\left(\pi r^2\right) = \boxed{7.40 \ \mu\text{Wb}}$$

(b) $\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA = \left(\frac{\mu_0 N I}{\ell}\right)\left[\pi\left(r_2^2 - r_1^2\right)\right]$

$$\Phi_B = \left[\frac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)(300)(12.0 \text{ A})}{(0.300 \text{ m})}\right]\pi\left[(8.00)^2 - (4.00)^2\right]\left(10^{-3} \text{ m}\right)^2 = \boxed{2.27 \ \mu\text{Wb}}$$

***P30.42** The field can be uniform in magnitude. Gauss's law for magnetism implies that magnetic field lines never start or stop. If the field is uniform in direction, the lines are parallel and their density stays constant along any one bundle of lines. Thus the magnitude of the field has the same value at all points along a line in the direction of the field. The magnitude of the field could vary over a plane perpendicular to the lines, or it could be constant throughout the volume.

Section 30.6 Magnetism in Matter

P30.43 The magnetic moment of one electron is taken as one Bohr magneton μ_B. Let x represent the number of electrons per atom contributing and n the number of atoms per unit volume. Then $nx\mu_B$ is the magnetic moment per volume and the magnetic field (in the absence of any currents in wires) is $B = \mu_0 n x \mu_B = 2.00$ T.

Then $x = \dfrac{B}{\mu_0 \mu_B n} = \dfrac{2.00 \text{ T}}{\left(8.50 \times 10^{28} \text{ m}^{-3}\right)\left(9.27 \times 10^{-24} \text{ N} \cdot \text{m/T}\right)\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)} = \boxed{2.02}$

Section 30.7 **The Magnetic Field of the Earth**

P30.44 (a) $B_h = B_{\text{coil}} = \dfrac{\mu_0 NI}{2R} = \dfrac{\left(4\pi\times10^{-7}\right)(5.00)(0.600)}{0.300} = \boxed{12.6\ \mu\text{T}}$

(b) $B_h = B\sin\phi \rightarrow B = \dfrac{B_h}{\sin\phi} = \dfrac{12.6\ \mu\text{T}}{\sin 13.0°} = \boxed{56.0\ \mu\text{T}}$

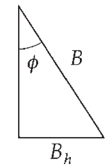


FIG. P30.44

P30.45 (a) Number of unpaired electrons $= \dfrac{8.00\times10^{22}\ \text{A}\cdot\text{m}^2}{9.27\times10^{-24}\ \text{A}\cdot\text{m}^2} = \boxed{8.63\times10^{45}}$.

Each iron atom has two unpaired electrons, so the number of iron atoms required is $\frac{1}{2}\left(8.63\times10^{45}\right)$.

(b) $\text{Mass} = \dfrac{\left(4.31\times10^{45}\ \text{atoms}\right)\left(7\,900\ \text{kg/m}^3\right)}{\left(8.50\times10^{28}\ \text{atoms/m}^3\right)} = \boxed{4.01\times10^{20}\ \text{kg}}$

***P30.46** (a) Gravitational field and magnetic field are vectors; atmospheric pressure is a scalar.

(b) At 42° north latitude, 76° west longitude, 260 m above sea level: Gravitational field is 9.803 m/s^2 down. From the Coast and Geodetic Survey, the magnetic field is 54 μT at 12° west of geographic north and 69° below the horizontal. Atmospheric pressure is 98 kPa.

(c) The atmosphere is held on by gravitation, but otherwise the effects are all separate. The magnetic field could be produced by permanent magnetization of a cold iron-nickel deposit within the Earth, so it need not be associated with present-day action of gravitation.

Additional Problems

P30.47 Consider a longitudinal filament of the strip of width dr as shown in the sketch. The contribution to the field at point P due to the current dI in the element dr is

$$dB = \frac{\mu_0 dI}{2\pi r}$$

where $$dI = I\left(\frac{dr}{w}\right)$$

$$\vec{\mathbf{B}} = \int d\vec{\mathbf{B}} = \int_b^{b+w} \frac{\mu_0 I dr}{2\pi wr}\hat{\mathbf{k}} = \boxed{\frac{\mu_0 I}{2\pi w}\ln\left(1+\frac{w}{b}\right)\hat{\mathbf{k}}}$$

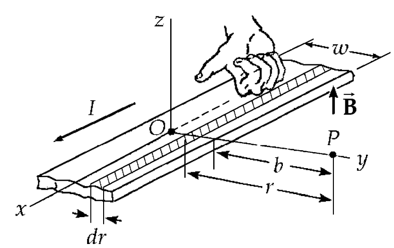


FIG. P30.47

P30.48 $B = \dfrac{\mu_0 IR^2}{2\left(R^2+R^2\right)^{3/2}} = \dfrac{\mu_0 I}{2^{5/2}R}$ $\qquad I = \dfrac{2^{5/2}BR}{\mu_0} = \dfrac{2^{5/2}\left(7.00\times10^{-5}\ \text{T}\right)\left(6.37\times10^6\ \text{m}\right)}{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)}$

so $\boxed{I = 2.01\times10^9\ \text{A}}$ toward the west

P30.49 At a point at distance x from the left end of the bar, current I_2 creates magnetic field $\vec{\mathbf{B}} = \dfrac{\mu_0 I_2}{2\pi\sqrt{h^2+x^2}}$ to the left and above the horizontal at angle θ where $\tan\theta = \dfrac{x}{h}$. This field exerts force on an element of the rod of length dx

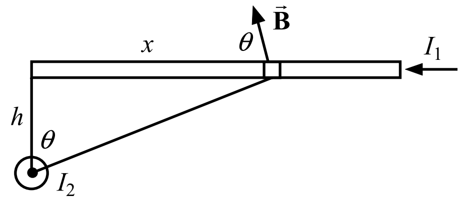


FIG. P30.49

$$d\vec{\mathbf{F}} = I_1\vec{\ell}\times\vec{\mathbf{B}} = I_1\frac{\mu_0 I_2 dx}{2\pi\sqrt{h^2+x^2}}\underset{\text{right hand rule}}{\sin\theta}$$

$$= \frac{\mu_0 I_1 I_2 dx}{2\pi\sqrt{h^2+x^2}}\frac{x}{\sqrt{h^2+x^2}} \text{ into the page}$$

$$d\vec{\mathbf{F}} = \frac{\mu_0 I_1 I_2 x dx}{2\pi\left(h^2+x^2\right)}\left(-\hat{\mathbf{k}}\right)$$

The whole force is the sum of the forces on all of the elements of the bar:

$$\vec{\mathbf{F}} = \int_{x=0}^{\ell}\frac{\mu_0 I_1 I_2 x dx}{2\pi\left(h^2+x^2\right)}\left(-\hat{\mathbf{k}}\right) = \frac{\mu_0 I_1 I_2\left(-\hat{\mathbf{k}}\right)}{4\pi}\int_0^{\ell}\frac{2x dx}{h^2+x^2} = \frac{\mu_0 I_1 I_2\left(-\hat{\mathbf{k}}\right)}{4\pi}\ln\left(h^2+x^2\right)\Big|_0^{\ell}$$

$$= \frac{\mu_0 I_1 I_2\left(-\hat{\mathbf{k}}\right)}{4\pi}\left[\ln\left(h^2+\ell^2\right)-\ln h^2\right] = \frac{10^{-7}\text{ N}(100\text{ A})(200\text{ A})\left(-\hat{\mathbf{k}}\right)}{\text{A}^2}\ln\left[\frac{(0.5\text{ cm})^2+(10\text{ cm})^2}{(0.5\text{ cm})^2}\right]$$

$$= 2\times10^{-3}\text{ N}\left(-\hat{\mathbf{k}}\right)\ln 401 = \boxed{1.20\times10^{-2}\text{ N}\left(-\hat{\mathbf{k}}\right)}$$

P30.50 Suppose you have two 100-W headlights running from a 12-V battery, with the whole $\dfrac{200\text{ W}}{12\text{ V}} = 17\text{ A}$ current going through the switch 60 cm from the compass. Suppose the dashboard contains little iron, so $\mu \approx \mu_0$. Model the current as straight. Then,

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\right)17}{2\pi(0.6)}\boxed{\sim10^{-5}\text{ T}}$$

If the local geomagnetic field is 5×10^{-5} T, this is $\boxed{\sim10^{-1}\text{ times as large,}}$ enough to affect the compass noticeably.

P30.51 On the axis of a current loop, the magnetic field is given by $B = \dfrac{\mu_0 I R^2}{2\left(x^2+R^2\right)^{3/2}}$

where in this case $I = \dfrac{q}{(2\pi/\omega)}$. The magnetic field is directed away from the center, with a magnitude of

$$B = \frac{\mu_0\omega R^2 q}{4\pi\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0(20.0)(0.100)^2\left(10.0\times10^{-6}\right)}{4\pi\left[(0.050\,0)^2+(0.100)^2\right]^{3/2}} = \boxed{1.43\times10^{-10}\text{ T}}$$

P30.52 On the axis of a current loop, the magnetic field is given by $B = \dfrac{\mu_0 I R^2}{2\left(x^2+R^2\right)^{3/2}}$

where in this case $I = \dfrac{q}{(2\pi/\omega)}$. Therefore, $B = \dfrac{\mu_0\omega R^2 q}{4\pi\left(x^2+R^2\right)^{3/2}}$

when $x = \dfrac{R}{2}$ then $B = \dfrac{\mu_0\omega R^2 q}{4\pi\left(\frac{5}{4}R^2\right)^{3/2}} = \boxed{\dfrac{\mu_0 q\omega}{2.5\sqrt{5}\pi R}}$

P30.53 (a) Use twice the equation for the field created by a current loop

$$B_x = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

If each coil has N turns, the field is just N times larger.

$$B = B_{x1} + B_{x2} = \frac{N\mu_0 I R^2}{2}\left[\frac{1}{\left(x^2+R^2\right)^{3/2}} + \frac{1}{\left[\left(R-x\right)^2 + R^2\right]^{3/2}}\right]$$

$$\boxed{B = \frac{N\mu_0 I R^2}{2}\left[\frac{1}{\left(x^2+R^2\right)^{3/2}} + \frac{1}{\left(2R^2 + x^2 - 2xR\right)^{3/2}}\right]}$$

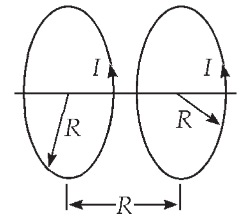


FIG. P30.53

(b) $$\frac{dB}{dx} = \frac{N\mu_0 I R^2}{2}\left[-\frac{3}{2}(2x)\left(x^2+R^2\right)^{-5/2} - \frac{3}{2}\left(2R^2+x^2-2xR\right)^{-5/2}\left(2x-2R\right)\right]$$

Substituting $x = \frac{R}{2}$ and canceling terms, $\boxed{\frac{dB}{dx} = 0}$.

$$\frac{d^2B}{dx^2} = \frac{-3N\mu_0 I R^2}{2}\left[\left(x^2+R^2\right)^{-5/2} - 5x^2\left(x^2+R^2\right)^{-7/2} + \left(2R^2+x^2-2xR\right)^{-5/2} - 5\left(x-R\right)^2\left(2R^2+x^2-2xR\right)^{-7/2}\right]$$

Again substituting $x = \frac{R}{2}$ and canceling terms, $\boxed{\frac{d^2B}{dx^2} = 0}$.

P30.54 "Helmholtz pair" → separation distance = radius

$$B = \frac{2\mu_0 I R^2}{2\left[\left(R/2\right)^2 + R^2\right]^{3/2}} = \frac{\mu_0 I R^2}{\left[\frac{1}{4}+1\right]^{3/2} R^3} = \frac{\mu_0 I}{1.40R} \text{ for 1 turn}$$

For N turns in each coil, $B = \frac{\mu_0 N I}{1.40R} = \frac{\left(4\pi\times10^{-7}\right)100\left(10.0\right)}{1.40\left(0.500\right)} = \boxed{1.80\times10^{-3}\text{ T}}$

P30.55 Consider first a solid cylindrical rod of radius R carrying current toward you, uniformly distributed over its cross-sectional area. To find the field at distance r from its center we consider a circular loop of radius r:

$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{inside}}$$

$$B2\pi r = \mu_0 \pi r^2 J \qquad B = \frac{\mu_0 J r}{2} \qquad \vec{\mathbf{B}} = \frac{\mu_0 \mathbf{J}}{2}\hat{\mathbf{k}}\times\mathbf{r}$$

Now the total field at P inside the saddle coils is the field due to a solid rod carrying current toward you, centered at the head of vector $\vec{\mathbf{a}}$, plus the field of a solid rod centered at the tail of vector $\vec{\mathbf{a}}$ carrying current away from you.

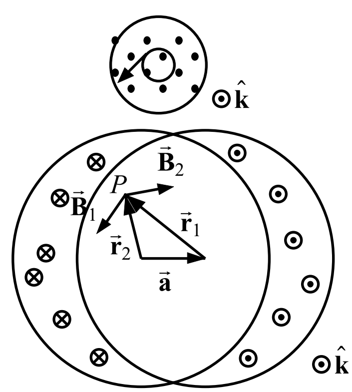


FIG. P30.55

$$\vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \frac{\mu_0 J}{2}\hat{\mathbf{k}}\times\vec{\mathbf{r}}_1 + \frac{\mu_0 J}{2}\left(-\hat{\mathbf{k}}\right)\times\vec{\mathbf{r}}_2$$

Now note $\vec{\mathbf{a}} + \vec{\mathbf{r}}_1 = \vec{\mathbf{r}}_2$

$$\vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \frac{\mu_0 J}{2}\hat{\mathbf{k}}\times\vec{\mathbf{r}}_1 - \frac{\mu_0 J}{2}\hat{\mathbf{k}}\times\left(\vec{\mathbf{a}}+\vec{\mathbf{r}}_1\right) = \frac{\mu_0 J}{2}\vec{\mathbf{a}}\times\hat{\mathbf{k}} = \boxed{\frac{\mu_0 J a}{2}\text{ down in the diagram}}$$

P30.56 From Problem 33, the upper sheet creates field

$\vec{\mathbf{B}} = \frac{\mu_0 J_s}{2}\hat{\mathbf{k}}$ above it and $\frac{\mu_0 J_s}{2}\left(-\hat{\mathbf{k}}\right)$ below it.

Consider a patch of the sheet of width w parallel to the z axis and length d parallel to the x axis. The charge on it σwd passes a point in time $\frac{d}{v}$, so

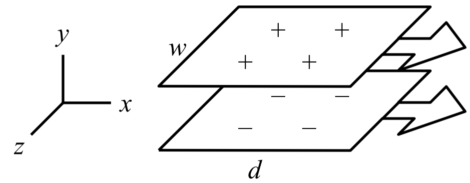


FIG. P30.56

the current it constitutes is $\frac{q}{t} = \frac{\sigma wdv}{d}$ and the linear current density is $J_s = \frac{\sigma wv}{w} = \sigma v$. Then the magnitude of the magnetic field created by the upper sheet is $\frac{1}{2}\mu_0\sigma v$. Similarly, the lower sheet in its motion toward the right constitutes current toward the left. It creates magnetic field $\frac{1}{2}\mu_0\sigma v\left(-\hat{\mathbf{k}}\right)$ above it and $\frac{1}{2}\mu_0\sigma v\hat{\mathbf{k}}$ below it. We choose to write down answer (b) first.

(b) Above both sheets and below both, their equal-magnitude fields add to $\boxed{\text{zero}}$.

(a) Between the plates, their fields add to

$$\mu_0\sigma v\left(-\hat{\mathbf{k}}\right) = \boxed{\mu_0\sigma v \text{ away from you horizontally}}.$$

(c) The upper plate exerts no force on itself. The field of the lower plate, $\frac{1}{2}\mu_0\sigma v\left(-\hat{\mathbf{k}}\right)$ will exert a force on the current in the w- by d-section, given by

$$I\vec{\ell}\times\vec{\mathbf{B}} = \sigma wvd\hat{\mathbf{i}}\times\frac{1}{2}\mu_0\sigma v\left(-\hat{\mathbf{k}}\right) = \frac{1}{2}\mu_0\sigma^2v^2wd\hat{\mathbf{j}}$$

The force per area is $\frac{1}{2}\frac{\mu_0\sigma^2v^2wd}{wd}\hat{\mathbf{j}} = \boxed{\frac{1}{2}\mu_0\sigma^2v^2 \text{ up}}$.

(d) The electrical force on our section of the upper plate is $q\vec{\mathbf{E}}_{\text{lower}} = \sigma\ell w\frac{\sigma}{2\epsilon_0}\left(-\hat{\mathbf{j}}\right) = \frac{\ell w\sigma^2}{2\epsilon_0}\left(-\hat{\mathbf{j}}\right)$.

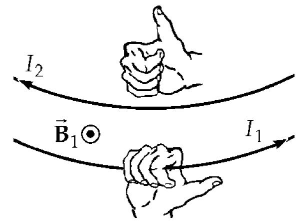
The electrical force per area is $\frac{\ell w\sigma^2}{2\epsilon_0\ell w}$ down $= \frac{\sigma^2}{2\epsilon_0}$ down. To have $\frac{1}{2}\mu_0\sigma^2v^2 = \frac{\sigma^2}{2\epsilon_0}$ we require

$$v = \frac{1}{\sqrt{\mu_0\epsilon_0}} = \frac{1}{\sqrt{4\pi\times10^{-7}\,(\text{Tm/A})(\text{N/TAm})\,8.85\times10^{-12}\,(\text{C}^2/\text{Nm}^2)(\text{As/C})^2}}$$

$$= \boxed{3.00\times10^8 \text{ m/s}}$$

This is the speed of light, not a possible speed for a metal plate.

***P30.57** Model the two wires as straight parallel wires (!). From the treatment of this situation in the chapter text we have

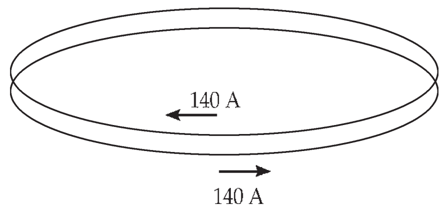


(a) $$F_B = \frac{\mu_0 I^2 \ell}{2\pi a}$$

$$F_B = \frac{(4\pi \times 10^{-7})(140)^2 (2\pi)(0.100)}{2\pi(1.00 \times 10^{-3})}$$

$$= \boxed{2.46 \text{ N}} \text{ upward}$$

(b) The magnetic field at the center of the loop or on its axis is much weaker than the magnetic field just outside the wire. The wire has negligible curvature on the scale of 1 mm, so we model the lower loop as a long straight wire to find the field it creates at the location of the upper wire.



(c) $$a_{\text{loop}} = \frac{2.46 \text{ N} - m_{\text{loop}} g}{m_{\text{loop}}} = \boxed{107 \text{ m/s}^2} \text{ upward}$$

FIG. P30.57

P30.58 (a) In $d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi r^2} I d\vec{\mathbf{s}} \times \hat{\mathbf{r}}$, the moving charge constitutes a bit of current as in $I = nqvA$. For a positive charge the direction of $d\vec{\mathbf{s}}$ is the direction of $\vec{\mathbf{v}}$, so $d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi r^2} nqA(ds)\vec{\mathbf{v}} \times \hat{\mathbf{r}}$. Next, $A(ds)$ is the volume occupied by the moving charge, and $nA(ds) = 1$ for just one charge. Then,

$$\boxed{\vec{\mathbf{B}} = \frac{\mu_0}{4\pi r^2} q\vec{\mathbf{v}} \times \hat{\mathbf{r}}}$$

(b) $$B = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1.60 \times 10^{-19} \text{ C})(2.00 \times 10^7 \text{ m/s})}{4\pi(1.00 \times 10^{-3})^2} \sin 90.0° = \boxed{3.20 \times 10^{-13} \text{ T}}$$

(c) $$F_B = q\left|\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right| = (1.60 \times 10^{-19} \text{ C})(2.00 \times 10^7 \text{ m/s})(3.20 \times 10^{-13} \text{ T}) \sin 90.0°$$

$$F_B = \boxed{1.02 \times 10^{-24} \text{ N directed away from the first proton}}$$

(d) $$F_e = qE = \frac{k_e q_1 q_2}{r^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(1.00 \times 10^{-3})^2}$$

$$F_e = \boxed{2.30 \times 10^{-22} \text{ N directed away from the first proton}}$$

Both forces act together. The electrical force is stronger by two orders of magnitude. It is productive to think about how it would look to an observer in a reference frame moving along with one proton or the other.

P30.59 (a) $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(24.0\ \text{A})}{2\pi(0.017\ 5\ \text{m})} = \boxed{2.74 \times 10^{-4}\ \text{T}}$

(b) At point C, conductor AB produces a field $\frac{1}{2}(2.74 \times 10^{-4}\ \text{T})(-\hat{\mathbf{j}})$, conductor DE produces a field of $\frac{1}{2}(2.74 \times 10^{-4}\ \text{T})(-\hat{\mathbf{j}})$, BD produces no field, and AE produces negligible field. The total field at C is $\boxed{2.74 \times 10^{-4}\ \text{T}(-\hat{\mathbf{j}})}$.

(c) $\vec{\mathbf{F}}_B = I\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}} = (24.0\ \text{A})(0.035\ 0\ \text{m}\hat{\mathbf{k}}) \times \left[5(2.74 \times 10^{-4}\ \text{T})(-\hat{\mathbf{j}})\right] = \boxed{(1.15 \times 10^{-3}\ \text{N})\hat{\mathbf{i}}}$

(d) $\vec{\mathbf{a}} = \dfrac{\sum \vec{\mathbf{F}}}{m} = \dfrac{(1.15 \times 10^{-3}\ \text{N})\hat{\mathbf{i}}}{3.0 \times 10^{-3}\ \text{kg}} = \boxed{(0.384\ \text{m/s}^2)\hat{\mathbf{i}}}$

(e) The bar is already so far from AE that it moves through nearly constant magnetic field. The force acting on the bar is constant, and therefore the bar's $\boxed{\text{acceleration is constant}}$.

(f) $v_f^2 = v_i^2 + 2ax = 0 + 2(0.384\ \text{m/s}^2)(1.30\ \text{m})$, so $\vec{\mathbf{v}}_f = \boxed{(0.999\ \text{m/s})\hat{\mathbf{i}}}$

P30.60 Each turn creates field at the center $\dfrac{\mu_0 I}{2R}$. Together they create field

$$\frac{\mu_0 I}{2}\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_{50}}\right) = \frac{4\pi \times 10^{-7}\ \text{Tm}I}{2\ \text{A}}\left(\frac{1}{5.05} + \frac{1}{5.15} + \cdots + \frac{1}{9.95}\right)\frac{1}{10^{-2}\ \text{m}}$$

$$= \mu_0 I(50/\text{m})6.93 = \boxed{347\mu_0 I/\text{m}}$$

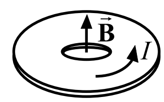


FIG. P30.60

P30.61 The central wire creates field $\vec{\mathbf{B}} = \dfrac{\mu_0 I_1}{2\pi R}$ counterclockwise. The curved portions of the loop feels no force since $\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}} = 0$ there. The straight portions both feel $I\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}$ forces to the right, amounting to $\vec{\mathbf{F}}_B = I_2 2L \dfrac{\mu_0 I_1}{2\pi R} = \boxed{\dfrac{\mu_0 I_1 I_2 L}{\pi R}\ \text{to the right}}$.

P30.62 (a) From an equation in the chapter text, the magnetic field produced by one loop at the center of the second loop is given by $B = \dfrac{\mu_0 I R^2}{2x^3} = \dfrac{\mu_0 I(\pi R^2)}{2\pi x^3} = \dfrac{\mu_0 \mu}{2\pi x^3}$ where the magnetic moment of either loop is $\mu = I(\pi R^2)$. Therefore,

$$|F_x| = \mu \frac{dB}{dx} = \mu\left(\frac{\mu_0 \mu}{2\pi}\right)\left(\frac{3}{x^4}\right) = \frac{3\mu_0(\pi R^2 I)^2}{2\pi x^4} = \boxed{\frac{3\pi}{2}\frac{\mu_0 I^2 R^4}{x^4}}.$$

(b) $|F_x| = \dfrac{3\pi}{2}\dfrac{\mu_0 I^2 R^4}{x^4} = \dfrac{3\pi}{2}\dfrac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(10.0\ \text{A})^2(5.00 \times 10^{-3}\ \text{m})^4}{(5.00 \times 10^{-2}\ \text{m})^4} = \boxed{5.92 \times 10^{-8}\ \text{N}}$

P30.63 By symmetry of the arrangement, the magnitude of the net magnetic field at point P is $B = 8B_{0x}$ where B_0 is the contribution to the field due to current in an edge length equal to $\frac{L}{2}$. In order to calculate B_0, we use the Biot-Savart law and consider the plane of the square to be the yz-plane with point P on the x-axis. The contribution to the magnetic field at point P due to a current element of length dz and located a distance z along the axis is given by the integral form of the Biot-Savart law as

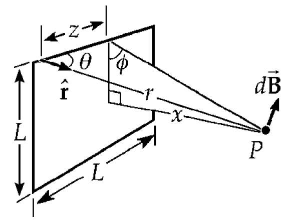


FIG. P30.63

$$\vec{\mathbf{B}}_0 = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{\boldsymbol{\ell}}\times\hat{\mathbf{r}}}{r^2}$$

From the figure we see that

$$r = \sqrt{x^2 + \left(L^2/4\right) + z^2} \text{ and } \left|d\vec{\boldsymbol{\ell}}\times\hat{\mathbf{r}}\right| = dz\sin\theta = dz\sqrt{\frac{\left(L^2/4\right)+x^2}{\left(L^2/4\right)+x^2+z^2}}$$

By symmetry all components of the field $\vec{\mathbf{B}}$ at P cancel except the components along x (perpendicular to the plane of the square); and

$$B_{0x} = B_0\cos\phi \quad \text{where} \quad \cos\phi = \frac{L/2}{\sqrt{\left(L^2/4\right)+x^2}}$$

Therefore, $\vec{\mathbf{B}}_{0x} = \frac{\mu_0 I}{4\pi}\int_0^{L/2}\frac{\sin\theta\cos\phi\, dz}{r^2}$ and $B = 8B_{0x}$

Using the expressions given above for $\sin\theta\cos\phi$, and r, we find

$$B = \frac{\mu_0 I L^2}{2\pi\left(x^2+\left(L^2/4\right)\right)\sqrt{x^2+\left(L^2/2\right)}}$$

P30.64 There is no contribution from the straight portion of the wire since $d\vec{\mathbf{s}}\times\hat{\mathbf{r}} = 0$. For the field of the spiral,

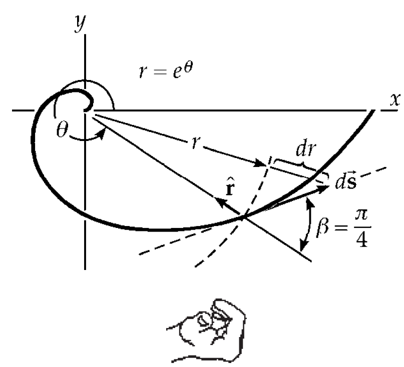


FIG. P30.64

$$d\vec{\mathbf{B}} = \frac{\mu_0 I}{(4\pi)}\frac{\left(d\vec{\mathbf{s}}\times\hat{\mathbf{r}}\right)}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi}\frac{\left|d\vec{\mathbf{s}}\right|\sin\theta\left|\hat{\mathbf{r}}\right|}{r^2} = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi}\left(\sqrt{2}dr\right)\left[\sin\left(\frac{3\pi}{4}\right)\right]\frac{1}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi} r^{-2}dr = -\frac{\mu_0 I}{4\pi}\left(r^{-1}\right)\Big|_{\theta=0}^{2\theta}$$

Substitute $r = e^\theta$: $B = -\frac{\mu_0 I}{4\pi}\left[e^{-\theta}\right]_0^{2\pi} = -\frac{\mu_0 I}{4\pi}\left[e^{-2\pi} - e^0\right] =$

$\boxed{\frac{\mu_0 I}{4\pi}\left(1-e^{-2\pi}\right)}$ out of the page.

P30.65 Consider the sphere as being built up of little rings of radius r, centered on the rotation axis. The contribution to the field from each ring is

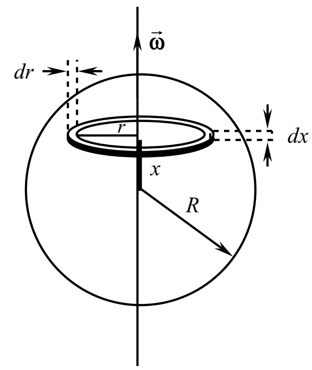


FIG. P30.65

$$dB=\frac{\mu_0 r^2 dI}{2\left(x^2+r^2\right)^{3/2}} \quad \text{where} \quad dI=\frac{dQ}{t}=\frac{\omega dQ}{2\pi}$$

$$dQ=\rho dV=\rho\left(2\pi r dr\right)\left(dx\right)$$

$$dB=\frac{\mu_0\rho\omega r^3 dr dx}{2\left(x^2+r^2\right)^{3/2}} \quad \text{where} \quad \rho=\frac{Q}{\left(4/3\right)\pi R^3}$$

$$B=\int_{x=-R}^{+R}\int_{r=0}^{\sqrt{R^2-x^2}}\frac{\mu_0\rho\omega}{2}\frac{r^3 dr dx}{\left(x^2+r^2\right)^{3/2}}$$

Let $v=r^2+x^2$, $dv=2r dr$, and $r^2=v-x^2$.

$$B=\int_{x=-R}^{+R}\int_{v=x^2}^{R^2}\frac{\mu_0\rho\omega}{2}\frac{\left(v-x^2\right)dv}{2v^{3/2}}dx=\frac{\mu_0\rho\omega}{4}\int_{x=-R}^{R}\left[\int_{v=x^2}^{R^2}v^{-1/2}dv-x^2\int_{v=x^2}^{R^2}v^{-3/2}dv\right]dx$$

$$B=\frac{\mu_0\rho\omega}{4}\int_{x=-R}^{R}\left[2v^{1/2}\Big|_{x^2}^{R^2}+\left(2x^2\right)v^{-1/2}\Big|_{x^2}^{R^2}\right]dx=\frac{\mu_0\rho\omega}{4}\int_{x=-R}^{R}\left[2\left(R-|x|\right)+2x^2\left(\frac{1}{R}-\frac{1}{|x|}\right)\right]dx$$

$$B=\frac{\mu_0\rho\omega}{4}\int_{-R}^{R}\left[2\frac{x^2}{R}-4|x|+2R\right]dx=\frac{2\mu_0\rho\omega}{4}\int_{0}^{R}\left[2\frac{x^2}{R}-4x+2R\right]dx$$

$$B=\frac{2\mu_0\rho\omega}{4}\left(\frac{2R^3}{3R}-\frac{4R^2}{2}+2R^2\right)=\boxed{\frac{\mu_0\rho\omega R^2}{3}}$$

P30.66 Consider the sphere as being built up of little rings of radius r, centered on the rotation axis. The current associated with each rotating ring of charge is

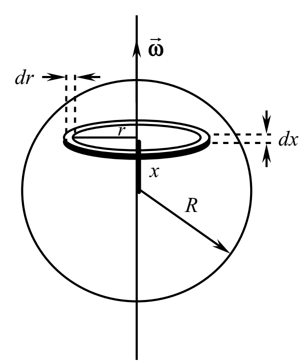


FIG. P30.66

$$dI=\frac{dQ}{t}=\frac{\omega}{2\pi}\left[\rho\left(2\pi r dr\right)\left(dx\right)\right]$$

The magnetic moment contributed by this ring is

$$d\mu=A\left(dI\right)=\pi r^2\frac{\omega}{2\pi}\left[\rho\left(2\pi r dr\right)\left(dx\right)\right]=\pi\omega\rho r^3 dr dx$$

$$\mu=\pi\omega\rho\int_{x=-R}^{+R}\left[\int_{r=0}^{\sqrt{R^2-x^2}}r^3 dr\right]dx=\pi\omega\rho\int_{x=-R}^{+R}\frac{\left(\sqrt{R^2-x^2}\right)^4}{4}dx=\pi\omega\rho\int_{x=-R}^{+R}\frac{\left(R^2-x^2\right)^2}{4}dx$$

$$\mu=\frac{\pi\omega\rho}{4}\int_{x=-R}^{+R}\left(R^4-2R^2x^2+x^4\right)dx=\frac{\pi\omega\rho}{4}\left[R^4\left(2R\right)-2R^2\left(\frac{2R^3}{3}\right)+\frac{2R^5}{5}\right]$$

$$\mu=\frac{\pi\omega\rho}{4}R^5\left(2-\frac{4}{3}+\frac{2}{5}\right)=\frac{\pi\omega\rho R^5}{4}\left(\frac{16}{15}\right)=\boxed{\frac{4\pi\omega\rho R^5}{15}}\text{ up}$$

P30. 67 Note that the current I exists in the conductor with a current density $J=\frac{I}{A}$, where

$$A=\pi\left[a^2-\frac{a^2}{4}-\frac{a^2}{4}\right]=\frac{\pi a^2}{2}$$

Therefore $J=\frac{2I}{\pi a^2}$.

To find the field at either point P_1 or P_2, find B_s which would exist if the conductor were solid, using Ampère's law. Next, find B_1 and B_2 that *would* be due to the conductors of radius $\frac{a}{2}$ that *could* occupy the void where the holes exist. Then use the superposition principle and subtract the field that would be due to the part of the conductor where the holes exist from the field of the solid conductor.

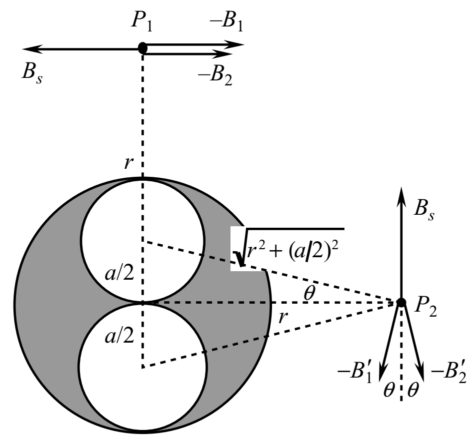


FIG. P30.67

(a) At point P_1, $B_s=\frac{\mu_0 J\left(\pi a^2\right)}{2\pi r}$, $B_1=\frac{\mu_0 J\pi(a/2)^2}{2\pi\left(r-(a/2)\right)}$, and $B_2=\frac{\mu_0 J\pi(a/2)^2}{2\pi\left(r+(a/2)\right)}$.

$$B=B_s-B_1-B_2=\frac{\mu J\pi a^2}{2\pi}\left[\frac{1}{r}-\frac{1}{4\left(r-(a/2)\right)}-\frac{1}{4\left(r+(a/2)\right)}\right]$$

$$B=\frac{\mu_0(2I)}{2\pi}\left[\frac{4r^2-a^2-2r^2}{4r\left(r^2-\left(a^2/4\right)\right)}\right]=\boxed{\frac{\mu_0 I}{\pi r}\left[\frac{2r^2-a^2}{4r^2-a^2}\right]\text{ directed to the left}}$$

(b) At point P_2, $B_s=\frac{\mu_0 J\left(\pi a^2\right)}{2\pi r}$ and $B_1'=B_2'=\frac{\mu_0 J\,\pi(a/2)^2}{2\pi\sqrt{r^2+(a/2)^2}}$.

The horizontal components of B_1' and B_2' cancel while their vertical components add.

$$B=B_s-B_1'\cos\theta-B_2'\cos\theta=\frac{\mu_0 J\left(\pi a^2\right)}{2\pi r}-2\left(\frac{\mu_0 J\,\pi a^2/4}{2\pi\sqrt{r^2+\left(a^2/4\right)}}\right)\frac{r}{\sqrt{r^2+\left(a^2/4\right)}}$$

$$B=\frac{\mu_0 J\left(\pi a^2\right)}{2\pi r}\left[1-\frac{r^2}{2\left(r^2+\left(a^2/4\right)\right)}\right]=\frac{\mu_0(2I)}{2\pi r}\left[1-\frac{2r^2}{4r^2+a^2}\right]$$

$$=\boxed{\frac{\mu_0 I}{\pi r}\left[\frac{2r^2+a^2}{4r^2+a^2}\right]\text{ directed toward the top of the page}}$$

ANSWERS TO EVEN PROBLEMS

P30.2 200 nT

P30.4 $\left(1+\frac{1}{\pi}\right)\frac{\mu_0 I}{2R}$ into the page

P30.6 See the solution.

P30.8 $\left(\frac{1}{\pi}+\frac{1}{4}\right)\frac{\mu_0 I}{2r}$ into the page

P30.10 262 nT into the page

P30.12 (a) 3.84×10^{-21} N up. See the solution. (b) 21.4 μm. This distance is negligible compared to 50 m, so the electron does move in a uniform field. (c) 134 revolutions.

P30.14 $\frac{\mu_0 I\left(a^2+d^2-d\sqrt{a^2+d^2}\right)}{2\pi ad\sqrt{a^2+d^2}}$ into the page

P30.16 4.97 mm

P30.18 (a) 10.0 μT out of the page (b) 80.0 μN toward wire 1 (c) 16.0 μT into the page

(d) 80.0 μN toward wire 2

P30.20 (a) It is possible in just one way. (b) wire 3 must be 12.0 cm to the left of wire 1, carrying (c) current 2.40 A down

P30.22 (a) 10.0 μm. (b) Yes. If we tried to use wires with diameter 10 μm, they would feel the force only momentarily after we turn on the current, until they melt. We can use wide, thin sheets of copper, perhaps plated onto glass, and perhaps with water cooling, to have the forces act continuously.

P30.24 5.40 cm

P30.26 (a) 400 cm (b) 7.50 nT (c) 1.26 m (d) zero

P30.28 (a) 3.60 T (b) 1.94 T

P30.30 500 A

P30.32 (a) See the solution. (b) zero; zero (c) $\vec{\mathbf{B}} = \frac{32\times10^{-7}\, z\ \mathrm{T\cdot m}}{9\times10^{-4}\ \mathrm{m}^2+z^2}\hat{\mathbf{j}}$ (d) at $d = 3.00$ cm (e) 53.3 $\hat{\mathbf{j}}$ μT

P30.34 (a) Make the wire as long and thin as possible without melting when it carries the 5-A current. Then the solenoid can have many turns. (b) As small in radius as possible with your experiment fitting inside. Then with a smaller circumference the wire can form a solenoid with more turns.

P30.36 (a) $\frac{\mu_0 IN}{2\ell}\left[\frac{a}{\sqrt{a^2+R^2}}-\frac{a-\ell}{\sqrt{(a-\ell)^2+R^2}}\right]$ (b) See the solution.

P30.38 207 W

P30.40 (a) $-B\pi R^2\cos\theta$ (b) $B\pi R^2\cos\theta$

P30.42 The field can be uniform in magnitude. Gauss's law for magnetism implies that magnetic field lines never start or stop. If the field is uniform in direction, the lines are parallel and their density stays constant along any one bundle of lines. Thus the magnitude of the field has the same value at all points along a line in the direction of the field. The magnitude of the field could vary over a plane perpendicular to the lines, or it could be constant throughout the volume.

P30.44 (a) 12.6 μT (b) 56.0 μT

P30.46 (a) Gravitational field and magnetic field are vectors; atmospheric pressure is a scalar. (b) At 42° north latitude, 76° west longitude, 260 m above sea level: Gravitational field is 9.803 m/s^2 down. From the Coast and Geodetic Survey, the magnetic field is 54 μT at 12° west of geographic north and 69° below the horizontal. Atmospheric pressure is 98 kPa. The atmosphere is held on by gravitation, but otherwise the effects are all separate. The magnetic field could be produced by permanent magnetization of a cold iron-nickel deposit within the Earth, so it need not be associated with present-day action of gravitation.

P30.48 2.01 GA west

P30.50 $\sim 10^{-5}$ T, enough to affect the compass noticeably

P30.52 $\dfrac{\mu_0 q\omega}{2.5\sqrt{5}\pi R}$

P30.54 1.80 mT

P30.56 (a) $\mu_0\sigma v$ horizontally away from you (b) 0 (c) $\frac{1}{2}\mu_0\sigma^2 v^2$ up (d) 3.00×10^8 m/s

P30.58 (a) See the solution. (b) 3.20×10^{-13} T (c) 1.02×10^{-24} N away from the first proton (d) 2.30×10^{-22} N away from the first proton

P30.60 $347\mu_0 I/\text{m}$ perpendicular to the coil

P30.62 (a) See the solution. (b) 59.2 nN

P30.64 $(\mu_0 I/4\pi)(1 - e^{-2\pi})$ out of the plane of the paper

P30.66 $\dfrac{4}{15}\pi\omega\rho R^5$ upward

31

Faraday's Law

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q31.1 Magnetic flux measures the "flow" of the magnetic field through a given area of a loop—even though the field does not actually flow. By changing the size of the loop, or the orientation of the loop and the field, one can change the magnetic flux through the loop, but the magnetic field will not change.

***Q31.2** The emf is given by the negative of the time derivative of the magnetic flux. We pick out the steepest downward slope at instant F as marking the moment of largest emf. Next comes A. At B and at D the graph line is horizontal so the emf is zero. At E the flux graph slopes gently upward so the emf is slightly negative. At C the emf has its greatest negative value. The answer is then F > A > B = D = 0 > E > C.

***Q31.3** (i) c (ii) b. The magnetic flux is $\Phi_B = BA\cos\theta$. Therefore the flux is a maximum when $\vec{\mathbf{B}}$ is perpendicular to the loop of wire and zero when there is no component of magnetic field perpendicular to the loop. The flux is zero when the loop is turned so that the field lies in the plane of its area.

***Q31.4** (i) Answer (c). The magnetic flux through the coil is constant in time, so the induced emf is zero. (ii) Answer (a). Positive test charges in the leading and trailing sides of the square experience a $\vec{\mathbf{F}} = q\left(\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$ force that is in direction (to the right) × (perpendicularly into the plane away from you) = toward the top of the square. The charges migrate upward to give positive charge to the top of the square until there is a downward electric field large enough to prevent more charge separation.

Q31.5 By the magnetic force law $\vec{\mathbf{F}} = q\left(\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$: the positive charges in the moving bar will feel a magnetic force in direction (right) × (perpendicularly out of the page) = downward toward the bottom end of the bar. These charges will move downward and therefore clockwise in the circuit. If the bar is moving to the left, the positive charge in the bar will flow upward and therefore counterclockwise in the circuit.

***Q31.6** (i) No. A magnetic force acts within the front and back edges of the coil, but produces no current and has no influence on the forward motion of the coil.

(ii) Yes. An induced current exists in the bar, which we can attribute either to an induced emf in the loop or to magnetic force on free charges in the bar. Then a backward magnetic force acts on the current and an external force must counterbalance it to maintain steady motion.

(iii) No. A magnetic force acts within the bar, but produces no current and has no influence on the forward motion of the bar.

Q31.7 As water falls, it gains speed and kinetic energy. It then pushes against turbine blades, transferring its energy to the rotor coils of a large AC generator. The rotor of the generator turns within a strong magnetic field. Because the rotor is spinning, the magnetic flux through its turns changes in time as $\Phi_B = BA\cos\omega t$. Generated in the rotor is an induced emf of $\mathcal{E} = \frac{-Nd\Phi_B}{dt}$. This induced emf is the voltage driving the current in our electric power lines.

Q31.8 Yes. The induced eddy currents on the surface of the aluminum will slow the descent of the aluminum. In a strong field the plate may fall very slowly.

***Q31.9** Answer (b). The rate of changes of flux of the external magnetic field in the turns of the coil is doubled, to double the maximum induced emf.

Q31.10 The increasing counterclockwise current in the solenoid coil produces an upward magnetic field that increases rapidly. The increasing upward flux of this field through the ring induces an emf to produce clockwise current in the ring. The magnetic field of the solenoid has a radially outward component at each point on the ring. This field component exerts upward force on the current in the ring there. The whole ring feels a total upward force larger than its weight.

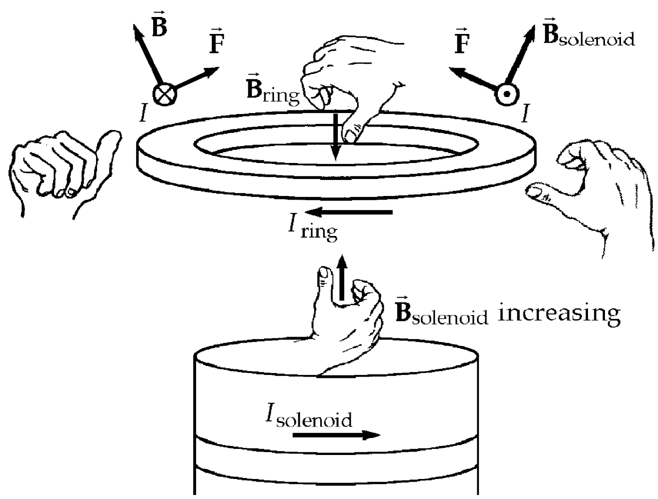


FIG. Q31.10

Q31.11 Oscillating current in the solenoid produces an always-changing magnetic field. Vertical flux through the ring, alternately increasing and decreasing, produces current in it with a direction that is alternately clockwise and counterclockwise. The current through the ring's resistance converts electrically transmitted energy into internal energy at the rate I^2R.

***Q31.12** (i) Answer (a). The south pole of the magnet produces an upward magnetic field that increases as the magnet approaches the loop. The loop opposes change by making its own downward magnetic field; it carries current clockwise, which goes to the left through the resistor.

(ii) Answer (b). The north pole of the magnet produces an upward magnetic field. The loop sees decreasing upward flux as the magnet falls away, and tries to make an upward magnetic field of its own by carrying current counterclockwise, to the right in the resistor.

(iii) Answer (c). The north pole of the magnet creates some downward flux through the section of the coil straight below it, but the south pole creates an equal amount of upward flux. The net flux through the coil due to the magnet is always zero so the induced emf is zero and the current is zero.

***Q31.13** (i) Answer (b). The battery makes counterclockwise current I_1 in the primary coil, so its magnetic field $\vec{\mathbf{B}}_1$ is to the right and increasing just after the switch is closed. The secondary coil will oppose the change with a leftward field $\vec{\mathbf{B}}_2$, which comes from an induced clockwise current I_2 that goes to the right in the resistor. The upper pair of hands in the diagram represent this effect.

(ii) Answer (d). At steady state the primary magnetic field is unchanging, so no emf is induced in the secondary.

(iii) Answer (a). The primary's field is to the right and decreasing as the switch is opened. The secondary coil opposes this decrease by making its own field to the right, carrying counterclockwise current to the left in the resistor. The lower pair of hands diagrammed represent this chain of events.

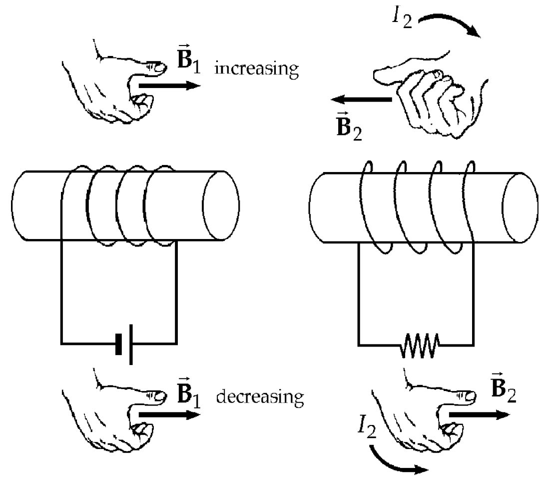


FIG. Q31.13

***Q31.14** A positive electric charge carried around a circular electric field line in the direction of the field gains energy from the field every step of the way. It can be a test charge imagined to exist in vacuum or it can be an actual free charge participating in a current driven by an induced emf. By doing net work on an object carried around a closed path to its starting point, the magnetically-induced electric field exerts by definition a nonconservative force. We can get a larger and larger voltage just by looping a wire around into a coil with more and more turns.

SOLUTIONS TO PROBLEMS

Section 31.1 **Faraday's Law of Induction**

Section 31.3 **Lenz's Law**

***P31.1** (a) $\mathcal{E} = -N\dfrac{\Delta BA\cos\theta}{\Delta t} = -1\pi r^2\left(\dfrac{B_f - B_i}{\Delta t}\right)\cos\theta = -\left[\pi(0.00160\text{ m})^2\right]\left(\dfrac{1.5\text{ T}-0}{0.120\text{ s}}\right)1$

$$= -(8.04\times10^{-6}\text{ m}^2)12.5\text{ T/s}$$

$$= \boxed{101\ \mu\text{V tending to produce clockwise current as seen from above}}$$

(b) The rate of change of magnetic field in this case is (–0.5 T – 1.5 T)/0.08 s = 25 T/s. It is twice as large in magnitude and in the opposite sense from the rate of change in case (a), so the emf is also $\boxed{\text{twice as large in magnitude and in the opposite sense}}$.

P31.2 $$|\mathcal{E}| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{\Delta\left(\vec{\mathbf{B}}\cdot\vec{\mathbf{A}}\right)}{\Delta t} = \frac{(2.50\text{ T}-0.500\text{ T})\left(8.00\times10^{-4}\text{ m}^2\right)}{1.00\text{ s}}\left(\frac{1\text{ N}\cdot\text{s}}{1\text{ T}\cdot\text{C}\cdot\text{m}}\right)\left(\frac{1\text{ V}\cdot\text{C}}{1\text{ N}\cdot\text{m}}\right)$$

$|\mathcal{E}| = 1.60\text{ mV}$ and $I_{\text{loop}} = \dfrac{\mathcal{E}}{R} = \dfrac{1.60\text{ mV}}{2.00\ \Omega} = \boxed{0.800\text{ mA}}$

P31.3 $\mathcal{E} = -N\dfrac{\Delta BA\cos\theta}{\Delta t} = -NB\pi r^2\left(\dfrac{\cos\theta_f - \cos\theta_i}{\Delta t}\right)$

$= -25.0\left(50.0\times10^{-6}\ \text{T}\right)\left[\pi(0.500\ \text{m})^2\right]\left(\dfrac{\cos180° - \cos0°}{0.200\ \text{s}}\right)$

$\mathcal{E} = \boxed{+9.82\ \text{mV}}$

***P31.4** (a) We evaluate the average emf:

$$\mathcal{E} = -N\frac{\Delta BA\cos\theta}{\Delta t} = -N\pi r^2\left(\frac{B_f - B_i}{\Delta t}\right)\cos\theta = -12\left[\pi(0.0210\ \text{m})^2\right]\left(\frac{0 - 0.11\ \text{T}}{0.180\ \text{s}}\right)\cos0°$$
$$= 0.0102\ \text{V}$$

The average induced current will then be 0.0102 V/2.3 Ω = 4.42 mA. If the meter has a sufficiently short response time, it will register the current. The average current may even run the meter offscale by a factor of 4.42, so you might wish to slow down the motion of the coil.

(b) Positive. The coil sees decreasing external magnetic flux toward you, so it makes some flux of its own in this direction by carrying counterclockwise current, that enters the red terminal of the ammeter.

P31.5 (a) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -A\dfrac{dB}{dt} = \boxed{\dfrac{AB_{\max}}{\tau}e^{-t/\tau}}$

(b) $\mathcal{E} = \dfrac{\left(0.160\ \text{m}^2\right)(0.350\ \text{T})}{2.00\ \text{s}}e^{-4.00/2.00} = \boxed{3.79\ \text{mV}}$

(c) At $t = 0$ $\mathcal{E} = \boxed{28.0\ \text{mV}}$

***P31.6** $\mathcal{E} = -N\dfrac{d\Phi_B}{dt} = -N\dfrac{d}{dt}(BA\cos\theta)$

$$\mathcal{E} = -NB\cos\theta\left(\frac{\Delta A}{\Delta t}\right) = -200\left(50.0\times10^{-6}\ \text{T}\right)(\cos62.0°)\left(\frac{39.0\times10^{-4}\ \text{m}^2}{1.80\ \text{s}}\right) = \boxed{-10.2\ \mu\text{V}}$$

P31.7 Noting unit conversions from $\vec{\mathbf{F}} = q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$ and $U = qV$, the induced voltage is

$$\mathcal{E} = -N\frac{d\left(\vec{\mathbf{B}}\cdot\vec{\mathbf{A}}\right)}{dt} = -N\left(\frac{0 - B_iA\cos\theta}{\Delta t}\right) = \frac{+200(1.60\ \text{T})\left(0.200\ \text{m}^2\right)\cos0°}{20.0\times10^{-3}\ \text{s}}\left(\frac{1\ \text{N}\cdot\text{s}}{1\ \text{T}\cdot\text{C}\cdot\text{m}}\right)\left(\frac{1\ \text{V}\cdot\text{C}}{\text{N}\cdot\text{m}}\right)$$
$$= 3\,200\ \text{V}$$

$$I = \frac{\mathcal{E}}{R} = \frac{3\,200\ \text{V}}{20.0\ \Omega} = \boxed{160\ \text{A}}$$

P31.8 (a) $d\Phi_B = \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \frac{\mu_0 I}{2\pi x} L dx$: $\Phi_B = \int_h^{h+w} \frac{\mu_0 IL}{2\pi} \frac{dx}{x} = \boxed{\frac{\mu_0 IL}{2\pi} \ln\left(\frac{h+w}{h}\right)}$

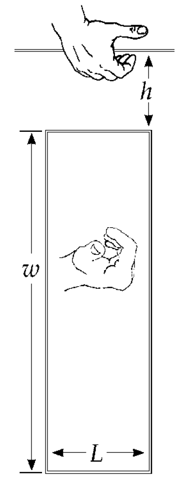


FIG. P31.8

(b) $\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 IL}{2\pi} \ln\left(\frac{h+w}{h}\right)\right] = -\left[\frac{\mu_0 L}{2\pi} \ln\left(\frac{h+w}{h}\right)\right]\frac{dI}{dt}$

$$\mathcal{E} = -\frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(1.00\ \text{m})}{2\pi} \ln\left(\frac{1.00+10.0}{1.00}\right)(10.0\ \text{A/s})$$
$$= \boxed{-4.80\ \mu\text{V}}$$

The long wire produces magnetic flux into the page through the rectangle, shown by the first hand in the figure to the right.

As the magnetic flux increases, the rectangle produces its own magnetic field out of the page, which it does by carrying $\boxed{\text{counterclockwise}}$ current (second hand in the figure).

P31.9 $|\mathcal{E}| = \frac{d(BA)}{dt} = 0.500\mu_0 nA \frac{dI}{dt} = 0.480 \times 10^{-3}\ \text{V}$

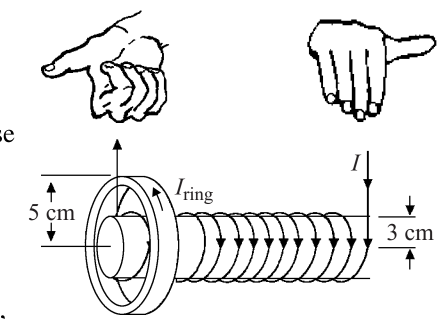


FIG. P31.9

(a) $I_{\text{ring}} = \frac{\mathcal{E}}{R} = \frac{4.80 \times 10^{-4}}{3.00 \times 10^{-4}} = \boxed{1.60\ \text{A}}$ counterclockwise

(b) $B_{\text{ring}} = \frac{\mu_0 I}{2r_{\text{ring}}} = \boxed{20.1\ \mu\text{T}}$

(c) The coil's field points into the coil, and is increasing,

so $\boxed{B_{\text{ring}} \text{ points away from the center of the coil, or left in the textbook picture}}$.

P31.10 $|\mathcal{E}| = \frac{d(BA)}{dt} = 0.500\mu_0 nA \frac{dI}{dt} = 0.500\mu_0 n\pi r_2^2 \frac{\Delta I}{\Delta t}$

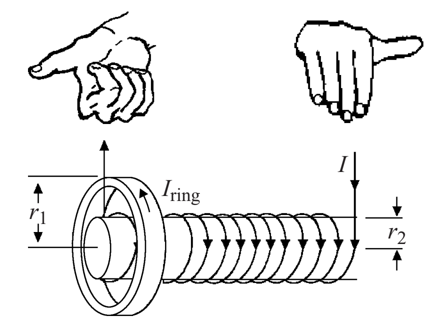


FIG. P31.10

(a) $I_{\text{ring}} = \frac{\mathcal{E}}{R} = \boxed{\frac{\mu_0 n\pi r_2^2}{2R} \frac{\Delta I}{\Delta t}}$ counterclockwise

(b) $B = \frac{\mu_0 I}{2r_1} = \boxed{\frac{\mu_0^2 n\pi r_2^2}{4r_1 R} \frac{\Delta I}{\Delta t}}$

(c) The coil's field points into the coil, and is increasing,

so $\boxed{B_{\text{ring}} \text{ points away from the center of the coil, or left in the textbook picture}}$.

P31.11 $\Phi_B = (\mu_0 nI) A_{\text{solenoid}}$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\mu_0 n\left(\pi r_{\text{solenoid}}^2\right)\frac{dI}{dt}$$

$$\mathcal{E} = -15.0\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.00\times10^{3}\ \text{m}^{-1}\right)\pi(0.020\,0\ \text{m})^2(600\ \text{A/s})\cos(120t)$$

$$\boxed{\mathcal{E} = -14.2\cos(120t)\ \text{mV}}$$

***P31.12** (a) $\mathcal{E} = -20(130)4\pi\times10^{-7}(0-6[1]/13\times10^{-6})\pi(0.03)^2 1/0.8 = \boxed{5.33\ \text{V}}$

(b) A flat compact circular coil with 130 turns and radius 40.0 cm carries current 3.00 A counterclockwise. The current is smoothly reversed to become 3.00 A clockwise after 13.0 μs. At the center of this primary coil is a secondary coil in the same plane, with 20 turns and radius 3.00 cm. Find the emf induced in the secondary.

P31.13 For a counterclockwise trip around the left-hand loop, with $B = At$

$$\frac{d}{dt}\left[At\left(2a^2\right)\cos 0°\right] - I_1(5R) - I_{PQ}R = 0$$

and for the right-hand loop,

$$\frac{d}{dt}\left[Ata^2\right] + I_{PQ}R - I_2(3R) = 0$$

where $I_{PQ} = I_1 - I_2$ is the upward current in QP.

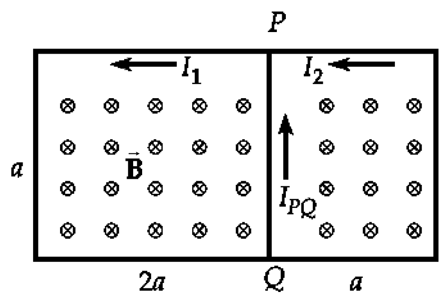


FIG. P31.13

Thus, $2Aa^2 - 5R\left(I_{PQ} + I_2\right) - I_{PQ}R = 0$

and $Aa^2 + I_{PQ}R = I_2(3R)$

$$2Aa^2 - 6RI_{PQ} - \frac{5}{3}\left(Aa^2 + I_{PQ}R\right) = 0$$

$$I_{PQ} = \frac{Aa^2}{23R}$$ upward, and since $R = (0.100\ \Omega/\text{m})(0.650\ \text{m}) = 0.0650\ \Omega$

$$I_{PQ} = \frac{\left(1.00\times10^{-3}\ \text{T/s}\right)(0.650\ \text{m})^2}{23(0.065\,0\ \Omega)} = \boxed{283\ \mu\text{A upward}}.$$

P31.14 $|\mathcal{E}| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = N\left(\frac{dB}{dt}\right)A = N(0.010\,0 + 0.080\,0t)A$

At $t = 5.00$ s, $|\mathcal{E}| = 30.0(0.410\ \text{T/s})\left[\pi(0.040\,0\ \text{m})^2\right] = \boxed{61.8\ \text{mV}}$

P31.15 $B = \mu_0 nI = \mu_0 n(30.0\ \text{A})\left(1-e^{-1.60t}\right)$

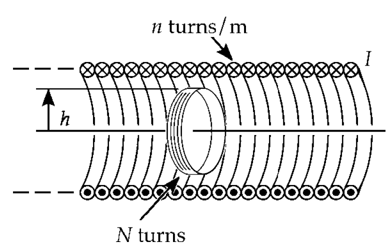


FIG. P31.15

$$\Phi_B = \int B\,dA = \mu_0 n(30.0\ \text{A})\left(1-e^{-1.60t}\right)\int dA$$

$$\Phi_B = \mu_0 n(30.0\ \text{A})\left(1-e^{-1.60t}\right)\pi R^2$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\mu_0 n(30.0\ \text{A})\pi R^2(1.60)e^{-1.60t}$$

$$\mathcal{E} = -(250)\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(400\ \text{m}^{-1}\right)(30.0\ \text{A})\left[\pi(0.060\,0\ \text{m})^2\right]1.60\ \text{s}^{-1}e^{-1.60t}$$

$$\mathcal{E} = \boxed{(68.2\ \text{mV})e^{-1.60t}\ \text{counterclockwise}}$$

P31.16 (a) Suppose, first, that the central wire is long and straight. The enclosed current of unknown amplitude creates a circular magnetic field around it, with the magnitude of the field given by Ampère's law.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I: \qquad B = \frac{\mu_0 I_{max} \sin \omega t}{2\pi R}$$

at the location of the Rogowski coil, which we assume is centered on the wire. This field passes perpendicularly through each turn of the toroid, producing flux

$$\vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = \frac{\mu_0 I_{max} A \sin \omega t}{2\pi R}$$

The toroid has $2\pi Rn$ turns. As the magnetic field varies, the emf induced in it is

$$\mathcal{E} = -N \frac{d}{dt} \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = -2\pi Rn \frac{\mu_0 I_{max} A}{2\pi R} \frac{d}{dt} \sin \omega t = -\mu_0 I_{max} nA\omega \cos \omega t$$

This is an alternating voltage with amplitude $\mathcal{E}_{max} = \mu_0 nA\omega I_{max}$. Measuring the amplitude determines the size I_{max} of the central current. Our assumptions that the central wire is long and straight and passes perpendicularly through the center of the Rogowski coil are all unnecessary.

(b) If the wire is not centered, the coil will respond to stronger magnetic fields on one side, but to correspondingly weaker fields on the opposite side. The emf induced in the coil is proportional to the line integral of the magnetic field around the circular axis of the toroid. Ampère's law says that this line integral depends only on the amount of current the coil encloses. It does not depend on the shape or location of the current within the coil, or on any currents outside the coil.

P31.17 $$\mathcal{E} = \frac{d}{dt}\left(NB\ell^2 \cos\theta\right) = \frac{N\ell^2 \Delta B \cos\theta}{\Delta t}$$

$$\ell = \sqrt{\frac{\mathcal{E}\,\Delta t}{N\Delta B \cos\theta}} = \sqrt{\frac{\left(80.0\times10^{-3}\text{ V}\right)(0.400\text{ s})}{(50)\left(600\times10^{-6}\text{ T} - 200\times10^{-6}\text{ T}\right)\cos(30.0°)}} = 1.36\text{ m}$$

Length $= 4\ell N = 4(1.36\text{ m})(50) = \boxed{272\text{ m}}$

P31.18 In a toroid, all the flux is confined to the inside of the toroid.

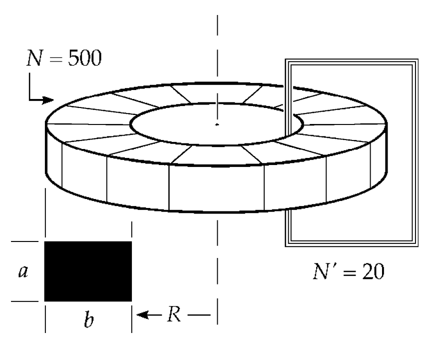


FIG. P31.18

$$B = \frac{\mu_0 NI}{2\pi r} = \frac{500\mu_0 I}{2\pi r}$$

$$\Phi_B = \int B\,dA = \frac{500\mu_0 I_{max}}{2\pi} \sin \omega t \int \frac{a\,dr}{r}$$

$$\Phi_B = \frac{500\mu_0 I_{max}}{2\pi} a \sin \omega t \ln\left(\frac{b+R}{R}\right)$$

$$\mathcal{E} = N_2 \frac{d\Phi_B}{dt} = 20\left(\frac{500\mu_0 I_{max}}{2\pi}\right)\omega a \ln\left(\frac{b+R}{R}\right)\cos \omega t$$

$$\mathcal{E} = \frac{10^4}{2\pi}\left(4\pi\times10^{-7}\text{ N/A}^2\right)(50.0\text{ A})(377\text{ rad/s})(0.020\,0\text{ m})\ln\left(\frac{(3.00+4.00)\text{ cm}}{4.00\text{ cm}}\right)\cos \omega t$$

$$= \boxed{(0.422\text{ V})\cos \omega t}$$

P31.19 The upper loop has area $\pi(0.05\text{ m})^2 = 7.85\times10^{-3}\text{ m}^2$. The induced emf in it is

$$\mathcal{E} = -N\frac{d}{dt}BA\cos\theta = -1A\cos 0°\frac{dB}{dt} = -7.85\times10^{-3}\text{ m}^2(2\text{ T/s}) = -1.57\times10^{-2}\text{ V}$$

The minus sign indicates that it tends to produce counterclockwise current, to make its own magnetic field out of the page. Similarly, the induced emf in the lower loop is

$$\mathcal{E} = -NA\cos\theta\frac{dB}{dt} = -\pi(0.09\text{ m})^2\,2\text{ T/s} = -5.09\times10^{-2}\text{ V} = +5.09\times10^{-2}\text{ V to produce}$$

$\boxed{\text{counterclockwise current in the lower loop, which becomes clockwise current in the upper loop}}$.

The net emf for current in this sense around the figure 8 is

$5.09\times10^{-2}\text{ V} - 1.57\times10^{-2}\text{ V} = 3.52\times10^{-2}\text{ V}$.

It pushes current in this sense through series resistance $[2\pi(0.05\text{ m}) + 2\pi(0.09\text{ m})]3\ \Omega/\text{m} = 2.64\ \Omega$.

The current is $I = \frac{\mathcal{E}}{R} = \frac{3.52\times10^{-2}\text{ V}}{2.64\ \Omega} = \boxed{13.3\text{ mA}}$.

Section 31.2 **Motional emf**

Section 31.3 **Lenz's Law**

P31.20 (a) For maximum induced emf, with positive charge at the top of the antenna,

$\vec{\mathbf{F}}_+ = q_+(\vec{\mathbf{v}}\times\vec{\mathbf{B}})$, so the auto must move $\boxed{east}$.

(b) $\mathcal{E} = B\ell v = (5.00\times10^{-5}\text{ T})(1.20\text{ m})\left(\frac{65.0\times10^3\text{ m}}{3\,600\text{ s}}\right)\cos 65.0° = \boxed{4.58\times10^{-4}\text{ V}}$

***P31.21** (a) $\mathcal{E} = B\ell v = (1.2\times10^{-6}\text{ T})(14.0\text{ m})(70\text{ m/s}) = \boxed{1.18\times10^{-3}\text{ V}}$. A free positive test charge in the wing feels a magnetic force in direction $\vec{\mathbf{v}}\times\vec{\mathbf{B}}$ = north × down = west. Then it migrates west to make the left-hand wingtip positive.

(b) No change. The charges in the horizontally-moving wing respond only to the vertical component of the Earth's field.

(c) No. If we tried to connect the wings into a circuit with the light bulb, we would run an extra insulated wire along the wing. With the wing it would form a one-turn coil, in which the emf is zero as the coil moves in a uniform field.

P31.22 $I = \frac{\mathcal{E}}{R} = \frac{B\ell v}{R} = 2.5(1.2)v/6 = 0.5$

$\boxed{v = 1.00\text{ m/s}}$

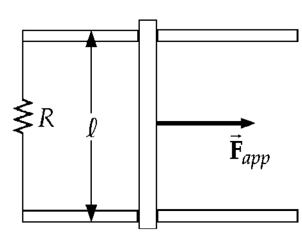


FIG. P31.22

P31.23 (a) $\left|\vec{\mathbf{F}}_B\right| = I\left|\vec{\ell}\times\vec{\mathbf{B}}\right| = I\ell B$

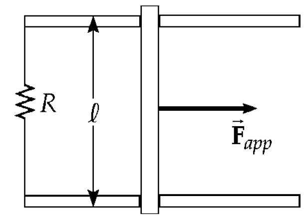


FIG. P31.23

When $I = \dfrac{\mathcal{E}}{R}$

and $\mathcal{E} = B\ell v$

we get $F_B = \dfrac{B\ell v}{R}(\ell B) = \dfrac{B^2\ell^2 v}{R} = \dfrac{(2.50)^2(1.20)^2(2.00)}{6.00}$

$= 3.00 \text{ N}.$

The applied force is $\boxed{3.00 \text{ N to the right}}$.

(b) $\mathcal{P} = I^2R = \dfrac{B^2\ell^2v^2}{R} = 6.00 \text{ W}$ or $\mathcal{P} = Fv = \boxed{6.00 \text{ W}}$

***P31.24** $F_B = I\ell B$ and $\mathcal{E} = B\ell v$

$I = \dfrac{\mathcal{E}}{R} = \dfrac{B\ell v}{R}$ so $B = \dfrac{IR}{\ell v}$

(a) $F_B = \dfrac{I^2\ell R}{\ell v}$ and $I = \sqrt{\dfrac{F_B v}{R}} = \boxed{0.500 \text{ A}}$

(b) $I^2R = \boxed{2.00 \text{ W}}$

(c) For constant force, $\mathcal{P} = \vec{\mathbf{F}}\cdot\vec{\mathbf{v}} = (1.00 \text{ N})(2.00 \text{ m/s}) = \boxed{2.00 \text{ W}}$.

(d) The powers computed in parts (b) and (c) are mathematically equal. More profoundly, they are physically identical. Each bit of energy delivered to the circuit mechanically immediately goes through being electrically transmitted to the resistor and there becomes additional internal energy. Counting the 2 W twice is like counting your lunch money twice.

P31.25 Observe that the homopolar generator has no commutator and produces a voltage constant in time: DC with no ripple. In time dt, the disk turns by angle $d\theta = \omega dt$. The outer brush slides over distance $rd\theta$. The radial line to the outer brush sweeps over area

$$dA = \frac{1}{2}rrd\theta = \frac{1}{2}r^2\omega dt$$

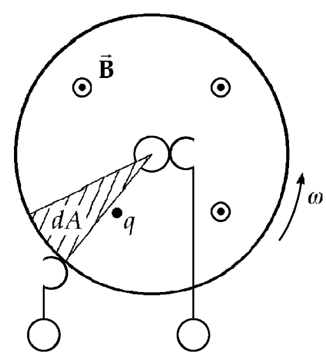


FIG. P31.25

The emf generated is $\mathcal{E} = -N\dfrac{d}{dt}\vec{\mathbf{B}}\cdot\vec{\mathbf{A}}$

$$\mathcal{E} = -(1)B\cos 0°\frac{dA}{dt} = -B\left(\frac{1}{2}r^2\omega\right)$$

(We could think of this as following from the result of the example in the chapter text about the helicopter blade.)
The magnitude of the emf is

$$|\mathcal{E}| = B\left(\frac{1}{2}r^2\omega\right) = (0.9 \text{ N}\cdot\text{s/C}\cdot\text{m})\left[\frac{1}{2}(0.4 \text{ m})^2(3\,200 \text{ rev/min})\right]\left(\frac{2\pi \text{ rad/rev}}{60 \text{ s/min}}\right)$$

$|\mathcal{E}| = \boxed{24.1 \text{ V}}$

A free positive charge q shown, turning with the disk, feels a magnetic force $q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$ radially outward. Thus the $\boxed{\text{outer contact is positive}}$.

P31.26 The speed of waves on the wire is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{267\ \text{N}\cdot\text{m}}{3\times10^{-3}\ \text{kg}}} = 298\ \text{m/s}$$

In the simplest standing-wave vibration state,

$$d_{NN} = 0.64\ \text{m} = \frac{\lambda}{2} \qquad \lambda = 1.28\ \text{m}$$

and $$f = \frac{v}{\lambda} = \frac{298\ \text{m/s}}{1.28\ \text{m}} = 233\ \text{Hz}$$

(a) The changing flux of magnetic field through the circuit containing the wire will drive current to the left in the wire as it moves up and to the right as it moves down. The emf will have this same frequency of $\boxed{233\ \text{Hz}}$.

(b) The vertical coordinate of the center of the wire is described by $x = A\cos\omega t = (1.5\ \text{cm})\cos(2\pi 233 t/\text{s})$.

Its velocity is $v = \dfrac{dx}{dt} = -(1.5\ \text{cm})(2\pi\, 233/\text{s})\sin(2\pi 233\ t/\text{s})$.

Its maximum speed is $1.5\ \text{cm}(2\pi)233/\text{s} = 22.0\ \text{m/s}$.

The induced emf is $\mathcal{E} = -B\ell v$, with amplitude

$$\mathcal{E}_{\max} = B\ell v_{\max} = 4.50\times10^{-3}\ \text{T}(0.02\ \text{m})22\ \text{m/s} = \boxed{1.98\times10^{-3}\ \text{V}}.$$

P31.27 $\omega = (2.00\ \text{rev/s})(2\pi\ \text{rad/rev}) = 4.00\pi\ \text{rad/s}$

$$\mathcal{E} = \frac{1}{2}B\omega\ell^2 = \boxed{2.83\ \text{mV}}$$

P31.28 (a) $\vec{\mathbf{B}}_{\text{ext}} = B_{\text{ext}}\hat{\mathbf{i}}$ and B_{ext} decreases; therefore, the induced field is $\vec{\mathbf{B}}_0 = B_0\hat{\mathbf{i}}$ (to the right) and the current in the resistor is directed $\boxed{\text{to the right}}$.

(b) $\vec{\mathbf{B}}_{\text{ext}} = B_{\text{ext}}\left(-\hat{\mathbf{i}}\right)$ increases; therefore, the induced field $\vec{\mathbf{B}}_0 = B_0\left(+\hat{\mathbf{i}}\right)$ is to the right, and the current in the resistor is directed $\boxed{\text{out of the plane}}$ in the textbook picture and to the right in the diagram here.

(c) $\vec{\mathbf{B}}_{\text{ext}} = B_{\text{ext}}\left(-\hat{\mathbf{k}}\right)$ into the paper and B_{ext} decreases; therefore, the induced field is $\vec{\mathbf{B}}_0 = B_0\left(-\hat{\mathbf{k}}\right)$ into the paper, and the current in the resistor is directed $\boxed{\text{to the right}}$.

(d) By the magnetic force law, $\vec{\mathbf{F}}_B = q\left(\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)$. Therefore, a positive charge will move to the top of the bar if $\vec{\mathbf{B}}$ is $\boxed{\text{into the paper}}$.

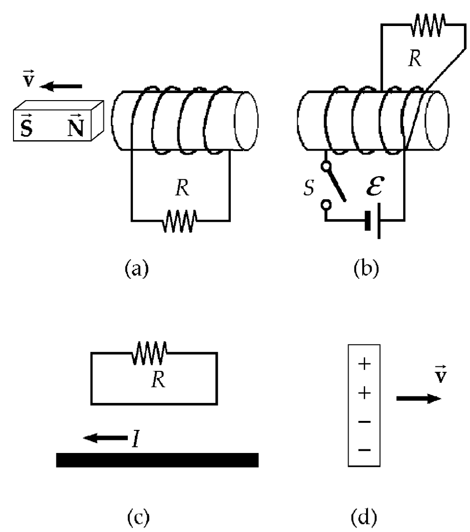


FIG. P31.28

P31.29 (a) The force on the side of the coil entering the field (consisting of N wires) is

$$F = N(ILB) = N(IwB)$$

The induced emf in the coil is

$$|\mathcal{E}| = N\frac{d\Phi_B}{dt} = N\frac{d(Bwx)}{dt} = NBwv$$

so the current is $I = \frac{|\mathcal{E}|}{R} = \frac{NBwv}{R}$ counterclockwise.

The force on the leading side of the coil is then:

$$F = N\left(\frac{NBwv}{R}\right)wB = \boxed{\frac{N^2B^2w^2v}{R} \text{ to the left}}$$

(b) Once the coil is entirely inside the field,

$\Phi_B = NBA = \text{constant}$,

so $\quad \mathcal{E} = 0,\ I = 0, \quad$ and $\quad F = \boxed{0}$

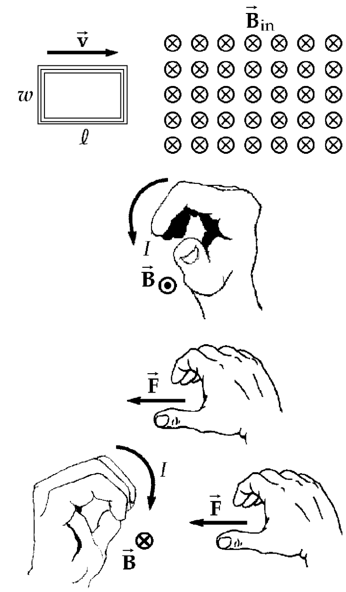


FIG. P31.29

(c) As the coil starts to leave the field, the flux *decreases* at the rate Bwv, so the magnitude of the current is the same as in part (a), but now the current is clockwise. Thus, the force exerted on the trailing side of the coil is:

$$F = \boxed{\frac{N^2B^2w^2v}{R} \text{ to the left again}}$$

P31.30 Look in the direction of *ba*. The bar magnet creates a field into the page, and the field increases. The loop will create a field out of the page by carrying a counterclockwise current. Therefore, current must flow from b to a through the resistor. Hence, $V_a - V_b$ will be $\boxed{\text{negative}}$.

P31.31 Name the currents as shown in the diagram:

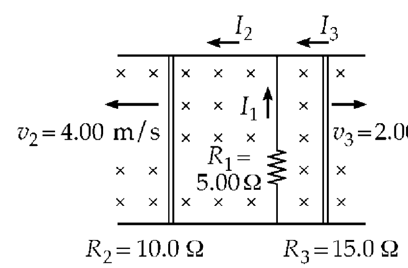


FIG. P31.31

Left loop: $\quad +Bdv_2 - I_2R_2 - I_1R_1 = 0$

Right loop: $\quad +Bdv_3 - I_3R_3 + I_1R_1 = 0$

At the junction: $\quad I_2 = I_1 + I_3$

Then, $\quad Bdv_2 - I_1R_2 - I_3R_2 - I_1R_1 = 0$

$$I_3 = \frac{Bdv_3}{R_3} + \frac{I_1R_1}{R_3}$$

So, $\quad Bdv_2 - I_1(R_1 + R_2) - \frac{Bdv_3R_2}{R_3} - \frac{I_1R_1R_2}{R_3} = 0$

$$I_1 = Bd\left(\frac{v_2R_3 - v_3R_2}{R_1R_2 + R_1R_3 + R_2R_3}\right) \text{ upward}$$

$$I_1 = (0.010\,0\ \text{T})(0.100\ \text{m})\left[\frac{(4.00\ \text{m/s})(15.0\ \Omega) - (2.00\ \text{m/s})(10.0\ \Omega)}{(5.00\ \Omega)(10.0\ \Omega) + (5.00\ \Omega)(15.0\ \Omega) + (10.0\ \Omega)(15.0\ \Omega)}\right] = \boxed{145\ \mu\text{A}}$$

upward.

Section 31.4 **Induced emf and Electric Fields**

P31.32 (a) $\frac{dB}{dt} = 6.00t^2 - 8.00t$ $\qquad |\mathcal{E}| = \frac{d\Phi_B}{dt}$

At $t = 2.00$ s,

$$E = \frac{\pi R^2 (dB/dt)}{2\pi r_2} = \frac{8.00\pi(0.0250)^2}{2\pi(0.0500)}$$

$$F = qE = \boxed{8.00\times10^{-21}\ \text{N}}$$

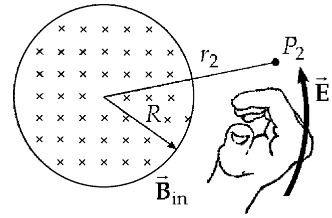


FIG. P31.32

(b) When $6.00t^2 - 8.00t = 0,$ $\qquad t = \boxed{1.33\ \text{s}}$

P31.33 $\frac{dB}{dt} = 0.060\,0t$ $\qquad |\mathcal{E}| = \frac{d\Phi_B}{dt} = \pi r_1^2 \frac{dB}{dt} = 2\pi r_1 E$

At $t = 3.00$ s,

$$E = \left(\frac{\pi r_1^2}{2\pi r_1}\right)\frac{dB}{dt} = \frac{0.020\,0\ \text{m}}{2}\left(0.060\,0\ \text{T/s}^2\right)(3.00\ \text{s})\left(\frac{1\ \text{N}\cdot\text{s}}{1\ \text{T}\cdot\text{C}\cdot\text{m}}\right)$$

$$\vec{\mathbf{E}} = \boxed{1.80\times10^{-3}\ \text{N/C perpendicular to } r_1 \text{ and counterclockwise}}$$

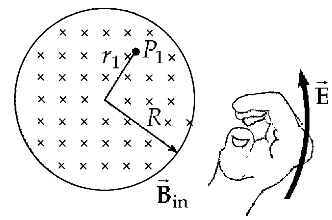


FIG. P31.33

P31.34 (a) $\oint \vec{\mathbf{E}}\cdot d\vec{\ell} = \left|\frac{d\Phi_B}{dt}\right|$

$2\pi r E = \left(\pi r^2\right)\frac{dB}{dt}$ $\qquad$ so $\qquad E = \boxed{(9.87\ \text{mV/m})\cos(100\pi t)}$

(b) The E field is always opposite to increasing B. Therefore it is $\boxed{\text{clockwise}}$.

Section 31.5 **Generators and Motors**

P31.35 (a) $\mathcal{E}_{max} = NAB\omega = (1\,000)(0.100)(0.200)(120\pi) = \boxed{7.54\ \text{kV}}$

(b) $\mathcal{E}(t) = NBA\omega\sin\omega t = NBA\omega\sin\theta$

$|\mathcal{E}|$ is maximal when $|\sin\theta| = 1$

or $\theta = \pm\frac{\pi}{2}$

so the $\boxed{\text{plane of coil is parallel to } \vec{\mathbf{B}}}$.

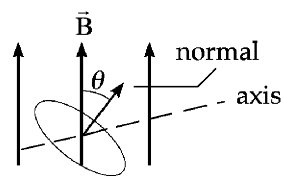


FIG. P31.35

P31.36 For the alternator, $\omega = (3\,000\ \text{rev/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 314\ \text{rad/s}$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -250\frac{d}{dt}\left[\left(2.50\times10^{-4}\ \text{T}\cdot\text{m}^2\right)\cos(314t/\text{s})\right] = +250\left(2.50\times10^{-4}\ \text{T}\cdot\text{m}^2\right)(314/\text{s})\sin(314t)$$

(a) $\boxed{\mathcal{E} = (19.6\ \text{V})\sin(314t)}$

(b) $\boxed{\mathcal{E}_{max} = 19.6\ \text{V}}$

P31.37 $B=\mu_0 nI=\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(200\text{ m}^{-1}\right)(15.0\text{ A})=3.77\times10^{-3}\text{ T}$

For the small coil, $\Phi_B=N\vec{\mathbf{B}}\cdot\vec{\mathbf{A}}=NBA\cos\omega t=NB\left(\pi r^2\right)\cos\omega t$.

Thus, $\mathcal{E}=-\dfrac{d\Phi_B}{dt}=NB\pi r^2\omega\sin\omega t$

$$\mathcal{E}=(30.0)\left(3.77\times10^{-3}\text{ T}\right)\pi(0.080\,0\text{ m})^2\left(4.00\pi\text{ s}^{-1}\right)\sin(4.00\pi t)=\boxed{(28.6\text{ mV})\sin(4.00\pi t)}$$

P31.38 As the magnet rotates, the flux through the coil varies sinusoidally in time with $\Phi_B=0$ at $t=0$. Choosing the flux as positive when the field passes from left to right through the area of the coil, the flux at any time may be written as $\Phi_B=-\Phi_{\max}\sin\omega t$ so the induced emf is given by

$$\mathcal{E}=-\frac{d\Phi_B}{dt}=\omega\Phi_{\max}\cos\omega t.$$

The current in the coil is then

$$I=\frac{\mathcal{E}}{R}=\frac{\omega\Phi_{\max}}{R}\cos\omega t=\boxed{I_{\max}\cos\omega t}.$$

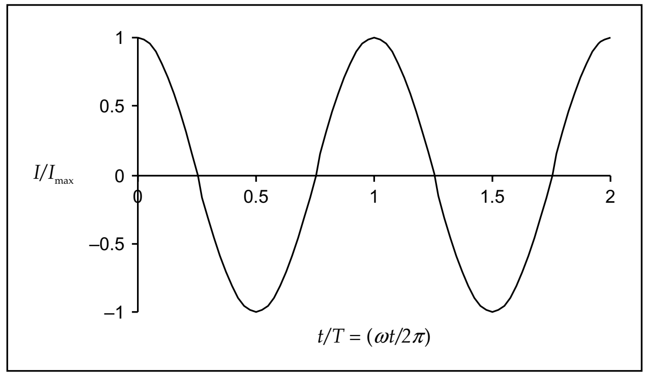
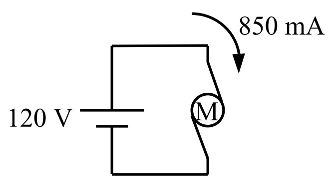
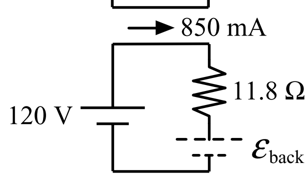


FIG. P31.38

P31.39



To analyze the actual circuit, we model it as



(a) The loop rule gives $+120\text{ V}-0.85\text{ A}(11.8\ \Omega)-\mathcal{E}_{\text{back}}=0$ $\quad\mathcal{E}_{\text{back}}=\boxed{110\text{ V}}$

(b) The resistor is the device changing electrical work input into internal energy:
$\mathcal{P}=I^2R=(0.85\text{ A})^2(11.8\ \Omega)=\boxed{8.53\text{ W}}$

(c) With no motion, the motor does not function as a generator, and $\mathcal{E}_{\text{back}}=0$. Then
$120\text{ V}-I_c(11.8\ \Omega)=0\qquad I_c=10.2\text{ A}$
$\mathcal{P}_c=I_c^2R=(10.2\text{ A})^2(11.8\ \Omega)=\boxed{1.22\text{ kW}}$

P31.40 (a) $\Phi_B = BA\cos\theta = BA\cos\omega t = (0.800\text{ T})(0.010\,0\text{ m}^2)\cos 2\pi(60.0)t$

$$= \boxed{(8.00\text{ mT}\cdot\text{m}^2)\cos(377t)}$$

(b) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = \boxed{(3.02\text{ V})\sin(377t)}$

(c) $I = \dfrac{\mathcal{E}}{R} = \boxed{(3.02\text{ A})\sin(377t)}$

(d) $\mathcal{P} = I^2R = \boxed{(9.10\text{ W})\sin^2(377t)}$

(e) $\mathcal{P} = Fv = \tau\omega$ so $\tau = \dfrac{\mathcal{P}}{\omega} = \boxed{(24.1\text{ mN}\cdot\text{m})\sin^2(377t)}$

Section 31.6 **Eddy Currents**

P31.41 The current in the magnet creates an  upward magnetic field, so the N and S poles on the solenoid core are shown correctly. On the rail in front of the brake, the upward flux of $\vec{\mathbf{B}}$ increases as the coil approaches, so a current is induced here to create a downward magnetic field. This is  clockwise current, so the S pole on the rail is shown correctly. On the rail behind the brake, the upward magnetic flux is decreasing. The induced current in the rail will produce upward magnetic field by being  counterclockwise as the picture correctly shows.

P31.42 (a) Consider an annulus of radius *r*, width *dr*, height *b*, and resistivity ρ. Around its circumference, a voltage is induced according to

$$\mathcal{E} = -N\frac{d}{dt}\vec{\mathbf{B}}\cdot\vec{\mathbf{A}} = -1\frac{d}{dt}B_{\max}(\cos\omega t)\pi r^2 = +B_{\max}\pi r^2\omega\sin\omega t$$

The resistance around the loop is $\dfrac{\rho\ell}{A_x} = \dfrac{\rho(2\pi r)}{bdr}$

The eddy current in the ring is $dI = \dfrac{\mathcal{E}}{\text{resistance}} = \dfrac{B_{\max}\pi r^2\omega(\sin\omega t)bdr}{\rho(2\pi r)} = \dfrac{B_{\max}rb\omega dr\sin\omega t}{2\rho}$

The instantaneous power is $d\mathcal{P}_i = \mathcal{E}\,dI = \dfrac{B_{\max}^2\pi r^3 b\omega^2 dr\sin^2\omega t}{2\rho}$

The time average of the function $\sin^2\omega t = \dfrac{1}{2} - \dfrac{1}{2}\cos 2\omega t$ is $\dfrac{1}{2} - 0 = \dfrac{1}{2}$

so the time-averaged power delivered to the annulus is

$$d\mathcal{P} = \frac{B_{\max}^2\pi r^3 b\omega^2 dr}{4\rho}$$

The power delivered to the disk is $\mathcal{P} = \int d\mathcal{P} = \int_0^R \dfrac{B_{\max}^2\pi b\omega^2}{4\rho}r^3 dr$

$$\mathcal{P} = \frac{B_{\max}^2\pi b\omega^2}{4\rho}\left(\frac{R^4}{4} - 0\right) = \boxed{\frac{\pi B_{\max}^2 R^4 b\omega^2}{16\rho}}$$

(b) When $B_{\max}$ gets two times larger, $B_{\max}^2$ and $\mathcal{P}$ get $\boxed{4}$ times larger.

(c) When *f* and $\omega = 2\pi f$ double, ω^2 and $\mathcal{P}$ get $\boxed{4}$ times larger.

(d) When *R* doubles, R^4 and $\mathcal{P}$ become $2^4 = \boxed{16}$ times larger.

P31.43 (a) At terminal speed,

$$Mg = F_B = IwB = \left(\frac{\mathcal{E}}{R}\right)wB = \left(\frac{Bwv_T}{R}\right)wB = \frac{B^2w^2v_T}{R}$$

or $\boxed{v_T = \dfrac{MgR}{B^2\omega^2}}$

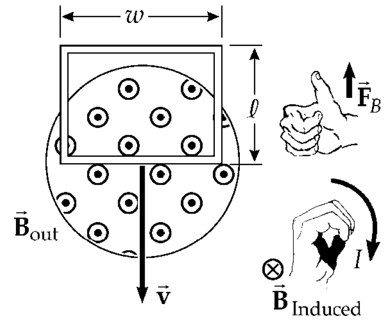


FIG. P31.43

(b) The emf is directly proportional to v_T, but the current is inversely proportional to *R*. A large *R* means a small current at a given speed, so the loop must travel faster to get $F_B = mg$.

(c) At a given speed, the current is directly proportional to the magnetic field. But the force is proportional to the product of the current and the field. For a small *B*, the speed must increase to compensate for both the small *B* and also the current, so $v_T \propto 1/B^2$.

Additional Problems

***P31.44** (a) The circuit encloses increasing flux of magnetic field into the page, so it tries to make its own field out of the page, by carrying $\boxed{\text{counterclockwise}}$ current.

(b) $I = \mathcal{E}/R = B\ell v/R = (0.4\text{ T } 0.8\text{ m } 15\text{ m/s})/48\ \Omega = \boxed{100\text{ mA}}$

(c) The magnetic field exerts a backward magnetic force on the induced current. With the values in (b), this force is $I\ell B = 0.1\text{ A } 0.8\text{ m } 0.4\text{ T} = 0.032\text{ N}$, much less than 0.6 N. The speed of the bar increases until the backward magnetic force exerted on the current in the bar is equal to 0.6 N. The terminal speed is given by $0.6\text{ N} = I\ell B = (B\ell)^2v/R$.

Then $v = 0.6\text{ N } R/(B\ell)^2 = 0.6\text{ N } 48\ \Omega/(0.4\text{ T } 0.8\text{ m})^2 = \boxed{281\text{ m/s}}$.

(d) $Fv = 0.6\text{ N } 281\text{ m/s} = \boxed{169\text{ W}}$

(e) The terminal speed becomes $\boxed{\text{larger}}$. The bar must move faster to generate a larger emf to produce enough current in the larger resistance to feel the 0.6-N magnetic force.

(f) The power delivered to the circuit by the agent moving the bar, and then converted into internal energy by the resistor, is described by $\mathcal{P} = Fv = F^2R/B^2\ell^2$. Thus the power is directly proportional to the resistance and becomes $\boxed{\text{larger}}$ as the bulb heats up.

P31.45 $\mathcal{E} = -N\dfrac{d}{dt}(BA\cos\theta) = -N\left(\pi r^2\right)\cos 0°\left(\dfrac{dB}{dt}\right)$

$$\mathcal{E} = -(30.0)\left[\pi\left(2.70\times10^{-3}\text{ m}\right)^2\right](1)\frac{d}{dt}\left[50.0\text{ mT} + (3.20\text{ mT})\sin\left(2\pi\left[523t\text{ s}^{-1}\right]\right)\right]$$

$$\mathcal{E} = -(30.0)\left[\pi\left(2.70\times10^{-3}\text{ m}\right)^2\right]\left(3.20\times10^{-3}\text{ T}\right)\left[2\pi\left(523\text{ s}^{-1}\right)\cos\left(2\pi\left[523t\text{ s}^{-1}\right]\right)\right]$$

$$\mathcal{E} = \boxed{-\left(7.22\times10^{-3}\text{ V}\right)\cos\left[2\pi\left(523t\text{ s}^{-1}\right)\right]}$$

P31.46 $\mathcal{E} = -N\dfrac{\Delta}{\Delta t}(BA\cos\theta) = -N\left(\pi r^2\right)\cos^\circ\dfrac{\Delta B}{\Delta t} = -1\left(0.005\ 00\ \text{m}^2\right)(1)\left(\dfrac{1.50\ \text{T} - 5.00\ \text{T}}{20.0\times10^{-3}\ \text{s}}\right) = 0.875\ \text{V}$

(a) $I = \dfrac{\mathcal{E}}{R} = \dfrac{0.875\ \text{V}}{0.020\ 0\ \Omega} = \boxed{43.8\ \text{A}}$

(b) $\mathcal{P} = \mathcal{E}I = (0.875\ \text{V})(43.8\ \text{A}) = \boxed{38.3\ \text{W}}$

P31.47 (a) Doubling the number of turns has this effect:

Amplitude doubles and period unchanged

(b) Doubling the angular velocity has this effect:

doubles the amplitude and cuts the period in half

(c) Doubling the angular velocity while reducing the number of turns to one half the original value has this effect:

Amplitude unchanged and period is cut in half

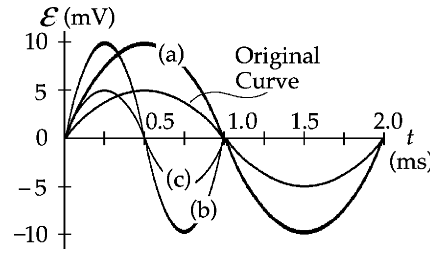


FIG. P31.47

P31.48 In the loop on the left, the induced emf is

$|\mathcal{E}| = \dfrac{d\Phi_B}{dt} = A\dfrac{dB}{dt} = \pi(0.100\ \text{m})^2(100\ \ \text{T/s}) = \pi\ \text{V}$

and it attempts to produce a counterclockwise current in this loop.

In the loop on the right, the induced emf is

$|\mathcal{E}| = \dfrac{d\Phi_B}{dt} = \pi(0.150\ \text{m})^2(100\ \ \text{T/s}) = 2.25\pi\ \text{V}$

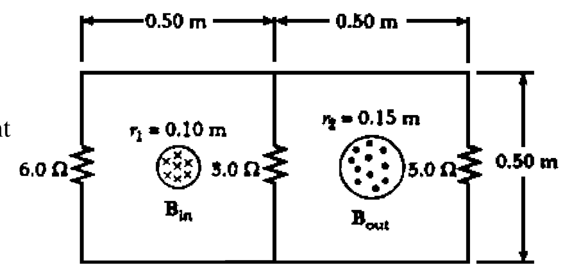


FIG. P31.48

and it attempts to produce a clockwise current. Assume that I_1 flows down through the 6.00-Ω resistor, I_2 flows down through the 5.00-Ω resistor, and that I_3 flows up through the 3.00-Ω resistor.

From Kirchhoff's junction rule: $I_3 = I_1 + I_2$ (1)

Using the loop rule on the left loop: $6.00I_1 + 3.00I_3 = \pi$ (2)

Using the loop rule on the right loop: $5.00I_2 + 3.00I_3 = 2.25\pi$ (3)

Solving these three equations simultaneously, $I_3 = (\pi - 3I_3)/6 + (2.25\ \pi - 3I_3)/5$

$I_1 = \boxed{0.062\ 3\ \text{A}}$, $I_2 = \boxed{0.860\ \text{A}}$, and $I_3 = \boxed{0.923\ \text{A}}$.

P31.49 The emf induced between the ends of the moving bar is

$$\mathcal{E} = B\ell v = (2.50\ \text{T})(0.350\ \text{m})(8.00\ \text{m/s}) = 7.00\ \text{V}$$

The left-hand loop contains decreasing flux away from you, so the induced current in it will be clockwise, to produce its own field directed away from you. Let I_1 represent the current flowing upward through the 2.00-Ω resistor. The right-hand loop will carry counterclockwise current. Let I_3 be the upward current in the 5.00-Ω resistor.

(a) Kirchhoff's loop rule then gives: $+7.00\ \text{V} - I_1(2.00\ \Omega) = 0$ $\quad I_1 = \boxed{3.50\ \text{A}}$

and $+7.00\ \text{V} - I_3(5.00\ \Omega) = 0$ $\quad I_3 = \boxed{1.40\ \text{A}}$

(b) The total power converted in the resistors of the circuit is

$$\mathcal{P} = \mathcal{E}I_1 + \mathcal{E}I_3 = \mathcal{E}(I_1 + I_3) = (7.00\ \text{V})(3.50\ \text{A} + 1.40\ \text{A}) = \boxed{34.3\ \text{W}}$$

(c) *Method 1*: The current in the sliding conductor is downward with value $I_2 = 3.50\ \text{A} + 1.40\ \text{A} = 4.90\ \text{A}$. The magnetic field exerts a force of $F_m = I\ell B = (4.90\ \text{A})(0.350\ \text{m})(2.50\ \text{T}) = 4.29\ \text{N}$ directed toward the right on this conductor. An outside agent must then exert a force of $\boxed{4.29\ \text{N}}$ to the left to keep the bar moving.

Method 2: The agent moving the bar must supply the power according to $\mathcal{P} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} = Fv\cos 0°$. The force required is then:

$$F = \frac{\mathcal{P}}{v} = \frac{34.3\ \text{W}}{8.00\ \text{m/s}} = \boxed{4.29\ \text{N}}$$

P31.50 $I = \dfrac{\mathcal{E} + \mathcal{E}_{\text{induced}}}{R}$ and $\mathcal{E}_{\text{induced}} = -\dfrac{d}{dt}(BA)$

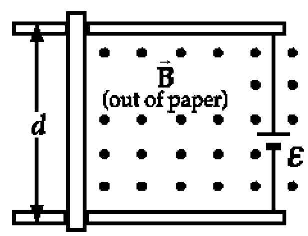


FIG. P31.50

$$F = m\frac{dv}{dt} = IBd$$

$$\frac{dv}{dt} = \frac{IBd}{m} = \frac{Bd}{mR}(\mathcal{E} + \mathcal{E}_{\text{induced}})$$

$$\frac{dv}{dt} = \frac{Bd}{mR}(\mathcal{E} - Bvd)$$

To solve the differential equation, let $u = \mathcal{E} - Bvd$

$$\frac{du}{dt} = -Bd\frac{dv}{dt}$$

$$-\frac{1}{Bd}\frac{du}{dt} = \frac{Bd}{mR}u$$

so

$$\int_{u_0}^{u} \frac{du}{u} = -\int_0^t \frac{(Bd)^2}{mR}dt$$

Integrating from $t = 0$ to $t = t$, $\quad \ln\dfrac{u}{u_0} = -\dfrac{(Bd)^2}{mR}t$

or $\quad \dfrac{u}{u_0} = e^{-B^2d^2t/mR}$

Since $v = 0$ when $t = 0$, $\quad u_0 = \mathcal{E}$

and $\quad u = \mathcal{E} - Bvd$

$$\mathcal{E} - Bvd = \mathcal{E}e^{-B^2d^2t/mR}$$

Therefore, $\quad \boxed{v = \dfrac{\mathcal{E}}{Bd}\left(1 - e^{-B^2d^2t/mR}\right)}$

P31.51 Suppose we wrap twenty turns of wire into a flat compact circular coil of diameter 3 cm. Suppose we use a bar magnet to produce field 10^{-3} T through the coil in one direction along its axis. Suppose we then flip the magnet to reverse the flux in 10^{-1} s. The average induced emf is then

$$\bar{\mathcal{E}} = -N\frac{\Delta\Phi_B}{\Delta t} = -N\frac{\Delta[BA\cos\theta]}{\Delta t} = -NB\left(\pi r^2\right)\left(\frac{\cos 180° - \cos 0°}{\Delta t}\right)$$

$$\bar{\mathcal{E}} = -(20)\left(10^{-3}\text{ T}\right)\pi(0.0150\text{ m})^2\left(\frac{-2}{10^{-1}\text{ s}}\right) \boxed{\sim 10^{-4}\text{ V}}$$

P31.52 (a) $I = \frac{dq}{dt} = \frac{\mathcal{E}}{R}$ where $\mathcal{E} = -N\frac{d\Phi_B}{dt}$ so $\int dq = \frac{N}{R}\int_{\Phi_1}^{\Phi_2} d\Phi_B$

and the charge passing any point in the circuit will be $|Q| = \frac{N}{R}(\Phi_2 - \Phi_1)$.

(b) $Q = \frac{N}{R}\left[BA\cos 0 - BA\cos\left(\frac{\pi}{2}\right)\right] = \frac{BAN}{R}$

so $B = \frac{RQ}{NA} = \frac{(200\ \Omega)\left(5.00\times 10^{-4}\text{ C}\right)}{(100)\left(40.0\times 10^{-4}\text{ m}^2\right)} = \boxed{0.250\text{ T}}$

P31.53 $I = \frac{\mathcal{E}}{R} = \frac{B}{R}\frac{|A|}{\Delta t}$

so $q = I\Delta t = \frac{(15.0\ \mu\text{T})(0.200\text{ m})^2}{0.500\ \Omega} = \boxed{1.20\ \mu\text{C}}$

P31.54 The enclosed flux is $\Phi_B = BA = B\pi r^2$

The particle moves according to $\sum\vec{\mathbf{F}} = m\vec{\mathbf{a}}$: $qvB\sin 90° = \frac{mv^2}{r}$

$$r = \frac{mv}{qB}$$

Then $\Phi_B = \frac{B\pi m^2 v^2}{q^2 B^2}$

(a) $v = \sqrt{\frac{\Phi_B q^2 B}{\pi m^2}} = \sqrt{\frac{\left(15\times 10^{-6}\text{ T}\cdot\text{m}^2\right)\left(30\times 10^{-9}\text{ C}\right)^2(0.6\text{ T})}{\pi\left(2\times 10^{-16}\text{ kg}\right)^2}} = \boxed{2.54\times 10^5\text{ m/s}}$

(b) Energy for the particle-electric field system is conserved in the firing process:

$U_i = K_f$: $q\Delta V = \frac{1}{2}mv^2$

$$\Delta V = \frac{mv^2}{2q} = \frac{\left(2\times 10^{-16}\text{ kg}\right)\left(2.54\times 10^5\text{ m/s}\right)^2}{2\left(30\times 10^{-9}\text{ C}\right)} = \boxed{215\text{ V}}$$

P31.55 (a) $\mathcal{E} = B\ell v = 0.360\text{ V}$ $\quad I = \dfrac{\mathcal{E}}{R} = \boxed{0.900\text{ A}}$

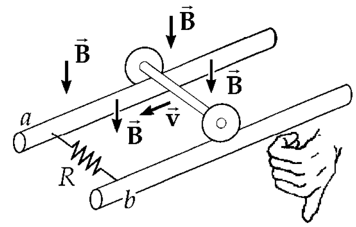


FIG. P31.55

(b) $F_B = I\ell B = \boxed{0.108\text{ N}}$

(c) Since the magnetic flux $\vec{\mathbf{B}}\cdot\vec{\mathbf{A}}$ is in effect decreasing, the induced current flow through R is from b to a. $\boxed{\text{Point } b}$ is at higher potential.

(d) $\boxed{\text{No}}$. Magnetic flux will increase through a loop to the left of ab. Here counterclockwise current will flow to produce upward magnetic field. The current in R is still from b to a.

P31.56 $\mathcal{E} = B\ell v$ at a distance r from wire

$$|\mathcal{E}| = \left(\frac{\mu_0 I}{2\pi r}\right)\ell v$$

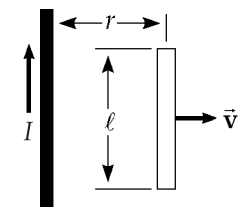


FIG. P31.56

P31.57 $\mathcal{E} = -\dfrac{d}{dt}(NBA) = -1\left(\dfrac{dB}{dt}\right)\pi a^2 = \pi a^2 K$

(a) $Q = C\mathcal{E} = \boxed{C\pi a^2 K}$

(b) $\vec{\mathbf{B}}$ into the paper is decreasing; therefore, current will attempt to counteract this. Positive charge will go to $\boxed{\text{upper plate}}$.

(c) The changing magnetic field through the enclosed area $\boxed{\text{induces an electric field}}$, surrounding the $\vec{\mathbf{B}}$-field, and this pushes on charges in the wire.

***P31.58** (a) We would need to know whether the field is increasing or decreasing.

(b) $\mathcal{P} = \mathcal{E}I = \mathcal{E}^2/R = \left(N\dfrac{dB}{dt}\pi r^2\cos 0°\right)^2 \Big/ R$

$$R = \left(N\frac{dB}{dt}\pi r^2\right)^2 \Big/ \mathcal{P} = \frac{\left[220(0.020\text{ T/s})\pi(0.12\text{ m})^2\right]^2}{160\text{ W}} = \boxed{248\ \mu\Omega}$$

Higher resistance would reduce the power delivered.

P31.59 The flux through the coil is $\Phi_B = \vec{\mathbf{B}}\cdot\vec{\mathbf{A}} = BA\cos\theta = BA\cos\omega t$. The induced emf is

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -NBA\frac{d(\cos\omega t)}{dt} = NBA\omega\sin\omega t$$

(a) $\mathcal{E}_{\max} = NBA\omega = 60.0(1.00\text{ T})(0.100\times 0.200\text{ m}^2)(30.0\text{ rad/s}) = \boxed{36.0\text{ V}}$

(b) $\dfrac{d\Phi_B}{dt} = \dfrac{\mathcal{E}}{N}$, thus $\left|\dfrac{d\Phi_B}{dt}\right|_{\max} = \dfrac{\mathcal{E}_{\max}}{N} = \dfrac{36.0\text{ V}}{60.0} = 0.600\text{ V} = \boxed{0.600\text{ Wb/s}}$

(c) At $t = 0.050\,0$ s, $\omega t = 1.50$ rad and

$$\mathcal{E} = \mathcal{E}_{\max}\sin(1.50\text{ rad}) = (36.0\text{ V})\sin(1.50\text{ rad}) = \boxed{35.9\text{ V}}$$

(d) The torque on the coil at any time is in magnitude

$$\tau = \left|\vec{\mu}\times\vec{\mathbf{B}}\right| = \left|NI\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = (NAB)I|\sin\omega t| = \left(\frac{\mathcal{E}_{\max}}{\omega}\right)\left(\frac{\mathcal{E}}{R}\right)|\sin\omega t|$$

When $\mathcal{E} = \mathcal{E}_{\max}$, $\sin\omega t = 1.00$ and $\tau = \dfrac{\mathcal{E}_{\max}^2}{\omega R} = \dfrac{(36.0\text{ V})^2}{(30.0\text{ rad/s})(10.0\ \Omega)} = \boxed{4.32\text{ N}\cdot\text{m}}$.

P31.60 (a) We use $\mathcal{E} = -N\dfrac{\Delta\Phi_B}{\Delta t}$, with $N = 1$.

Taking $a = 5.00 \times 10^{-3}$ m to be the radius of the washer, and $h = 0.500$ m,

$$\Delta\Phi_B = B_2A - B_1A = A(B_2 - B_1) = \pi a^2\left(\frac{\mu_0 I}{2\pi(h+a)} - \frac{\mu_0 I}{2\pi a}\right)$$

$$= \frac{a^2\mu_0 I}{2}\left(\frac{1}{h+a} - \frac{1}{a}\right) = \frac{-\mu_0 ahI}{2(h+a)}$$

The time for the washer to drop a distance *h* (from rest) is: $\Delta t = \sqrt{\dfrac{2h}{g}}$.

Therefore, $\mathcal{E} = \dfrac{\mu_0 ahI}{2(h+a)\Delta t} = \dfrac{\mu_0 ahI}{2(h+a)}\sqrt{\dfrac{g}{2h}} = \dfrac{\mu_0 aI}{2(h+a)}\sqrt{\dfrac{gh}{2}}$

and $\mathcal{E} = \dfrac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(5.00 \times 10^{-3}\ \text{m})(10.0\ \text{A})}{2(0.500\ \text{m} + 0.005\,00\ \text{m})}\sqrt{\dfrac{(9.80\ \text{m/s}^2)(0.500\ \text{m})}{2}}$

$= \boxed{97.4\ \text{nV}}$

(b) Since the magnetic flux going through the washer (into the plane of the paper) is decreasing in time, a current will form in the washer so as to oppose that decrease. Therefore, the current will flow in a $\boxed{\text{clockwise direction}}$.

P31.61 Find an expression for the flux through a rectangular area "swept out" by the bar in time *t*. The magnetic field at a distance *x* from wire is

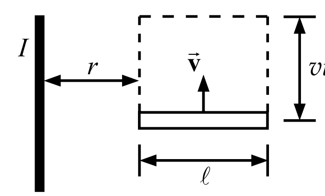


FIG. P31.61

$B = \dfrac{\mu_0 I}{2\pi x}$ and $\Phi_B = \int B\,dA$. Therefore,

$\Phi_B = \dfrac{\mu_0 Ivt}{2\pi}\displaystyle\int_r^{r+\ell}\dfrac{dx}{x}$ where *vt* is the distance the bar has moved in time *t*.

Then, $|\mathcal{E}| = \dfrac{d\Phi_B}{dt} = \boxed{\dfrac{\mu_0 Iv}{2\pi}\ln\left(1 + \dfrac{\ell}{r}\right)}$.

P31.62 The magnetic field at a distance *x* from a long wire is $B = \dfrac{\mu_0 I}{2\pi x}$. We find an expression for the flux through the loop.

$$d\Phi_B = \frac{\mu_0 I}{2\pi x}(\ell dx) \quad \text{so} \quad \Phi_B = \frac{\mu_0 I\ell}{2\pi}\int_r^{r+w}\frac{dx}{x} = \frac{\mu_0 I\ell}{2\pi}\ln\left(1 + \frac{w}{r}\right)$$

Therefore, $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = \dfrac{\mu_0 I\ell v}{2\pi r}\dfrac{w}{(r+w)}$ and $I = \dfrac{\mathcal{E}}{R} = \boxed{\dfrac{\mu_0 I\ell v}{2\pi Rr}\dfrac{w}{(r+w)}}$

P31.63 We are given $\Phi_B = \left(6.00t^3 - 18.0t^2\right)\ \text{T}\cdot\text{m}^2$

and $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -18.0t^2 + 36.0t$

Maximum $\mathcal{E}$ occurs when $\dfrac{d\mathcal{E}}{dt} = -36.0t + 36.0 = 0$

which gives $t = 1.00\ \text{s}$

Therefore, the maximum current (at $t = 1.00\ \text{s}$) is $I = \dfrac{\mathcal{E}}{R} = \dfrac{(-18.0 + 36.0)\ \text{V}}{3.00\ \Omega} = \boxed{6.00\ \text{A}}$

P31.64 For the suspended mass, M: $\sum F = Mg - T = Ma$.

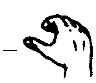
For the sliding bar, m: $\sum F = T - I\ell B = ma$, where $I = \dfrac{\mathcal{E}}{R} = \dfrac{B\ell v}{R}$

$Mg - \dfrac{B^2\ell^2 v}{R} = (m+M)a$ or $a = \dfrac{dv}{dt} = \dfrac{Mg}{m+M} - \dfrac{B^2\ell^2 v}{R(M+m)}$

$\displaystyle\int_0^v \frac{dv}{(\alpha - \beta v)} = \int_0^t dt$ where $\alpha = \dfrac{Mg}{M+m}$ and $\beta = \dfrac{B^2\ell^2}{R(M+m)}$

Therefore, the velocity varies with time as $v = \dfrac{\alpha}{\beta}\left(1 - e^{-\beta t}\right) = \boxed{\dfrac{MgR}{B^2\ell^2}\left[1 - e^{-B^2\ell^2 t/R(M+m)}\right]}$

P31.65 Suppose the field is vertically down. When an electron is moving away from you the force on it is in the direction given by

$q\vec{\mathbf{v}} \times \vec{\mathbf{B}}_c$ as $-(\text{away}) \times \text{down} = -$  $= -\text{left} = \text{right}$.

Therefore, the electrons circulate clockwise.

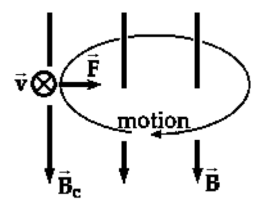


FIG. P31.65

(a) As the downward field increases, an emf is induced to produce some current that in turn produces an upward field. This current is directed  counterclockwise, carried by negative electrons moving clockwise. Therefore the original electron motion speeds up.

(b) At the circumference, we have $\sum F_c = ma_c$: $|q|vB_c \sin 90° = \dfrac{mv^2}{r}$

$mv = |q|rB_c$

The increasing magnetic field $\vec{\mathbf{B}}_{\text{av}}$ in the area enclosed by the orbit produces a tangential electric field according to

$\left|\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}\right| = \left|-\dfrac{d}{dt}\vec{\mathbf{B}}_{av} \cdot \vec{\mathbf{A}}\right| \qquad E(2\pi r) = \pi r^2 \dfrac{dB_{av}}{dt} \qquad E = \dfrac{r}{2}\dfrac{dB_{av}}{dt}$

An electron feels a tangential force according to $\sum F_t = ma_t$: $|q|E = m\dfrac{dv}{dt}$

Then $|q|\dfrac{r}{2}\dfrac{dB_{av}}{dt} = m\dfrac{dv}{dt}$ $\qquad |q|\dfrac{r}{2}B_{av} = mv = |q|rB_c$

and $B_{av} = 2B_c$

***P31.66** (a) The induced emf is $\mathcal{E} = B\ell v$ where $B = \frac{\mu_0 I}{2\pi y}$, $v_f = v_i + gt = (9.80 \text{ m/s}^2)t$, and

$$y_f = y_i - \frac{1}{2}gt^2 = 0.800 \text{ m} - (4.90 \text{ m/s}^2)t^2$$

$$\mathcal{E} = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(200 \text{ A})}{2\pi\left[0.800 \text{ m} - (4.90 \text{ m/s}^2)t^2\right]}(0.300 \text{ m})(9.80 \text{ m/s}^2)t = \boxed{\frac{(1.18\times 10^{-4})t}{\left[0.800 - 4.90t^2\right]} \text{ V}}$$

(b) The emf is zero at $t = 0$.

(c) The emf diverges to infinity at 0.404 s.

(d) At $t = 0.300$ s, $\mathcal{E} = \frac{(1.18\times 10^{-4})(0.300)}{\left[0.800 - 4.90(0.300)^2\right]} \text{ V} = \boxed{98.3\ \mu\text{V}}$.

P31.67 The magnetic field produced by the current in the straight wire is perpendicular to the plane of the coil at all points within the coil. The magnitude of the field is $B = \frac{\mu_0 I}{2\pi r}$. Thus, the flux linkage is

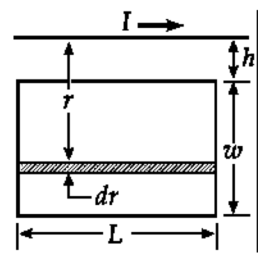


FIG. P31.67

$$N\Phi_B = \frac{\mu_0 NIL}{2\pi}\int_h^{h+w}\frac{dr}{r} = \frac{\mu_0 NI_{max}L}{2\pi}\ln\left(\frac{h+w}{h}\right)\sin(\omega t + \phi)$$

Finally, the induced emf is

$$\mathcal{E} = -\frac{\mu_0 NI_{max}L\omega}{2\pi}\ln\left(1 + \frac{w}{h}\right)\cos(\omega t + \phi)$$

$$\mathcal{E} = -\frac{(4\pi\times 10^{-7})(100)(50.0)(0.200 \text{ m})(200\pi \text{ s}^{-1})}{2\pi}\ln\left(1 + \frac{5.00 \text{ cm}}{5.00 \text{ cm}}\right)\cos(\omega t + \phi)$$

$$\mathcal{E} = \boxed{-(87.1 \text{ mV})\cos(200\pi t + \phi)}$$

The term $\sin(\omega t + \phi)$ in the expression for the current in the straight wire does not change appreciably when ωt changes by 0.10 rad or less. Thus, the current does not change appreciably during a time interval

$$\Delta t < \frac{0.10}{(200\pi \text{ s}^{-1})} = 1.6\times 10^{-4} \text{ s}$$

We define a critical length $c\Delta t = (3.00\times 10^8 \text{ m/s})(1.6\times 10^{-4} \text{ s}) = 4.8\times 10^4$ m equal to the distance to which field changes could be propagated during an interval of 1.6×10^{-4} s. This length is so much larger than any dimension of the coil or its distance from the wire that, although we consider the straight wire to be infinitely long, we can also safely ignore the field propagation effects in the vicinity of the coil. Moreover, the phase angle can be considered to be constant along the wire in the vicinity of the coil.

If the angular frequency ω were much larger, say, $200\pi\times 10^5 \text{ s}^{-1}$, the corresponding critical length would be only 48 cm. In this situation propagation effects would be important and the above expression for $\mathcal{E}$ would require modification. As a general rule we can consider field propagation effects for circuits of laboratory size to be negligible for frequencies, $f = \frac{\omega}{2\pi}$, that are less than about 10^6 Hz.

ANSWERS TO EVEN PROBLEMS

P31.2 0.800 mA

P31.4 (a) If the meter has a sufficiently short response time, it will register the current. The average current may even run the meter offscale by a factor of 4.42, so you might wish to slow down the motion of the coil. (b) Positive. The coil sees decreasing external magnetic flux toward you, so it makes some flux of its own in this direction by carrying counterclockwise current, that enters the red terminal of the ammeter.

P31.6 $-10.2\ \mu\text{V}$

P31.8 (a) $(\mu_0 IL/2\pi)\ln(1 + w/h)$ (b) $-4.80\ \mu\text{V}$; current is counterclockwise

P31.10 (a) $\dfrac{\mu_0 n\pi r_2^2}{2R}\dfrac{\Delta I}{\Delta t}$ counterclockwise (b) $\dfrac{\mu_0^2 n\pi r_2^2}{4r_1R}\dfrac{\Delta I}{\Delta t}$ (c) left

P31.12 (a) 5.33 V (b) A flat compact circular coil with 130 turns and radius 40.0 cm carries current 3.00 A counterclockwise. The current is smoothly reversed to become 3.00 A clockwise after 13.0 μs. At the center of this primary coil is a secondary coil in the same plane, with 20 turns and radius 3.00 cm. Find the emf induced in the secondary.

P31.14 61.8 mV

P31.16 (a) See the solution. (b) If the wire is not centered, the coil will respond to stronger magnetic fields on one side, but to correspondingly weaker fields on the opposite side. The emf induced in the coil is proportional to the line integral of the magnetic field around the circular axis of the toroid. Ampère's law says that this line integral depends only on the amount of current the coil encloses. It does not depend on the shape or location of the current within the coil, or on any currents outside the coil.

P31.18 $(0.422\ \text{V})\cos \omega t$

P31.20 (a) eastward (b) $458\ \mu\text{V}$

P31.22 1.00 m/s

P31.24 (a) 500 mA (b) 2.00 W (c) 2.00 W (d) They are physically identical.

P31.26 (a) 233 Hz (b) 1.98 mV

P31.28 (a) to the right (b) out of the plane of the paper (c) to the right (d) into the paper

P31.30 Negative; see the solution.

P31.32 (a) 8.00×10^{-21} N downward perpendicular to r_1 (b) 1.33 s

P31.34 (a) $(9.87\ \text{mV/m})\cos(100\pi t)$ (b) clockwise

P31.36 (a) $(19.6\ \text{V})\sin(314t)$ (b) 19.6 V

P31.38 See the solution.

P31.40 (a) 8.00 Wb cos(377*t*) (b) 3.02 V sin(377*t*) (c) 3.02 A sin(377*t*) (d) 9.10 W$\sin^2(377t)$
(e) 24.1 mN·m $\sin^2(377t)$

P31.42 (a) $\dfrac{\pi B_{\max}^2 R^4 b\omega^2}{16\rho}$ (b) 4 times larger (c) 4 times larger (d) 16 times larger

P31.44 (a) counterclockwise (b) 100 mA (c) The speed of the bar increases until the backward magnetic force exerted on the current in the bar is equal to 0.6 N. The terminal speed is 281 m/s
(d) 169 W (e) The terminal speed becomes larger. The bar must move faster to generate a larger emf to produce enough current in the larger resistance to feel the 0.6-N magnetic force.
(f) The power delivered to the circuit by the agent moving the bar, and then converted into internal energy by the resistor, is described by $\mathcal{P} = Fv = F^2R/B^2\ell^2$. Thus the power is directly proportional to the resistance.

P31.46 (a) 43.8 A (b) 38.3 W

P31.48 62.3 mA down through 6.00 Ω, 860 mA down through 5.00 Ω, 923 mA up through 3.00 Ω

P31.50 See the solution.

P31.52 (a) See the solution. (b) 0.250 T

P31.54 (a) 254 km/s (b) 215 V

P31.56 See the solution.

P31.58 (a) We would need to know whether the field is increasing or decreasing. (b) 248 $\mu\Omega$. Higher resistance would reduce the power delivered.

P31.60 (a) 97.4 nV (b) clockwise

P31.62 $\dfrac{\mu_0 I\ell v}{2\pi Rr}\dfrac{w}{(r+w)}$

P31.64 $\dfrac{MgR}{B^2\ell^2}\left[1-e^{-B^2\ell^2 t/R(M+m)}\right]$

P31.66 (a) $118t\ \mu\text{V}/(0.8-4.9t^2)$ where *t* is in seconds. (b) The emf is zero at $t = 0$. (c) The emf diverges to infinity at 0.404. (d) 98.3 μV

32

Inductance

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q32.1 The coil has an inductance regardless of the nature of the current in the circuit. Inductance depends only on the coil geometry and its construction. Since the current is constant, the self-induced emf in the coil is zero, and the coil does not affect the steady-state current. (We assume the resistance of the coil is negligible.)

Q32.2 The inductance of a coil is determined by (a) the geometry of the coil and (b) the "contents" of the coil. This is similar to the parameters that determine the capacitance of a capacitor and the resistance of a resistor. With an inductor, the most important factor in the geometry is the number of turns of wire, or turns per unit length. By the "contents" we refer to the material in which the inductor establishes a magnetic field, notably the magnetic properties of the core around which the wire is wrapped.

***Q32.3** The emf across an inductor is zero whenever the current is constant, large or small. Answer (d).

***Q32.4** The fine wire has considerable resistance, so a few seconds is many time constants. The final current is not affected by the inductance of the coil. Answer (c).

***Q32.5** The inductance of a solenoid is proportional to the number of turns squared, to the cross-sectional area, and to the reciprocal of the length. Coil A has twice as many turns with the same length of wire, so its circumference must be half as large as that of coil B. Its radius is half as large and its area one quarter as large. For coil A the inductance will be different by the factor $2^2(1/4)(1/2) = 1/2$. Answer (e).

Q32.6 When it is being opened. When the switch is initially standing open, there is no current in the circuit. Just after the switch is then closed, the inductor tends to maintain the zero-current condition, and there is very little chance of sparking. When the switch is standing closed, there is current in the circuit. When the switch is then opened, the current rapidly decreases. The induced emf is created in the inductor, and this emf tends to maintain the original current. Sparking occurs as the current bridges the air gap between the contacts of the switch.

***Q32.7** Just before the switch is thrown, the voltage across the twelve-ohm resistor is very nearly 12 V. Just after the switch is thrown, the current is nearly the same, maintained by the inductor. The voltage across the 1 200-Ω resistor is then much more than 12 V. By Kirchhoff's loop rule, the voltage across the coil is larger still: $\Delta V_L > \Delta V_{1\,200\,\Omega} > 12.0\text{ V} > \Delta V_{12\,\Omega}$.

***Q32.8** (i) (a) The bulb glows brightly right away, and then more and more faintly as the capacitor charges up. (b) The bulb gradually gets brighter and brighter, changing rapidly at first and then more and more slowly. (c) The bulb gradually gets brighter and brighter. (d) The bulb glows brightly right away, and then more and more faintly as the inductor starts carrying more and more current.

(ii) (a) The bulb goes out immediately. (b) The bulb glows for a moment as a spark jumps across the switch. (c) The bulb stays lit for a while, gradually getting fainter and fainter. (d) The bulb suddenly glows brightly. Then its brightness decreases to zero, changing rapidly at first and then more and more slowly.

***Q32.9** The wire's magnetic field goes in circles around it. We want this field to "shine" perpendicularly through the area of the coil. Answer (c).

Q32.10 A physicist's list of constituents of the universe in 1829 might include matter, light, heat, the stuff of stars, charge, momentum, and several other entries. Our list today might include the quarks, electrons, muons, tauons, and neutrinos of matter; gravitons of gravitational fields; photons of electric and magnetic fields; W and Z particles; gluons; energy; momentum; angular momentum; charge; baryon number; three different lepton numbers; upness; downness; strangeness; charm; topness; and bottomness. Alternatively, the relativistic interconvertibility of mass and energy, and of electric and magnetic fields, can be used to make the list look shorter. Some might think of the conserved quantities energy, momentum, … bottomness as properties of matter, rather than as things with their own existence. The idea of a field is not due to Henry, but rather to Faraday, to whom Henry personally demonstrated self-induction. Still the thesis stated in the question has an important germ of truth. Henry precipitated a basic change if he did not cause it. The biggest difference between the two lists is that the 1829 list does not include fields and today's list does.

***Q32.11** The energy stored in the magnetic field of an inductor is proportional to the square of the current. Doubling I makes $U = \frac{1}{2}LI^2$ get four times larger. Answer (a).

***Q32.12** Cutting the number of turns in half makes the inductance four times smaller. Doubling the current would by itself make the stored energy four times larger, to just compensate. Answer (b).

Q32.13 The energy stored in a capacitor is proportional to the square of the electric field, and the energy stored in an induction coil is proportional to the square of the magnetic field. The capacitor's energy is proportional to its capacitance, which depends on its geometry and the dielectric material inside. The coil's energy is proportional to its inductance, which depends on its geometry and the core material. On the other hand, we can think of Henry's discovery of self-inductance as fundamentally new. Before a certain school vacation at the Albany Academy about 1830, one could visualize the universe as consisting of only one thing, matter. All the forms of energy then known (kinetic, gravitational, elastic, internal, electrical) belonged to chunks of matter. But the energy that temporarily maintains a current in a coil after the battery is removed is not energy that belongs to any bit of matter. This energy is vastly larger than the kinetic energy of the drifting electrons in the wires. This energy belongs to the magnetic field around the coil. Beginning in 1830, Nature has forced us to admit that the universe consists of matter and also of fields, massless and invisible, known only by their effects.

***Q32.14** (a) The instant after the switch is closed, the situation is as shown in the circuit diagram of Figure (a). The requested quantities are:

$$\boxed{I_L = 0,\ I_C = \frac{\mathcal{E}_0}{R},\ I_R = \frac{\mathcal{E}_0}{R}}$$

$$\boxed{\Delta V_L = \mathcal{E}_0,\ \Delta V_C = 0,\ \Delta V_R = \mathcal{E}_0}$$

(b) After the switch has been closed a long time, the steady-state conditions shown in Figure (b) will exist. The currents and voltages are:

$$\boxed{I_L = 0,\ I_C = 0,\ I_R = 0}$$

$$\boxed{\Delta V_L = 0,\ \Delta V_C = \mathcal{E}_0,\ \Delta V_R = 0}$$

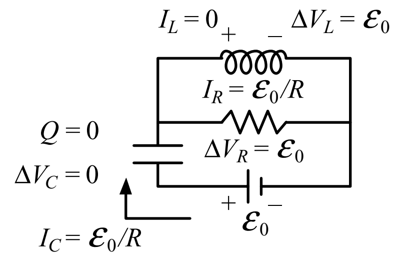


Figure (a)

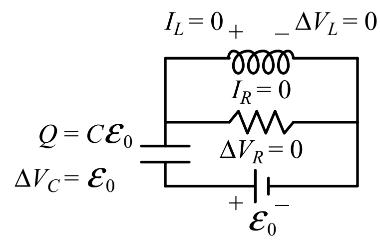


Figure (b)

FIG. Q32.14

***Q32.15** (i) Answer (a). The mutual inductance of two loops in free space—that is, ignoring the use of cores—is a maximum if the loops are coaxial. In this way, the maximum flux of the primary loop will pass through the secondary loop, generating the largest possible emf given the changing magnetic field due to the first.

(ii) Answer (c). The mutual inductance is a minimum if the magnetic field of the first coil lies in the plane of the second coil, producing no flux through the area the second coil encloses.

Q32.16 When the capacitor is fully discharged, the current in the circuit is a maximum. The inductance of the coil is making the current continue to flow. At this time the magnetic field of the coil contains all the energy that was originally stored in the charged capacitor. The current has just finished discharging the capacitor and is proceeding to charge it up again with the opposite polarity.

Q32.17 If $R > \sqrt{\frac{4L}{C}}$, then the oscillator is overdamped—it will not oscillate. If $R < \sqrt{\frac{4L}{C}}$, then the oscillator is underdamped and can go through several cycles of oscillation before the radiated signal falls below background noise.

Q32.18 An object cannot exert a net force on itself. An object cannot create momentum out of nothing. A coil can induce an emf in itself. When it does so, the actual forces acting on charges in different parts of the loop add as vectors to zero. The term electromotive force does not refer to a force, but to a voltage.

SOLUTIONS TO PROBLEMS

Section 32.1 **Self-Induction and Inductance**

P32.1 $$\bar{\mathcal{E}} = -L\frac{\Delta I}{\Delta t} = (-2.00\text{ H})\left(\frac{0 - 0.500\text{ A}}{0.010\,0\text{ s}}\right)\left(\frac{1\text{ V}\cdot\text{s}}{1\text{ H}\cdot\text{A}}\right) = \boxed{100\text{ V}}$$

P32.2 Treating the telephone cord as a solenoid, we have:

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(70.0)^2\,\pi\left(6.50\times10^{-3}\text{ m}\right)^2}{0.600\text{ m}} = \boxed{1.36\ \mu\text{H}}$$

***P32.3** $\mathcal{E} = -L\dfrac{dI}{dt} = -L\dfrac{d}{dt}(I_{\max}\sin\omega t) = -L\omega I_{\max}\cos\omega t = -(10.0\times10^{-3})(120\pi)(5.00)\cos\omega t$

$$\mathcal{E} = -(6.00\pi)\cos(120\pi t) = \boxed{-(18.8\text{ V})\cos(377t)}$$

P32.4 From $|\mathcal{E}| = L\left(\dfrac{\Delta I}{\Delta t}\right)$, we have $L = \dfrac{\mathcal{E}}{(\Delta I/\Delta t)} = \dfrac{24.0\times10^{-3}\text{ V}}{10.0\text{ A/s}} = 2.40\times10^{-3}\text{ H}$

From $L = \dfrac{N\Phi_B}{I}$, we have $\Phi_B = \dfrac{LI}{N} = \dfrac{(2.40\times10^{-3}\text{ H})(4.00\text{ A})}{500} = \boxed{19.2\ \mu\text{T}\cdot\text{m}^2}$

P32.5 $L = \dfrac{\mu_0 N^2 A}{\ell} = \dfrac{\mu_0(420)^2(3.00\times10^{-4})}{0.160} = 4.16\times10^{-4}\text{ H}$

$$\mathcal{E} = -L\frac{dI}{dt} \to \frac{dI}{dt} = \frac{-\mathcal{E}}{L} = \frac{-175\times10^{-6}\text{ V}}{4.16\times10^{-4}\text{ H}} = \boxed{-0.421\text{ A/s}}$$

P32.6 $|\mathcal{E}| = L\dfrac{dI}{dt} = (90.0\times10^{-3})\dfrac{d}{dt}(t^2 - 6t)\text{ V}$

(a) At $t = 1.00$ s, $\mathcal{E} = \boxed{360\text{ mV}}$

(b) At $t = 4.00$ s, $\mathcal{E} = \boxed{180\text{ mV}}$

(c) $\mathcal{E} = (90.0\times10^{-3})(2t - 6) = 0$

when $\boxed{t = 3.00\text{ s}}$

P32.7 (a) $B = \mu_0 nI = \mu_0\left(\dfrac{450}{0.120}\right)(0.040\,0\text{ A}) = \boxed{188\ \mu\text{T}}$

(b) $\Phi_B = BA = \boxed{3.33\times10^{-8}\text{ T}\cdot\text{m}^2}$

(c) $L = \dfrac{N\Phi_B}{I} = \boxed{0.375\text{ mH}}$

(d) $\boxed{B \text{ and } \Phi_B \text{ are proportional to current; } L \text{ is independent of current.}}$

P32.8 $L = \dfrac{N\Phi_B}{I} = \dfrac{NBA}{I} \approx \dfrac{NA}{I}\cdot\dfrac{\mu_0 NI}{2\pi R} = \boxed{\dfrac{\mu_0 N^2 A}{2\pi R}}$

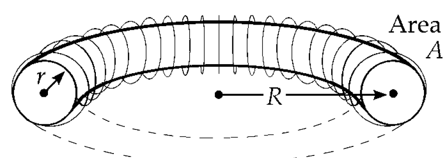


FIG. P32.8

P32.9 $\mathcal{E} = \mathcal{E}_0 e^{-kt} = -L\dfrac{dI}{dt}$

$$dI = -\frac{\mathcal{E}_0}{L}e^{-kt}\,dt$$

If we require $I \to 0$ as $t \to \infty$, the solution is $I = \dfrac{\mathcal{E}_0}{kL}e^{-kt} = \dfrac{dq}{dt}$

$$Q = \int I\,dt = \int_0^\infty \frac{\mathcal{E}_0}{kL}e^{-kt}\,dt = -\frac{\mathcal{E}_0}{k^2 L} \qquad \boxed{|Q| = \frac{\mathcal{E}_0}{k^2 L}}$$

Section 32.2 *RL* Circuits

P32.10 Taking $\tau = \frac{L}{R}$, $I = I_i e^{-t/\tau}$: $\frac{dI}{dt} = I_i e^{-t/\tau}\left(-\frac{1}{\tau}\right)$

$IR + L\frac{dI}{dt} = 0$ will be true if $I_i R\, e^{-t/\tau} + L\left(I_i e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right) = 0$

Because $\tau = \frac{L}{R}$, we have agreement with $0 = 0$.

P32.11 (a) At time t, $I(t) = \frac{\mathcal{E}\left(1 - e^{-t/\tau}\right)}{R}$

where $\tau = \frac{L}{R} = 0.200\text{ s}$

After a long time, $I_{\max} = \frac{\mathcal{E}\left(1 - e^{-\infty}\right)}{R} = \frac{\mathcal{E}}{R}$

At $I(t) = 0.500 I_{\max}$ $(0.500)\frac{\mathcal{E}}{R} = \frac{\mathcal{E}\left(1 - e^{-t/0.200\text{ s}}\right)}{R}$

so $0.500 = 1 - e^{-t/0.200\text{ s}}$

Isolating the constants on the right, $\ln\left(e^{-t/0.200\text{ s}}\right) = \ln(0.500)$

and solving for t, $-\frac{t}{0.200\text{ s}} = -0.693$

or $t = \boxed{0.139\text{ s}}$

(b) Similarly, to reach 90% of $I_{\max}$, $0.900 = 1 - e^{-t/\tau}$

and $t = -\tau\ln(1 - 0.900)$

Thus, $t = -(0.200\text{ s})\ln(0.100) = \boxed{0.461\text{ s}}$

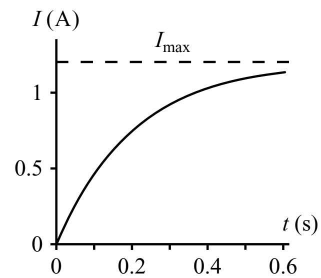


FIG. P32.11

***P32.12** The current increases from 0 to asymptotically approach 500 mA. In case (a) the current jumps up essentially instantaneously. In case (b) it increases with a longer time constant, and in case (c) the increase is still slower.

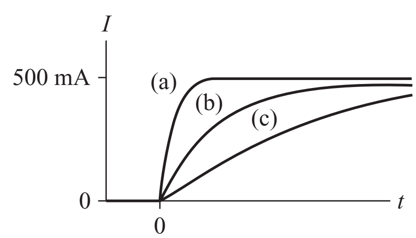


FIG. P32.12

P32.13 (a) $\tau = \frac{L}{R} = 2.00 \times 10^{-3}\text{ s} = \boxed{2.00\text{ ms}}$

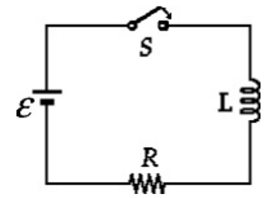


FIG. P32.13

(b) $I = I_{\max}\left(1 - e^{-t/\tau}\right) = \left(\frac{6.00\text{ V}}{4.00\ \Omega}\right)\left(1 - e^{-0.250/2.00}\right) = \boxed{0.176\text{ A}}$

(c) $I_{\max} = \frac{\mathcal{E}}{R} = \frac{6.00\text{ V}}{4.00\ \Omega} = \boxed{1.50\text{ A}}$

(d) $0.800 = 1 - e^{-t/2.00\text{ ms}} \rightarrow t = -(2.00\text{ ms})\ln(0.200) = \boxed{3.22\text{ ms}}$

P32.14 $I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) = \frac{120}{9.00}\left(1 - e^{-1.80/7.00}\right) = 3.02\text{ A}$

$\Delta V_R = IR = (3.02)(9.00) = 27.2\text{ V}$

$\Delta V_L = \mathcal{E} - \Delta V_R = 120 - 27.2 = \boxed{92.8\text{ V}}$

P32.15 (a) $\Delta V_R = IR = (8.00\ \Omega)(2.00\text{ A}) = 16.0\text{ V}$

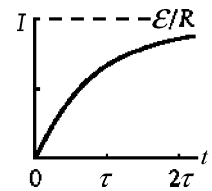


FIG. P32.15

and $\Delta V_L = \mathcal{E} - \Delta V_R = 36.0\text{ V} - 16.0\text{ V} = 20.0\text{ V}$

Therefore, $\frac{\Delta V_R}{\Delta V_L} = \frac{16.0\text{ V}}{20.0\text{ V}} = \boxed{0.800}$

(b) $\Delta V_R = IR = (4.50\text{ A})(8.00\ \Omega) = 36.0\text{ V}$

$\Delta V_L = \mathcal{E} - \Delta V_R = \boxed{0}$

P32.16 After a long time, $12.0\text{ V} = (0.200\text{ A})R$. Thus, $R = 60.0\ \Omega$. Now, $\tau = \frac{L}{R}$ gives

$L = \tau R = \left(5.00 \times 10^{-4}\text{ s}\right)(60.0\text{ V/A}) = \boxed{30.0\text{ mH}}$

P32.17 $I = I_{\max}\left(1 - e^{-t/\tau}\right)$: $\quad \frac{dI}{dt} = -I_{\max}\left(e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right)$

$\tau = \frac{L}{R} = \frac{15.0\text{ H}}{30.0\ \Omega} = 0.500\text{ s}$: $\quad \frac{dI}{dt} = \frac{R}{L} I_{\max} e^{-t/\tau}$ and $I_{\max} = \frac{\mathcal{E}}{R}$

(a) $t = 0$: $\frac{dI}{dt} = \frac{R}{L} I_{\max} e^0 = \frac{\mathcal{E}}{L} = \frac{100\text{ V}}{15.0\text{ H}} = \boxed{6.67\text{ A/s}}$

(b) $t = 1.50\text{ s}$: $\frac{dI}{dt} = \frac{\mathcal{E}}{L} e^{-t/\tau} = (6.67\text{ A/s}) e^{-1.50/(0.500)} = (6.67\text{ A/s}) e^{-3.00} = \boxed{0.332\text{ A/s}}$

P32.18 Name the currents as shown. By Kirchhoff's laws:

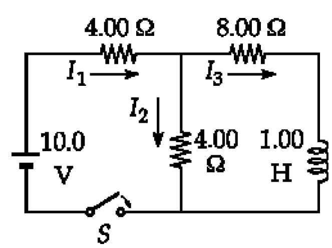


FIG. P32.18

$I_1 = I_2 + I_3$ (1)

$+10.0\text{ V} - 4.00 I_1 - 4.00 I_2 = 0$ (2)

$+10.0\text{ V} - 4.00 I_1 - 8.00 I_3 - (1.00)\frac{dI_3}{dt} = 0$ (3)

continued on next page

From (1) and (2), $+10.0-4.00I_1-4.00I_1+4.00I_3=0$

and $I_1=0.500I_3+1.25\text{ A}$

Then (3) becomes $10.0\text{ V}-4.00\left(0.500I_3+1.25\text{ A}\right)-8.00I_3-(1.00)\dfrac{dI_3}{dt}=0$

$$(1.00\text{ H})\left(\frac{dI_3}{dt}\right)+(10.0\ \Omega)I_3=5.00\text{ V}$$

We solve the differential equation using equations from the chapter text:

$$I_3(t)=\left(\frac{5.00\text{ V}}{10.0\ \Omega}\right)\left[1-e^{-(10.0\ \Omega)t/1.00\text{ H}}\right]=\boxed{(0.500\text{ A})\left[1-e^{-10t/\text{s}}\right]}$$

$$I_1=1.25+0.500I_3=\boxed{1.50\text{ A}-(0.250\text{ A})e^{-10t/\text{s}}}$$

P32.19 (a) Using $\tau=RC=\dfrac{L}{R}$, we get $R=\sqrt{\dfrac{L}{C}}=\sqrt{\dfrac{3.00\text{ H}}{3.00\times10^{-6}\text{ F}}}=1.00\times10^{3}\ \Omega=\boxed{1.00\text{ k}\Omega}$.

(b) $\tau=RC=\left(1.00\times10^{3}\ \Omega\right)\left(3.00\times10^{-6}\text{ F}\right)=3.00\times10^{-3}\text{ s}=\boxed{3.00\text{ ms}}$

P32.20 For $t\le0$, the current in the inductor is $\boxed{\text{zero}}$. At $t=0$, it starts to grow from zero toward 10.0 A with time constant

$$\tau=\frac{L}{R}=\frac{(10.0\text{ mH})}{(100\ \Omega)}=1.00\times10^{-4}\text{ s}$$

For $0\le t\le200\,\mu\text{s}$, $I=I_{\max}\left(1-e^{-t/\tau}\right)=\boxed{(10.0\text{ A})\left(1-e^{-10\,000t/\text{s}}\right)}$.

At $t=200\ \mu\text{s}$, $I=(10.00\text{ A})\left(1-e^{-2.00}\right)=8.65\text{ A}$.

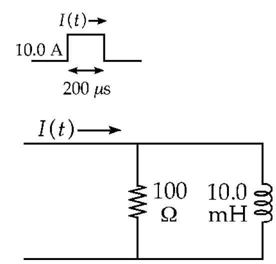


FIG. P32.20

Thereafter, it decays exponentially as $I=I_ie^{-t'/\tau}$, so for $t\ge200\ \mu\text{s}$,

$$I=(8.65\text{ A})e^{-10\,000(t-200\ \mu\text{s})/\text{s}}=(8.65\text{ A})e^{-10\,000t/\text{s}+2.00}=\left(8.65e^{2.00}\text{ A}\right)e^{-10\,000t/\text{s}}=\boxed{(63.9\text{ A})e^{-10\,000t/\text{s}}}$$

P32.21 $\tau=\dfrac{L}{R}=\dfrac{0.140}{4.90}=28.6\text{ ms}$

$$I_{\max}=\frac{\mathcal{E}}{R}=\frac{6.00\text{ V}}{4.90\ \Omega}=1.22\text{ A}$$

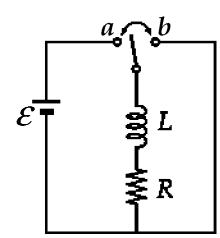


FIG. P32.21

(a) $I=I_{\max}\left(1-e^{-t/\tau}\right)$ so $0.220=1.22\left(1-e^{-t/\tau}\right)$

$e^{-t/\tau}=0.820$: $t=-\tau\ln(0.820)=\boxed{5.66\text{ ms}}$

(b) $I=I_{\max}\left(1-e^{-10.0/0.028\,6}\right)=(1.22\text{ A})\left(1-e^{-350}\right)=\boxed{1.22\text{ A}}$

(c) $I=I_{\max}e^{-t/\tau}$ and $0.160=1.22e^{-t/\tau}$

so $t=-\tau\ln(0.131)=\boxed{58.1\text{ ms}}$

P32.22 (a) For a series connection, both inductors carry equal currents at every instant, so $\frac{dI}{dt}$ is the same for both. The voltage across the pair is

$$L_{eq}\frac{dI}{dt} = L_1\frac{dI}{dt} + L_2\frac{dI}{dt} \qquad \text{so} \qquad \boxed{L_{eq} = L_1 + L_2}$$

(b) $L_{eq}\frac{dI}{dt} = L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt} = \Delta V_L$ where $I = I_1 + I_2$ and $\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$

Thus, $\frac{\Delta V_L}{L_{eq}} = \frac{\Delta V_L}{L_1} + \frac{\Delta V_L}{L_2}$ and $\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}}$

(c) $L_{eq}\frac{dI}{dt} + R_{eq}I = L_1\frac{dI}{dt} + IR_1 + L_2\frac{dI}{dt} + IR_2$

Now I and $\frac{dI}{dt}$ are separate quantities under our control, so functional equality requires both $\boxed{L_{eq} = L_1 + L_2 \quad \text{and} \quad R_{eq} = R_1 + R_2}$.

(d) $\Delta V = L_{eq}\frac{dI}{dt} + R_{eq}I = L_1\frac{dI_1}{dt} + R_1 I_1 = L_2\frac{dI_2}{dt} + R_2 I_2$ where $I = I_1 + I_2$ and $\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$.

We may choose to keep the currents constant in time. Then, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

We may choose to make the current swing through 0. Then, $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$

This equivalent coil with resistance will be equivalent to the pair of real inductors for all other currents as well.

Section 32.3 Energy in a Magnetic Field

P32.23 $L = \mu_0 \frac{N^2 A}{\ell} = \mu_0 \frac{(68.0)^2\left[\pi\left(0.600\times10^{-2}\right)^2\right]}{0.080\,0} = 8.21\ \mu\text{H}$

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\left(8.21\times10^{-6}\ \text{H}\right)\left(0.770\ \text{A}\right)^2 = \boxed{2.44\ \mu\text{J}}$$

P32.24 (a) The magnetic energy density is given by

$$u = \frac{B^2}{2\mu_0} = \frac{(4.50\ \text{T})^2}{2\left(1.26\times10^{-6}\ \text{T}\cdot\text{m/A}\right)} = \boxed{8.06\times10^6\ \text{J/m}^3}$$

(b) The magnetic energy stored in the field equals u times the volume of the solenoid (the volume in which B is non-zero).

$$U = uV = \left(8.06\times10^6\ \text{J/m}^3\right)\left[(0.260\ \text{m})\pi(0.031\,0\ \text{m})^2\right] = \boxed{6.32\ \text{kJ}}$$

P32.25 $u = \epsilon_0\frac{E^2}{2} = \boxed{44.2\ \text{nJ/m}^3}$ $\qquad u = \frac{B^2}{2\mu_0} = \boxed{995\ \mu\text{J/m}^3}$

P32.26 $\int_0^\infty e^{-2Rt/L}dt = -\frac{L}{2R}\int_0^\infty e^{-2Rt/L}\left(\frac{-2Rdt}{L}\right) = -\frac{L}{2R}e^{-2Rt/L}\Big|_0^\infty = -\frac{L}{2R}\left(e^{-\infty}-e^{0}\right) = \frac{L}{2R}(0-1) = \boxed{\frac{L}{2R}}$

***P32.27** (a) $\mathcal{P} = I\Delta V = 3\text{A } 22\text{ V} = \boxed{66.0\text{ W}}$

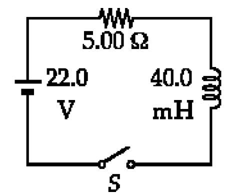


FIG. P32.27

(b) $\mathcal{P} = I\Delta V_R = I^2R = (3\text{A})^2\, 5\ \Omega = \boxed{45.0\text{ W}}$

(c) When the current is 3.00 A, Kirchhoff's loop rule reads
$+22.0\text{ V} - (3.00\text{ A})(5.00\ \Omega) - \Delta V_L = 0.$

Then $\Delta V_L = 7.00\text{ V}$

The power being stored in the inductor is

$$I\Delta V_L = (3.00\text{ A})(7.00\text{ V}) = \boxed{21.0\text{ W}}$$

(d) At all instants after the connection is made, the battery power is equal to the sum of the power delivered to the resistor and the power delivered to the magnetic field. Just after $t = 0$ the resistor power is nearly zero, and the battery power is nearly all going into the magnetic field. Long after the connection is made, the magnetic field is absorbing no more power and the battery power is going into the resistor.

P32.28 From the equation derived in the text, $I = \frac{\mathcal{E}}{R}\left(1-e^{-Rt/L}\right)$

(a) The maximum current, after a long time t, is $I = \frac{\mathcal{E}}{R} = 2.00\text{ A}$

At that time, the inductor is fully energized and $\mathcal{P} = I(\Delta V) = (2.00\text{ A})(10.0\text{ V}) = \boxed{20.0\text{ W}}$.

(b) $\mathcal{P}_{\text{lost}} = I^2R = (2.00\text{ A})^2(5.00\ \Omega) = \boxed{20.0\text{ W}}$

(c) $\mathcal{P}_{\text{inductor}} = I\left(\Delta V_{\text{drop}}\right) = \boxed{0}$

(d) $U = \frac{LI^2}{2} = \frac{(10.0\text{ H})(2.00\text{ A})^2}{2} = \boxed{20.0\text{ J}}$

P32.29 The total magnetic energy is the volume integral of the energy density, $u = \frac{B^2}{2\mu_0}$.

Because B changes with position, u is not constant. For $B = B_0\left(\frac{R}{r}\right)^2$, $u = \left(\frac{B_0^2}{2\mu_0}\right)\left(\frac{R}{r}\right)^4$.

Next, we set up an expression for the magnetic energy in a spherical shell of radius r and thickness dr. Such a shell has a volume $4\pi r^2 dr$, so the energy stored in it is

$$dU = u\left(4\pi r^2 dr\right) = \left(\frac{2\pi B_0^2R^4}{\mu_0}\right)\frac{dr}{r^2}$$

We integrate this expression for $r = R$ to $r = \infty$ to obtain the total magnetic energy outside the sphere. This gives

$$U = \boxed{\frac{2\pi B_0^2R^3}{\mu_0}} = \frac{2\pi\left(5.00\times10^{-5}\text{ T}\right)^2\left(6.00\times10^{6}\text{ m}\right)^3}{\left(1.26\times10^{-6}\text{ T}\cdot\text{m/A}\right)} = \boxed{2.70\times10^{18}\text{ J}}$$

Section 32.4 **Mutual Inductance**

P32.30 $I_1(t) = I_{\max} e^{-\alpha t} \sin \omega t$ with $I_{\max} = 5.00$ A, $\alpha = 0.025\,0\ \text{s}^{-1}$, and $\omega = 377$ rad/s

$$\frac{dI_1}{dt} = I_{\max} e^{-\alpha t}(-\alpha \sin \omega t + \omega \cos \omega t)$$

At $t = 0.800$ s,
$$\frac{dI_1}{dt} = (5.00\ \text{A/s})e^{-0.020\,0}\left[-(0.025\,0)\sin(0.800(377)) + 377\cos(0.800(377))\right]$$

$$\frac{dI_1}{dt} = 1.85 \times 10^3\ \text{A/s}$$

Thus, $\mathcal{E}_2 = -M\dfrac{dI_1}{dt}$: $\quad M = \dfrac{-\mathcal{E}_2}{dI_1/dt} = \dfrac{+3.20\ \text{V}}{1.85 \times 10^3\ \text{A/s}} = \boxed{1.73\ \text{mH}}$

P32.31 $\mathcal{E}_2 = -M\dfrac{dI_1}{dt} = -(1.00 \times 10^{-4}\ \text{H})(1.00 \times 10^4\ \text{A/s})\cos(1\,000t)$

$(\mathcal{E}_2)_{\max} = \boxed{1.00\ \text{V}}$

P32.32 Assume the long wire carries current *I*. Then the magnitude of the magnetic field it generates at distance *x* from the wire is $B = \dfrac{\mu_0 I}{2\pi x}$, and this field passes perpendicularly through the plane of the loop. The flux through the loop is

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int B\,dA = \int B(\ell\,dx) = \frac{\mu_0 I \ell}{2\pi}\int_{0.400\ \text{mm}}^{1.70\ \text{mm}} \frac{dx}{x} = \frac{\mu_0 I \ell}{2\pi}\ln\left(\frac{1.70}{0.400}\right)$$

The mutual inductance between the wire and the loop is then

$$M = \frac{N_2 \Phi_{12}}{I_1} = \frac{N_2 \mu_0 I \ell}{2\pi I}\ln\left(\frac{1.70}{0.400}\right) = \frac{N_2 \mu_0 \ell}{2\pi}(1.45) = \frac{1(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(2.70 \times 10^{-3}\ \text{m})}{2\pi}(1.45)$$

$M = 7.81 \times 10^{-10}\ \text{H} = \boxed{781\ \text{pH}}$

P32.33 (a) $M = \dfrac{N_B \Phi_{BA}}{I_A} = \dfrac{700(90.0 \times 10^{-6})}{3.50} = \boxed{18.0\ \text{mH}}$

(b) $L_A = \dfrac{\Phi_A}{I_A} = \dfrac{400(300 \times 10^{-6})}{3.50} = \boxed{34.3\ \text{mH}}$

(c) $\mathcal{E}_B = -M\dfrac{dI_A}{dt} = -(18.0\ \text{mH})(0.500\ \text{A/s}) = \boxed{-9.00\ \text{mV}}$

P32.34 (a) Solenoid 1 creates nearly uniform field everywhere inside it, given by $\mu_0 N_1 I/\ell$
The flux though one turn of solenoid 2 is $\mu_0 \pi R_2^2 N_1 I/\ell$
The emf induced in solenoid 2 is $-(\mu_0 \pi R_2^2 N_1 N_2/\ell)(dI/dt)$
The mutual inductance is $\mu_0 \pi R_2^2 N_1 N_2/\ell$

(b) Solenoid 2 creates nearly uniform field everywhere inside it, given by $\mu_0 N_2 I/\ell$ and nearly zero field outside.
The flux though one turn of solenoid 1 is $\mu_0 \pi R_2^2 N_2 I/\ell$
The emf induced in solenoid 1 is $-(\mu_0 \pi R_2^2 N_1 N_2/\ell)(dI/dt)$
The mutual inductance is $\mu_0 \pi R_2^2 N_1 N_2/\ell$

(c) The mutual inductances are the same. This is one example of von Neumann's rule, mentioned in the next problem.

P32.35 The large coil produces this field at the center of the small coil: $\dfrac{N_1\mu_0 I_1 R_1^2}{2\left(x^2+R_1^2\right)^{3/2}}$. The field is normal to the area of the small coil and nearly uniform over this area, so it produces flux

$\Phi_{12} = \dfrac{N_1\mu_0 I_1 R_1^2}{2\left(x^2+R_1^2\right)^{3/2}}\pi R_2^2$ through the face area of the small coil. When current I_1 varies, this is the emf induced in the small coil:

$$\mathcal{E}_2 = -N_2\frac{d}{dt}\frac{N_1\mu_0 R_1^2 \pi R_2^2}{2\left(x^2+R_1^2\right)^{3/2}} I_1 = -\frac{N_1 N_2 \pi\mu_0 R_1^2 R_2^2}{2\left(x^2+R_1^2\right)^{3/2}}\frac{dI_1}{dt} = -M\frac{dI_1}{dt} \quad \text{so} \quad \boxed{M = \frac{N_1 N_2 \pi\mu_0 R_1^2 R_2^2}{2\left(x^2+R_1^2\right)^{3/2}}}$$

P32.36 With $I = I_1 + I_2$, the voltage across the pair is:

$$\Delta V = -L_1\frac{dI_1}{dt} - M\frac{dI_2}{dt} = -L_2\frac{dI_2}{dt} - M\frac{dI_1}{dt} = -L_{eq}\frac{dI}{dt}$$

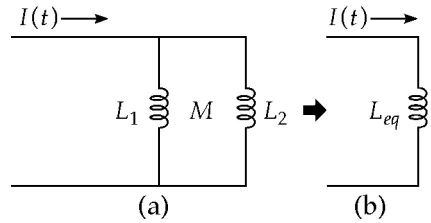


FIG. P32.36

So, $$-\frac{dI_1}{dt} = \frac{\Delta V}{L_1} + \frac{M}{L_1}\frac{dI_2}{dt}$$

and $$-L_2\frac{dI_2}{dt} + \frac{M(\Delta V)}{L_1} + \frac{M^2}{L_1}\frac{dI_2}{dt} = \Delta V$$

$$\left(-L_1L_2 + M^2\right)\frac{dI_2}{dt} = \Delta V\left(L_1 - M\right)$$

By substitution, $$-\frac{dI_2}{dt} = \frac{\Delta V}{L_2} + \frac{M}{L_2}\frac{dI_1}{dt}$$

leads to $$\left(-L_1L_2 + M^2\right)\frac{dI_1}{dt} = \Delta V\left(L_2 - M\right)$$

Adding [1] to [2], $$\left(-L_1L_2 + M^2\right)\frac{dI}{dt} = \Delta V\left(L_1 + L_2 - 2M\right)$$

So, $$L_{eq} = -\frac{\Delta V}{dI/dt} = \boxed{\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}}$$

Section 32.5 **Oscillations in an *LC* Circuit**

P32.37 At different times, $(U_C)_{max} = (U_L)_{max}$ so $\left[\frac{1}{2}C(\Delta V)^2\right]_{max} = \left(\frac{1}{2}LI^2\right)_{max}$

$$I_{max} = \sqrt{\frac{C}{L}}(\Delta V)_{max} = \sqrt{\frac{1.00\times 10^{-6}\text{ F}}{10.0\times 10^{-3}\text{ H}}}(40.0\text{ V}) = \boxed{0.400\text{ A}}$$

P32.38 $\left[\frac{1}{2}C(\Delta V)^2\right]_{max} = \left(\frac{1}{2}LI^2\right)_{max}$ so $(\Delta V_C)_{max} = \sqrt{\frac{L}{C}}I_{max} = \sqrt{\frac{20.0\times 10^{-3}\text{ H}}{0.500\times 10^{-6}\text{ F}}}(0.100\text{ A}) = \boxed{20.0\text{ V}}$

P32.39 When the switch has been closed for a long time, battery, resistor, and coil carry constant current $I_{\max} = \dfrac{\mathcal{E}}{R}$. When the switch is opened, current in battery and resistor drops to zero, but the coil carries this same current for a moment as oscillations begin in the LC loop.

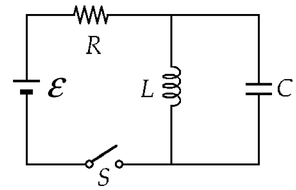


FIG. P32.39

We interpret the problem to mean that the voltage amplitude of these oscillations is ΔV, in $\dfrac{1}{2}C(\Delta V)^2 = \dfrac{1}{2}LI_{\max}^2$.

Then, $L = \dfrac{C(\Delta V)^2}{I_{\max}^2} = \dfrac{C(\Delta V)^2 R^2}{\mathcal{E}^2} = \dfrac{(0.500\times10^{-6}\text{ F})(150\text{ V})^2(250\ \Omega)^2}{(50.0\text{ V})^2} = \boxed{0.281\text{ H}}$.

P32.40 (a) $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(0.082\,0\text{ H})(17.0\times10^{-6}\text{ F})}} = \boxed{135\text{ Hz}}$

(b) $Q = Q_{\max}\cos\omega t = (180\ \mu\text{C})\cos(847\times0.001\,00) = \boxed{119\ \mu\text{C}}$

(c) $I = \dfrac{dQ}{dt} = -\omega Q_{\max}\sin\omega t = -(847)(180)\sin(0.847) = \boxed{-114\text{ mA}}$

P32.41 This radio is a radiotelephone on a ship, according to frequency assignments made by international treaties, laws, and decisions of the National Telecommunications and Information Administration.

The resonance frequency is $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

Thus, $C = \dfrac{1}{(2\pi f_0)^2 L} = \dfrac{1}{\left[2\pi(6.30\times10^6\text{ Hz})\right]^2(1.05\times10^{-6}\text{ H})} = \boxed{608\text{ pF}}$

P32.42 (a) $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(0.100\text{ H})(1.00\times10^{-6}\text{ F})}} = \boxed{503\text{ Hz}}$

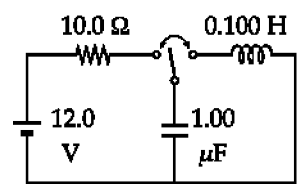


FIG. P32.42

(b) $Q = C\mathcal{E} = (1.00\times10^{-6}\text{ F})(12.0\text{ V}) = \boxed{12.0\ \mu\text{C}}$

(c) $\dfrac{1}{2}C\mathcal{E}^2 = \dfrac{1}{2}LI_{\max}^2$

$$I_{\max} = \mathcal{E}\sqrt{\frac{C}{L}} = 12\text{ V}\sqrt{\frac{1.00\times10^{-6}\text{ F}}{0.100\text{ H}}} = \boxed{37.9\text{ mA}}$$

(d) At all times $U = \dfrac{1}{2}C\mathcal{E}^2 = \dfrac{1}{2}(1.00\times10^{-6}\text{ F})(12.0\text{ V})^2 = \boxed{72.0\ \mu\text{J}}$

P32.43 $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(3.30\text{ H})(840\times10^{-12}\text{ F})}} = 1.899\times10^4\text{ rad/s}$

$Q = Q_{\max}\cos\omega t,\quad I = \dfrac{dQ}{dt} = -\omega Q_{\max}\sin\omega t$

(a) $U_C = \dfrac{Q^2}{2C} = \dfrac{\left(\left[105\times10^{-6}\right]\cos\left[(1.899\times10^4\text{ rad/s})(2.00\times10^{-3}\text{ s})\right]\right)^2}{2(840\times10^{-12})} = \boxed{6.03\text{ J}}$

continued on next page

(b) $U_L = \frac{1}{2}LI^2 = \frac{1}{2}L\omega^2 Q_{\max}^2 \sin^2(\omega t) = \frac{Q_{\max}^2 \sin^2(\omega t)}{2C}$

$$U_L = \frac{\left(105\times10^{-6}\text{ C}\right)^2 \sin^2\left[\left(1.899\times10^4\text{ rad/s}\right)\left(2.00\times10^{-3}\text{ s}\right)\right]}{2\left(840\times10^{-12}\text{ F}\right)} = \boxed{0.529\text{ J}}$$

(c) $U_{\text{total}} = U_C + U_L = \boxed{6.56\text{ J}}$

Section 32.6 **The *RLC* Circuit**

P32.44 (a) $\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \sqrt{\frac{1}{\left(2.20\times10^{-3}\right)\left(1.80\times10^{-6}\right)} - \left(\frac{7.60}{2\left(2.20\times10^{-3}\right)}\right)^2} = 1.58\times10^4\text{ rad/s}$

Therefore, $f_d = \frac{\omega_d}{2\pi} = \boxed{2.51\text{ kHz}}$

(b) $R_c = \sqrt{\frac{4L}{C}} = \boxed{69.9\ \Omega}$

P32.45 (a) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.500)\left(0.100\times10^{-6}\right)}} = \boxed{4.47\text{ krad/s}}$

(b) $\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} = \boxed{4.36\text{ krad/s}}$

(c) $\frac{\Delta\omega}{\omega_0} = \boxed{2.53\%\text{ lower}}$

P32.46 Choose to call positive current clockwise in Figure 32.15. It drains charge from the capacitor according to $I = -\frac{dQ}{dt}$. A clockwise trip around the circuit then gives

$$+\frac{Q}{C} - IR - L\frac{dI}{dt} = 0$$

$$+\frac{Q}{C} + \frac{dQ}{dt}R + L\frac{d}{dt}\frac{dQ}{dt} = 0,$$ identical with Equation 32.28.

P32.47 (a) $Q = Q_{\max}e^{-Rt/2L}\cos\omega_d t$ so $I_{\max} \propto e^{-Rt/2L}$

$0.500 = e^{-Rt/2L}$ and $\frac{Rt}{2L} = -\ln(0.500)$

$$t = -\frac{2L}{R}\ln(0.500) = \boxed{0.693\left(\frac{2L}{R}\right)}$$

(b) $U_0 \propto Q_{\max}^2$ and $U = 0.500U_0$ so $Q = \sqrt{0.500}Q_{\max} = 0.707Q_{\max}$

$$t = -\frac{2L}{R}\ln(0.707) = \boxed{0.347\left(\frac{2L}{R}\right)}$$ (half as long)

Additional Problems

P32.48 (a) Let Q represent the magnitude of the opposite charges on the plates of a parallel plate capacitor, the two plates having area A and separation d. The negative plate creates electric field $\vec{\mathbf{E}} = \dfrac{Q}{2\epsilon_0 A}$ toward itself. It exerts on the positive plate force $\vec{\mathbf{F}} = \dfrac{Q^2}{2\epsilon_0 A}$ toward the negative plate. The total field between the plates is $\dfrac{Q}{\epsilon_0 A}$. The energy density is

$u_E = \dfrac{1}{2}\epsilon_0 E^2 = \dfrac{1}{2}\epsilon_0 \dfrac{Q^2}{\epsilon_0^2 A^2} = \dfrac{Q^2}{2\epsilon_0 A^2}$. Modeling this as a negative or inward pressure,

we have for the force on one plate $F = PA = \dfrac{Q^2}{2\epsilon_0 A^2}$, in agreement with our first analysis.

(b) The lower of the two current sheets shown creates above it magnetic field $\vec{\mathbf{B}} = \dfrac{\mu_0 J_s}{2}\left(-\hat{\mathbf{k}}\right)$. Let ℓ and w represent the length and width of each sheet. The upper sheet carries current $J_s w$ and feels force

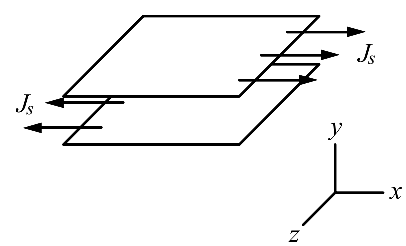


FIG. P32.48(b)

$$\vec{\mathbf{F}} = I\vec{\ell}\times\vec{\mathbf{B}} = J_s w\ell \frac{\mu_0 J_s}{2}\hat{\mathbf{i}}\times\left(-\hat{\mathbf{k}}\right) = \frac{\mu_0 w\ell J_s^2}{2}\hat{\mathbf{j}}.$$

The force per area is $P = \dfrac{F}{\ell w} = \boxed{\dfrac{\mu_0 J_s^2}{2}}$.

(c) Between the two sheets the total magnetic field is $\dfrac{\mu_0 J_s}{2}\left(-\hat{\mathbf{k}}\right) + \dfrac{\mu_0 J_s}{2}\left(-\hat{\mathbf{k}}\right) = \mu_0 J_s \hat{\mathbf{k}}$, with magnitude $\boxed{B = \mu_0 J_s}$. Outside the space they enclose, the fields of the separate sheets are in opposite directions and add to $\boxed{\text{zero}}$.

(d) $u_B = \dfrac{1}{2\mu_0}B^2 = \dfrac{\mu_0^2 J_s^2}{2\mu_0} = \boxed{\dfrac{\mu_0 J_s^2}{2}}$

(e) This energy density agrees with the magnetic pressure found in part (b).

P32.49 (a) $\mathcal{E}_L = -L\dfrac{dI}{dt} = -(1.00\text{ mH})\dfrac{d(20.0t)}{dt} = \boxed{-20.0\text{ mV}}$

(b) $Q = \displaystyle\int_0^t I dt = \int_0^t (20.0t)\,dt = 10.0t^2$

$$\Delta V_C = \frac{-Q}{C} = \frac{-10.0t^2}{1.00\times 10^{-6}\text{ F}} = \boxed{-\left(10.0\text{ MV/s}^2\right)t^2}$$

(c) When $\dfrac{Q^2}{2C} \geq \dfrac{1}{2}LI^2$, or $\dfrac{\left(-10.0t^2\right)^2}{2\left(1.00\times 10^{-6}\right)} \geq \dfrac{1}{2}\left(1.00\times 10^{-3}\right)(20.0t)^2$, then

$100t^4 \geq \left(400\times 10^{-9}\right)t^2$. The earliest time this is true is at $t = \sqrt{4.00\times 10^{-9}}\text{ s} = \boxed{63.2\ \mu\text{s}}$.

P32.50 (a) $\mathcal{E}_L = -L\frac{dI}{dt} = -L\frac{d}{dt}(Kt) = \boxed{-LK}$

(b) $I = \frac{dQ}{dt}$, so $Q = \int_0^t I dt = \int_0^t Kt dt = \frac{1}{2}Kt^2$

and $\Delta V_C = \frac{-Q}{C} = \boxed{-\frac{Kt^2}{2C}}$

(c) When $\frac{1}{2}C(\Delta V_C)^2 = \frac{1}{2}LI^2$, $\frac{1}{2}C\left(\frac{K^2t^4}{4C^2}\right) = \frac{1}{2}L(K^2t^2)$

Thus $t = \boxed{2\sqrt{LC}}$

P32.51 $\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2C}\left(\frac{Q}{2}\right)^2 + \frac{1}{2}LI^2$ so $I = \sqrt{\frac{3Q^2}{4CL}}$

The flux through each turn of the coil is $\Phi_B = \frac{LI}{N} = \boxed{\frac{Q}{2N}\sqrt{\frac{3L}{C}}}$

where N is the number of turns.

***P32.52** (a) The inductor has no voltage across it. It behaves as a $\boxed{\text{short circuit}}$. The battery sees equivalent resistance $4\ \Omega + (1/4\ \Omega + 1/8\ \Omega)^{-1} = 6.67\ \Omega$. The battery current is $10\ \text{V}/6.67\ \Omega = 1.50\ \text{A}$. The voltage across the parallel combination of resistors is $10\ \text{V} - 1.50\ \text{A}\ 4\ \Omega = 4\ \text{V}$. The current in the 8-Ω resistor and the inductor is $4\text{V}/8\Omega = \boxed{500\ \text{mA}}$.

(b) $U = (1/2)\,LI^2 = (1/2)\ 1\ \text{H}(0.5\ \text{A})^2 = \boxed{125\ \text{mJ}}$

(c) The energy becomes 125 mJ of additional internal energy in the 8-Ω resistor and the 4-Ω resistor in the middle branch.

(d) The current decreases from 500 mA toward zero, showing exponential decay with a time constant of $L/R = 1\ \text{H}/12\ \Omega = 83.3\ \text{ms}$.

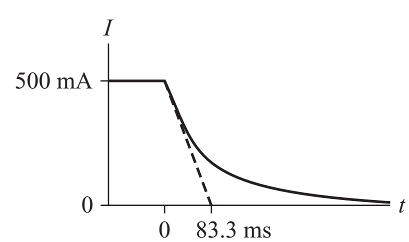


FIG. P32.52(d)

***P32.53** (a) Just after the circuit is connected, the potential difference across the resistor is 0 and the emf across the coil is 24.0 V.

(b) After several seconds, the potential difference across the resistor is 24.0 V and that across the coil is 0.

(c) The resistor voltage and inductor voltage always add to 24 V. The resistor voltage increases monotonically, so the two voltages are equal to each other, both being 12.0 V, just once. The time is given by $12\ \text{V} = IR = R\mathcal{E}/R(1 - e^{-Rt/L}) = 24\ \text{V}(1 - e^{-6\Omega t/0.005\ \text{H}})$.

This is $0.5 = e^{-1\,200\,t}$ or $1\,200\,t = \ln 2$ giving $t = \boxed{0.578\ \text{ms after the circuit is connected}}$.

(d) As the current decays the potential difference across the resistor is always equal to the emf across the coil. It decreases from 24.0 V to zero.

***P32.54** We have $9\text{ V} = 2\text{ A}\,R + L\,(0.5\text{ A/s})$ and $5\text{ V} = 2\text{ A}\,R + L\,(-0.5\text{ A/s})$

Solving simultaneously, $9\text{ V} - 5\text{ V} = L(1\text{ A/s})$ so $\boxed{L = 4.00\text{ H}}$ and $7\text{ V} = 2\text{ A}\,R$ so $\boxed{R = 3.50\ \Omega}$

***P32.55** Between $t = 0$ and $t = 1$ ms, the rate of change of current is 2 A/s, so the induced voltage $\Delta V_{ab} = -L\,dI/dt$ is –100 mV. Between $t = 1$ ms and $t = 2$ ms, the induced voltage is zero. Between $t = 2$ ms and $t = 3$ ms the induced voltage is –50 mV. Between $t = 3$ ms and $t = 5$ ms, the rate of change of current is (–3/2) A/s, and the induced voltage is +75 mV.

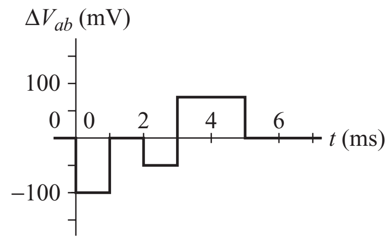


FIG. P32.55

***P32.56** (a) $\omega = (LC)^{-1/2} = (0.032\text{ V}\cdot\text{s/A}\ 0.000\ 5\text{ C/V})^{-1/2} = \boxed{250\text{ rad/s}}$

(b) $$\omega = \left(\frac{1}{LC} - \left[\frac{R}{2L}\right]^2\right)^{1/2} = \left(\frac{1}{1.6\times10^{-5}\ \text{s}^2} - \left[\frac{4\ \Omega}{2\cdot 0.032\ \text{V}\cdot\text{s/A}}\right]^2\right)^{1/2} = \boxed{242\text{ rad/s}}$$

(c) $$\omega = \left(\frac{1}{LC} - \left[\frac{R}{2L}\right]^2\right)^{1/2} = \left(\frac{1}{1.6\times10^{-5}\ \text{s}^2} - \left[\frac{15\ \Omega}{2\cdot 0.032\ \text{V}\cdot\text{s/A}}\right]^2\right)^{1/2} = \boxed{87.0\text{ rad/s}}$$

(d) $$\omega = \left(\frac{1}{LC} - \left[\frac{R}{2L}\right]^2\right)^{1/2} = \left(\frac{1}{1.6\times10^{-5}\ \text{s}^2} - \left[\frac{17\ \Omega}{2\cdot 0.032\ \text{V}\cdot\text{s/A}}\right]^2\right)^{1/2}$$ gives an imaginary answer. In parts (a), (b), and (c) the calculated angular frequency is experimentally verifiable. Experimentally, in part (d) $\boxed{\text{no oscillations occur}}$. The circuit is overdamped.

***P32.57** $B = \dfrac{\mu_0 NI}{2\pi r}$

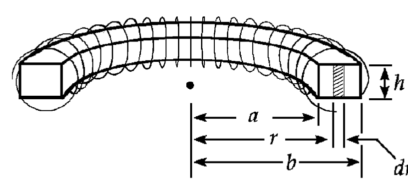


FIG. P32.57

(a) $$\Phi_B = \int B\,dA = \int_a^b \frac{\mu_0 NI}{2\pi r}h\,dr = \frac{\mu_0 NIh}{2\pi}\int_a^b \frac{dr}{r}$$

$$= \frac{\mu_0 NIh}{2\pi}\ln\left(\frac{b}{a}\right)$$

$$L = \frac{N\Phi_B}{I} = \boxed{\frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)}$$

(b) $$L = \frac{\mu_0 (500)^2 (0.010\ 0)}{2\pi}\ln\left(\frac{12.0}{10.0}\right) = \boxed{91.2\ \mu\text{H}}$$

(c) $$L_{\text{appx}} = \frac{\mu_0 N^2}{2\pi}\left(\frac{A}{R}\right) = \frac{\mu_0 (500)^2}{2\pi}\left(\frac{2.00\times10^{-4}\ \text{m}^2}{0.110}\right) = \boxed{90.9\ \mu\text{H}}$$, This approximate result is only 0.3% different from the precise result.

P32.58 (a) At the center, $B = \dfrac{N\mu_0 IR^2}{2\left(R^2+0^2\right)^{3/2}} = \dfrac{N\mu_0 I}{2R}$

So the coil creates flux through itself $\Phi_B = BA\cos\theta = \dfrac{N\mu_0 I}{2R}\pi R^2\cos 0° = \dfrac{\pi}{2}N\mu_0 IR$

When the current it carries changes, $\mathcal{E}_L = -N\dfrac{d\Phi_B}{dt} \approx -N\left(\dfrac{\pi}{2}\right)N\mu_0 R\dfrac{dI}{dt} = -L\dfrac{dI}{dt}$

so $\boxed{L \approx \dfrac{\pi}{2}N^2\mu_0 R}$

(b) $2\pi r = 3(0.3\text{ m})$ so $r \approx 0.14\text{ m}$

$$L \approx \frac{\pi}{2}\left(1^2\right)\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(0.14\text{ m}) = 2.8\times10^{-7}\text{ H}$$

$\boxed{L \sim 100\text{ nH}}$

(c) $\dfrac{L}{R} = \dfrac{2.8\times10^{-7}\text{ V}\cdot\text{s/A}}{270\text{ V/A}} = 1.0\times10^{-9}\text{ s}$ $\boxed{\dfrac{L}{R}\sim 1\text{ ns}}$

P32.59 Left-hand loop: $\mathcal{E} - (I+I_2)R_1 - I_2R_2 = 0$

Outside loop: $\mathcal{E} - (I+I_2)R_1 - L\dfrac{dI}{dt} = 0$

Eliminating I_2 gives $\mathcal{E}' - IR' - L\dfrac{dI}{dt} = 0$

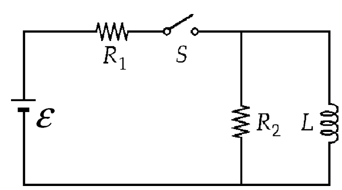


FIG. P32.59

This is of the same form as the differential equation 32.6 in the chapter text for a simple *RL* circuit, so its solution is of the same form as the equation 32.7 for the current in the circuit:

$$I(t) = \frac{\mathcal{E}'}{R'}\left(1-e^{-R't/L}\right)$$

But $R' = \dfrac{R_1R_2}{R_1+R_2}$ and $\mathcal{E}' = \dfrac{R_2\mathcal{E}}{R_1+R_2}$, so $\dfrac{\mathcal{E}'}{R'} = \dfrac{\mathcal{E}R_2/(R_1+R_2)}{R_1R_2/(R_1+R_2)} = \dfrac{\mathcal{E}}{R_1}$

Thus $I(t) = \dfrac{\mathcal{E}}{R_1}\left(1-e^{-R't/L}\right)$

P32.60 From Ampère's law, the magnetic field at distance $r \le R$ is found as:

$$B(2\pi r) = \mu_0 J\left(\pi r^2\right) = \mu_0\left(\frac{I}{\pi R^2}\right)\left(\pi r^2\right),\ \text{or}\ B = \frac{\mu_0 Ir}{2\pi R^2}$$

The magnetic energy per unit length within the wire is then

$$\frac{U}{\ell} = \int_0^R \frac{B^2}{2\mu_0}(2\pi r\,dr) = \frac{\mu_0 I^2}{4\pi R^4}\int_0^R r^3\,dr = \frac{\mu_0 I^2}{4\pi R^4}\left(\frac{R^4}{4}\right) = \boxed{\frac{\mu_0 I^2}{16\pi}}$$

This is independent of the radius of the wire.

P32.61 (a) While steady-state conditions exist, a 9.00 mA flows clockwise around the right loop of the circuit. Immediately after the switch is opened, a 9.00 mA current will flow around the outer loop of the circuit. Applying Kirchhoff's loop rule to this loop gives:

$$+\mathcal{E}_0 - \left[(2.00 + 6.00)\times 10^3\ \Omega\right]\left(9.00\times 10^{-3}\ \text{A}\right) = 0$$

$$+\mathcal{E}_0 = \boxed{72.0\ \text{V with end } b \text{ at the higher potential}}$$

(b)

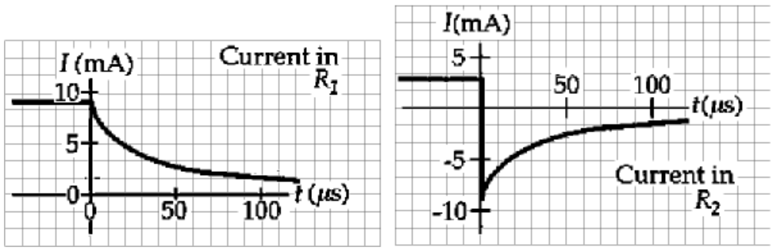


FIG. P32.61(b)

(c) After the switch is opened, the current around the outer loop decays as $I = I_i e^{-Rt/L}$ with $I_{max} = 9.00$ mA, $R = 8.00\ \text{k}\Omega$, and $L = 0.400$ H.

Thus, when the current has reached a value $I = 2.00$ mA, the elapsed time is:

$$t = \left(\frac{L}{R}\right)\ln\left(\frac{I_i}{I}\right) = \left(\frac{0.400\ \text{H}}{8.00\times 10^3\ \Omega}\right)\ \ln\ \left(\frac{9.00}{2.00}\right) = 7.52\times 10^{-5}\ \text{s} = \boxed{75.2\ \mu\text{s}}$$

P32.62 (a) It has a magnetic field, and it stores energy, so $L = \frac{2U}{I^2}$ is non-zero.

(b) Every field line goes through the rectangle between the conductors.

(c) $\Phi = LI$ so

$$L = \frac{\Phi}{I} = \frac{1}{I}\int_{y=a}^{w-a} B\,dA$$

$$L = \frac{1}{I}\int_a^{w-a} x\,dy\left(\frac{\mu_0 I}{2\pi y} + \frac{\mu_0 I}{2\pi(w-y)}\right) = \frac{2}{I}\int \frac{\mu_0 I x}{2\pi y}dy = \frac{2\mu_0 x}{2\pi}\ln y\Big|_a^{w-a}$$

Thus

$$L = \frac{\mu_0 x}{\pi}\ln\left(\frac{w-a}{a}\right)$$

P32.63 When the switch is closed, as shown in Figure (a), the current in the inductor is I:

$$12.0 - 7.50I - 10.0 = 0 \rightarrow I = 0.267\ \text{A}$$

When the switch is opened, the initial current in the inductor remains at 0.267 A.

$IR = \Delta V$:

$$(0.267\ \text{A})R \le 80.0\ \text{V}$$

$$\boxed{R \le 300\ \Omega}$$

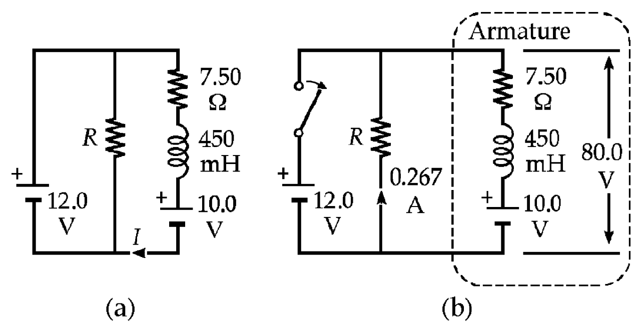


FIG. P32.63

P32.64 For an *RL* circuit,

$I(t) = I_i e^{-(R/L)t}$: $\quad \frac{I(t)}{I_i} = 1 - 10^{-9} = e^{-(R/L)t} \cong 1 - \frac{R}{L}t$

$\frac{R}{L}t = 10^{-9}$ so $\quad R_{max} = \frac{(3.14\times 10^{-8})(10^{-9})}{(2.50\ \text{yr})(3.16\times 10^7\ \text{s/yr})} = \boxed{3.97\times 10^{-25}\ \Omega}$

(If the ring were of purest copper, of diameter 1 cm, and cross-sectional area $1\ \text{mm}^2$, its resistance would be at least $10^{-6}\ \Omega$.)

P32.65 (a) $U_B = \frac{1}{2}LI^2 = \frac{1}{2}(50.0\text{ H})(50.0\times10^3\text{ A})^2 = \boxed{6.25\times10^{10}\text{ J}}$

(b) Two adjacent turns are parallel wires carrying current in the same direction. Since the loops have such large radius, a one-meter section can be regarded as straight.

Then one wire creates a field of $B = \frac{\mu_0 I}{2\pi r}$

This causes a force on the next wire of $F = I\ell B\sin\theta$

giving $F = I\ell\frac{\mu_0 I}{2\pi r}\sin 90° = \frac{\mu_0 \ell I^2}{2\pi r}$

Evaluating the force, $F = (4\pi\times10^{-7}\text{ N/A}^2)\frac{(1.00\text{ m})(50.0\times10^3\text{ A})^2}{2\pi(0.250\text{ m})} = \boxed{2\,000\text{ N}}$

P32.66 $\mathcal{P} = I\Delta V \qquad I = \frac{\mathcal{P}}{\Delta V} = \frac{1.00\times10^9\text{ W}}{200\times10^3\text{ V}} = 5.00\times10^3\text{ A}$

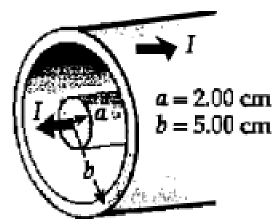


FIG. P32.66

From Ampère's law, $B(2\pi r) = \mu_0 I_{\text{enclosed}}$ or $B = \frac{\mu_0 I_{\text{enclosed}}}{2\pi r}$

(a) At $r = a = 0.020\,0\text{ m}$, $I_{\text{enclosed}} = 5.00\times10^3\text{ A}$

and $B = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(5.00\times10^3\text{ A})}{2\pi(0.020\,0\text{ m})}$

$= 0.050\,0\text{ T} = \boxed{50.0\text{ mT}}$

(b) At $r = b = 0.050\,0\text{ m}$, $I_{\text{enclosed}} = I = 5.00\times10^3\text{ A}$

and $B = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(5.00\times10^3\text{ A})}{2\pi(0.050\,0\text{ m})} = 0.020\,0\text{ T} = \boxed{20.0\text{ mT}}$

(c) $U = \int u\,dV = \int_a^b \frac{[B(r)]^2(2\pi r\ell\,dr)}{2\mu_0} = \frac{\mu_0 I^2\ell}{4\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 I^2\ell}{4\pi}\ln\left(\frac{b}{a}\right)$

$U = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(5.00\times10^3\text{ A})^2(1\,000\times10^3\text{ m})}{4\pi}\ln\left(\frac{5.00\text{ cm}}{2.00\text{ cm}}\right)$

$= 2.29\times10^6\text{ J} = \boxed{2.29\text{ MJ}}$

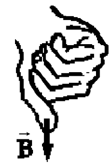
(d) The magnetic field created by the inner conductor exerts a force of repulsion on the current in the outer sheath. The strength of this field, from part (b), is 20.0 mT. Consider a small rectangular section of the outer cylinder of length ℓ and width w.

It carries a current of $(5.00\times10^3\text{ A})\left(\frac{w}{2\pi(0.050\,0\text{ m})}\right)$

and experiences an outward force

$$F = I\ell B\sin\theta = \frac{(5.00\times10^3\text{ A})w}{2\pi(0.050\,0\text{ m})}\ell(20.0\times10^{-3}\text{ T})\sin 90.0°$$

The pressure on it is $P = \frac{F}{A} = \frac{F}{w\ell} = \frac{(5.00\times10^3\text{ A})(20.0\times10^{-3}\text{ T})}{2\pi(0.050\,0\text{ m})} = \boxed{318\text{ Pa}}$



P32.67 (a) $B = \dfrac{\mu_0 NI}{\ell} = \dfrac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(1\,400)(2.00\ \text{A})}{1.20\ \text{m}} = \boxed{2.93 \times 10^{-3}\ \text{T (upward)}}$

(b) $u = \dfrac{B^2}{2\mu_0} = \dfrac{(2.93 \times 10^{-3}\ \text{T})^2}{2(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})} = (3.42\ \text{J/m}^3)\left(\dfrac{1\ \text{N}\cdot\text{m}}{1\ \text{J}}\right)$

$= 3.42\ \text{N/m}^2 = \boxed{3.42\ \text{Pa}}$

(c) To produce a downward magnetic field, the surface of the superconductor must carry a $\boxed{\text{clockwise}}$ current.

(d) The vertical component of the field of the solenoid exerts an inward force on the superconductor. The total horizontal force is zero. Over the top end of the solenoid, its field diverges and has a radially outward horizontal component. This component exerts upward force on the clockwise superconductor current. The total force on the core is $\boxed{\text{upward}}$. You can think of it as a force of repulsion between the solenoid with its north end pointing up, and the core, with its north end pointing down.

(e) $F = PA = (3.42\ \text{Pa})\left[\pi(1.10 \times 10^{-2}\ \text{m})^2\right] = \boxed{1.30 \times 10^{-3}\ \text{N}}$

Note that we have not proved that energy density is pressure. In fact, it is not in some cases. Chapter 21 proved that the pressure is two-thirds of the translational energy density in an ideal gas.

ANSWERS TO EVEN PROBLEMS

P32.2 1.36 μH

P32.4 19.2 μWb

P32.6 (a) 360 mV (b) 180 mV (c) $t = 3.00$ s

P32.8 See the solution.

P32.10 See the solution.

P32.12 See the solution.

P32.14 92.8 V

P32.16 30.0 mH

P32.18 $(500\ \text{mA})(1 - e^{-10t/\text{s}})$, $1.50\ \text{A} - (0.250\ \text{A})\, e^{-10t/\text{s}}$

P32.20 0 for $t < 0$; $(10\ \text{A})(1 - e^{-10\,000t})$ for $0 < t < 200\ \mu\text{s}$; $(63.9\ \text{A})\, e^{-10\,000t}$ for $t > 200\ \mu\text{s}$

P32.22 (a), (b), and (c) See the solution. (d) Yes; see the solution.

P32.24 (a) $8.06\ \text{MJ/m}^3$ (b) 6.32 kJ

P32.26 See the solution.

P32.28 (a) 20.0 W (b) 20.0 W (c) 0 (d) 20.0 J

P32.30 1.73 mH

P32.32 781 pH

P32.34 (a) and (b) $\mu_0 \pi R_2^2 N_1 N_2/\ell$ (c) They are the same.

P32.36 $(L_1 L_2 - M^2)/(L_1 + L_2 - 2M)$

P32.38 20.0 V

P32.40 (a) 135 Hz (b) 119 μC (c) –114 mA

P32.42 (a) 503 Hz (b) 12.0 μC (c) 37.9 mA (d) 72.0 μJ

P32.44 (a) 2.51 kHz (b) 69.9 Ω

P32.46 See the solution.

P32.48 (b) $\mu_0 J_s^2/2$ away from the other sheet (c) $\mu_0 J_s$ and zero (d) $\mu_0 J_s^2/2$

P32.50 (a) $\mathcal{E}_L = -LK$ (b) $\Delta V_c = \dfrac{-Kt^2}{2C}$ (c) $t = 2\sqrt{LC}$

P32.52 (a) a short circuit; 500 mA (b) 125 mJ (c) The energy becomes 125 mJ of additional internal energy in the 8-Ω resistor and the 4-Ω resistor in the middle branch. (d) See the solution. The current decreases from 500 mA toward zero, showing exponential decay with a time constant of 83.3 ms.

P32.54 L = 4.00 H and R = 3.50 Ω

P32.56 (a) 250 rad/s (b) 242 rad/s (c) 87.0 rad/s (d) In parts (a), (b), and (c) the calculated angular frequency is experimentally verifiable. In part (d) the equation for ω gives an imaginary answer. Experimentally, no oscillations occur when the circuit is overdamped.

P32.58 (a) $L \approx (\pi/2)N^2\mu_0 R$ (b) ~100 nH (c) ~1 ns

P32.60 See the solution.

P32.62 (a) It creates a magnetic field. (b) The long narrow rectangular area between the conductors encloses all of the magnetic flux.

P32.64 3.97×10^{-25} Ω

P32.66 (a) 50.0 mT (b) 20.0 mT (c) 2.29 MJ (d) 318 Pa

33

Alternating Current Circuits

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q33.1** (i) Answer (d). $\Delta V_{avg} = \dfrac{\Delta V_{\max}}{2}$

(ii) Answer (c). The average of the squared voltage is

$\left([\Delta V]^2\right)_{\text{avg}} = \dfrac{\left(\Delta V_{\max}\right)^2}{2}$. Then its square root is

$\Delta V_{rms} = \dfrac{\Delta V_{\max}}{\sqrt{2}}$

***Q33.2** Answer (c). AC ammeters and voltmeters read rms values. With an oscilloscope you can read a maximum voltage, or test whether the average is zero.

***Q33.3** (i) Answer (f). The voltage varies between +170 V and −170 V.
(ii) Answer (d).
(iii) $170\text{V}/\sqrt{2} = 120$ V. Answer (c).

Q33.4 The capacitive reactance is proportional to the inverse of the frequency. At higher and higher frequencies, the capacitive reactance approaches zero, making a capacitor behave like a wire. As the frequency goes to zero, the capacitive reactance approaches infinity—the resistance of an open circuit.

Q33.5 The second letter in each word stands for the circuit element. For an inductor *L*, the emf $\mathcal{E}$ leads the current *I*—thus *ELI*. For a capacitor *C*, the current leads the voltage across the device. In a circuit in which the capacitive reactance is larger than the inductive reactance, the current leads the source emf—thus *ICE*.

Q33.6 The voltages are not added in a scalar form, but in a vector form, as shown in the phasor diagrams throughout the chapter. Kirchhoff's loop rule is true at any instant, but the voltages across different circuit elements are not simultaneously at their maximum values. Do not forget that an inductor can induce an emf in itself and that the voltage across it is 90° *ahead* of the current in the circuit in phase.

Q33.7 In an *RLC* series circuit, the phase angle depends on the source frequency. At very low frequency the capacitor dominates the impedance and the phase angle is near −90°. The phase angle is zero at the resonance frequency, where the inductive and capacitive reactances are equal. At very high frequencies ϕ approaches +90°.

***Q33.8** (i) Inductive reactance doubles when frequency doubles. Answer (f).
(ii) Capacitive reactance is cut in half when frequency doubles. Answer (b).
(iii) The resistance remains unchanged. Answer (d).

***Q33.9** At resonance the inductive reactance and capacitive reactance cancel out. Answer (c).

***Q33.10** At resonance the inductive reactance and capacitive reactance add to zero. $\tan^{-1}(X_L - X_C)/R = 0$. Answer (c).

***Q33.11** (a) The circuit is in resonance. (b) 10 Ω/20 Ω = 0.5 (c) The resistance of the load could be increased to make a greater *fraction* of the emf's power go to the load. Then the emf would put out a lot less power and less power would reach the load.

Q33.12 The person is doing work at a rate of $\mathcal{P} = Fv\cos\theta$. One can consider the emf as the "force" that moves the charges through the circuit, and the current as the "speed" of the moving charges. The $\cos\theta$ factor measures the effectiveness of the cause in producing the effect. Theta is an angle in real space for the vacuum cleaner and phi is the analogous angle of phase difference between the emf and the current in the circuit.

***Q33.13** The resonance is high-Q, so at 1 000 Hz both X_L and X_C are equal and much larger than R. Now X_C at 500 Hz is twice as large as at 1 kHz. And X_L at 1.5 kHz is 1.5 times larger than at 1 kHz. Again, X_C at 1 500 Hz is two-thirds as large as at 1 kHz. And X_L at 500 Hz is half as large as at 1 kHz. The resistance does not change with frequency. The ranking is then a > f > b = e > c > d > g = h = i.

Q33.14 In 1881, an assassin shot President James Garfield. The bullet was lost in his body. Alexander Graham Bell invented the metal detector in an effort to save the President's life. The coil is preserved in the Smithsonian Institution. The detector was thrown off by metal springs in Garfield's mattress, a new invention itself. Surgeons went hunting for the bullet in the wrong place. Garfield died.

Q33.15 No. A voltage is only induced in the secondary coil if the flux through the core changes in time.

Q33.16 The Q factor determines the selectivity of the radio receiver. For example, a receiver with a very low Q factor will respond to a wide range of frequencies and might pick up several adjacent radio stations at the same time. To discriminate between 102.5 MHz and 102.7 MHz requires a high-Q circuit. Typically, lowering the resistance in the circuit is the way to get a higher quality resonance.

***Q33.17** In its intended use, the transformer takes in energy by electric transmission at 12 kV and puts out nearly the same energy by electric transmission at 120 V. With the small generator putting energy into the secondary side of the transformer at 120 V, the primary side has 12 kV induced across it. It is deadly dangerous for the repairman.

SOLUTIONS TO PROBLEMS

Section 33.1 AC Sources

Section 33.2 Resistors in an AC Circuit

P33.1 $$\Delta v(t) = \Delta V_{\max}\sin(\omega t) = \sqrt{2}\Delta V_{\text{rms}}\sin(\omega t) = 200\sqrt{2}\sin\left[2\pi(100t)\right] = \boxed{(283\text{ V})\sin(628t)}$$

P33.2 $$\Delta V_{\text{rms}} = \frac{170\text{ V}}{\sqrt{2}} = 120\text{ V}$$

(a) $$\mathcal{P} = \frac{(\Delta V_{\text{rms}})^2}{R} \rightarrow R = \frac{(120\text{ V})^2}{75.0\text{ W}} = \boxed{193\ \Omega}$$

(b) $$R = \frac{(120\text{ V})^2}{100\text{ W}} = \boxed{144\ \Omega}$$

P33.3 Each meter reads the rms value.

$$\Delta V_{\text{rms}} = \frac{100\text{ V}}{\sqrt{2}} = \boxed{70.7\text{ V}}$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{R} = \frac{70.7\text{ V}}{24.0\ \Omega} = \boxed{2.95\text{ A}}$$

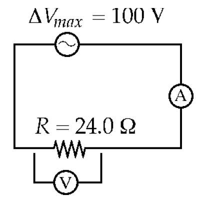


FIG. P33.3

P33.4 (a) $\Delta v_R = \Delta V_{\max} \sin\omega t$

$\Delta v_R = 0.250(\Delta V_{\max})$, so, $\sin\omega t = 0.250$, or $\omega t = \sin^{-1}(0.250)$.

The smallest angle for which this is true is $\omega t = 0.253$ rad. Thus, if $t = 0.010\,0$ s,
$\omega = \dfrac{0.253\text{ rad}}{0.010\,0\text{ s}} = \boxed{25.3\ \text{rad/s}}$.

(b) The second time when $\Delta v_R = 0.250(\Delta V_{\max})$, $\omega t = \sin^{-1}(0.250)$ again. For this occurrence, $\omega t = \pi - 0.253$ rad $= 2.89$ rad (to understand why this is true, recall the identity $\sin(\pi - \theta) = \sin\theta$ from trigonometry). Thus,

$$t = \frac{2.89\text{ rad}}{25.3\ \text{rad/s}} = \boxed{0.114\text{ s}}$$

P33.5 $i_R = I_{\max}\sin\omega t$ becomes $0.600 = \sin(\omega\, 0.007\,00)$

Thus, $(0.007\,00)\omega = \sin^{-1}(0.600) = 0.644$

and $\omega = 91.9\ \text{rad/s} = 2\pi f$ so $\boxed{f = 14.6\text{ Hz}}$

P33.6 $\Delta V_{\max} = 15.0\text{ V}$ and $R_{\text{total}} = 8.20\ \Omega + 10.4\ \Omega = 18.6\ \Omega$

$$I_{\max} = \frac{\Delta V_{\max}}{R_{\text{total}}} = \frac{15.0\text{ V}}{18.6\ \Omega} = 0.806\text{ A} = \sqrt{2} I_{\text{rms}}$$

$$\mathcal{P}_{\text{speaker}} = I_{\text{rms}}^2 R_{\text{speaker}} = \left(\frac{0.806\text{ A}}{\sqrt{2}}\right)^2 (10.4\ \Omega) = \boxed{3.38\text{ W}}$$

Section 33.3 **Inductors in an AC Circuit**

P33.7 (a) $X_L = \dfrac{\Delta V_{\max}}{I_{\max}} = \dfrac{100}{7.50} = 13.3\ \Omega$

$$L = \frac{X_L}{\omega} = \frac{13.3}{2\pi(50.0)} = 0.042\,4\text{ H} = \boxed{42.4\text{ mH}}$$

(b) $X_L = \dfrac{\Delta V_{\max}}{I_{\max}} = \dfrac{100}{2.50} = 40.0\ \Omega$

$$\omega = \frac{X_L}{L} = \frac{40.0}{42.4 \times 10^{-3}} = \boxed{942\ \text{rad/s}}$$

P33.8 At 50.0 Hz, $X_L = 2\pi(50.0\text{ Hz})L = 2\pi(50.0\text{ Hz})\left(\dfrac{X_L|_{60.0\text{ Hz}}}{2\pi(60.0\text{ Hz})}\right) = \dfrac{50.0}{60.0}(54.0\ \Omega) = 45.0\ \Omega$

$$I_{max} = \frac{\Delta V_{max}}{X_L} = \frac{\sqrt{2}(\Delta V_{rms})}{X_L} = \frac{\sqrt{2}(100\text{ V})}{45.0\ \Omega} = \boxed{3.14\text{ A}}$$

P33.9 $i_L(t) = \dfrac{\Delta V_{max}}{\omega L}\sin\left(\omega t - \dfrac{\pi}{2}\right) = \dfrac{(80.0\text{ V})\sin[(65.0\pi)(0.015\,5) - \pi/2]}{(65.0\pi\text{ rad/s})(70.0\times10^{-3}\text{ H})}$

$$i_L(t) = (5.60\text{ A})\sin(1.59\text{ rad}) = \boxed{5.60\text{ A}}$$

P33.10 $\omega = 2\pi f = 2\pi(60.0/\text{s}) = 377\text{ rad/s}$

$$X_L = \omega L = (377/\text{s})(0.020\,0\text{ V}\cdot\text{s/A}) = 7.54\ \Omega$$

$$I_{rms} = \frac{\Delta V_{rms}}{X_L} = \frac{120\text{ V}}{7.54\ \Omega} = 15.9\text{ A}$$

$$I_{max} = \sqrt{2}I_{rms} = \sqrt{2}(15.9\text{ A}) = 22.5\text{ A}$$

$$i(t) = I_{max}\sin\omega t = (22.5\text{ A})\sin\left(\frac{2\pi(60.0)}{\text{s}}\cdot\frac{1\text{ s}}{180}\right) = (22.5\text{ A})\sin 120° = 19.5\text{ A}$$

$$U = \frac{1}{2}Li^2 = \frac{1}{2}(0.020\,0\text{ V}\cdot\text{s/A})(19.5\text{ A})^2 = \boxed{3.80\text{ J}}$$

P33.11 $L = \dfrac{N\Phi_B}{I}$ where Φ_B is the flux through each turn. $N\Phi_{B,max} = LI_{max} = \dfrac{X_L}{\omega}\dfrac{(\Delta V_{L,max})}{X_L}$

$$N\Phi_{B,max} = \frac{\sqrt{2}(\Delta V_{L,rms})}{2\pi f} = \frac{120\text{ V}\cdot\text{s}}{\sqrt{2}\pi(60.0)}\left(\frac{\text{T}\cdot\text{C}\cdot\text{m}}{\text{N}\cdot\text{s}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right)\left(\frac{\text{J}}{\text{V}\cdot\text{C}}\right) = \boxed{0.450\text{ T}\cdot\text{m}^2}$$

Section 33.4 **Capacitors in an AC Circuit**

P33.12 (a) $X_C = \dfrac{1}{2\pi f C}$: $\dfrac{1}{2\pi f(22.0\times10^{-6})} < 175\ \Omega$

$$\frac{1}{2\pi(22.0\times10^{-6})(175)} < f \qquad \boxed{f > 41.3\text{ Hz}}$$

(b) $X_C \propto \dfrac{1}{C}$, so $X(44) = \dfrac{1}{2}X(22)$: $\boxed{X_C < 87.5\ \Omega}$

P33.13 $I_{max} = \sqrt{2}I_{rms} = \dfrac{\sqrt{2}(\Delta V_{rms})}{X_C} = \sqrt{2}(\Delta V_{rms})2\pi f C$

(a) $I_{max} = \sqrt{2}(120\text{ V})2\pi(60.0/\text{s})(2.20\times10^{-6}\text{ C/V}) = \boxed{141\text{ mA}}$

(b) $I_{max} = \sqrt{2}(240\text{ V})2\pi(50.0/\text{s})(2.20\times10^{-6}\text{ F}) = \boxed{235\text{ mA}}$

P33.14 $Q_{max} = C(\Delta V_{max}) = C\left[\sqrt{2}(\Delta V_{rms})\right] = \boxed{\sqrt{2}C(\Delta V_{rms})}$

P33.15 $I_{max} = (\Delta V_{max})\omega C = (48.0\text{ V})(2\pi)(90.0\text{ s}^{-1})(3.70\times10^{-6}\text{ F}) = \boxed{100\text{ mA}}$

P33.16 $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi(60.0/\text{s})(1.00\times10^{-3}\ \text{C/V})} = 2.65\ \Omega$

$\Delta v_C(t) = \Delta V_{\max}\sin\omega t$, to be zero at $t = 0$

$$i_C = \frac{\Delta V_{\max}}{X_C}\sin(\omega t+\phi) = \frac{\sqrt{2}(120\ \text{V})}{2.65\ \Omega}\sin\left[2\pi\frac{60\ \text{s}^{-1}}{180\ \text{s}^{-1}} + 90.0°\right] = (64.0\ \text{A})\sin(120° + 90.0°)$$

$$= \boxed{-32.0\ \text{A}}$$

Section 33.5 **The *RLC* Series Circuit**

P33.17 (a) $X_L = \omega L = 2\pi(50.0)(400\times10^{-3}) = 126\ \Omega$

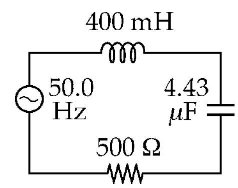


FIG. P33.17

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(50.0)(4.43\times10^{-6})} = 719\ \Omega$$

$$Z = \sqrt{R^2+(X_L-X_C)^2} = \sqrt{500^2+(126-719)^2} = 776\ \Omega$$

$$\Delta V_{\max} = I_{\max}Z = (250\times10^{-3})(776) = \boxed{194\ \text{V}}$$

(b) $\phi = \tan^{-1}\left(\dfrac{X_L-X_C}{R}\right) = \tan^{-1}\left(\dfrac{126-719}{500}\right) = \boxed{-49.9°}$. Thus, the $\boxed{\text{current leads the voltage}}$.

P33.18 $\omega L = \dfrac{1}{\omega C} \to \omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(57.0\times10^{-6})(57.0\times10^{-6})}} = 1.75\times10^4\ \text{rad/s}$

$$f = \frac{\omega}{2\pi} = \boxed{2.79\ \text{kHz}}$$

P33.19 (a) $X_L = \omega L = 2\pi(50.0\ \text{s}^{-1})(250\times10^{-3}\ \text{H}) = \boxed{78.5\ \Omega}$

(b) $X_C = \dfrac{1}{\omega C} = \left[2\pi(50.0\ \text{s}^{-1})(2.00\times10^{-6}\ \text{F})\right]^{-1} = \boxed{1.59\ \text{k}\Omega}$

(c) $Z = \sqrt{R^2+(X_L-X_C)^2} = \boxed{1.52\ \text{k}\Omega}$

(d) $I_{\max} = \dfrac{\Delta V_{\max}}{Z} = \dfrac{210\ \text{V}}{1.52\times10^3\ \Omega} = \boxed{138\ \text{mA}}$

(e) $\phi = \tan^{-1}\left[\dfrac{X_L-X_C}{R}\right] = \tan^{-1}(-10.1) = \boxed{-84.3°}$

P33.20 (a) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{68.0^2 + (16.0 - 101)^2} = \boxed{109\ \Omega}$

$X_L = \omega L = (100)(0.160) = 16.0\ \Omega$

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{(100)(99.0 \times 10^{-6})} = 101\ \Omega$

(b) $I_{\max} = \dfrac{\Delta V_{\max}}{Z} = \dfrac{40.0\ \text{V}}{109\ \Omega} = \boxed{0.367\ \text{A}}$

(c) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{16.0 - 101}{68.0} = -1.25$:

$\phi = -0.896\ \text{rad} = -51.3°$

$\boxed{I_{\max} = 0.367\ \text{A}}$ $\boxed{\omega = 100\ \text{rad/s}}$ $\boxed{\phi = -0.896\ \text{rad} = -51.3°}$

P33.21 $X_L = 2\pi f L = 2\pi(60.0)(0.460) = 173\ \Omega$

$X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(60.0)(21.0 \times 10^{-6})} = 126\ \Omega$

(a) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{173\ \Omega - 126\ \Omega}{150\ \Omega} = 0.314$

$\phi = 0.304\ \text{rad} = \boxed{17.4°}$

(b) Since $X_L > X_C$, ϕ is positive; so $\boxed{\text{voltage leads the current}}$.

P33.22 For the source-capacitor circuit, the rms source voltage is $\Delta V_s = (25.1\ \text{mA})X_C$. For the circuit with resistor, $\Delta V_s = (15.7\ \text{mA})\sqrt{R^2 + X_C^2} = (25.1\ \text{mA})X_C$. This gives $R = 1.247X_C$. For the circuit with ideal inductor, $\Delta V_s = (68.2\ \text{mA})|X_L - X_C| = (25.1\ \text{mA})X_C$. So $|X_L - X_C| = 0.368\,0X_C$. Now for the full circuit

$\Delta V_s = I\sqrt{R^2 + (X_L - X_C)^2}$

$(25.1\ \text{mA})X_C = I\sqrt{(1.247X_C)^2 + (0.368X_C)^2}$

$\boxed{I = 19.3\ \text{mA}}$

P33.23 $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(60.0\ \text{Hz})(20.0 \times 10^{-12}\ \text{F})} = 1.33 \times 10^8\ \Omega$

$Z = \sqrt{(50.0 \times 10^3\ \Omega)^2 + (1.33 \times 10^8\ \Omega)^2} \approx 1.33 \times 10^8\ \Omega$

$I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{Z} = \dfrac{5\,000\ \text{V}}{1.33 \times 10^8\ \Omega} = 3.77 \times 10^{-5}\ \text{A}$

$(\Delta V_{\text{rms}})_{\text{body}} = I_{\text{rms}} R_{\text{body}} = (3.77 \times 10^{-5}\ \text{A})(50.0 \times 10^3\ \Omega) = \boxed{1.88\ \text{V}}$

P33.24 $X_C = \frac{1}{\omega C} = \frac{1}{2\pi(50.0)(65.0\times10^{-6})} = 49.0\ \Omega$

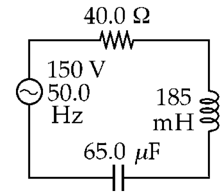


FIG. P33.24

$X_L = \omega L = 2\pi(50.0)(185\times10^{-3}) = 58.1\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(40.0)^2 + (58.1 - 49.0)^2} = 41.0\ \Omega$

$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{150}{41.0} = 3.66\ \text{A}$

(a) $\Delta V_R = I_{\max} R = (3.66)(40) = \boxed{146\ \text{V}}$

(b) $\Delta V_L = I_{\max} X_L = (3.66)(58.1) = 212.5 = \boxed{212\ \text{V}}$

(c) $\Delta V_C = I_{\max} X_C = (3.66)(49.0) = 179.1\ \text{V} = \boxed{179\ \text{V}}$

(d) $\Delta V_L - \Delta V_C = 212.5 - 179.1 = \boxed{33.4\ \text{V}}$

P33.25 $R = 300\ \Omega$

$X_L = \omega L = 2\pi\left(\frac{500}{\pi}\ \text{s}^{-1}\right)(0.200\ \text{H}) = 200\ \Omega$

$X_C = \frac{1}{\omega C} = \left[2\pi\left(\frac{500}{\pi}\ \text{s}^{-1}\right)(11.0\times10^{-6}\ \text{F})\right]^{-1} = 90.9\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = 319\ \Omega$ and

$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = 20.0°$

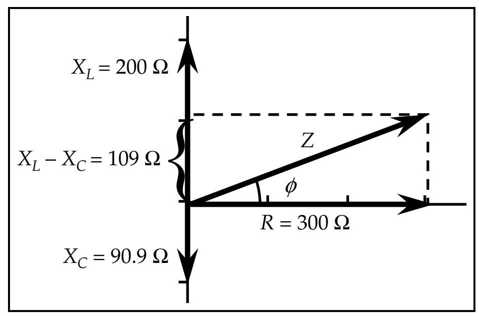


FIG. P33.25

***P33.26** Let X_c represent the initial capacitive reactance. Moving the plates to half their original separation doubles the capacitance and cuts $X_C = \frac{1}{\omega C}$ in half. For the current to double, the total impedance must be cut in half: $Z_i = 2Z_f$, $\sqrt{R^2 + (X_L - X_C)^2} = 2\sqrt{R^2 + \left(X_L - \frac{X_C}{2}\right)^2}$. With $X_L = R$, algebra then gives

$$R^2 + (R - X_C)^2 = 4\left(R^2 + \left(R - \frac{X_C}{2}\right)^2\right)$$

$$2R^2 - 2RX_C + X_C^2 = 8R^2 - 4RX_C + X_C^2$$

$$\boxed{X_C = 3R}$$

Section 33.6 **Power in an AC Circuit**

P33.27 $\omega = 1\,000\ \text{rad/s}$, $R = 400\ \Omega$, $C = 5.00\times10^{-6}\ \text{F}$, $L = 0.500\ \text{H}$

$\Delta V_{\max} = 100\ \text{V}$, $\omega L = 500\ \Omega$, $\left(\frac{1}{\omega C}\right) = 200\ \Omega$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{400^2 + 300^2} = 500\ \Omega$$

$$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{100}{500} = 0.200\ \text{A}$$

The average power dissipated in the circuit is $\mathcal{P} = I_{\text{rms}}^2 R = \left(\frac{I_{\max}^2}{2}\right)R$.

$$\mathcal{P} = \frac{(0.200\ \text{A})^2}{2}(400\ \Omega) = \boxed{8.00\ \text{W}}$$

P33.28 $Z = \sqrt{R^2 + (X_L - X_C)^2}$ or $(X_L - X_C) = \sqrt{Z^2 - R^2}$

$$(X_L - X_C) = \sqrt{(75.0\ \Omega)^2 - (45.0\ \Omega)^2} = 60.0\ \Omega$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{60.0\ \Omega}{45.0\ \Omega}\right) = 53.1°$$

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{210\ \text{V}}{75.0\ \Omega} = 2.80\ \text{A}$$

$$\mathcal{P} = (\Delta V_{\text{rms}}) I_{\text{rms}} \cos\phi = (210\ \text{V})(2.80\ \text{A})\cos(53.1°) = \boxed{353\ \text{W}}$$

P33.29 (a) $\mathcal{P} = I_{\text{rms}}(\Delta V_{\text{rms}})\cos\phi = (9.00)180\cos(-37.0°) = 1.29\times10^3\ \text{W}$

$\mathcal{P} = I_{\text{rms}}^2 R$ so $1.29\times10^3 = (9.00)^2 R$ and $R = \boxed{16.0\ \Omega}$

(b) $\tan\phi = \frac{X_L - X_C}{R}$ becomes $\tan(-37.0°) = \frac{X_L - X_C}{16}$: so $X_L - X_C = \boxed{-12.0\ \Omega}$

P33.30 $X_L = \omega L = 2\pi(60.0/\text{s})(0.025\,0\ \text{H}) = 9.42\ \Omega$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(20.0)^2 + (9.42)^2}\ \Omega = 22.1\ \Omega$$

(a) $I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{120\ \text{V}}{22.1\ \Omega} = \boxed{5.43\ \text{A}}$

(b) $\phi = \tan^{-1}\left(\frac{9.42}{20.0}\right) = 25.2°$ so power factor $= \cos\phi = \boxed{0.905}$

(c) We require $\phi = 0$. Thus, $X_L = X_C$: $9.42\ \Omega = \frac{1}{2\pi(60.0\ \text{s}^{-1})C}$

and $C = \boxed{281\ \mu\text{F}}$

(d) $\mathcal{P}_b = \mathcal{P}_d$ or $(\Delta V_{\text{rms}})_b (I_{\text{rms}})_b \cos\phi_b = \frac{(\Delta V_{\text{rms}})_d^2}{R}$

$$(\Delta V_{\text{rms}})_d = \sqrt{R(\Delta V_{\text{rms}})_b (I_{\text{rms}})_b \cos\phi_b} = \sqrt{(20.0\ \Omega)(120\ \text{V})(5.43\ \text{A})(0.905)} = \boxed{109\ \text{V}}$$

***P33.31** Consider a two-wire transmission line taking in power $\mathcal{P}$

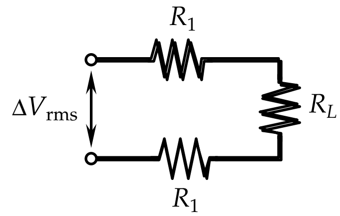


FIG. P33.31

$I_{\text{rms}} = \dfrac{\mathcal{P}}{\Delta V_{\text{rms}}}$. Then power loss $= I_{\text{rms}}^2 R_{\text{line}} = \dfrac{\mathcal{P}}{100}$.

Thus, $\left(\dfrac{\mathcal{P}}{\Delta V_{\text{rms}}}\right)^2 (2R_1) = \dfrac{\mathcal{P}}{100}$ or $R_1 = \dfrac{(\Delta V_{\text{rms}})^2}{200\mathcal{P}}$

$R_1 = \dfrac{\rho \ell}{A} = \dfrac{(\Delta V_{\text{rms}})^2}{200\mathcal{P}}$ or $A = \dfrac{\pi (2r)^2}{4} = \dfrac{200\rho \mathcal{P}\ell}{(\Delta V_{\text{rms}})^2}$

and the diameter is $2r = \sqrt{\dfrac{800\rho \mathcal{P}\ell}{\pi (\Delta V)^2}}$

(a) $2r = \sqrt{\dfrac{800(1.7\times 10^{-8}\ \Omega\text{m})\ 20\ 000\ \text{W}\ (18\ 000\ \text{m})}{\pi(\Delta V)^2}} = \boxed{39.5\ \text{V}\cdot\text{m}/\Delta V}$

(b) The diameter is inversely proportional to the potential difference.

(c) $2r = 39.5$ V·m/1 500 V $= \boxed{2.63\ \text{cm}}$

(d) $\Delta V = 39.5$ V·m/0.003 m $= \boxed{13.2\ \text{kV}}$

***P33.32** (a) $X_L = \omega L = 2\pi(60/\text{s})\ 0.1\ \text{H} = 37.7\ \Omega$
$Z = (100^2 + 37.7^2)^{1/2} = 107\ \Omega$
power factor $= \cos\phi = 100/107 = \boxed{0.936}$

(b) The power factor cannot in practice be made 1.00. If the inductor were removed or if the generator were replaced with a battery, so that either $L = 0$ or $f = 0$, the power factor would be 1, but we would not have a magnetic buzzer.

(c) We want resonance, with $\phi = 0$. We insert a capacitor in series with

$X_L = X_C$ so $37.7\ \Omega = 1\ \text{s}/2\pi C\ 60$ and $C = \boxed{70.4\ \mu\text{F}}$

P33.33 One-half the time, the left side of the generator is positive, the top diode conducts, and the bottom diode switches off. The power supply sees resistance

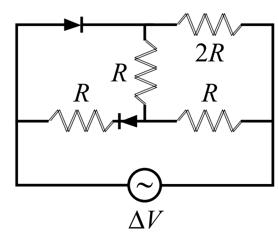


FIG. P33.33

$\left[\dfrac{1}{2R} + \dfrac{1}{2R}\right]^{-1} = R$ and the power is $\dfrac{(\Delta V_{\text{rms}})^2}{R}$.

The other half of the time the right side of the generator is positive, the upper diode is an open circuit, and the lower diode has zero resistance. The equivalent resistance is then

$R_{\text{eq}} = R + \left[\dfrac{1}{3R} + \dfrac{1}{R}\right]^{-1} = \dfrac{7R}{4}$ and $\mathcal{P} = \dfrac{(\Delta V_{\text{rms}})^2}{R_{\text{eq}}} = \dfrac{4(\Delta V_{\text{rms}})^2}{7R}$

The overall time average power is: $\dfrac{\left[(\Delta V_{\text{rms}})^2/R\right] + \left[4(\Delta V_{\text{rms}})^2/7R\right]}{2} = \boxed{\dfrac{11(\Delta V_{\text{rms}})^2}{14R}}$

Section 33.7 **Resonance in a Series *RLC* Circuit**

P33.34 (a) $f = \dfrac{1}{2\pi\sqrt{LC}}$

$$C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 \left(10^{10}/\text{s}\right)^2 400\times 10^{-12}\ \text{Vs}}\frac{\text{A}}{}\left(\frac{\text{C}}{\text{As}}\right) = \boxed{6.33\times 10^{-13}\ \text{F}}$$

(b) $C = \dfrac{\kappa \in_0 A}{d} = \dfrac{\kappa \in_0 \ell^2}{d}$

$$\ell = \left(\frac{Cd}{\kappa \in_0}\right)^{1/2} = \left(\frac{6.33\times 10^{-13}\ \text{F}\times 10^{-3}\ \text{mm}}{1\times 8.85\times 10^{-12}\ \text{F}}\right)^{1/2} = \boxed{8.46\times 10^{-3}\ \text{m}}$$

(c) $X_L = 2\pi f L = 2\pi\times 10^{10}/\text{s}\times 400\times 10^{-12}\ \text{Vs/A} = \boxed{25.1\ \Omega}$

P33.35 $\omega_0 = 2\pi\left(99.7\times 10^6\right) = 6.26\times 10^8\ \text{rad/s} = \dfrac{1}{\sqrt{LC}}$

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{\left(6.26\times 10^8\right)^2\left(1.40\times 10^{-6}\right)} = \boxed{1.82\ \text{pF}}$$

P33.36 $L = 20.0\ \text{mH},\ C = 1.00\times 10^{-7},\ R = 20.0\ \Omega,\ \Delta V_{\max} = 100\ \text{V}$

(a) The resonant frequency for a series *RLC* circuit is $f = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}} = \boxed{3.56\ \text{kHz}}$.

(b) At resonance, $I_{\max} = \dfrac{\Delta V_{\max}}{R} = \boxed{5.00\ \text{A}}$

(c) From Equation 33.38, $Q = \dfrac{\omega_0 L}{R} = \boxed{22.4}$

(d) $\Delta V_{L,\max} = X_L I_{\max} = \omega_0 L I_{\max} = \boxed{2.24\ \text{kV}}$

P33.37 The resonance frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}}$. Thus, if $\omega = 2\omega_0$,

$$X_L = \omega L = \left(\frac{2}{\sqrt{LC}}\right)L = 2\sqrt{\frac{L}{C}} \quad \text{and} \quad X_C = \frac{1}{\omega C} = \frac{\sqrt{LC}}{2C} = \frac{1}{2}\sqrt{\frac{L}{C}}$$

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + 2.25\left(\frac{L}{C}\right)} \quad \text{so} \quad I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + 2.25(L/C)}}$$

and the energy delivered in one period is $E = \mathcal{P}\Delta t$:

$$E = \frac{\left(\Delta V_{\text{rms}}\right)^2 R}{R^2 + 2.25(L/C)}\left(\frac{2\pi}{\omega}\right) = \frac{\left(\Delta V_{\text{rms}}\right)^2 RC}{R^2 C + 2.25L}\left(\pi\sqrt{LC}\right) = \frac{4\pi\left(\Delta V_{\text{rms}}\right)^2 RC\sqrt{LC}}{4R^2 C + 9.00L}$$

With the values specified for this circuit, this gives:

$$E = \frac{4\pi(50.0\ \text{V})^2(10.0\ \Omega)\left(100\times 10^{-6}\ \text{F}\right)^{3/2}\left(10.0\times 10^{-3}\ \text{H}\right)^{1/2}}{4(10.0\ \Omega)^2\left(100\times 10^{-6}\ \text{F}\right) + 9.00\left(10.0\times 10^{-3}\ \text{H}\right)} = \boxed{242\ \text{mJ}}$$

P33.38 The resonance frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}}$. Thus, if $\omega = 2\omega_0$,

$$X_L = \omega L = \left(\frac{2}{\sqrt{LC}}\right)L = 2\sqrt{\frac{L}{C}}$$ and $$X_C = \frac{1}{\omega C} = \frac{\sqrt{LC}}{2C} = \frac{1}{2}\sqrt{\frac{L}{C}}$$

Then $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + 2.25\left(\frac{L}{C}\right)}$ so $I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{Z} = \dfrac{\Delta V_{\text{rms}}}{\sqrt{R^2 + 2.25(L/C)}}$

and the energy delivered in one period is

$$E = \mathcal{P}\Delta t = \frac{(\Delta V_{\text{rms}})^2 R}{R^2 + 2.25(L/C)}\left(\frac{2\pi}{\omega}\right) = \frac{(\Delta V_{\text{rms}})^2 RC}{R^2C + 2.25L}\left(\pi\sqrt{LC}\right) = \boxed{\frac{4\pi(\Delta V_{\text{rms}})^2 RC\sqrt{LC}}{4R^2C + 9.00L}}$$

P33.39 For the circuit of Problem 20, $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(160\times10^{-3}\text{ H})(99.0\times10^{-6}\text{ F})}} = 251\text{ rad/s}$

$$Q = \frac{\omega_0 L}{R} = \frac{(251\text{ rad/s})(160\times10^{-3}\text{ H})}{68.0\ \Omega} = \boxed{0.591}$$

For the circuit of Problem 21, $Q = \dfrac{\omega_0 L}{R} = \dfrac{L}{R\sqrt{LC}} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{150\ \Omega}\sqrt{\dfrac{460\times10^{-3}\text{ H}}{21.0\times10^{-6}\text{ F}}} = \boxed{0.987}$

The $\boxed{\text{circuit of Problem 21}}$ has a sharper resonance.

Section 33.8 **The Transformer and Power Transmission**

P33.40 (a) $\Delta V_{2,\text{rms}} = \dfrac{1}{13}(120\text{ V}) = \boxed{9.23\text{ V}}$

(b) $\Delta V_{1,\text{rms}} I_{1,\text{rms}} = \Delta V_{2,\text{rms}} I_{2,\text{rms}}$

$(120\text{ V})(0.350\text{ A}) = (9.23\text{ V})I_{2,\text{rms}}$

$I_{2,\text{rms}} = \dfrac{42.0\text{ W}}{9.23\text{ V}} = \boxed{4.55\text{ A}}$ for a transformer with no energy loss.

(c) $\mathcal{P} = \boxed{42.0\text{ W}}$ from part (b).

P33.41 $(\Delta V_{\text{out}})_{\text{max}} = \dfrac{N_2}{N_1}(\Delta V_{\text{in}})_{\text{max}} = \left(\dfrac{2\,000}{350}\right)(170\text{ V}) = 971\text{ V}$

$(\Delta V_{\text{out}})_{\text{rms}} = \dfrac{(971\text{ V})}{\sqrt{2}} = \boxed{687\text{ V}}$

P33.42 (a) $(\Delta V_{2,\text{rms}}) = \dfrac{N_2}{N_1}(\Delta V_{1,\text{rms}})$ $\quad N_2 = \dfrac{(2\,200)(80)}{110} = \boxed{1\,600\text{ windings}}$

(b) $I_{1,\text{rms}}(\Delta V_{1,\text{rms}}) = I_{2,\text{rms}}(\Delta V_{2,\text{rms}})$ $\quad I_{1,\text{rms}} = \dfrac{(1.50)(2\,200)}{110} = \boxed{30.0\text{ A}}$

(c) $0.950 I_{1,\text{rms}}(\Delta V_{1,\text{rms}}) = I_{2,\text{rms}}(\Delta V_{2,\text{rms}})$ $\quad I_{1,\text{rms}} = \dfrac{(1.20)(2\,200)}{110(0.950)} = \boxed{25.3\text{ A}}$

P33.43 (a) $R = (4.50 \times 10^{-4}\ \Omega/\text{M})(6.44 \times 10^{5}\ \text{m}) = 290\ \Omega$ and $I_{\text{rms}} = \dfrac{\mathcal{P}}{\Delta V_{\text{rms}}} = \dfrac{5.00 \times 10^{6}\ \text{W}}{5.00 \times 10^{5}\ \text{V}} = 10.0\ \text{A}$

$\mathcal{P}_{\text{loss}} = I_{\text{rms}}^2 R = (10.0\ \text{A})^2 (290\ \Omega) = \boxed{29.0\ \text{kW}}$

(b) $\dfrac{\mathcal{P}_{\text{loss}}}{\mathcal{P}} = \dfrac{2.90 \times 10^{4}}{5.00 \times 10^{6}} = \boxed{5.80 \times 10^{-3}}$

(c) It is impossible to transmit so much power at such low voltage. Maximum power transfer occurs when load resistance equals the line resistance of 290 Ω, and is

$\dfrac{(4.50 \times 10^{3}\ \text{V})^2}{2 \cdot 2(290\ \Omega)} = 17.5$ kW, far below the required 5 000 kW.

Section 33.9 **Rectifiers and Filters**

P33.44 (a) Input power = 8 W

Useful output power $= I\Delta V = 0.3\ \text{A}(9\ \text{V}) = 2.7\ \text{W}$

$\text{efficiency} = \dfrac{\text{useful output}}{\text{total input}} = \dfrac{2.7\ \text{W}}{8\ \text{W}} = \boxed{0.34} = 34\%$

(b) Total input power = Total output power

$8\ \text{W} = 2.7\ \text{W} + \text{wasted power}$

$\text{wasted power} = \boxed{5.3\ \text{W}}$

(c) $E = \mathcal{P}\,\Delta t = 8\ \text{W}(6)(31\ \text{d})\left(\dfrac{86\,400\ \text{s}}{1\ \text{d}}\right)\left(\dfrac{1\ \text{J}}{1\ \text{Ws}}\right) = 1.29 \times 10^{8}\ \text{J}\left(\dfrac{\$0.135}{3.6 \times 10^{6}\ \text{J}}\right) = \boxed{\$4.8}$

P33.45 (a) The input voltage is $\Delta V_{\text{in}} = IZ = I\sqrt{R^2 + X_C^2} = I\sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}$. The output voltage is

$\Delta V_{\text{out}} = IR$. The gain ratio is $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{IR}{I\sqrt{R^2 + (1/\omega C)^2}} = \dfrac{R}{\sqrt{R^2 + (1/\omega C)^2}}$.

(b) As $\omega \to 0$, $\dfrac{1}{\omega C} \to \infty$ and $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} \to \boxed{0}$.

As $\omega \to \infty$, $\dfrac{1}{\omega C} \to 0$ and $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} \to \dfrac{R}{R} = \boxed{1}$.

(c) $\dfrac{1}{2} = \dfrac{R}{\sqrt{R^2 + (1/\omega C)^2}}$

$R^2 + \dfrac{1}{\omega^2 C^2} = 4R^2 \qquad \omega^2 C^2 = \dfrac{1}{3R^2} \qquad \omega = 2\pi f = \dfrac{1}{\sqrt{3}RC} \qquad \boxed{f = \dfrac{1}{2\pi\sqrt{3}RC}}$

P33.46 (a) The input voltage is $\Delta V_{\text{in}} = IZ = I\sqrt{R^2 + X_C^2} = I\sqrt{R^2 + (1/\omega C)^2}$. The output voltage is

$\Delta V_{\text{out}} = IX_C = \dfrac{I}{\omega C}$. The gain ratio is $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{I/\omega C}{I\sqrt{R^2 + (1/\omega C)^2}} = \dfrac{1/\omega C}{\sqrt{R^2 + (1/\omega C)^2}}$.

(b) As $\omega \to 0$, $\dfrac{1}{\omega C} \to \infty$ and R becomes negligible in comparison. Then $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} \to \dfrac{1/\omega C}{1/\omega C} = \boxed{1}$.

As $\omega \to \infty$, $\dfrac{1}{\omega C} \to 0$ and $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} \to \boxed{0}$.

(c) $\dfrac{1}{2} = \dfrac{1/\omega C}{\sqrt{R^2 + (1/\omega C)^2}}$ $\quad R^2 + \left(\dfrac{1}{\omega C}\right)^2 = \dfrac{4}{\omega^2 C^2}$ $\quad R^2\omega^2C^2 = 3$ $\quad \omega = 2\pi f = \dfrac{\sqrt{3}}{RC}$

$$\boxed{f = \frac{\sqrt{3}}{2\pi RC}}$$

P33.47 For this RC high-pass filter, $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \dfrac{R}{\sqrt{R^2 + X_C^2}}$.

(a) When $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = 0.500$,

then $\dfrac{0.500\ \Omega}{\sqrt{(0.500\ \Omega)^2 + X_C^2}} = 0.500$ or $X_C = 0.866\ \Omega$.

If this occurs at $f = 300$ Hz, the capacitance is

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(300\ \text{Hz})(0.866\ \Omega)}$$
$$= 6.13 \times 10^{-4}\ \text{F} = \boxed{613\ \mu\text{F}}$$

(b) With this capacitance and a frequency of 600 Hz,

$$X_C = \frac{1}{2\pi(600\ \text{Hz})(6.13 \times 10^{-4}\ \text{F})} = 0.433\ \Omega$$

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{0.500\ \Omega}{\sqrt{(0.500\ \Omega)^2 + (0.433\ \Omega)^2}} = \boxed{0.756}$$

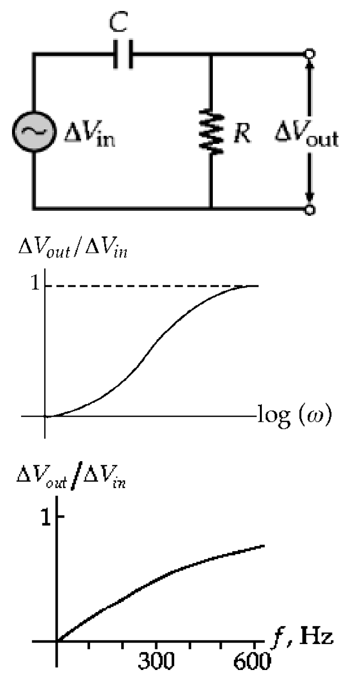


FIG. P33.47

P33.48 For the filter circuit, $\frac{\Delta V_{out}}{\Delta V_{in}} = \frac{X_C}{\sqrt{R^2 + X_C^2}}$.

(a) At $f = 600$ Hz, $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(600\text{ Hz})(8.00\times10^{-9}\text{ F})} = 3.32\times10^4\ \Omega$

and $\frac{\Delta V_{out}}{\Delta V_{in}} = \frac{3.32\times10^4\ \Omega}{\sqrt{(90.0\ \Omega)^2 + (3.32\times10^4\ \Omega)^2}} \approx \boxed{1.00}$

(b) At $f = 600$ kHz, $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(600\times10^3\text{ Hz})(8.00\times10^{-9}\text{ F})} = 33.2\ \Omega$

and $\frac{\Delta V_{out}}{\Delta V_{in}} = \frac{33.2\ \Omega}{\sqrt{(90.0\ \Omega)^2 + (33.2\ \Omega)^2}} = \boxed{0.346}$

P33.49 $\frac{\Delta v_{out}}{\Delta v_{in}} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$

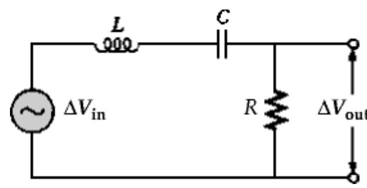


FIG. P33.49(a)

(a) At 200 Hz: $\frac{1}{4} = \frac{(8.00\ \Omega)^2}{(8.00\ \Omega)^2 + [400\pi L - 1/400\pi C]^2}$

At 4 000 Hz: $(8.00\ \Omega)^2 + \left[8\,000\pi L - \frac{1}{8\,000\pi C}\right]^2 = 4(8.00\ \Omega)^2$

At the low frequency, $X_L - X_C < 0$. This reduces to $400\pi L - \frac{1}{400\pi C} = -13.9\ \Omega$ [1]

For the high frequency half-voltage point, $8\,000\pi L - \frac{1}{8\,000\pi C} = +13.9\ \Omega$ [2]

Solving Equations (1) and (2) simultaneously gives $C = \boxed{54.6\ \mu\text{F}}$ and $L = \boxed{580\ \mu\text{H}}$

(b) When $X_L = X_C$, $\frac{\Delta v_{out}}{\Delta v_{in}} = \left(\frac{\Delta v_{out}}{\Delta v_{in}}\right)_{max} = \boxed{1.00}$

(c) $X_L = X_C$ requires $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(5.80\times10^{-4}\text{ H})(5.46\times10^{-5}\text{ F})}} = \boxed{894\text{ Hz}}$

(d) At 200 Hz, $\frac{\Delta v_{out}}{\Delta v_{in}} = \frac{R}{Z} = \frac{1}{2}$ and $X_C > X_L$,

so the phasor diagram is as shown:

$\phi = -\cos^{-1}\left(\frac{R}{Z}\right) = -\cos^{-1}\left(\frac{1}{2}\right)$ so

$\boxed{\Delta v_{out}\text{ leads }\Delta v_{in}\text{ by }60.0°}$.

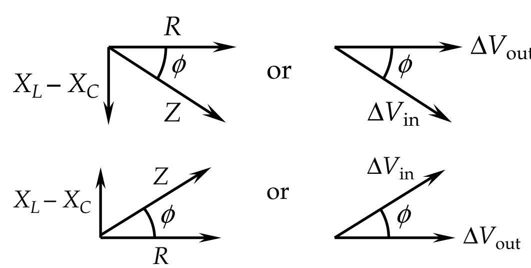


FIG. P33.49(d)

At f_0, $X_L = X_C$ so

$\boxed{\Delta v_{out}\text{ and }\Delta v_{in}\text{ have a phase difference of }0°}$.

At 4 000 Hz, $\frac{\Delta v_{out}}{\Delta v_{in}} = \frac{R}{Z} = \frac{1}{2}$ and $X_L - X_C > 0$

Thus, $\phi = \cos^{-1}\left(\frac{1}{2}\right) = 60.0°$

or $\boxed{\Delta v_{out}\text{ lags }\Delta v_{in}\text{ by }60.0°}$

continued on next page

(e) At 200 Hz and at 4 kHz,

$$\mathcal{P}=\frac{(\Delta v_{\text{out,rms}})^2}{R}=\frac{((1/2)\Delta v_{\text{in,rms}})^2}{R}=\frac{(1/2)[(1/2)\Delta v_{\text{in,max}}]^2}{R}=\frac{(10.0\text{ V})^2}{8(8.00\ \Omega)}=\boxed{1.56\text{ W}}$$

$$\text{At } f_0,\ \mathcal{P}=\frac{(\Delta v_{\text{out,rms}})^2}{R}=\frac{(\Delta v_{\text{in,rms}})^2}{R}=\frac{(1/2)[\Delta v_{\text{in,max}}]^2}{R}=\frac{(10.0\text{ V})^2}{2(8.00\ \Omega)}=\boxed{6.25\text{ W}}$$

(f) We take $Q=\dfrac{\omega_0 L}{R}=\dfrac{2\pi f_0 L}{R}=\dfrac{2\pi(894\text{ Hz})(5.80\times10^{-4}\text{ H})}{8.00\ \Omega}=\boxed{0.408}$

Additional Problems

P33.50 The equation for $\Delta v(t)$ during the first period (using $y=mx+b$) is:

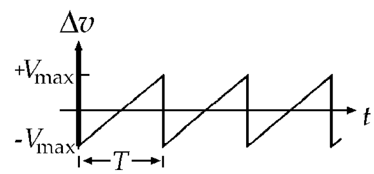


FIG. P33.50

$$\Delta v(t)=\frac{2(\Delta V_{\max})t}{T}-\Delta V_{\max}$$

$$\left[(\Delta v)^2\right]_{\text{avg}}=\frac{1}{T}\int_0^T[\Delta v(t)]^2\,dt=\frac{(\Delta V_{\max})^2}{T}\int_0^T\left[\frac{2}{T}t-1\right]^2 dt$$

$$\left[(\Delta v)^2\right]_{\text{avg}}=\frac{(\Delta V_{\max})^2}{T}\left(\frac{T}{2}\right)\frac{[2t/T-1]^3}{3}\Bigg|_{t=0}^{t=T}=\frac{(\Delta V_{\max})^2}{6}\left[(+1)^3-(-1)^3\right]=\frac{(\Delta V_{\max})^2}{3}$$

$$\Delta V_{\text{rms}}=\sqrt{\left[(\Delta v)^2\right]_{\text{avg}}}=\sqrt{\frac{(\Delta V_{\max})^2}{3}}=\boxed{\frac{\Delta V_{\max}}{\sqrt{3}}}$$

***P33.51** (a) $Z^2=R^2+(X_L-X_C)^2 \quad 760^2=400^2+(700-X_C)^2 \quad 417\,600=(700-X_C)^2$

There are two values for the square root. We can have $646.2=700-X_C$ or $-646.2=700-X_C$.

$\boxed{X_C \text{ can be } 53.8\ \Omega \text{ or it can be } 1.35\text{ k}\Omega.}$

(b) If we were below resonance, with inductive reactance 700 Ω and capacitive reactance 1.35 kΩ, raising the frequency would increase the power. We must be above resonance, with inductive reactance 700 Ω and $\boxed{\text{capacitive reactance } 53.8\ \Omega.}$

(c) $760^2=200^2+(700-X_C)^2 \quad 537\,600=(700-X_C)^2$ Here $+733=700-X_C$ has no solution so we must have $-733.2=700-X_C$ and $\boxed{X_C=1.43\text{ k}\Omega}$.

P33.52 The angular frequency is $\omega = 2\pi 60/\text{s} = 377/\text{s}$. When S is open, R, L, and C are in series with the source:

$$R^2 + (X_L - X_C)^2 = \left(\frac{\Delta V_s}{I}\right)^2 = \left(\frac{20\text{ V}}{0.183\text{ A}}\right)^2 = 1.194\times 10^4\ \Omega^2 \qquad (1)$$

When S is in position 1, a parallel combination of two R's presents equivalent resistance $\frac{R}{2}$, in series with L and C:

$$\left(\frac{R}{2}\right)^2 + (X_L - X_C)^2 = \left(\frac{20\text{ V}}{0.298\text{ A}}\right)^2 = 4.504\times 10^3\ \Omega^2 \qquad (2)$$

When S is in position 2, the current by passes the inductor. R and C are in series with the source:

$$R^2 + X_C^2 = \left(\frac{20\text{ V}}{0.137\text{ A}}\right)^2 = 2.131\times 10^4\ \Omega^2 \qquad (3)$$

Take equation (1) minus equation (2):

$$\frac{3}{4}R^2 = 7.440\times 10^3\ \Omega^2 \qquad \boxed{R = 99.6\ \Omega}$$

Only the positive root is physical. We have shown than only one resistance value is possible. Now equation (3) gives

$X_C = \left[2.131\times 10^4 - (99.6)^2\right]^{1/2}\ \Omega = 106.7\ \Omega = \frac{1}{\omega C}$ Only the positive root is physical and only one capacitance is possible.

$$C = (\omega X_C)^{-1} = \left[(377/\text{s})106.7\ \Omega\right]^{-1} = \boxed{2.49\times 10^{-5}\text{ F} = C}$$

Now equation (1) gives

$$X_L - X_C = \pm\left[1.194\times 10^4 - (99.6)^2\right]^{1/2}\ \Omega = \pm 44.99\ \Omega$$

$$X_L = 106.7\ \Omega + 44.99\ \Omega = 61.74\ \Omega \quad \text{or} \quad 151.7\ \Omega = \omega L$$

$$L = \frac{X_L}{\omega} = \boxed{0.164\text{ H or }0.402\text{ H} = L}$$

Two values for self-inductance are possible.

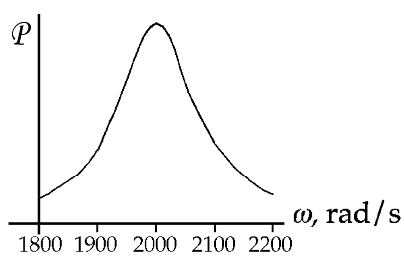
P33.53 $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.050\,0\text{ H})(5.00\times 10^{-6}\text{ F})}} = 2\,000\text{ s}^{-1}$$

so the operating angular frequency of the circuit is

$$\omega = \frac{\omega_0}{2} = 1\,000\text{ s}^{-1}$$

Using Equation 33.37, $\mathcal{P} = \frac{(\Delta V_{\text{rms}})^2 R\omega^2}{R^2\omega^2 + L^2(\omega^2 - \omega_0^2)^2}$

$$\mathcal{P} = \frac{(400)^2(8.00)(1\,000)^2}{(8.00)^2(1\,000)^2 + (0.050\,0)^2\left[(1.00 - 4.00)\times 10^6\right]^2} = \boxed{56.7\text{ W}}$$



$(Q \approx 12.5)$

FIG. P33.53

***P33.54** (a) At the resonance frequency X_L and X_C are equal. The certain frequency must be $\boxed{\text{higher}}$ than the resonance frequency for the inductive reactance to be the greater.

(b) $\boxed{\text{It is possible}}$ to determine the values for L and C, because we have three independent equations in the three unknowns L, C, and the unknown angular frequency ω.
The equations are
$2\,000^2 = 1/LC \quad 12 = \omega L \quad \text{and} \quad 8 = 1/\omega C$

(c) We eliminate $\omega = 12/L$ to have $8\,\omega C = 1 = 8(12/L)C = 96C/L$ so $L = 96C$
Then $4\,000\,000 = 1/96\,C^2$ so $\boxed{C = 51.0\ \mu\text{F and } L = 4.90\text{ mH}}$

***P33.55** The lowest-frequency standing-wave state is NAN. The distance between the clamps we represent as $d = d_{NN} = \dfrac{\lambda}{2}$. The speed of transverse waves on the string is $v = f\lambda = \sqrt{\dfrac{T}{\mu}} = f2d$.
The magnetic force on the wire oscillates at 60 Hz, so the wire will oscillate in resonance at 60 Hz.

$$\frac{T}{0.019\ \text{kg/m}} = (60/\text{s})^2\,4d^2 \qquad \boxed{T = \left(274\ \text{kg/m}\cdot\text{s}^2\right)d^2}$$

Any values of T and d related according to this expression will work, including $\boxed{\text{if } d = 0.200\text{ m} \quad T = 10.9\text{ N}}$. We did not need to use the value of the current and magnetic field. If we assume the subsection of wire in the field is 2 cm wide, we can find the rms value of the magnetic force:

$F_B = I\ell B\sin\theta = (9\text{ A})(0.02\text{ m})(0.015\,3T)\sin 90° = 2.75\text{ mN}$

So a small force can produce an oscillation of noticeable amplitude if internal friction is small.

***P33.56** $\phi = \tan^{-1}\left(\dfrac{\omega L - 1/\omega C}{R}\right)$ changes from $-90°$ for $\omega = 0$ to 0 at the resonance frequency to $+90°$ as ω goes to infinity.

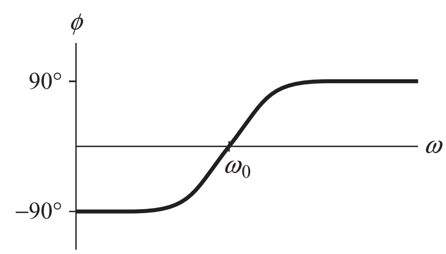


FIG. P33.56

The slope of the graph is $d\phi/d\omega$:

$$\frac{d\phi}{d\omega} = \frac{1}{1+\left(\dfrac{\omega L - 1/\omega C}{R}\right)^2}\frac{1}{R}\left(L - \frac{1}{C}(-1)\frac{1}{\omega^2}\right)$$ At resonance

we have $\omega_0 L = 1/\omega_0 C$ and $LC = 1/\omega_0^2$.

Substituting, the slope at the resonance point is

$$\left.\frac{d\phi}{d\omega}\right|_{\omega_0} = \frac{1}{1+0^2}\frac{1}{R}\left(L + \frac{1}{C}LC\right) = \frac{2L}{R} = \frac{2Q}{\omega_0}$$

P33.57 (a) When ωL is very large, the bottom branch carries negligible current. Also, $\dfrac{1}{\omega C}$ will be negligible compared to $200\ \Omega$ and $\dfrac{45.0\text{ V}}{200\ \Omega} = \boxed{225\text{ mA}}$ flows in the power supply and the top branch.

(b) Now $\dfrac{1}{\omega C} \to \infty$ and $\omega L \to 0$ so the generator and bottom branch carry $\boxed{450\text{ mA}}$.

P33.58 (a) With both switches closed, the current goes only through generator and resistor.

$$\boxed{i(t) = \frac{\Delta V_{\max}}{R}\cos\omega t}$$

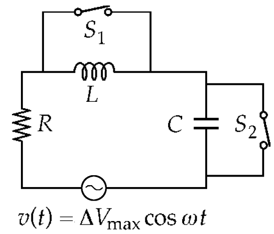


FIG. P33.58

(b) $$\boxed{\mathcal{P} = \frac{1}{2}\frac{(\Delta V_{\max})^2}{R}}$$

(c) $$\boxed{i(t) = \frac{\Delta V_{\max}}{\sqrt{R^2+\omega^2 L^2}}\cos\left[\omega t + \tan^{-1}\left(\frac{\omega L}{R}\right)\right]}$$

(d) For $$0 = \phi = \tan^{-1}\left(\frac{\omega_0 L - (1/\omega_0 C)}{R}\right)$$

We require $\omega_0 L = \dfrac{1}{\omega_0 C}$, so $\boxed{C = \dfrac{1}{\omega_0^2 L}}$

(e) At this resonance frequency, $Z = \boxed{R}$

(f) $$U = \frac{1}{2}C(\Delta V_C)^2 = \frac{1}{2}CI^2X_C^2$$

$$U_{\max} = \frac{1}{2}CI_{\max}^2X_C^2 = \frac{1}{2}C\frac{(\Delta V_{\max})^2}{R^2}\frac{1}{\omega_0^2C^2} = \boxed{\frac{(\Delta V_{\max})^2 L}{2R^2}}$$

(g) $$U_{\max} = \frac{1}{2}LI_{\max}^2 = \boxed{\frac{1}{2}L\frac{(\Delta V_{\max})^2}{R^2}}$$

(h) Now $\omega = 2\omega_0 = \dfrac{2}{\sqrt{LC}}$.

So $$\phi = \tan^{-1}\left(\frac{\omega L - (1/\omega C)}{R}\right) = \tan^{-1}\left(\frac{2\sqrt{L/C} - (1/2)\sqrt{L/C}}{R}\right) = \boxed{\tan^{-1}\left(\frac{3}{2R}\sqrt{\frac{L}{C}}\right)}.$$

(i) Now $\omega L = \dfrac{1}{2}\dfrac{1}{\omega C}$ $\qquad \omega = \boxed{\dfrac{1}{\sqrt{2LC}}} = \dfrac{\omega_0}{\sqrt{2}}$

P33.59 (a) $$I_{R,\text{rms}} = \frac{\Delta V_{\text{rms}}}{R} = \frac{100\text{ V}}{80.0\ \Omega} = \boxed{1.25\text{ A}}$$

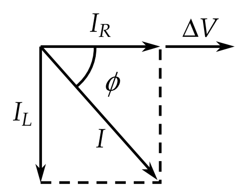


FIG. P33.59

(b) The total current will $\boxed{\text{lag}}$ the applied voltage as seen in the phasor diagram at the right.

$$I_{L,\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L} = \frac{100\text{ V}}{2\pi(60.0\text{ s}^{-1})(0.200\text{ H})} = 1.33\text{ A}$$

Thus, the phase angle is: $\phi = \tan^{-1}\left(\dfrac{I_{L,\text{rms}}}{I_{R,\text{rms}}}\right) = \tan^{-1}\left(\dfrac{1.33\text{ A}}{1.25\text{ A}}\right) = \boxed{46.7°}$.

P33.60 Suppose each of the 20 000 people uses an average power of 500 W. (This means 12 kWh per day, or \$36 per 30 days at 10¢per kWh.) Suppose the transmission line is at 20 kV. Then

$$I_{\text{rms}} = \frac{\mathcal{P}}{\Delta V_{\text{rms}}} = \frac{(20\,000)(500\text{ W})}{20\,000\text{ V}} \boxed{\sim 10^3\text{ A}}$$

If the transmission line had been at 200 kV, the current would be only $\boxed{\sim 10^2\text{ A}}$.

P33.61 $R = 200\ \Omega,\ L = 663\text{ mH},\ C = 26.5\ \mu\text{F},\ \omega = 377\text{ s}^{-1},\ \Delta V_{\max} = 50.0\text{ V}$

$$\omega L = 250\ \Omega,\ \left(\frac{1}{\omega C}\right) = 100\ \Omega,\ Z = \sqrt{R^2 + (X_L - X_C)^2} = 250\ \Omega$$

(a) $I_{\max} = \dfrac{\Delta V_{\max}}{Z} = \dfrac{50.0\text{ V}}{250\ \Omega} = \boxed{0.200\text{ A}}$

$\phi = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right) = \boxed{36.8°}$ (ΔV leads I)

(b) $\Delta V_{R,\max} = I_{\max} R = \boxed{40.0\text{ V}}$ at $\boxed{\phi = 0°}$

(c) $\Delta V_{C,\max} = \dfrac{I_{\max}}{\omega C} = \boxed{20.0\text{ V}}$ at $\boxed{\phi = -90.0°}$ (I leads ΔV)

(d) $\Delta V_{L,\max} = I_{\max}\omega L = \boxed{50.0\text{ V}}$ at $\boxed{\phi = +90.0°}$ (ΔV leads I)

P33.62 $L = 2.00\text{ H},\ C = 10.0\times 10^{-6}\text{ F},\ R = 10.0\ \Omega,\ \Delta v(t) = (100\sin\omega t)$

(a) The resonant frequency ω_0 produces the maximum current and thus the maximum power delivery to the resistor.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2.00)(10.0\times 10^{-6})}} = \boxed{224\text{ rad/s}}$$

(b) $\mathcal{P} = \dfrac{(\Delta V_{\max})^2}{2R} = \dfrac{(100)^2}{2(10.0)} = \boxed{500\text{ W}}$

(c) $I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{Z} = \dfrac{\Delta V_{\text{rms}}}{\sqrt{R^2 + (\omega L - (1/\omega C))^2}}$ and $(I_{\text{rms}})_{\max} = \dfrac{\Delta V_{\text{rms}}}{R}$

$I_{\text{rms}}^2 R = \dfrac{1}{2}(I_{\text{rms}}^2)_{\max} R$ or $\dfrac{(\Delta V_{\text{rms}})^2}{Z^2}R = \dfrac{1}{2}\dfrac{(\Delta V_{\text{rms}})^2}{R^2}R$

This occurs where $Z^2 = 2R^2$: $R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2 = 2R^2$

$\omega^4 L^2 C^2 - 2L\omega^2 C - R^2\omega^2 C^2 + 1 = 0$ or $L^2C^2\omega^4 - (2LC + R^2C^2)\omega^2 + 1 = 0$

$$\left[(2.00)^2(10.0\times 10^{-6})^2\right]\omega^4 - \left[2(2.00)(10.0\times 10^{-6}) + (10.0)^2(10.0\times 10^{-6})^2\right]\omega^2 + 1 = 0$$

Solving this quadratic equation, we find that $\omega^2 = 51\,130$, or $48\,894$

$\omega_1 = \sqrt{48\,894} = \boxed{221\text{ rad/s}}$ and $\omega_2 = \sqrt{51\,130} = \boxed{226\text{ rad/s}}$

P33.63 (a) From Equation 33.41, $\frac{N_1}{N_2}=\frac{\Delta V_1}{\Delta V_2}$.

Let input impedance $Z_1=\frac{\Delta V_1}{I_1}$ and the output impedance $Z_2=\frac{\Delta V_2}{I_2}$

so that $\frac{N_1}{N_2}=\frac{Z_1 I_1}{Z_2 I_2}$ But from Eq. 33.42, $\frac{I_1}{I_2}=\frac{\Delta V_2}{\Delta V_1}=\frac{N_2}{N_1}$

So, combining with the previous result we have $\boxed{\frac{N_1}{N_2}=\sqrt{\frac{Z_1}{Z_2}}}$.

(b) $\frac{N_1}{N_2}=\sqrt{\frac{Z_1}{Z_2}}=\sqrt{\frac{8\,000}{8.00}}=\boxed{31.6}$

P33.64 $\mathcal{P}=I_{\text{rms}}^2 R=\left(\frac{\Delta V_{\text{rms}}}{Z}\right)^2 R$, so $250\text{ W}=\frac{(120\text{ V})^2}{Z^2}(40.0\ \Omega)$: $Z=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$

$$250=\frac{(120)^2(40.0)}{(40.0)^2+\left[2\pi f(0.185)-\left[1/2\pi f\left(65.0\times10^{-6}\right)\right]\right]^2}$$ and

$$250=\frac{576\,000 f^2}{1\,600 f^2+\left(1.162\,4 f^2-2\,448.5\right)^2}$$

$$1=\frac{2\,304 f^2}{1\,600 f^2+1.351\,1 f^4-5\,692.3 f^2+5\,995\,300}$$ so $1.351\,1 f^4-6\,396.3 f^2+5\,995\,300=0$

$$f^2=\frac{6\,396.3\pm\sqrt{(6\,396.3)^2-4(1.351\,1)(5\,995\,300)}}{2(1.351\,1)}=3\,446.5\text{ or }1\,287.4$$

$f=\boxed{58.7\text{ Hz or }35.9\text{ Hz}}$ There are two answers because we could be above resonance or below resonance.

P33.65 $I_R=\frac{\Delta V_{\text{rms}}}{R}$; $I_L=\frac{\Delta V_{\text{rms}}}{\omega L}$; $I_C=\frac{\Delta V_{\text{rms}}}{(\omega C)^{-1}}$

(a) $I_{\text{rms}}=\sqrt{I_R^2+\left(I_C-I_L\right)^2}=\boxed{\Delta V_{\text{rms}}\sqrt{\left(\frac{1}{R^2}\right)+\left(\omega C-\frac{1}{\omega L}\right)^2}}$

(b) $\tan\phi=\frac{I_C-I_L}{I_R}=\Delta V_{\text{rms}}\left[\frac{1}{X_C}-\frac{1}{X_L}\right]\left(\frac{1}{\Delta V_{\text{rms}}/R}\right)$

$\boxed{\tan\phi=R\left[\frac{1}{X_C}-\frac{1}{X_L}\right]}$

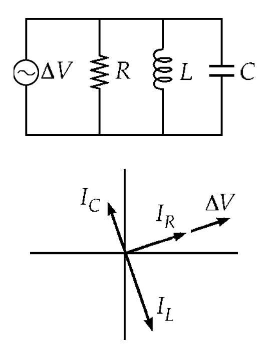


FIG. P33.65

***P33.66** An *RLC* series circuit, containing a 35.0-Ω resistor, a 205-mH inductor, a capacitor, and a power supply with rms voltage 200 V and frequency 100 Hz, carries rms current 4.00 A. Find the capacitance of the capacitor.

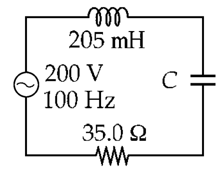


FIG. P33.66

We solve for C

$2500 = 35^2 + (129 - 1/628C)^2 \quad 1275 = (129 - 1/628C)^2$

There are two possibilities: $35.7 = 129 - 1/628C$ and $-35.7 = 129 - 1/628C$

$1/628C = 93.1$ or $1/628C = 164.5$

$$\boxed{C = \text{either } 17.1\ \mu\text{F or } 9.67\ \mu\text{F}}$$

P33.67 (a) $X_L = X_C = 1\,884\ \Omega$ when $f = 2\,000$ Hz

$$L = \frac{X_L}{2\pi f} = \frac{1\,884\ \Omega}{4\,000\pi\ \text{rad/s}} = 0.150\ \text{H} \quad \text{and}$$

$$C = \frac{1}{(2\pi f)X_C} = \frac{1}{(4\,000\pi\ \text{rad/s})(1\,884\ \Omega)} = 42.2\ \text{nF}$$

$$X_L = 2\pi f(0.150\ \text{H}) \qquad X_C = \frac{1}{(2\pi f)(4.22\times 10^{-8}\ \text{F})}$$

$$Z = \sqrt{(40.0\ \Omega)^2 + (X_L - X_C)^2}$$

f (Hz)	X_L (Ω)	X_C (Ω)	Z (Ω)
300	283	12 600	1 2300
600	565	6 280	5 720
800	754	4 710	3 960
1 000	942	3 770	2 830
1 500	1 410	2 510	1 100
2 000	1 880	1 880	40
3 000	2 830	1 260	1 570
4 000	3 770	942	2 830
6 000	5 650	628	5 020
10 000	9 420	377	9 040

(b) Impedance, Ω

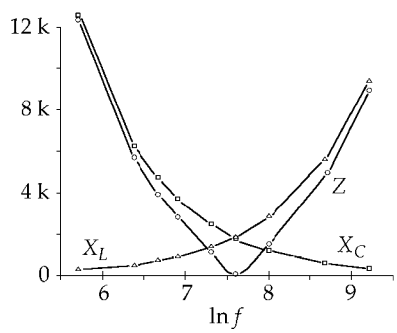


FIG. P33.67(b)

P33.68 $\omega_0 = \dfrac{1}{\sqrt{LC}} = 1.00 \times 10^6 \text{ rad/s}$

For each angular frequency, we find

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

then $I = \dfrac{1.00 \text{ V}}{Z}$

and $\mathcal{P} = I^2 (1.00\ \Omega)$

The full width at half maximum is:

$$\Delta f = \frac{\Delta\omega}{2\pi} = \frac{(1.000\,5 - 0.999\,5)\omega_0}{2\pi}$$

$$\Delta f = \frac{1.00 \times 10^3 \text{ s}^{-1}}{2\pi} = 159 \text{ Hz}$$

while

$$\frac{R}{2\pi L} = \frac{1.00\ \Omega}{2\pi(1.00 \times 10^{-3} \text{ H})} = 159 \text{ Hz}$$

$\dfrac{\omega}{\omega_0}$	$\omega L\ (\Omega)$	$\dfrac{1}{\omega C}\ (\Omega)$	$Z\ (\Omega)$	$\mathcal{P} = I^2 R$ (W)
0.9990	999.0	1001.0	2.24	0.19984
0.9991	999.1	1000.9	2.06	0.23569
0.9993	999.3	1000.7	1.72	0.33768
0.9995	999.5	1000.5	1.41	0.49987
0.9997	999.7	1000.3	1.17	0.73524
0.9999	999.9	1000.1	1.02	0.96153
1.0000	1000	1000.0	1.00	1.00000
1.0001	1000.1	999.9	1.02	0.96154
1.0003	1000.3	999.7	1.17	0.73535
1.0005	1000.5	999.5	1.41	0.50012
1.0007	1000.7	999.3	1.72	0.33799
1.0009	1000.9	999.1	2.06	0.23601
1.0010	1001	999.0	2.24	0.20016

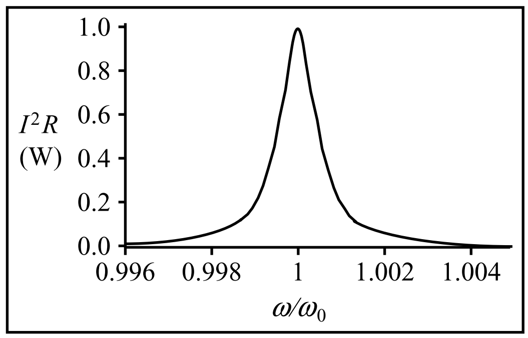


FIG. P33.68

***P33.69** (a) We can use $\sin A + \sin B = 2\sin\left(\dfrac{A}{2} + \dfrac{B}{2}\right)\cos\left(\dfrac{A}{2} - \dfrac{B}{2}\right)$ to find the sum of the two sine functions to be

$$E_1 + E_2 = (24.0 \text{ cm})\sin(4.5t + 35.0°)\cos 35.0°$$

$$E_1 + E_2 = (19.7 \text{ cm})\sin(4.5t + 35.0°)$$

Thus, the total wave has amplitude $\boxed{19.7 \text{ cm}}$ and has a constant phase difference of $\boxed{35.0°}$ from the first wave.

(b) In units of cm, the resultant phasor is

$$\vec{\mathbf{y}}_R = \vec{\mathbf{y}}_1 + \vec{\mathbf{y}}_2 = (12.0\hat{\mathbf{i}}) + (12.0\cos 70.0°\hat{\mathbf{i}} + 12.0\sin 70.0\hat{\mathbf{j}})$$
$$= 16.1\hat{\mathbf{i}} + 11.3\hat{\mathbf{j}}$$

$$\vec{\mathbf{y}}_R = \sqrt{(16.1)^2 + (11.3)^2} \text{ at } \tan^{-1}\left(\frac{11.3}{16.1}\right) = \boxed{19.7 \text{ cm at } 35.0°}$$

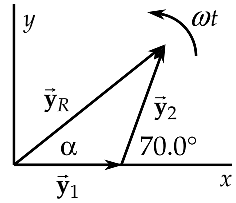


FIG. P33.69(b)

The answers are identical.

continued on next page

(c) $\vec{\mathbf{y}}_R = 12.0\cos 70.0°\hat{\mathbf{i}} + 12.0\sin 70.0°\hat{\mathbf{j}}$
$+15.5\cos 80.0°\hat{\mathbf{i}} - 15.5\sin 80.0°\hat{\mathbf{j}}$
$+17.0\cos 160°\hat{\mathbf{i}} + 17.0\sin 160°\hat{\mathbf{j}}$

$\vec{\mathbf{y}}_R = -9.18\hat{\mathbf{i}} + 1.83\hat{\mathbf{j}} = \boxed{9.36 \text{ cm at } 169°}$

The wave function of the total wave is
$y_R = (9.36 \text{ cm})\sin(15x - 4.5t + 169°)$.

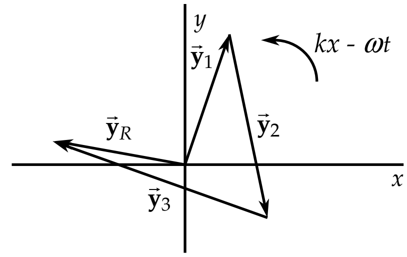


FIG. P33.69(c)

ANSWERS TO EVEN PROBLEMS

P33.2 (a) 193 Ω (b) 144 Ω

P33.4 (a) 25.3 rad/s (b) 0.114 s

P33.6 3.38 W

P33.8 3.14 A

P33.10 3.80 J

P33.12 (a) greater than 41.3 Hz (b) less than 87.5 Ω

P33.14 $\sqrt{2}C(\Delta V_{\text{rms}})$

P33.16 –32.0 A

P33.18 2.79 kHz

P33.20 (a) 109 Ω (b) 0.367 A (c) $I_{\max} = 0.367$ A, $\omega = 100$ rad/s, $\phi = -0.896$ rad

P33.22 19.3 mA

P33.24 (a) 146 V (b) 212 V (c) 179 V (d) 33.4 V

P33.26 Cutting the plate separation in half doubles the capacitance and cuts in half the capacitive reactance to $X_C/2$. The new impedance must be half as large as the old impedance for the new current to be doubled. For the new impedance we then have

$(R^2 + [R - X_C/2]^2)^{1/2} = 0.5(R^2 + [R - X_C]^2)^{1/2}$. Solving yields $X_C = 3R$.

P33.28 353 W

P33.30 (a) 5.43 A (b) 0.905 (c) 281 μF (d) 109 V

P33.32 (a) 0.936 (b) Not in practice. If the inductor were removed or if the generator were replaced with a battery, so that either $L = 0$ or $f = 0$, the power factor would be 1, but we would not have a magnetic buzzer. (c) 70.4 μF

P33.34 (a) 633 fF (b) 8.46 mm (c) 25.1 Ω

P33.36 (a) 3.56 kHz (b) 5.00 A (c) 22.4 (d) 2.24 kV

P33.38 $\dfrac{4\pi(\Delta V_{\text{rms}})^2 RC\sqrt{LC}}{4R^2C+9L}$

P33.40 (a) 9.23 V (b) 4.55 A (c) 42.0 W

P33.42 (a) 1 600 turns (b) 30.0 A (c) 25.3 A

P33.44 (a) 0.34 (b) 5.3 W (c) \$4.8

P33.46 (a) See the solution. (b) 1; 0 (c) $\dfrac{\sqrt{3}}{2\pi RC}$

P33.48 (a) 1.00 (b) 0.346

P33.50 See the solution.

P33.52 Only one value for R and only one value for C are possible. Two values for L are possible. $R = 99.6\ \Omega$, $C = 24.9\ \mu\text{F}$, and $L = 164$ mH or 402 mH

P33.54 (a) Higher. At the resonance frequency $X_L = X_C$. As the frequency increases, X_L goes up and X_C goes down. (b) It is. We have three independent equations in the three unknowns L, C, and the certain f. (c) $L = 4.90$ mH and $C = 51.0\ \mu\text{F}$

P33.56 See the solution.

P33.58 (a) $i(t) = \dfrac{\Delta V_{\text{max}}}{R}\cos\omega t$ (b) $\mathcal{P} = \dfrac{(\Delta V_{\text{max}})^2}{2R}$ (c) $i(t) = \dfrac{\Delta V_{\text{max}}}{\sqrt{R^2+\omega^2L^2}}\cos\left[\omega t + \tan^{-1}\left(\dfrac{\omega L}{R}\right)\right]$

(d) $C = \dfrac{1}{\omega_0^2 L}$ (e) $Z = R$ (f) $\dfrac{(\Delta V_{\text{max}})^2 L}{2R^2}$ (g) $\dfrac{(\Delta V_{\text{max}})^2 L}{2R^2}$ (h) $\tan^{-1}\left(\dfrac{3}{2R}\sqrt{\dfrac{L}{C}}\right)$ (i) $\dfrac{1}{\sqrt{2LC}}$

P33.60 $\sim 10^3$ A

P33.62 (a) 224 rad/s (b) 500 W (c) 221 rad/s and 226 rad/s

P33.64 The frequency could be either 58.7 Hz or 35.9 Hz. We can be either above or below resonance.

P33.66 An *RLC* series circuit, containing a 35.0-Ω resistor, a 205-mH inductor, a capacitor, and a power supply with rms voltage 200 V and frequency 100 Hz, carries rms current 4.00 A. Find the capacitance of the capacitor. Answer: It could be either 17.1 μF or 9.67 μF.

P33.68 See the solution.

34

Electromagnetic Waves

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

***Q34.1** Maxwell included a term in Ampère's law to account for the contributions to the magnetic field by changing electric fields, by treating those changing electric fields as "displacement currents."

***Q34.2** No, they do not. Specifically, Gauss's law in magnetism prohibits magnetic monopoles. If magnetic monopoles existed, then the magnetic field lines would not have to be closed loops, but could begin or terminate on a magnetic monopole, as they can in Gauss's law in electrostatics.

Q34.3 Radio waves move at the speed of light. They can travel around the curved surface of the Earth, bouncing between the ground and the ionosphere, which has an altitude that is small when compared to the radius of the Earth. The distance across the lower forty-eight states is approximately 5 000 km, requiring a transit time of $\frac{5\times10^6\text{ m}}{3\times10^8\text{ m/s}} \sim 10^{-2}\text{ s}$. To go halfway around the Earth takes only 0.07 s. In other words, a speech can be heard on the other side of the world before it is heard at the back of a large room.

Q34.4 Energy moves. No matter moves. You could say that electric and magnetic fields move, but it is nicer to say that the fields at one point stay at that point and oscillate. The fields vary in time, like sports fans in the grandstand when the crowd does the wave. The fields constitute the medium for the wave, and energy moves.

Q34.5 The changing magnetic field of the solenoid induces eddy currents in the conducting core. This is accompanied by I^2R conversion of electrically-transmitted energy into internal energy in the conductor.

***Q34.6** (i) According to $f = (2\pi)^{-1}\,(LC)^{-1/2}$, to make f half as large, the capacitance should be made four times larger. Answer (a).

(ii) Answer (b).

***Q34.7** Answer (e). Accelerating charge, changing electric field, or changing magnetic field can be the source of a radiated electromagnetic wave.

***Q34.8** (i) Answer (c). (ii) Answer (c). (iii) Answer (c). (iv) Answer (b). (v) Answer (b).

***Q34.9** (i) through (v) have the same answer (c).

Q34.10

Sound	*Light*
The world of sound extends to the top of the atmosphere and stops there; sound requires a material medium. Sound propagates by a chain reaction of density and pressure disturbances recreating each other. Sound in air moves at hundreds of meters per second. Audible sound has frequencies over a range of three decades (ten octaves) from 20 Hz to 20 kHz. Audible sound has wavelengths of ordinary size (1.7 cm to 17 m). Sound waves are longitudinal.	The universe of light fills the whole universe. Light moves through materials, but faster in a vacuum. Light propagates by a chain reaction of electric and magnetic fields recreating each other. Light in air moves at hundreds of millions of meters per second. Visible light has frequencies over a range of less than one octave, from 430 to 750 **Tera**hertz. Visible light has wavelengths of very small size (400 nm to 700 nm). Light waves are transverse.

Sound and light can both be reflected, refracted, or absorbed to produce internal energy. Both have amplitude and frequency set by the source, speed set by the medium, and wavelength set by both source and medium. Sound and light both exhibit the Doppler effect, standing waves, beats, interference, diffraction, and resonance. Both can be focused to make images. Both are described by wave functions satisfying wave equations. **Both carry energy.** If the source is small, their intensities both follow an inverse-square law. Both are waves.

Q34.11 The Poynting vector $\vec{\mathbf{S}}$ describes the energy flow associated with an electromagnetic wave. The direction of $\vec{\mathbf{S}}$ is along the direction of propagation and the magnitude of $\vec{\mathbf{S}}$ is the rate at which electromagnetic energy crosses a unit surface area perpendicular to the direction of $\vec{\mathbf{S}}$.

***Q34.12** (i) Answer (b). Electric and magnetic fields must both carry the same energy, so their amplitudes are proportional to each other. (ii) Answer (a). The intensity is proportional to the square of the amplitude.

***Q34.13** (i) Answer (c). Both the light intensity and the gravitational force follow inverse-square laws.

(ii) Answer (a). The smaller grain presents less face area and feels a smaller force due to light pressure.

Q34.14 Photons carry momentum. Recalling what we learned in Chapter 9, the impulse imparted to a particle that bounces elastically is twice that imparted to an object that sticks to a massive wall. Similarly, the impulse, and hence the pressure exerted by a photon reflecting from a surface must be twice that exerted by a photon that is absorbed.

Q34.15 Different stations have transmitting antennas at different locations. For best reception align your rabbit ears perpendicular to the straight-line path from your TV to the transmitting antenna. The transmitted signals are also polarized. The polarization direction of the wave can be changed by reflection from surfaces—including the atmosphere—and through Kerr rotation—a change in polarization axis when passing through an organic substance. In your home, the plane of polarization is determined by your surroundings, so antennas need to be adjusted to align with the polarization of the wave.

Q34.16 Consider a typical metal rod antenna for a car radio. The rod detects the electric field portion of the carrier wave. Variations in the amplitude of the incoming radio wave cause the electrons in the rod to vibrate with amplitudes emulating those of the carrier wave. Likewise, for frequency modulation, the variations of the frequency of the carrier wave cause constant-amplitude vibrations of the electrons in the rod but at frequencies that imitate those of the carrier.

***Q34.17** (i) Gamma rays have the shortest wavelength. The ranking is a < g < e < f < b < c < d.

(ii) Gamma rays have the highest frequency: d < c < b < f < e < g < a.

(iii) All electromagnetic waves have the same physical nature. a = b = c = d = e = f = g.

Q34.18 The frequency of EM waves in a microwave oven, typically 2.45 GHz, is chosen to be in a band of frequencies absorbed by water molecules. The plastic and the glass contain no water molecules. Plastic and glass have very different absorption frequencies from water, so they may not absorb any significant microwave energy and remain cool to the touch.

Q34.19 People of all the world's races have skin the same color in the infrared. When you blush or exercise or get excited, you stand out like a beacon in an infrared group picture. The brightest portions of your face show where you radiate the most. Your nostrils and the openings of your ear canals are bright; brighter still are just the pupils of your eyes.

Q34.20 Light bulbs and the toaster shine brightly in the infrared. Somewhat fainter are the back of the refrigerator and the back of the television set, while the TV screen is dark. The pipes under the sink show the same weak sheen as the walls until you turn on the faucets. Then the pipe on the right turns very black while that on the left develops a rich glow that quickly runs up along its length. The food on your plate shines; so does human skin, the same color for all races. Clothing is dark as a rule, but your bottom glows like a monkey's rump when you get up from a chair, and you leave behind a patch of the same blush on the chair seat. Your face shows you are lit from within, like a jack-o-lantern: your nostrils and the openings of your ear canals are bright; brighter still are just the pupils of your eyes.

Q34.21 12.2-cm waves have a frequency of 2.46 GHz. If the Q value of the phone is low (namely if it is cheap), and your microwave oven is not well shielded (namely, if it is also cheap), the phone can likely pick up interference from the oven. If the phone is well constructed and has a high Q value, then there should be no interference at all.

SOLUTIONS TO PROBLEMS

Section 34.1 Displacement Current and the General Form of Ampère's Law

***P34.1** (a) $$\frac{d\Phi_E}{dt} = \frac{dQ/dt}{\epsilon_0} = \frac{I}{\epsilon_0} = \frac{(0.100 \text{ A})}{8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2} = \boxed{11.3\times10^{9} \text{ V}\cdot\text{m/s}}$$

(b) $$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = I = \boxed{0.100 \text{ A}}$$

***P34.2** $$\frac{d\Phi_E}{dt} = \frac{d}{dt}(EA) = \frac{dQ/dt}{\epsilon_0} = \frac{I}{\epsilon_0}$$

(a) $$\frac{dE}{dt} = \frac{I}{\epsilon_0 A} = \boxed{7.19\times10^{11} \text{ V/m}\cdot\text{s}}$$

(b) $$\oint B\cdot ds = \epsilon_0 \mu_0 \frac{d\Phi_E}{dt} \text{ so } 2\pi rB = \epsilon_0 \mu_0 \frac{d}{dt}\left[\frac{Q}{\epsilon_0 A}\cdot \pi r^2\right]$$

$$B = \frac{\mu_0 Ir}{2A} = \frac{\mu_0 (0.200)\left(5.00\times10^{-2}\right)}{2\pi(0.100)^2} = \boxed{2.00\times10^{-7} \text{ T}}$$

***P34.3** We use the extended form of Ampère's law, Equation 34.7. Since no moving charges are present, $I = 0$ and we have

$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 \in_0 \frac{d\Phi_E}{dt}.$$

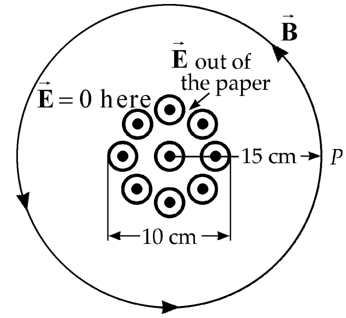


FIG. P34.3

In order to evaluate the integral, we make use of the symmetry of the situation. Symmetry requires that no particular direction from the center can be any different from any other direction. Therefore, there must be *circular symmetry* about the central axis. We know the magnetic field lines are circles about the axis. Therefore, as we travel around such a magnetic field circle, the magnetic field remains constant in magnitude. Setting aside until later the determination of the *direction* of $\vec{\mathbf{B}}$, we integrate $\oint \vec{\mathrm{B}} \cdot d\vec{l}$ around the circle

at $R = 0.15$ m

to obtain $2\pi RB$

Differentiating the expression $\Phi_E = AE$

we have $$\frac{d\Phi_E}{dt} = \left(\frac{\pi d^2}{4}\right)\frac{dE}{dt}$$

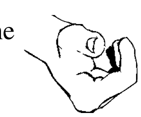
Thus, $$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = 2\pi RB = \mu_0 \in_0 \left(\frac{\pi d^2}{4}\right)\frac{dE}{dt}$$

Solving for B gives $$B = \frac{\mu_0 \in_0}{2\pi R}\left(\frac{\pi d^2}{4}\right)\frac{dE}{dt}$$

Substituting numerical values, $$B = \frac{\left(4\pi \times 10^{-7}\text{ H/m}\right)\left(8.85 \times 10^{-12}\text{ F/m}\right)\left[\pi (0.10\text{ m})^2\right](20\text{ V/m}\cdot\text{s})}{2\pi(0.15\text{ m})(4)}$$

$$B = \boxed{1.85 \times 10^{-18}\text{ T}}$$

In Figure P34.3, the direction of the *increase* of the electric field is out the plane of the paper. By the right-hand rule, this implies that the direction of $\vec{\mathbf{B}}$ is *counterclockwise*. Thus, the direction of $\vec{\mathbf{B}}$ at P is $\boxed{\text{upwards}}$.



Section 34.2 **Maxwell's Equations and Hertz's Discoveries**

P34.4 (a) The rod creates the same electric field that it would if stationary. We apply Gauss's law to a cylinder of radius $r = 20$ cm and length ℓ:

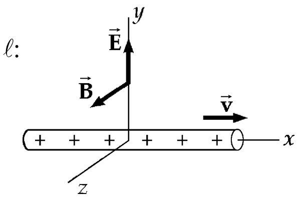


FIG. P34.4

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{inside}}}{\epsilon_0}$$

$$E(2\pi r l)\cos 0° = \frac{\lambda l}{\epsilon_0}$$

$$\vec{\mathbf{E}} = \frac{\lambda}{2\pi \epsilon_0 r} \text{ radially outward} = \frac{\left(35\times10^{-9}\ \text{C/m}\right)\ \text{N}\cdot\text{m}^2}{2\pi\left(8.85\times10^{-12}\ \text{C}^2\right)(0.2\ \text{m})}\hat{\mathbf{j}} = \boxed{3.15\times10^{3}\,\hat{\mathbf{j}}\ \text{N/C}}$$

(b) The charge in motion constitutes a current of $\left(35\times10^{-9}\ \text{C/m}\right)\left(15\times10^{6}\ \text{m/s}\right) = 0.525$ A. This current creates a magnetic field.

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(0.525\ \text{A})}{2\pi(0.2\ \text{m})}\hat{\mathbf{k}} = \boxed{5.25\times10^{-7}\,\hat{\mathbf{k}}\ \text{T}}$$

(c) The Lorentz force on the electron is $\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$

$$\vec{\mathbf{F}} = \left(-1.6\times10^{-19}\ \text{C}\right)\left(3.15\times10^{3}\,\hat{\mathbf{j}}\ \text{N/C}\right)$$
$$+\left(-1.6\times10^{-19}\ \text{C}\right)\left(240\times10^{6}\,\hat{\mathbf{i}}\ \text{m/s}\right)$$
$$\times\left(5.25\times10^{-7}\,\hat{\mathbf{k}}\ \frac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}}\right)$$
$$\vec{\mathbf{F}} = 5.04\times10^{-16}\left(-\hat{\mathbf{j}}\right)\ \text{N} + 2.02\times10^{-17}\left(+\hat{\mathbf{j}}\right)\ \text{N} = \boxed{4.83\times10^{-16}\left(-\hat{\mathbf{j}}\right)\ \text{N}}$$

***P34.5** $\vec{\mathbf{F}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$

$$\vec{\mathbf{a}} = \frac{e}{m}\left[\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}}\right] \text{ where } \vec{\mathbf{v}}\times\vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 200 & 0 & 0 \\ 0.200 & 0.300 & 0.400 \end{vmatrix} = -200(0.400)\hat{\mathbf{j}} + 200(0.300)\hat{\mathbf{k}}$$

$$\vec{\mathbf{a}} = \frac{1.60\times10^{-19}}{1.67\times10^{-27}}\left[50.0\hat{\mathbf{j}} - 80.0\hat{\mathbf{j}} + 60.0\hat{\mathbf{k}}\right] = 9.58\times10^{7}\left[-30.0\hat{\mathbf{j}} + 60.0\hat{\mathbf{k}}\right]$$

$$\vec{\mathbf{a}} = 2.87\times10^{9}\left[-\hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right]\ \text{m/s}^2 = \boxed{\left(-2.87\times10^{9}\,\hat{\mathbf{j}} + 5.75\times10^{9}\hat{\mathbf{k}}\right)\ \text{m/s}^2}$$

***P34.6** $\vec{\mathbf{F}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$ so $\vec{\mathbf{a}} = \frac{-e}{m}\left[\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}}\right]$ where $\vec{\mathbf{v}}\times\vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 10.0 & 0 & 0 \\ 0 & 0 & 0.400 \end{vmatrix} = -4.00\hat{\mathbf{j}}$

$$\vec{\mathbf{a}} = \frac{\left(-1.60\times10^{-19}\right)}{9.11\times10^{-31}}\left[2.50\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}} - 4.00\hat{\mathbf{j}}\right] = \left(-1.76\times10^{11}\right)\left[2.50\hat{\mathbf{i}} + 1.00\hat{\mathbf{j}}\right]$$

$$\vec{\mathbf{a}} = \boxed{\left(-4.39\times10^{11}\,\hat{\mathbf{i}} - 1.76\times10^{11}\,\hat{\mathbf{j}}\right)\text{m/s}^2}$$

Section 34.3 **Plane Electromagnetic Waves**

P34.7 (a) Since the light from this star travels at 3.00×10^8 m/s

the last bit of light will hit the Earth in $\dfrac{6.44\times10^{18}\ \text{m}}{3.00\times10^8\ \text{m/s}} = 2.15\times10^{10}\ \text{s} = 680$ years.

Therefore, it will disappear from the sky in the year $2\,007 + 680 = \boxed{2.69\times10^3\ \text{A.D.}}$
The star is 680 light-years away.

(b) $$\Delta t = \frac{\Delta x}{v} = \frac{1.496\times10^{11}\ \text{m}}{3\times10^8\ \text{m/s}} = \boxed{499\ \text{s}} = 8.31\ \text{min}$$

(c) $$\Delta t = \frac{\Delta x}{v} = \frac{2\left(3.84\times10^8\ \text{m}\right)}{3\times10^8\ \text{m/s}} = \boxed{2.56\ \text{s}}$$

(d) $$\Delta t = \frac{\Delta x}{v} = \frac{2\pi\left(6.37\times10^6\ \text{m}\right)}{3\times10^8\ \text{m/s}} = \boxed{0.133\ \text{s}}$$

(e) $$\Delta t = \frac{\Delta x}{v} = \frac{10\times10^3\ \text{m}}{3\times10^8\ \text{m/s}} = \boxed{3.33\times10^{-5}\ \text{s}}$$

P34.8 $$v = \frac{1}{\sqrt{\kappa\mu_0 \in_0}} = \frac{1}{\sqrt{1.78}}c = 0.750c = \boxed{2.25\times10^8\ \text{m/s}}$$

P34.9 (a) $f\lambda = c$ or $f(50.0\ \text{m}) = 3.00\times10^8\ \text{m/s}$

so $\boxed{f = 6.00\times10^6\ \text{Hz} = 6.00\ \text{MHz}}$

(b) $\dfrac{E}{B} = c$ or $\dfrac{22.0}{B_{max}} = 3.00\times10^8$

so $\vec{\mathbf{B}}_{max} = \boxed{-73.3\hat{\mathbf{k}}\ \text{nT}}$

(c) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{50.0} = 0.126\ \text{m}^{-1}$

and $\omega = 2\pi f = 2\pi\left(6.00\times10^6\ \text{s}^{-1}\right) = 3.77\times10^7$ rad/s

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_{max}\cos(kx-\omega t) = \boxed{-73.3\cos\left(0.126x - 3.77\times10^7 t\right)\hat{\mathbf{k}}\ \text{nT}}$$

P34.10 $\dfrac{E}{B} = c$ or $\dfrac{220}{B} = 3.00\times10^8$

so $B = 7.33\times10^{-7}\ \text{T} = \boxed{733\ \text{nT}}$

P34.11 (a) $B=\dfrac{E}{c}=\dfrac{100\text{ V/m}}{3.00\times10^{8}\text{ m/s}}=3.33\times10^{-7}\text{ T}=\boxed{0.333\ \mu\text{T}}$

(b) $\lambda=\dfrac{2\pi}{k}=\dfrac{2\pi}{1.00\times10^{7}\text{ m}^{-1}}=\boxed{0.628\ \mu\text{m}}$

(c) $f=\dfrac{c}{\lambda}=\dfrac{3.00\times10^{8}\text{ m/s}}{6.28\times10^{-7}\text{ m}}=\boxed{4.77\times10^{14}\text{ Hz}}$

P34.12 $E=E_{\max}\cos(kx-\omega t)$

$$\frac{\partial E}{\partial x}=-E_{\max}\sin(kx-\omega t)(k)$$
$$\frac{\partial E}{\partial t}=-E_{\max}\sin(kx-\omega t)(-\omega)$$
$$\frac{\partial^2 E}{\partial x^2}=-E_{\max}\cos(kx-\omega t)\left(k^2\right)$$
$$\frac{\partial^2 E}{\partial t^2}=-E_{\max}\cos(kx-\omega t)(-\omega)^2$$

We must show: $\dfrac{\partial E}{\partial x^2}=\mu_0\in_0\dfrac{\partial^2 E}{\partial t^2}$

That is, $-\left(k^2\right)E_{\max}\cos(kx-\omega t)=-\mu_0\in_0(-\omega)^2E_{\max}\cos(kx-\omega t)$

But this is true, because $\dfrac{k^2}{\omega^2}=\left(\dfrac{1}{f\lambda}\right)^2=\dfrac{1}{c^2}=\mu_0\in_0$

The proof for the wave of magnetic field follows precisely the same steps.

P34.13 In the fundamental mode, there is a single loop in the standing wave between the plates. Therefore, the distance between the plates is equal to half a wavelength.

$$\lambda=2L=2(2.00\text{ m})=4.00\text{ m}$$

Thus, $f=\dfrac{c}{\lambda}=\dfrac{3.00\times10^{8}\text{ m/s}}{4.00\text{ m}}=7.50\times10^{7}\text{ Hz}=\boxed{75.0\text{ MHz}}$

P34.14 $d_{\text{A to A}}=6\text{ cm}\pm5\%=\dfrac{\lambda}{2}$

$\lambda=12\text{ cm}\pm5\%$

$v=\lambda f=(0.12\text{ m}\pm5\%)\left(2.45\times10^{9}\text{ s}^{-1}\right)=\boxed{2.9\times10^{8}\text{ m/s}\pm5\%}$

Section 34.4 **Energy Carried by Electromagnetic Waves**

P34.15 $S = I = \frac{U}{At} = \frac{Uc}{V} = uc \qquad \frac{\text{Energy}}{\text{Unit Volume}} = u = \frac{I}{c} = \frac{1\,000\ \text{W/m}^2}{3.00\times10^8\ \text{m/s}} = \boxed{3.33\ \mu\text{J/m}^3}$

P34.16 $S_{\text{av}} = \frac{\overline{\mathcal{P}}}{4\pi r^2} = \frac{4.00\times10^3\ \text{W}}{4\pi(4.00\times1\,609\ \text{m})^2} = 7.68\ \mu\text{W/m}^2$

$E_{\max} = \sqrt{2\mu_0 c S_{\text{av}}} = 0.076\,1\ \text{V/m}$

$\Delta V_{\max} = E_{\max}L = (76.1\ \text{mV/m})(0.650\ \text{m}) = \boxed{49.5\ \text{mV (amplitude)}}$ or 35.0 mV (rms)

P34.17 $r = (5.00\ \text{mi})(1\,609\ \text{m/mi}) = 8.04\times10^3\ \text{m}$

$S = \frac{\overline{\mathcal{P}}}{4\pi r^2} = \frac{250\times10^3\ \text{W}}{4\pi(8.04\times10^3\ \text{W})^2} = \boxed{307\ \mu\text{W/m}^2}$

***P34.18** (a) $\frac{\mathcal{P}}{\text{area}} = \frac{\text{energy}}{\Delta t\cdot\text{area}} = \frac{600\ \text{kWh}}{(30\ \text{d})(13\ \text{m})(9.5\ \text{m})} = \frac{600\times10^3\ (\text{J/s})\text{h}}{30\ \text{d}(123.5\ \text{m}^2)}\left(\frac{1\ \text{d}}{24\ \text{h}}\right) = \boxed{6.75\ \text{W/m}^2}$

(b) The car uses gasoline at the rate $(55\ \text{mi/h})\left(\frac{\text{gal}}{25\ \text{mi}}\right)$. Its rate of energy conversion is

$\mathcal{P} = 44\times10^6\ \text{J/kg}\left(\frac{2.54\ \text{kg}}{1\ \text{gal}}\right)(55\ \text{mi/h})\left(\frac{\text{gal}}{25\ \text{mi}}\right)\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) = 6.83\times10^4\ \text{W}$. Its power-per-footprint-area is $\frac{\mathcal{P}}{\text{area}} = \frac{6.83\times10^4\ \text{W}}{2.10\ \text{m}(4.90\ \text{m})} = \boxed{6.64\times10^3\ \text{W/m}^2}$.

(c) For an automobile of typical weight and power to run on sunlight, it would have to carry a solar panel huge compared to its own size. Rather than running a conventional car, it is much more natural to use solar energy for agriculture, forestry, lighting, space heating, drying, water purification, water heating, and small appliances.

P34.19 Power output = (power input)(efficiency).

Thus, Power input $= \frac{\text{Power out}}{\text{eff}} = \frac{1.00\times10^6\ \text{W}}{0.300} = 3.33\times10^6\ \text{W}$

and $A = \frac{\mathcal{P}}{I} = \frac{3.33\times10^6\ \text{W}}{1.00\times10^3\ \text{W/m}^2} = \boxed{3.33\times10^3\ \text{m}^2}$

P34.20 $I = \frac{B_{\max}^2 c}{2\mu_0} = \frac{\mathcal{P}}{4\pi r^2}$

$B_{\max} = \sqrt{\left(\frac{\mathcal{P}}{4\pi r^2}\right)\left(\frac{2\mu_0}{c}\right)} = \sqrt{\frac{(10.0\times10^3)(2)(4\pi\times10^{-7})}{4\pi(5.00\times10^3)^2(3.00\times10^8)}} = \boxed{5.16\times10^{-10}\ \text{T}}$

Since the magnetic field of the Earth is approximately 5×10^{-5} T, the Earth's field is some 100 000 times stronger.

P34.21 (a) $\mathcal{P} = I^2 R = 150 \text{ W}$

$$A = 2\pi rL = 2\pi\left(0.900\times 10^{-3}\text{ m}\right)(0.080\ 0\text{ m}) = 4.52\times 10^{-4}\text{ m}^2$$

$$S = \frac{\mathcal{P}}{A} = \boxed{332\text{ kW/m}^2}\ \text{(points radially inward)}$$

(b) $$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0(1.00)}{2\pi\left(0.900\times 10^{-3}\right)} = \boxed{222\ \mu\text{T}}$$

$$E = \frac{\Delta V}{\Delta x} = \frac{IR}{L} = \frac{150\text{ V}}{0.080\ 0\text{ m}} = \boxed{1.88\text{ kV/m}}$$

Note: $$S = \frac{EB}{\mu_0} = 332\text{ kW/m}^2$$

P34.22 (a) $$I = \frac{E_{\max}^2}{2\mu_0 c} = \frac{\left(3\times 10^6\text{ V/m}\right)^2}{2\left(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A}\right)\left(3\times 10^8\text{ m/s}\right)}\left(\frac{\text{J}}{\text{V}\cdot\text{C}}\right)^2\left(\frac{\text{C}}{\text{A}\cdot\text{s}}\right)\left(\frac{\text{T}\cdot\text{C}\cdot\text{m}}{\text{N}\cdot\text{s}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right)$$

$$I = \boxed{1.19\times 10^{10}\text{ W/m}^2}$$

(b) $$\mathcal{P} = IA = \left(1.19\times 10^{10}\text{ W/m}^2\right)\pi\left(\frac{5\times 10^{-3}\text{ m}}{2}\right)^2 = \boxed{2.34\times 10^5\text{ W}}$$

P34.23 (a) $$\vec{\mathbf{E}}\cdot\vec{\mathbf{B}} = \left(80.0\hat{\mathbf{i}} + 32.0\hat{\mathbf{j}} - 64.0\hat{\mathbf{k}}\right)(\text{N/C})\cdot\left(0.200\hat{\mathbf{i}} + 0.080\ 0\hat{\mathbf{j}} + 0.290\hat{\mathbf{k}}\right)\mu\text{T}$$

$$\vec{\mathbf{E}}\cdot\vec{\mathbf{B}} = (16.0 + 2.56 - 18.56)\ \text{N}^2\cdot\text{s/C}^2\cdot\text{m} = \boxed{0}$$

(b) $$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} = \frac{\left[\left(80.0\hat{\mathbf{i}} + 32.0\hat{\mathbf{j}} - 64.0\hat{\mathbf{k}}\right)\text{ N/C}\right]\times\left[\left(0.200\hat{\mathbf{i}} + 0.080\ 0\hat{\mathbf{j}} + 0.290\hat{\mathbf{k}}\right)\mu\text{T}\right]}{4\pi\times 10^{-7}\text{ T}\cdot\text{m/A}}$$

$$\vec{\mathbf{S}} = \frac{\left(6.40\hat{\mathbf{k}} - 23.2\hat{\mathbf{j}} - 6.40\hat{\mathbf{k}} + 9.28\hat{\mathbf{i}} - 12.8\hat{\mathbf{j}} + 5.12\hat{\mathbf{i}}\right)\times 10^{-6}\text{ W/m}^2}{4\pi\times 10^{-7}}$$

$$\vec{\mathbf{S}} = \boxed{\left(11.5\hat{\mathbf{i}} - 28.6\hat{\mathbf{j}}\right)\text{ W/m}^2} = 30.9\text{ W/m}^2\text{ at }-68.2^\circ\text{ from the }+x\text{ axis}$$

P34.24 The energy put into the water in each container by electromagnetic radiation can be written as $e\mathcal{P}\Delta t = eIA\Delta t$ where e is the percentage absorption efficiency. This energy has the same effect as heat in raising the temperature of the water:

$$eIA\Delta t = mc\Delta T = \rho Vc\Delta T$$

$$\Delta T = \frac{eI\ell^2\Delta t}{\rho\ell^3 c} = \frac{eI\Delta t}{\rho\ell c}$$

where ℓ is the edge dimension of the container and c the specific heat of water. For the small container,

$$\Delta T = \frac{0.7\left(25\times 10^3\text{ W/m}^2\right)480\text{ s}}{\left(10^3\text{ kg/m}^3\right)(0.06\text{ m})4\ 186\text{ J/kg}\cdot{}^\circ\text{C}} = \boxed{33.4^\circ\text{C}}$$

For the larger,

$$\Delta T = \frac{0.91\left(25\text{ J/s}\cdot\text{m}^2\right)480\text{ s}}{\left(0.12/\text{m}^2\right)4\ 186\text{ J/}^\circ\text{C}} = \boxed{21.7^\circ\text{C}}$$

P34.25 (a) $B_{max} = \dfrac{E_{max}}{c}$: $B_{max} = \dfrac{7.00 \times 10^5 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = \boxed{2.33 \text{ mT}}$

(b) $I = \dfrac{E_{max}^2}{2\mu_0 c}$: $I = \dfrac{\left(7.00 \times 10^5\right)^2}{2\left(4\pi \times 10^{-7}\right)\left(3.00 \times 10^8\right)} = \boxed{650 \text{ MW/m}^2}$

(c) $I = \dfrac{\mathcal{P}}{A}$: $\mathcal{P} = IA = \left(6.50 \times 10^8 \text{ W/m}^2\right)\left[\dfrac{\pi}{4}\left(1.00 \times 10^{-3} \text{ m}\right)^2\right] = \boxed{510 \text{ W}}$

P34.26 (a) $E = cB = \left(3.00 \times 10^8 \text{ m/s}\right)\left(1.80 \times 10^{-6} \text{ T}\right) = \boxed{540 \text{ V/m}}$

(b) $u_{av} = \dfrac{B^2}{\mu_0} = \dfrac{\left(1.80 \times 10^{-6}\right)^2}{4\pi \times 10^{-7}} = \boxed{2.58 \ \mu\text{J/m}^3}$

(c) $S_{av} = cu_{av} = \left(3.00 \times 10^8\right)\left(2.58 \times 10^{-6}\right) = \boxed{773 \text{ W/m}^2}$

***P34.27** (a) We assume that the starlight moves through space without any of it being absorbed. The radial distance is

$$20 \text{ ly} = 20c(1 \text{ yr}) = 20\left(3 \times 10^8 \text{ m/s}\right)\left(3.156 \times 10^7 \text{ s}\right) = 1.89 \times 10^{17} \text{ m}$$

$$I = \frac{\mathcal{P}}{4\pi r^2} = \frac{4 \times 10^{28} \text{ W}}{4\pi\left(1.89 \times 10^{17} \text{ m}\right)^2} = \boxed{8.88 \times 10^{-8} \text{ W/m}^2}$$

(b) The Earth presents the projected target area of a flat circle:

$$\mathcal{P} = IA = \left(8.88 \times 10^{-8} \text{ W/m}^2\right)\pi\left(6.37 \times 10^6 \text{ m}\right)^2 = \boxed{1.13 \times 10^7 \text{ W}}$$

Section 34.5 **Momentum and Radiation Pressure**

***P34.28** (a) The radiation pressure is $\dfrac{2\left(1\,370 \text{ W/m}^2\right)}{3.00 \times 10^8 \text{ m/s}^2} = 9.13 \times 10^{-6} \text{ N/m}^2$.

Multiplying by the total area, $A = 6.00 \times 10^5 \text{ m}^2$ gives: $F = \boxed{5.48 \text{ N}}$.

(b) The acceleration is: $a = \dfrac{F}{m} = \dfrac{5.48 \text{ N}}{6\,000 \text{ kg}} = \boxed{9.13 \times 10^{-4} \text{ m/s}^2}$

(c) It will arrive at time t where $d = \dfrac{1}{2}at^2$

or $t = \sqrt{\dfrac{2d}{a}} = \sqrt{\dfrac{2\left(3.84 \times 10^8 \text{ m}\right)}{\left(9.13 \times 10^{-4} \text{ m/s}^2\right)}} = 9.17 \times 10^5 \text{ s} = \boxed{10.6 \text{ days}}$

P34.29 For complete absorption, $P = \dfrac{S}{c} = \dfrac{25.0}{3.00 \times 10^8} = \boxed{83.3 \text{ nPa}}$.

***P34.30** (a) The magnitude of the momentum transferred to the assumed totally reflecting surface in time t is $p = 2T_{ER}/c = 2SAt/c$. Then the vector momentum is

$$\vec{\mathbf{p}} = 2\vec{\mathbf{S}}At/c = 2(6\ \hat{\mathbf{i}}\ \text{W/m}^2)(40\times10^{-4}\ \text{m}^2)(1\ \text{s})/(3\times10^8\ \text{m/s})$$

$$= \boxed{1.60\times10^{-10}\,\hat{\mathbf{i}}\ \text{kg}\cdot\text{m/s each second}}$$

(b) The pressure on the assumed totally reflecting surface is $P = 2S/c$. Then the force is

$PA\hat{\mathbf{i}} = 2SA\hat{\mathbf{i}}/c = 2(6\ \text{W/m}^2)(40\times10^{-4}\ \text{m}^2)(1\ \text{s})/(3\times10^8\ \text{m/s}) = \boxed{1.60\times10^{-10}\,\hat{\mathbf{i}}\ \text{N}}$

(c) The answers are the same. Force is the time rate of momentum transfer.

P34.31 $I = \dfrac{\mathcal{P}}{\pi r^2} = \dfrac{E_{\max}^2}{2\mu_0 c}$

(a) $E_{\max} = \sqrt{\dfrac{\mathcal{P}(2\mu_0 c)}{\pi r^2}} = \boxed{1.90\ \text{kN/C}}$

(b) $\dfrac{15\times10^{-3}\ \text{J/s}}{3.00\times10^8\ \text{m/s}}(1.00\ \text{m}) = \boxed{50.0\ \text{pJ}}$

(c) $p = \dfrac{U}{c} = \dfrac{5\times10^{-11}}{3.00\times10^8} = \boxed{1.67\times10^{-19}\ \text{kg}\cdot\text{m/s}}$

***P34.32** (a) The light pressure on the absorbing Earth is $P = \dfrac{S}{c} = \dfrac{I}{c}$.

The force is $F = PA = \dfrac{I}{c}\left(\pi R^2\right) = \dfrac{(1370\ \text{W/m}^2)\pi(6.37\times10^6\ \text{m})^2}{3.00\times10^8\ \text{m/s}} = \boxed{5.82\times10^8\ \text{N}}$ away from the Sun.

(b) The attractive gravitational force exerted on Earth by the Sun is

$$F_g = \frac{GM_S M_M}{r_M^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times10^{30}\ \text{kg}\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left(1.496\times10^{11}\ \text{m}\right)^2}$$

$$= 3.55\times10^{22}\ \text{N}$$

which is $\boxed{6.10\times10^{13}\ \text{times stronger and in the opposite direction}}$ compared to the repulsive force in part (a).

***P34.33** (a) If $\mathcal{P}_S$ is the total power radiated by the Sun, and r_E and r_M are the radii of the orbits of the planets Earth and Mars, then the intensities of the solar radiation at these planets are:

$$I_E = \frac{\mathcal{P}_S}{4\pi r_E^2}$$

and $$I_M = \frac{\mathcal{P}_S}{4\pi r_M^2}$$

Thus, $$I_M = I_E\left(\frac{r_E}{r_M}\right)^2 = \left(1\,370\text{ W/m}^2\right)\left(\frac{1.496\times10^{11}\text{ m}}{2.28\times10^{11}\text{ m}}\right)^2 = \boxed{590\text{ W/m}^2}$$

(b) Mars intercepts the power falling on its circular face:

$$\mathcal{P}_M = I_M\left(\pi R_M^2\right) = \left(590\text{ W/m}^2\right)\left[\pi\left(3.37\times10^6\text{ m}\right)^2\right] = \boxed{2.10\times10^{16}\text{ W}}$$

(c) If Mars behaves as a perfect absorber, it feels pressure $P = \frac{S_M}{c} = \frac{I_M}{c}$

and force $$F = PA = \frac{I_M}{c}\left(\pi R_M^2\right) = \frac{\mathcal{P}_M}{c} = \frac{2.10\times10^{16}\text{ W}}{3.00\times10^8\text{ m/s}} = \boxed{7.01\times10^7\text{ N}}$$

(d) The attractive gravitational force exerted on Mars by the Sun is

$$F_g = \frac{GM_SM_M}{r_M^2} = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times10^{30}\text{ kg}\right)\left(6.42\times10^{23}\text{ kg}\right)}{\left(2.28\times10^{11}\text{ m}\right)^2}$$

$$= 1.64\times10^{21}\text{ N}$$

which is $\boxed{\sim10^{13}\text{ times stronger}}$ than the repulsive force of part (c).

(e) The relationship between the gravitational force and the light-pressure force is similar at very different distances because both forces follow inverse-square laws. The force ratios are not identical for the two planets because of their different radii and densities.

P34.34 The radiation pressure on the disk is $P = \frac{S}{c} = \frac{I}{c} = \frac{F}{A} = \frac{F}{\pi r^2}$.

Then $F = \frac{\pi r^2 I}{c}$.

Take torques about the hinge: $\sum \tau = 0$

$$H_x(0) + H_y(0) - mgr\sin\theta + \frac{\pi r^2 Ir}{c} = 0$$

$$\theta = \sin^{-1}\frac{\pi r^2 I}{mgc} = \sin^{-1}\frac{\pi(0.4\text{ m})^2\,10^7\text{ W s}^2\text{ s}}{(0.024\text{ kg})\text{ m}^2(9.8\text{ m})\left(3\times10^8\text{ m}\right)}\left(\frac{1\text{ kg m}^2}{1\text{ W s}^3}\right)$$

$$= \sin^{-1}0.0712 = \boxed{4.09°}$$

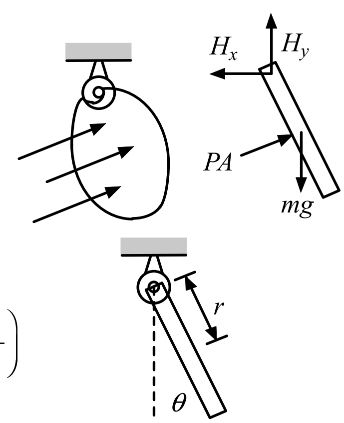


FIG. P34.34

Section 34.6 Production of Electromagnetic Waves by an Antenna

P34.35 $\lambda = \frac{c}{f} = 536\text{ m}$ so $h = \frac{\lambda}{4} = \boxed{134\text{ m}}$

$\lambda = \frac{c}{f} = 188\text{ m}$ so $h = \frac{\lambda}{4} = \boxed{46.9\text{ m}}$

P34.36 $\mathcal{P} = \frac{(\Delta V)^2}{R}$ or $\mathcal{P} \propto (\Delta V)^2$

$\Delta V = (-)E_y \cdot \Delta y = E_y \cdot \ell \cos\theta$

$\Delta V \propto \cos\theta$ so $\mathcal{P} \propto \cos^2\theta$

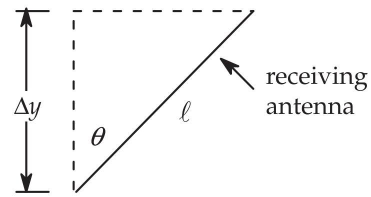


FIG. P34.36

(a) $\theta = 15.0°$: $\mathcal{P} = \mathcal{P}_{max}\cos^2(15.0°) = 0.933\mathcal{P}_{max} = \boxed{93.3\%}$

(b) $\theta = 45.0°$: $\mathcal{P} = \mathcal{P}_{max}\cos^2(45.0°) = 0.500\mathcal{P}_{max} = \boxed{50.0\%}$

(c) $\theta = 90.0°$: $\mathcal{P} = \mathcal{P}_{max}\cos^2(90.0°) = \boxed{0}$

P34.37 (a) Constructive interference occurs when $d\cos\theta = n\lambda$ for some integer n.

$$\cos\theta = n\frac{\lambda}{d} = n\left(\frac{\lambda}{\lambda/2}\right) = 2n$$

$$n = 0, \pm 1, \pm 2, \ldots$$

$\therefore$ $\boxed{\text{strong signal @ } \theta = \cos^{-1} 0 = 90°, 270°}$

(b) Destructive interference occurs when

$$d\cos\theta = \left(\frac{2n+1}{2}\right)\lambda: \qquad \cos\theta = 2n+1$$

$\therefore$ $\boxed{\text{weak signal @ } \theta = \cos^{-1}(\pm 1) = 0°, 180°}$

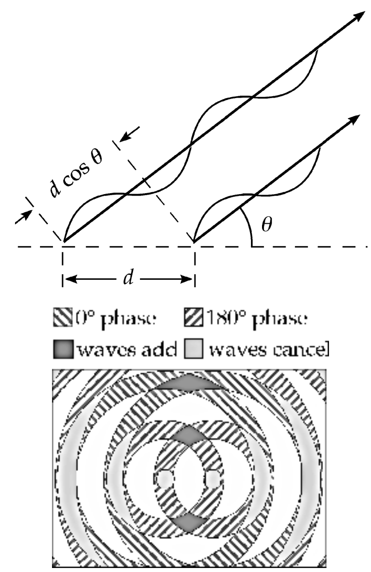


FIG. P34.37

P34.38 For the proton, $\Sigma F = ma$ yields $qvB\sin 90.0° = \frac{mv^2}{R}$

The period of the proton's circular motion is therefore: $T = \frac{2\pi R}{v} = \frac{2\pi m}{qB}$

The frequency of the proton's motion is $f = \frac{1}{T}$

The charge will radiate electromagnetic waves at this frequency, with $\lambda = \frac{c}{f} = cT = \boxed{\frac{2\pi mc}{qB}}$

P34.39 (a) The magnetic field $\vec{\mathbf{B}} = \frac{1}{2}\mu_0 J_{max}\cos(kx - \omega t)\hat{\mathbf{k}}$ applies for $x > 0$, since it describes a wave moving in the $\hat{\mathbf{i}}$ direction. The electric field direction must satisfy $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ as $\hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{k}}$ so the direction of the electric field is $\hat{\mathbf{j}}$ when the cosine is positive. For its magnitude we have $E = cB$, so altogether we have $\boxed{\vec{\mathbf{E}} = \frac{1}{2}\mu_0 c J_{max}\cos(kx - \omega t)\hat{\mathbf{j}}}$.

continued on next page

(b) $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} = \frac{1}{\mu_0}\frac{1}{4}\mu_0^2 c J_{max}^2 \cos^2(kx-\omega t)\hat{\mathbf{i}}$

$$\boxed{\vec{\mathbf{S}} = \frac{1}{4}\mu_0 c J_{max}^2 \cos^2(kx-\omega t)\hat{\mathbf{i}}}$$

(c) The intensity is the magnitude of the Poynting vector averaged over one or more cycles. The average of the cosine-squared function is $\frac{1}{2}$, so $\boxed{I = \frac{1}{8}\mu_0 c J_{max}^2}$.

(d) $J_{max} = \sqrt{\frac{8I}{\mu_0 c}} = \sqrt{\frac{8(570\ \text{W/m}^2)}{4\pi\times10^{-7}\ (\text{Tm/A})3\times10^8\ \text{m/s}}} = \boxed{3.48\ \text{A/m}}$

Section 34.7 The Spectrum of Electromagnetic Waves

P34.40 From the electromagnetic spectrum chart and accompanying text discussion, the following identifications are made:

Frequency, f	Wavelength, $\lambda = \frac{c}{f}$	Classification
$2\ \text{Hz} = 2\times10^0\ \text{Hz}$	150 Mm	Radio
$2\ \text{kHz} = 2\times10^3\ \text{Hz}$	150 km	Radio
$2\ \text{MHz} = 2\times10^6\ \text{Hz}$	150 m	Radio
$2\ \text{GHz} = 2\times10^9\ \text{Hz}$	15 cm	Microwave
$2\ \text{THz} = 2\times10^{12}\ \text{Hz}$	150 μm	Infrared
$2\ \text{PHz} = 2\times10^{15}\ \text{Hz}$	150 nm	Ultraviolet
$2\ \text{EHz} = 2\times10^{18}\ \text{Hz}$	150 pm	X-ray
$2\ \text{ZHz} = 2\times10^{21}\ \text{Hz}$	150 fm	Gamma ray
$2\ \text{YHz} = 2\times10^{24}\ \text{Hz}$	150 am	Gamma ray

Wavelengh, λ	Frequency, $f = \frac{c}{\lambda}$	Classification
$2\ \text{km} = 2\times10^3\ \text{m}$	1.5×10^5 Hz	Radio
$2\ \text{m} = 2\times10^0\ \text{m}$	1.5×10^8 Hz	Radio
$2\ \text{mm} = 2\times10^{-3}\ \text{m}$	1.5×10^{11} Hz	Microwave
$2\ \mu\text{m} = 2\times10^{-6}\ \text{m}$	1.5×10^{14} Hz	Infrared
$2\ \text{nm} = 2\times10^{-9}\ \text{m}$	1.5×10^{17} Hz	Ultraviolet or X-ray
$2\ \text{pm} = 2\times10^{-12}\ \text{m}$	1.5×10^{20} Hz	X-ray or Gamma ray
$2\ \text{fm} = 2\times10^{-15}\ \text{m}$	1.5×10^{23} Hz	Gamma ray
$2\ \text{am} = 2\times10^{-18}\ \text{m}$	1.5×10^{26} Hz	Gamma ray

P34.41 (a) $f\lambda = c$ gives $(5.00\times10^{19}\text{ Hz})\lambda = 3.00\times10^{8}\text{ m/s}$:

$\boxed{\lambda = 6.00\times10^{-12}\text{ m} = 6.00\text{ pm}}$

(b) $f\lambda = c$ gives $(4.00\times10^{9}\text{ Hz})\lambda = 3.00\times10^{8}\text{ m/s}$:

$\boxed{\lambda = 0.075\text{ m} = 7.50\text{ cm}}$

P34.42 (a) $f = \dfrac{c}{\lambda} = \dfrac{3\times10^{8}\text{ m/s}}{1.7\text{ m}}\ \boxed{\sim10^{8}\text{ Hz}}\ \boxed{\text{radio wave}}$

(b) 1 000 pages, 500 sheets, is about 3 cm thick so one sheet is about 6×10^{-5} m thick.

$f = \dfrac{3.00\times10^{8}\text{ m/s}}{6\times10^{-5}\text{ m}}\ \boxed{\sim10^{13}\text{ Hz}}\ \boxed{\text{infrared}}$

***P34.43** (a) Channel 4: $f_{\min} = 66\text{ MHz}$ $\lambda_{\max} = \boxed{4.55\text{ m}}$

$f_{\max} = 72\text{ MHz}$ $\lambda_{\min} = \boxed{4.17\text{ m}}$

(b) Channel 6: $f_{\min} = 82\text{ MHz}$ $\lambda_{\max} = \boxed{3.66\text{ m}}$

$f_{\max} = 88\text{ MHz}$ $\lambda_{\min} = \boxed{3.41\text{ m}}$

(c) Channel 8: $f_{\min} = 180\text{ MHz}$ $\lambda_{\max} = \boxed{1.67\text{ m}}$

$f_{\max} = 186\text{ MHz}$ $\lambda_{\min} = \boxed{1.61\text{ m}}$

P34.44 The time for the radio signal to travel 100 km is: $\Delta t_r = \dfrac{100\times10^{3}\text{ m}}{3.00\times10^{8}\text{ m/s}} = 3.33\times10^{-4}\text{ s}$

The sound wave travels 3.00 m across the room in: $\Delta t_s = \dfrac{3.00\text{ m}}{343\text{ m/s}} = 8.75\times10^{-3}\text{ s}$

Therefore, $\boxed{\text{listeners 100 km away}}$ will receive the news before the people in the newsroom by a total time difference of $\Delta t = 8.75\times10^{-3}\text{ s} - 3.33\times10^{-4}\text{ s} = 8.41\times10^{-3}\text{ s}$.

P34.45 The wavelength of an ELF wave of frequency 75.0 Hz is $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\text{ m/s}}{75.0\text{ Hz}} = 4.00\times10^{6}\text{ m}$.

The length of a quarter-wavelength antenna would be $L = 1.00\times10^{6}\text{ m} = \boxed{1.00\times10^{3}\text{ km}}$

or $L = (1\,000\text{ km})\left(\dfrac{0.621\text{ mi}}{1.00\text{ km}}\right) = \boxed{621\text{ mi}}$

Thus, while the project may be theoretically possible, it is not very practical.

Additional Problems

P34.46 $\omega = 2\pi f = 6.00\pi\times10^{9}\text{ s}^{-1} = 1.88\times10^{10}\text{ s}^{-1}$

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \frac{6.00\pi\times10^{9}\text{ s}^{-1}}{3.00\times10^{8}\text{ m/s}} = 20.0\pi = 62.8\text{ m}^{-1} \qquad B_{\max} = \frac{E}{c} = \frac{300\text{ V/m}}{3.00\times10^{8}\text{ m/s}} = 1.00\ \mu\text{T}$$

$$\boxed{E = (300\ \text{V/m})\cos\left(62.8x - 1.88\times10^{10}t\right)} \qquad \boxed{B = (1.00\ \mu\text{T})\cos\left(62.8x - 1.88\times10^{10}t\right)}$$

***P34.47** (a) $\mathcal{P} = SA$: $\quad \mathcal{P} = \left(1\,370\ \text{W/m}^2\right)\left[4\pi\left(1.496\times10^{11}\ \text{m}\right)^2\right] = \boxed{3.85\times10^{26}\ \text{W}}$

(b) $$S = \frac{cB_{\max}^2}{2\mu_0} \quad \text{so} \quad B_{\max} = \sqrt{\frac{2\mu_0 S}{c}} = \sqrt{\frac{2\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(1\,370\ \text{W/m}^2\right)}{3.00\times10^{8}\ \text{m/s}}} = \boxed{3.39\ \mu\text{T}}$$

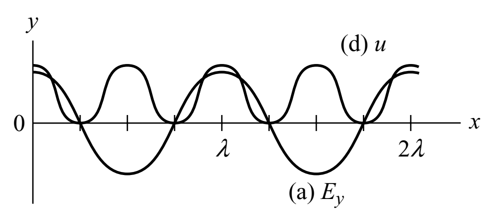
$$S = \frac{E_{\max}^2}{2\mu_0 c} \quad \text{so} \quad E_{\max} = \sqrt{2\mu_0 cS} = \sqrt{2\left(4\pi\times10^{-7}\right)\left(3.00\times10^{8}\right)\left(1\,370\right)} = \boxed{1.02\ \text{kV/m}}$$

***P34.48** Suppose you cover a 1.7 m × 0.3 m section of beach blanket. Suppose the elevation angle of the Sun is 60°. Then the target area you fill in the Sun's field of view is

$$(1.7\text{ m})(0.3\text{ m})\cos 30^\circ = 0.4\text{ m}^2$$

Now $I = \frac{\mathcal{P}}{A} = \frac{U}{At}$ $\qquad U = IAt = \left(1\,370\ \text{W/m}^2\right)\left[(0.6)(0.5)\left(0.4\ \text{m}^2\right)\right](3\,600\ \text{s})\ \boxed{\sim10^{6}\ \text{J}}$

***P34.49** (a)



(b) $u_E = \frac{1}{2}\in_0 E^2 = \boxed{\frac{1}{2}\in_0 E_{\max}^2\cos^2(kx)}$

(c) $$u_B = \frac{1}{2\mu_0}B^2 = \frac{1}{2\mu_0}B_{\max}^2\cos^2(kx) = \frac{1}{2\mu_0}\frac{E_{\max}^2}{c^2}\cos^2(kx) = \frac{\mu_0\in_0}{2\mu_0}E_{\max}^2\cos^2(kx) = \boxed{u_E}$$

(d) $u = u_E + u_B = \boxed{\in_0 E_{\max}^2\cos^2(kx)}$

(e) $$E_\lambda = \int_0^\lambda \in_0 E_{\max}^2\cos^2(kx)A\,dx = \int_0^\lambda \in_0 E_{\max}^2\left[\tfrac{1}{2}+\tfrac{1}{2}\cos(2kx)\right]A\,dx$$

$$= \tfrac{1}{2}\in_0 E_{\max}^2 A\,x\Big|_0^\lambda + \frac{\in_0 E_{\max}^2 A}{4k}\sin(2kx)\Big|_0^\lambda = \tfrac{1}{2}\in_0 E_{\max}^2 A\lambda + \frac{\in_0 E_{\max}^2 A}{4k}\left[\sin(4\pi)-\sin(0)\right]$$

$$= \boxed{\tfrac{1}{2}\in_0 E_{\max}^2 A\lambda}$$

(f) $$I = \frac{E_\lambda}{AT} = \frac{\in_0 E_{\max}^2 A\lambda}{2AT} = \boxed{\tfrac{1}{2}\in_0 c E_{\max}^2}$$

This result agrees with equation 34.24, $I = \frac{E_{\max}^2}{2\mu_0 c} = \frac{cE_{\max}^2}{2\mu_0 c^2} = \frac{cE_{\max}^2\mu_0\in_0}{2\mu_0}$.

P34.50 (a) $F_{\text{grav}} = \dfrac{GM_S m}{R^2} = \left(\dfrac{GM_S}{R^2}\right)\left[\rho\left(\dfrac{4}{3}\pi r^3\right)\right]$

where M_S = mass of Sun, r = radius of particle, and R = distance from Sun to particle.

Since $F_{rad} = \dfrac{S\pi r^2}{c}$,

$$\frac{F_{rad}}{F_{\text{grav}}} = \left(\frac{1}{r}\right)\left(\frac{3SR^2}{4cGM_S\rho}\right) \propto \frac{1}{r}$$

(b) From the result found in part (a), when $F_{\text{grav}} = F_{rad}$,

we have $r = \dfrac{3SR^2}{4cGM_S\rho}$

$$r = \frac{3\left(214\ \text{W/m}^2\right)\left(3.75\times10^{11}\ \text{m}\right)^2}{4\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times10^{30}\ \text{kg}\right)\left(1\,500\ \text{kg/m}^3\right)\left(3.00\times10^8\ \text{m/s}\right)}$$

$= \boxed{3.78\times10^{-7}\ \text{m}}$

P34.51 (a) $B_{\max} = \dfrac{E_{\max}}{c} = \boxed{6.67\times10^{-16}\ \text{T}}$

(b) $S_{\text{av}} = \dfrac{E_{\max}^2}{2\mu_0 c} = \boxed{5.31\times10^{-17}\ \text{W/m}^2}$

(c) $\mathcal{P} = S_{\text{av}}A = \boxed{1.67\times10^{-14}\ \text{W}}$

(d) $F = PA = \left(\dfrac{S_{\text{av}}}{c}\right)A = \boxed{5.56\times10^{-23}\ \text{N}}$ (approximately the weight of 3 000 hydrogen atoms!)

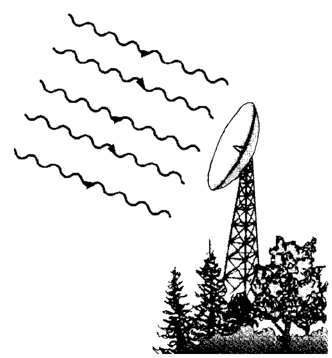


FIG. P34.51

***P34.52** (a) In $E = q/4\pi\epsilon_0 r^2$, the net flux is q/ϵ_0, so

$\vec{\mathbf{E}} = \Phi\,\hat{\mathbf{r}}/4\pi r^2 = (487\ \text{N}\cdot\text{m}^2/\text{C})\,\hat{\mathbf{r}}/4\pi r^2 = \boxed{(38.8/r^2)\,\hat{\mathbf{r}}\ \text{N}\cdot\text{m}^2/\text{C}}$

(b) The radiated intensity is $I = \mathcal{P}/4\pi r^2 = E^2_{\max}/2\mu_0 c$. Then

$E_{\max} = (\mathcal{P}\mu_0 c/2\pi)^{1/2}/r$

$= [(25\ \text{N}\cdot\text{m/s})(4\pi\times10^{-7}\ \text{T}\cdot\text{m}/2\pi\ \text{A})(3\times10^8\ \text{m/s})(1\ \text{N}\cdot\text{s}/1\ \text{T}\cdot\text{C}\cdot\text{m})(1\ \text{A}\cdot\text{s}/1\ \text{C})]^{1/2}/r$

$= \boxed{(38.7/r)\ \text{N}\cdot\text{m/C}}$

(c) For 3×10^6 N/C = (38.7 N·m/C)/r we find $r = \boxed{12.9\ \mu\text{m}}$, but the expression in part (b) does not apply if this point is inside the source.

(d) In the radiated wave, the field amplitude is inversely proportional to distance. As the distance doubles, the amplitude is cut in half and the intensity is reduced by a factor of 4. In the static case, the field is inversely proportional to the square of distance. As the distance doubles, the field is reduced by a factor of 4. The intensity of radiated energy is everywhere zero.

P34.53 $u = \frac{1}{2}\epsilon_0 E_{\max}^2$ $\quad E_{\max} = \sqrt{\frac{2u}{\epsilon_0}} = \boxed{95.1\ \text{mV/m}}$

P34.54 The area over which we model the antenna as radiating is the lateral surface of a cylinder,

$$A = 2\pi r\ell = 2\pi\left(4.00\times10^{-2}\ \text{m}\right)(0.100\ \text{m}) = 2.51\times10^{-2}\ \text{m}^2$$

(a) The intensity is then: $S = \frac{\mathcal{P}}{A} = \frac{0.600\ \text{W}}{2.51\times10^{-2}\ \text{m}^2} = \boxed{23.9\ \text{W/m}^2}$.

(b) The standard is

$$0.570\ \text{mW/cm}^2 = 0.570\left(\text{mW/cm}^2\right)\left(\frac{1.00\times10^{-3}\ \text{W}}{1.00\ \text{mW}}\right)\left(\frac{1.00\times10^4\ \text{cm}^2}{1.00\ \text{m}^2}\right) = 5.70\ \text{W/m}^2$$

While it is on, the telephone is over the standard by $\frac{23.9\ \text{W/m}^2}{5.70\ \text{W/m}^2} = \boxed{4.19\ \text{times}}$.

P34.55 (a) $B_{\max} = \frac{E_{\max}}{c} = \frac{175\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = \boxed{5.83\times10^{-7}\ \text{T}}$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.015\,0\ \text{m}} = \boxed{419\ \text{rad/m}}$ $\quad \omega = kc = \boxed{1.26\times10^{11}\ \text{rad/s}}$

Since $\vec{\mathbf{S}}$ is along x, and $\vec{\mathbf{E}}$ is along y, $\vec{\mathbf{B}}$ must be in $\boxed{\text{the } z \text{ direction}}$. (That is, $\vec{\mathbf{S}} \propto \vec{\mathbf{E}}\times\vec{\mathbf{B}}$.)

(b) $S_{av} = \frac{E_{\max}B_{\max}}{2\mu_0} = 40.6\ \text{W/m}^2$ $\quad \vec{\mathbf{S}}_{av} = \boxed{\left(40.6\ \text{W/m}^2\right)\hat{\mathbf{i}}}$

(c) $P_r = \frac{2S}{c} = \boxed{2.71\times10^{-7}\ \text{N/m}^2}$

(d) $a = \frac{\sum F}{m} = \frac{PA}{m} = \frac{\left(2.71\times10^{-7}\ \text{N/m}^2\right)\left(0.750\ \text{m}^2\right)}{0.500\ \text{kg}} = 4.06\times10^{-7}\ \text{m/s}^2$

$\vec{\mathbf{a}} = \boxed{\left(406\ \text{nm/s}^2\right)\hat{\mathbf{i}}}$

***P34.56** Of the intensity $\quad S = 1\,370\ \text{W/m}^2$

the 38.0% that is reflected exerts a pressure $\quad P_1 = \frac{2S_r}{c} = \frac{2(0.380)S}{c}$

The absorbed light exerts pressure $\quad P_2 = \frac{S_a}{c} = \frac{0.620S}{c}$

Altogether the pressure at the subsolar point on Earth is

(a) $P_{\text{total}} = P_1 + P_2 = \frac{1.38S}{c} = \frac{1.38\left(1\,370\ \text{W/m}^2\right)}{3.00\times10^8\ \text{m/s}} = \boxed{6.30\times10^{-6}\ \text{Pa}}$

(b) $\frac{P_a}{P_{\text{total}}} = \frac{1.01\times10^5\ \text{N/m}^2}{6.30\times10^{-6}\ \text{N/m}^2} = \boxed{1.60\times10^{10}\ \text{times smaller than atmospheric pressure}}$

P34.57 (a) $P = \frac{F}{A} = \frac{I}{c}$ $\qquad F = \frac{IA}{c} = \frac{\mathcal{P}}{c} = \frac{100 \text{ J/s}}{3.00 \times 10^8 \text{ m/s}} = 3.33 \times 10^{-7} \text{ N} = (110 \text{ kg})a$

$a = 3.03 \times 10^{-9} \text{ m/s}^2$ and $\qquad x = \frac{1}{2}at^2$

$$t = \sqrt{\frac{2x}{a}} = 8.12 \times 10^4 \text{ s} = \boxed{22.6 \text{ h}}$$

(b) $0 = (107 \text{ kg})v - (3.00 \text{ kg})(12.0 \text{ m/s} - v) = (107 \text{ kg})v - 36.0 \text{ kg} \cdot \text{m/s} + (3.00 \text{ kg})v$

$$v = \frac{36.0}{110} = 0.327 \text{ m/s} \qquad t = \boxed{30.6 \text{ s}}$$

P34.58 The mirror intercepts power $\mathcal{P} = I_1 A_1 = (1.00 \times 10^3 \text{ W/m}^2)\left[\pi (0.500 \text{ m})^2\right] = 785 \text{ W}$.
In the image,

(a) $I_2 = \frac{\mathcal{P}}{A_2}$: $\qquad I_2 = \frac{785 \text{ W}}{\pi (0.020\,0 \text{ m})^2} = \boxed{625 \text{ kW/m}^2}$

(b) $I_2 = \frac{E_{\max}^2}{2\mu_0 c}$ so $\qquad E_{\max} = \sqrt{2\mu_0 c I_2} = \sqrt{2(4\pi \times 10^{-7})(3.00 \times 10^8)(6.25 \times 10^5)}$

$$= \boxed{21.7 \text{ kN/C}}$$

$$B_{\max} = \frac{E_{\max}}{c} = \boxed{72.4 \ \mu\text{T}}$$

(c) $0.400 \ \mathcal{P} \Delta t = mc\Delta T$

$$0.400(785 \text{ W})\Delta t = (1.00 \text{ kg})(4\,186 \text{ J/kg} \cdot {}^\circ\text{C})(100^\circ\text{C} - 20.0^\circ\text{C})$$

$$\Delta t = \frac{3.35 \times 10^5 \text{ J}}{314 \text{ W}} = 1.07 \times 10^3 \text{ s} = \boxed{17.8 \text{ min}}$$

P34.59 Think of light going up and being absorbed by the bead which presents a face area πr_b^2

The light pressure is $P = \frac{S}{c} = \frac{I}{c}$.

(a) $F_\ell = \frac{I\pi r_b^2}{c} = mg = \rho \frac{4}{3}\pi r_b^3 g$ $\qquad$ and $\qquad I = \frac{4\rho g c}{3}\left(\frac{3m}{4\pi\rho}\right)^{1/3} = \boxed{8.32 \times 10^7 \text{ W/m}^2}$

(b) $\mathcal{P} = IA = (8.32 \times 10^7 \text{ W/m}^2)\pi(2.00 \times 10^{-3} \text{ m})^2 = \boxed{1.05 \text{ kW}}$

P34.60 Think of light going up and being absorbed by the bead, which presents face area πr_b^2 .

If we take the bead to be perfectly absorbing, the light pressure is $P = \frac{S_{\text{av}}}{c} = \frac{I}{c} = \frac{F_\ell}{A}$.

(a) $F_\ell = F_g$

so $$I = \frac{F_\ell c}{A} = \frac{F_g c}{A} = \frac{mgc}{\pi r_b^2}$$

From the definition of density, $\rho = \frac{m}{V} = \frac{m}{(4/3)\pi r_b^3}$

so $$\frac{1}{r_b} = \left(\frac{(4/3)\pi \rho}{m}\right)^{1/3}$$

Substituting for r_b , $$I = \frac{mgc}{\pi}\left(\frac{4\pi \rho}{3m}\right)^{2/3} = gc\left(\frac{4\rho}{3}\right)^{2/3}\left(\frac{m}{\pi}\right)^{1/3} = \boxed{\frac{4\rho gc}{3}\left(\frac{3m}{4\pi \rho}\right)^{1/3}}$$

(b) $\mathcal{P} = IA$ $$\mathcal{P} = \boxed{\frac{4\pi r^2 \rho gc}{3}\left(\frac{3m}{4\pi\rho}\right)^{1/3}}$$

P34.61 (a) On the right side of the equation, $\frac{\text{C}^2\left(\text{m/s}^2\right)^2}{\left(\text{C}^2/\text{N}\cdot\text{m}^2\right)\left(\text{m/s}\right)^3} = \frac{\text{N}\cdot\text{m}^2\cdot\text{C}^2\cdot\text{m}^2\cdot\text{s}^3}{\text{C}^2\cdot\text{s}^4\cdot\text{m}^3} = \frac{\text{N}\cdot\text{m}}{\text{s}} = \frac{\text{J}}{\text{s}} = \text{W}.$

(b) $F = ma = qE$ or $$a = \frac{qE}{m} = \frac{\left(1.60\times10^{-19}\ \text{C}\right)\left(100\ \text{N/C}\right)}{9.11\times10^{-31}\ \text{kg}} = \boxed{1.76\times10^{13}\ \text{m/s}^2}$$

The radiated power is then: $$\mathcal{P} = \frac{q^2a^2}{6\pi \in_0 c^3} = \frac{\left(1.60\times10^{-19}\right)^2\left(1.76\times10^{13}\right)^2}{6\pi\left(8.85\times10^{-12}\right)\left(3.00\times10^8\right)^3}$$
$$= \boxed{1.75\times10^{-27}\ \text{W}}$$

(c) $F = ma_c = m\left(\frac{v^2}{r}\right) = qvB$ so $v = \frac{qBr}{m}$

The proton accelerates at $$a = \frac{v^2}{r} = \frac{q^2B^2r}{m^2} = \frac{\left(1.60\times10^{-19}\right)^2(0.350)^2(0.500)}{\left(1.67\times10^{-27}\right)^2}$$
$$= 5.62\times10^{14}\ \text{m/s}^2$$

The proton then radiates $$\mathcal{P} = \frac{q^2a^2}{6\pi \in_0 c^3} = \frac{\left(1.60\times10^{-19}\right)^2\left(5.62\times10^{14}\right)^2}{6\pi\left(8.85\times10^{-12}\right)\left(3.00\times10^8\right)^3} = \boxed{1.80\times10^{-24}\ \text{W}}$$

P34.62 $f = 90.0\text{ MHz},\ E_{\max} = 2.00\times10^{-3}\text{ V/m} = 200\text{ mV/m}$

(a) $\lambda = \dfrac{c}{f} = \boxed{3.33\text{ m}}$

$T = \dfrac{1}{f} = 1.11\times10^{-8}\text{ s} = \boxed{11.1\text{ ns}}$

$B_{\max} = \dfrac{E_{\max}}{c} = 6.67\times10^{-12}\text{ T} = \boxed{6.67\text{ pT}}$

(b) $\boxed{\vec{\mathbf{E}} = (2.00\text{ mV/m})\cos 2\pi\left(\dfrac{x}{3.33\text{ m}} - \dfrac{t}{11.1\text{ ns}}\right)\hat{\mathbf{j}}}$

$\boxed{\vec{\mathbf{B}} = (6.67\text{ pT})\hat{\mathbf{k}}\cos 2\pi\left(\dfrac{x}{3.33\text{ m}} - \dfrac{t}{11.1\text{ ns}}\right)}$

(c) $I = \dfrac{E_{\max}^2}{2\mu_0 c} = \dfrac{(2.00\times10^{-3})^2}{2(4\pi\times10^{-7})(3.00\times10^{8})} = \boxed{5.31\times10^{-9}\text{ W/m}^2}$

(d) $I = cu_{av}$ so $u_{av} = \boxed{1.77\times10^{-17}\text{ J/m}^3}$

(e) $P = \dfrac{2I}{c} = \dfrac{(2)(5.31\times10^{-9})}{3.00\times10^{8}} = \boxed{3.54\times10^{-17}\text{ Pa}}$

P34.63 (a) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -\dfrac{d}{dt}(BA\cos\theta)$ $\mathcal{E} = -A\dfrac{d}{dt}(B_{\max}\cos\omega t\cos\theta) = AB_{\max}\omega(\sin\omega t\cos\theta)$

$\mathcal{E}(t) = 2\pi f B_{\max} A\sin 2\pi f\,t\cos\theta$ $\mathcal{E}(t) = 2\pi^2 r^2 f B_{\max}\cos\theta\sin 2\pi f\,t$

Thus, $\boxed{\mathcal{E}_{\max} = 2\pi^2 r^2 f\,B_{\max}\cos\theta}$

where θ is the angle between the magnetic field and the normal to the loop.

(b) If $\vec{\mathbf{E}}$ is vertical, $\vec{\mathbf{B}}$ is horizontal, so $\boxed{\text{the plane of the loop should be vertical}}$

and $\boxed{\text{the plane should contain the line of sight of the transmitter}}$.

P34.64 (a) $m=\rho V=\rho\frac{1}{2}\frac{4}{3}\pi r^3$

$$r=\left(\frac{6m}{\rho 4\pi}\right)^{1/3}=\left(\frac{6(8.7\text{ kg})}{(990\text{ kg/m}^3)4\pi}\right)^{1/3}=\boxed{0.161\text{ m}}$$

(b) $A=\frac{1}{2}4\pi r^2=2\pi(0.161\text{ m})^2=\boxed{0.163\text{ m}^2}$

(c) $I=e\sigma T^4=0.970\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)(304\text{ K})^4=\boxed{470\text{ W/m}^2}$

(d) $\mathcal{P}=IA=\left(470\text{ W/m}^2\right)0.163\text{ m}^2=\boxed{76.8\text{ W}}$

(e) $I=\frac{E_{\max}^2}{2\mu_0 c}$

$$E_{\max}=\left(2\mu_0 cI\right)^{1/2}=\left[\left(8\pi\times10^{-7}\text{ Tm/A}\right)\left(3\times10^8\text{ m/s}\right)\left(470\text{ W/m}^2\right)\right]^{1/2}=\boxed{595\text{ N/C}}$$

(f) $E_{\max}=cB_{\max}$

$$B_{\max}=\frac{595\text{ N/C}}{3\times10^8\text{ m/s}}=\boxed{1.98\ \mu\text{T}}$$

(g) The sleeping cats are uncharged and nonmagnetic. They carry no macroscopic current. They are a source of infrared radiation. They glow not by visible-light emission but by infrared emission.

(h) Each kitten has radius $r_k=\left(\frac{6(0.8)}{990\times4\pi}\right)^{1/3}=0.072\,8\text{ m}$ and radiating area $2\pi(0.072\,8\text{ m})^2=0.033\,3\text{ m}^2$. Eliza has area $2\pi\left(\frac{6(5.5)}{990\times4\pi}\right)^{2/3}=0.120\text{ m}$. The total glowing area is $0.120\text{ m}^2+4\left(0.033\,3\text{ m}^2\right)=0.254\text{ m}^2$ and has power output

$$\mathcal{P}=IA=\left(470\text{ W/m}^2\right)0.254\text{ m}^2=\boxed{119\text{ W}}$$

P34.65 (a) At steady state, $\mathcal{P}_{\text{in}}=\mathcal{P}_{\text{out}}$ and the power radiated out is $\mathcal{P}_{\text{out}}=e\sigma AT^4$.

Thus, $0.900\left(1\,000\text{ W/m}^2\right)A=0.700\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)AT^4$

or $T=\left[\frac{900\text{ W/m}^2}{0.700\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)}\right]^{1/4}=\boxed{388\text{ K}}=115°\text{C}$

(b) The box of horizontal area A presents projected area $A\sin50.0°$ perpendicular to the sunlight. Then by the same reasoning,

$$0.900\left(1\,000\text{ W/m}^2\right)A\sin50.0°=0.700\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)AT^4$$

or $T=\left[\frac{\left(900\text{ W/m}^2\right)\sin50.0°}{0.700\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)}\right]^{1/4}=\boxed{363\text{ K}}=90.0°\text{C}$

P34.66 We take R to be the planet's distance from its star. The planet, of radius r, presents a $\boxed{\text{projected area } \pi r^2}$ perpendicular to the starlight. $\boxed{\text{It radiates over area } 4\pi r^2.}$

At steady-state, $\mathcal{P}_{\text{in}} = \mathcal{P}_{\text{out}}$: $\quad eI_{\text{in}}\left(\pi r^2\right) = e\sigma\left(4\pi r^2\right)T^4$

$$e\left(\frac{6.00\times10^{23}\text{ W}}{4\pi R^2}\right)\left(\pi r^2\right) = e\sigma\left(4\pi r^2\right)T^4 \text{ so that } 6.00\times10^{23}\text{ W} = 16\pi\sigma R^2 T^4$$

$$R = \sqrt{\frac{6.00\times10^{23}\text{ W}}{16\pi\sigma T^4}} = \sqrt{\frac{6.00\times10^{23}\text{ W}}{16\pi\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)(310\text{ K})^4}} = \boxed{4.77\times10^9\text{ m} = 4.77\text{ Gm}}$$

ANSWERS TO EVEN PROBLEMS

P34.2 (a) 7.19×10^{11} V/m·s (b) 200 nT

P34.4 (a) $3.15\hat{\mathbf{j}}$ kN/C (b) 525 nT$\hat{\mathbf{k}}$ (c) $-483\hat{\mathbf{j}}$ aN

P34.6 $\left(-4.39\hat{\mathbf{i}} - 1.76\hat{\mathbf{j}}\right)10^{11}\text{ m/s}^2$

P34.8 2.25×10^8 m/s

P34.10 733 nT

P34.12 See the solution.

P34.14 $2.9\times10^8\text{ m/s}\pm5\%$

P34.16 49.5 mV

P34.18 (a) 6.75 W/m^2 (b) 6.64 kW/m^2 (c) A powerful automobile running on sunlight would have to carry on its roof a solar panel huge compared to the size of the car. Agriculture and forestry for food and fuels, space heating of large and small buildings, water heating, and heating for drying and many other processes are clear present and potential applications of solar energy.

P34.20 516 pT, $\sim10^5$ times stronger than the Earth's field

P34.22 (a) 11.9 GW/m^2 (b) 234 kW

P34.24 33.4°C for the smaller container and 21.7°C for the larger

P34.26 (a) 540 V/m (b) 2.58 μJ/m^3 (c) 773 W/m^2

P34.28 (a) 5.48 N away from the Sun (b) 913 μm/s^2 away from the Sun (c) 10.6 d

P34.30 (a) $1.60\times10^{-10}\ \hat{\mathbf{i}}$ kg·m/s each second (b) $1.60\times10^{-10}\ \hat{\mathbf{i}}$ N (c) The answers are the same. Force is the time rate of momentum transfer.

P34.32 (a) 582 MN away from the Sun (b) The gravitational force is 6.10×10^{13} times stronger and in the opposite direction.

P34.34 4.09°

P34.36 (a) 93.3% (b) 50.0% (c) 0

P34.38 $\dfrac{2\pi m_p c}{eB}$

P34.40 radio, radio, radio, radio or microwave, infrared, ultraviolet, x-ray, γ-ray, γ-ray; radio, radio, microwave, infrared, ultraviolet or x-ray, x- or γ-ray, γ-ray, γ-ray

P34.42 (a) $\sim 10^8$ Hz radio wave (b) $\sim 10^{13}$ Hz infrared light

P34.44 The radio audience gets the news 8.41 ms sooner.

P34.46 $E = (300\ \text{V/m})\cos(62.8x - 1.88\times 10^{10} t)$ $B = (1.00\ \mu\text{T})\cos(62.8x - 1.88\times 10^{10} t)$

P34.48 $\sim 10^6$ J

P34.50 (a) See the solution. (b) 378 nm

P34.52 (a) $\vec{\mathbf{E}} = (38.8/r^2)\,\hat{\mathbf{r}}$ N·m²/C (b) $E_{max} = (38.7/r)\ (\text{W}\cdot\text{T}\cdot\text{m}^2/\text{A}\cdot\text{s})^{1/2} = (38.7/r)$ N·m/C
(c) 12.9 μm, but the expression in part (b) does not apply if this point is inside the source.
(d) In the radiated wave, the field amplitude is inversely proportional to distance. As the distance doubles, the amplitude is cut in half and the intensity is reduced by a factor of 4. In the static case, the field is inversely proportional to the square of distance. As the distance doubles, the field is reduced by a factor of 4. The intensity of radiated energy is everywhere zero in the static case.

P34.54 (a) $23.9\ \text{W/m}^2$ (b) 4.19 times the standard

P34.56 (a) 6.30 μPa (b) 1.60×10^{10} times less than atmospheric pressure

P34.58 (a) $625\ \text{kW/m}^2$ (b) 21.7 kN/C and 72.4 μT (c) 17.8 min

P34.60 (a) $\left(\dfrac{16m\rho^2}{9\pi}\right)^{1/3} gc$ (b) $\left(\dfrac{16\pi^2 m\rho^2}{9}\right)^{1/3} r^2 gc$

P34.62 (a) 3.33 m, 11.1 ns, 6.67 pT (b) $\vec{\mathbf{E}} = (2.00\ \text{mV/m})\cos 2\pi\left(\dfrac{x}{3.33\ \text{m}} - \dfrac{t}{11.1\ \text{ns}}\right)\hat{\mathbf{j}}$;

$\vec{\mathbf{B}} = (6.67\ \text{pT})\hat{\mathbf{k}}\cos 2\pi\left(\dfrac{x}{3.33\ \text{m}} - \dfrac{t}{11.1\ \text{ns}}\right)$ (c) $5.31\ \text{nW/m}^2$ (d) $1.77\times 10^{-17}\ \text{J/m}^3$

(e) 3.54×10^{-17} Pa

P34.64 (a) 16.1 cm (b) 0.163 m² (c) $470\ \text{W/m}^2$ (d) 76.8 W (e) 595 N/C (f) 1.98 μT
(g) The cats are nonmagnetic and carry no macroscopic charge or current. Oscillating charges within molecules make them emit infrared radiation. (h) 119 W

P34.66 πr^2; $4\pi r^2$ where r is the radius of the planet; 4.77 Gm

35

The Nature of Light and the Laws of Geometric Optics

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q35.1 Light travels through a vacuum at a speed of 300 000 km per second. Thus, an image we see from a distant star or galaxy must have been generated some time ago. For example, the star Altair is 16 light-years away; if we look at an image of Altair today, we know only what was happening 16 years ago. This may not initially seem significant, but astronomers who look at other galaxies can gain an idea of what galaxies looked like when they were significantly younger. Thus, it actually makes sense to speak of "looking backward in time."

***Q35.2** 10^4 m/(3×10^8 m/s) is 33 μs. Answer (c).

***Q35.3** We consider the quantity λ / d. The smaller it is, the better the ray approximation works. In (a) it is like 0.34 m/1 m ≈ 0.3. In (b) we can have 0.7 μm/2 mm ≈ 0.000 3. In (c), 0.4 μm/2 mm ≈ 0.000 2. In (d), 300 m/1 m ≈ 300. In (e) 1 nm/1 mm ≈ 0.000 001. The ranking is then e, c, b, a, d.

Q35.4 With a vertical shop window, streetlights and his own reflection can impede the window shopper's clear view of the display. The tilted shop window can put these reflections out of the way. Windows of airport control towers are also tilted like this, as are automobile windshields.

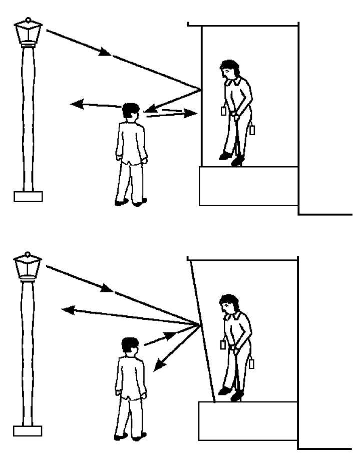


FIG. Q35.4

Q35.5 We assume that you and the child are always standing close together. For a flat wall to make an echo of a sound that you make, you must be standing along a normal to the wall. You must be on the order of 100 m away, to make the transit time sufficiently long that you can hear the echo separately from the original sound. Your sound must be loud enough so that you can hear it even at this considerable range. In the picture, the dashed rectangle represents an area in which you can be standing. The arrows represent rays of sound.

Now suppose two vertical perpendicular walls form an inside corner that you can see. Some of the sound you radiate horizontally will be headed generally toward the corner. It will reflect from both walls with high efficiency to reverse in direction and come back to you. You can stand anywhere reasonably far away to hear a retroreflected echo of sound you produce.

If the two walls are not perpendicular, the inside corner will not produce retroreflection. You will generally hear no echo of your shout or clap.

If two perpendicular walls have a reasonably narrow gap between them at the corner, you can still hear a clear echo. It is not the corner line itself that retroreflects the sound, but the perpendicular walls on both sides of the corner. Diagram (b) applies also in this case.

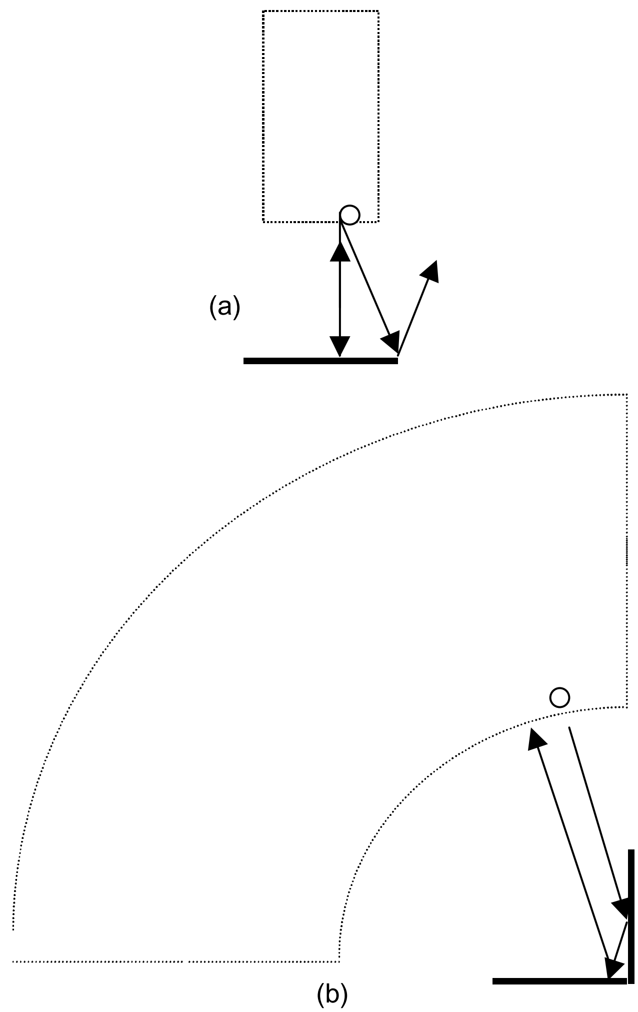


FIG. Q35.5

Q35.6 The stealth fighter is designed so that adjacent panels are not joined at right angles, to prevent any retroreflection of radar signals. This means that radar signals directed at the fighter will not be channeled back toward the detector by reflection. Just as with sound, radar signals can be treated as *diverging* rays, so that any ray that is by chance reflected back to the detector will be too weak in intensity to distinguish from background noise. This author is still waiting for the automotive industry to utilize this technology.

***Q35.7** Snell originally stated his law in terms of cosecants. From $v = c/n$ and $\sin\theta = 1/\csc\theta$ and $\lambda = c/nf$ with c and f constant between media, we conclude that a, b, and c are all correct statements.

Q35.8 An echo is an example of the reflection of sound. Hearing the noise of a distant highway on a cold morning, when you cannot hear it after the ground warms up, is an example of acoustical refraction. You can use a rubber inner tube inflated with helium as an acoustical lens to concentrate sound in the way a lens can focus light. At your next party, see if you can experimentally find the approximate focal point!

***Q35.9** (a) Yes. (b) No. (c) Yes. (d) No. If the light moves into a medium of higher refractive index, its wavelength decreases. The frequency remains constant. The speed diminishes by a factor equal to the index of refraction. If its angle of incidence is 0°, it will continue in the same direction.

Q35.10 If a laser beam enters a sugar solution with a concentration gradient (density and index of refraction increasing with depth) then the laser beam will be progressively bent downward (toward the normal) as it passes into regions of greater index of refraction.

***Q35.11** (a) Yes. It must be traveling in the medium in which it moves slower, water, to undergo total internal reflection.

(b) Yes. It must be traveling in the medium in which it moves slower, air, to undergo total internal reflection.

Q35.12 Diamond has higher index of refraction than glass and consequently a smaller critical angle for total internal reflection. A brilliant-cut diamond is shaped to admit light from above, reflect it totally at the converging facets on the underside of the jewel, and let the light escape only at the top. Glass will have less light internally reflected.

Q35.13 Highly silvered mirrors reflect about 98% of the incident light. With a 2-mirror periscope, that results in approximately a 4% decrease in intensity of light as the light passes through the periscope. This may not seem like much, but in low-light conditions, that lost light may mean the difference between being able to distinguish an enemy armada or an iceberg from the sky beyond. Using prisms results in total internal reflection, meaning that 100% of the incident light is reflected through the periscope. That is the "total" in total internal reflection.

***Q35.14** The light with the greater change in speed will have the larger deviation. Since the glass has a higher index than the surrounding air, A travels slower in the glass.

Q35.15 Immediately around the dark shadow of my head, I see a halo brighter than the rest of the dewy grass. It is called the *heiligenschein*. Cellini believed that it was a miraculous sign of divine favor pertaining to him alone. Apparently none of the people to whom he showed it told him that they could see halos around their own shadows but not around Cellini's. Thoreau knew that each person had his own halo. He did not draw any ray diagrams but assumed that it was entirely natural. Between Cellini's time and Thoreau's, the Enlightenment and Newton's explanation of the rainbow had happened. Today the effect is easy to see whenever your shadow falls on a retroreflecting traffic sign, license plate, or road stripe. When a bicyclist's shadow falls on a paint stripe marking the edge of the road, her halo races along with her. It is a shame that few people are sufficiently curious observers of the natural world to have noticed the phenomenon.

Q35.16 At the altitude of the plane the surface of the Earth need not block off the lower half of the rainbow. Thus, the full circle can be seen. You can see such a rainbow by climbing on a stepladder above a garden sprinkler in the middle of a sunny day. Set the sprinkler for fine mist. Do not let the slippery children fall from the ladder.

***Q35.17** Light from the lamps along the edges of the sheet enters the plastic. Then it is totally internally reflected by the front and back faces of the plastic, wherever the plastic has an interface with air. If the refractive index of the grease is intermediate between 1.55 and 1.00, some of this light can leave the plastic into the grease and leave the grease into the air. The surface of the grease is rough, so the grease can send out light in all directions. The customer sees the grease shining against a black background. The spotlight method of producing the same effect is much less efficient. With it, much of the light from the spotlight is absorbed by the blackboard. The refractive index of the grease must be less than 1.55. Perhaps the best choice would be $\sqrt{1.55 \times 1.00} = 1.24$.

***Q35.18** Answer (c). We want a big difference between indices of refraction to have total internal reflection under the widest range of conditions.

Q35.19 A mirage occurs when light changes direction as it moves between batches of air having different indices of refraction because they have different densities at different temperatures. When the sun makes a blacktop road hot, an apparent wet spot is bright due to refraction of light from the bright sky. The light, originally headed a little below the horizontal, always bends up as it first enters and then leaves sequentially hotter, lower-density, lower-index layers of air closer to the road surface.

SOLUTIONS TO PROBLEMS

Section 35.1 **The Nature of Light**

Section 35.2 **Measurements of the Speed of Light**

***P35.1** The Moon's radius is 1.74×10^6 m and the Earth's radius is 6.37×10^6 m. The total distance traveled by the light is:

$$d = 2\left(3.84\times10^8\text{ m} - 1.74\times10^6\text{ m} - 6.37\times10^6\text{ m}\right) = 7.52\times10^8\text{ m}$$

This takes 2.51 s, so $v = \dfrac{7.52\times10^8\text{ m}}{2.51\text{ s}} = 2.995\times10^8\text{ m/s} = \boxed{299.5\text{ Mm/s}}$. The sizes of the objects need to be taken into account. Otherwise the answer would be too large by 2%.

P35.2 $\Delta x = ct$; $\quad c = \dfrac{\Delta x}{t} = \dfrac{2\left(1.50\times10^8\text{ km}\right)\left(1\,000\text{ m/km}\right)}{(22.0\text{ min})(60.0\text{ s/min})} = 2.27\times10^8\text{ m/s} = \boxed{227\text{ Mm/s}}$

P35.3 The experiment is most convincing if the wheel turns fast enough to pass outgoing light through one notch and returning light through the next: $t = \dfrac{2\ell}{c}$

$$\theta = \omega t = \omega\left(\frac{2\ell}{c}\right) \quad \text{so} \quad \omega = \frac{c\theta}{2\ell} = \frac{\left(2.998\times10^8\right)\left[2\pi/(720)\right]}{2\left(11.45\times10^3\right)} = \boxed{114\text{ rad/s}}$$

The returning light would be blocked by a tooth at one-half the angular speed, giving another data point.

Section 35.3 **The Ray Approximation in Geometric Optics**

Section 35.4 **The Wave Under Reflection**

Section 35.5 **The Wave Under Refraction**

P35.4 (a) Let AB be the originally horizontal ceiling, BC its originally vertical normal, AD the new ceiling, and DE its normal. Then angle $BAD = \phi$. By definition DE is perpendicular to AD and BC is perpendicular to AB. Then the angle between DE extended and BC is ϕ because angles are equal when their sides are perpendicular, right side to right side and left side to left side.

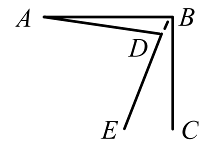


FIG. P35.4 (a)

(b) Now $CBE = \phi$ is the angle of incidence of the vertical light beam. Its angle of reflection is also ϕ. The angle between the vertical incident beam and the reflected beam is 2ϕ.

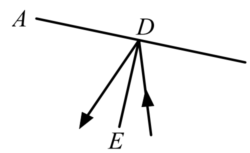


FIG. P35.4 (b)

(c) $\tan 2\phi = \dfrac{1.40\text{ cm}}{720\text{ cm}} = 0.001\,94$ $\boxed{\phi = 0.055\,7°}$

P35.5 (a) From geometry, $1.25\text{ m} = d\sin 40.0°$

so $d = \boxed{1.94\text{ m}}$

(b) $\boxed{50.0° \text{ above the horizontal}}$

or parallel to the incident ray.

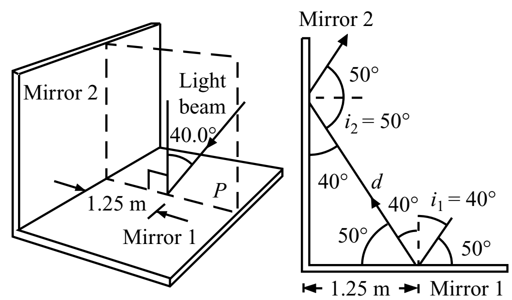


FIG. P35.5

P35.6 (a) **Method One:**

The incident ray makes angle $\alpha = 90° - \theta_1$ with the first mirror. In the picture, the law of reflection implies that

$$\theta_1 = \theta_1'$$

Then $\beta = 90° - \theta_1' = 90 - \theta_1 = \alpha$

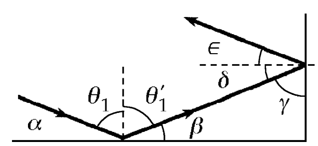


FIG. P35.6

In the triangle made by the mirrors and the ray passing between them,

$$\beta + 90° + \gamma = 180°$$

$$\gamma = 90° - \beta$$

Further, $\delta = 90° - \gamma = \beta = \alpha$

and $\in = \delta = \alpha$

Thus the final ray makes the same angle with the first mirror as did the incident ray. Its direction is opposite to the incident ray.

Method Two:

The vector velocity of the incident light has a component v_y perpendicular to the first mirror and a component v_x perpendicular to the second. The v_y component is reversed upon the first

continued on next page

reflection, which leaves v_x unchanged. The second reflection reverses v_x and leaves v_y unchanged. The doubly reflected ray then has velocity opposite to the incident ray.

(b) The incident ray has velocity $v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$. Each reflection reverses one component and leaves the other two unchanged. After all the reflections, the light has velocity $-v_x\hat{\mathbf{i}} - v_y\hat{\mathbf{j}} - v_z\hat{\mathbf{k}}$, opposite to the incident ray.

P35.7 Let d represent the perpendicular distance from the person to the mirror. The distance between lamp and person measured parallel to the mirror can be written in two ways: $2d\tan\theta + d\tan\theta = d\tan\phi$. The condition on the distance traveled by the light

is $\dfrac{2d}{\cos\phi} = \dfrac{2d}{\cos\theta} + \dfrac{d}{\cos\theta}$. We have the two

equations $3\tan\theta = \tan\phi$ and $2\cos\theta = 3\cos\phi$. To eliminate ϕ we write

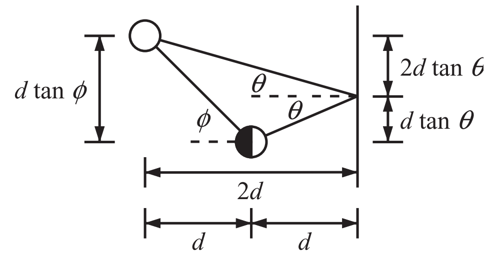


FIG. P35.7

$$\frac{9\sin^2\theta}{\cos^2\theta} = \frac{\sin^2\phi}{\cos^2\phi} \qquad 4\cos^2\theta = 9\cos^2\phi$$

$$9\cos^2\phi\sin^2\theta = \cos^2\theta\left(1-\cos^2\phi\right)$$

$$4\cos^2\theta\sin^2\theta = \cos^2\theta\left(1-\frac{4}{9}\cos^2\theta\right)$$

$$4\sin^2\theta = 1-\frac{4}{9}\left(1-\sin^2\theta\right) \qquad 36\sin^2\theta = 9-4+4\sin^2\theta$$

$$\sin^2\theta = \frac{5}{32} \qquad \theta = \boxed{23.3°}$$

***P35.8** The excess time the second pulse spends in the ice is 6.20 m/[(3.00 × 10^8 m/s)/1.309] = $\boxed{27.1 \text{ ns}}$

P35.9 Using Snell's law, $\sin\theta_2 = \dfrac{n_1}{n_2}\sin\theta_1$

$$\theta_2 = \boxed{25.5°}$$

$$\lambda_2 = \frac{\lambda_1}{n_1} = \boxed{442 \text{ nm}}$$

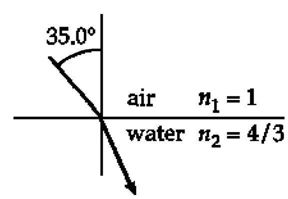


FIG. P35.9

***P35.10** The law of refraction $n_1 \sin\theta_1 = n_2 \sin\theta_2$ can be put into the more general form

$$\frac{c}{v_1}\sin\theta_1 = \frac{c}{v_2}\sin\theta_2$$

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

In this form it applies to all kinds of waves that move through space.

$$\frac{\sin 3.5°}{343\ \text{m/s}} = \frac{\sin\theta_2}{1\,493\ \text{m/s}}$$

$$\sin\theta_2 = 0.266$$

$$\theta_2 = \boxed{15.4°}$$

The wave keeps constant frequency in

$$f = \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$

$$\lambda_2 = \frac{v_2\lambda_1}{v_1} = \frac{1\,493\ \text{m/s}(0.589\ \text{m})}{343\ \text{m/s}} = \boxed{2.56\ \text{m}}$$

The light wave slows down as it moves from air into water but the sound speeds up by a large factor. The light wave bends toward the normal and its wavelength shortens, but the sound wave bends away from the normal and its wavelength increases.

P35.11 $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\sin\theta_1 = 1.333\sin 45°$$

$$\sin\theta_1 = (1.33)(0.707) = 0.943$$

$$\theta_1 = 70.5° \to \boxed{19.5°\ \text{above the horizon}}$$

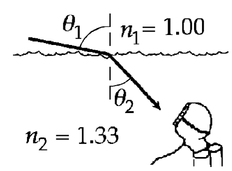


FIG. P35.11

P35.12 (a) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}\ \text{m/s}}{6.328\times10^{-7}\ \text{m}} = \boxed{4.74\times10^{14}\ \text{Hz}}$

(b) $\lambda_{\text{glass}} = \dfrac{\lambda_{\text{air}}}{n} = \dfrac{632.8\ \text{nm}}{1.50} = \boxed{422\ \text{nm}}$

(c) $v_{\text{glass}} = \dfrac{c_{\text{air}}}{n} = \dfrac{3.00\times10^{8}\ \text{m/s}}{1.50} = 2.00\times10^{8}\ \text{m/s} = \boxed{200\ \text{Mm/s}}$

P35.13 We find the angle of incidence:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$1.333\sin\theta_1 = 1.52\sin 19.6°$$

$$\theta_1 = 22.5°$$

The angle of reflection of the beam in water is then also $\boxed{22.5°}$.

***P35.14** (a) As measured from the diagram, the incidence angle is 60°, and the refraction angle is 35°.

From Snell's law, $\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$, then $\frac{\sin 35°}{\sin 60°} = \frac{v_2}{c}$ and the speed of light in the block is $\boxed{2.0\times10^8\ \text{m/s}}$.

(b) The frequency of the light does not change upon refraction. Knowing the wavelength in a vacuum, we can use the speed of light in a vacuum to determine the frequency: $c = f\lambda$,

thus $3.00\times10^8 = f\left(632.8\times10^{-9}\right)$, so the frequency is $\boxed{474.1\text{ THz}}$.

(c) To find the wavelength of light in the block, we use the same wave speed relation, $v = f\lambda$,

so $2.0\times10^8 = \left(4.741\times10^{14}\right)\lambda$, so $\lambda_{\text{glass}} = \boxed{420\text{ nm}}$.

P35.15 (a) Flint Glass: $v = \frac{c}{n} = \frac{3.00\times10^8\ \text{m/s}}{1.66} = 1.81\times10^8\ \text{m/s} = \boxed{181\ \text{Mm/s}}$

(b) Water: $v = \frac{c}{n} = \frac{3.00\times10^8\ \text{m/s}}{1.333} = 2.25\times10^8\ \text{m/s} = \boxed{225\ \text{Mm/s}}$

(c) Cubic Zirconia: $v = \frac{c}{n} = \frac{3.00\times10^8\ \text{m/s}}{2.20} = 1.36\times10^8\ \text{m/s} = \boxed{136\ \text{Mm/s}}$

***P35.16** From Snell's law, $\sin\theta = \left(\frac{n_{\text{medium}}}{n_{\text{liver}}}\right)\sin 50.0°$

But $\frac{n_{\text{medium}}}{n_{\text{liver}}} = \frac{c/v_{\text{medium}}}{c/v_{\text{liver}}} = \frac{v_{\text{liver}}}{v_{\text{medium}}} = 0.900$

so $\theta = \sin^{-1}\left[(0.900)\sin 50.0°\right] = 43.6°$

From the law of reflection,

$d = \frac{12.0\text{ cm}}{2} = 6.00\text{ cm}$, and

$h = \frac{d}{\tan\theta} = \frac{6.00\text{ cm}}{\tan(43.6°)} = \boxed{6.30\text{ cm}}$

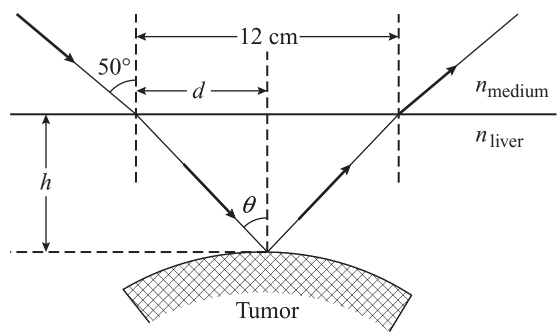


FIG. P35.16

P35.17 $n_1\sin\theta_1 = n_2\sin\theta_2$: $\theta_2 = \sin^{-1}\left(\frac{n_1\sin\theta_1}{n_2}\right)$

$$\theta_2 = \sin^{-1}\left\{\frac{1.00\sin 30°}{1.50}\right\} = \boxed{19.5°}$$

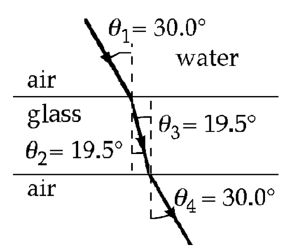


FIG. P35.17

θ_2 and θ_3 are alternate interior angles formed by the ray cutting parallel normals.

So, $\theta_3 = \theta_2 = \boxed{19.5°}$

$1.50\sin\theta_3 = 1.00\sin\theta_4$

$\theta_4 = \boxed{30.0°}$

P35.18 $\sin\theta_1 = n_w \sin\theta_2$

$$\sin\theta_2 = \frac{1}{1.333}\sin\theta_1 = \frac{1}{1.333}\sin(90.0° - 28.0°) = 0.662$$

$$\theta_2 = \sin^{-1}(0.662) = 41.5°$$

$$h = \frac{d}{\tan\theta_2} = \frac{3.00 \text{ m}}{\tan 41.5°} = \boxed{3.39 \text{ m}}$$

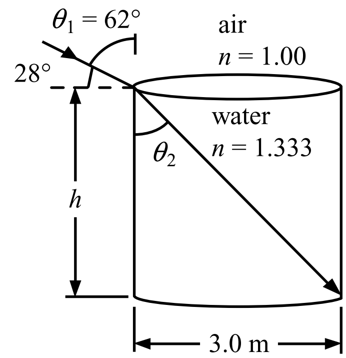


FIG. P35.18

P35.19 At entry, $n_1 \sin\theta_1 = n_2 \sin\theta_2$

or $1.00 \sin 30.0° = 1.50 \sin\theta_2$

$\theta_2 = 19.5°$

The distance h the light travels in the medium is given by

$$\cos\theta_2 = \frac{2.00 \text{ cm}}{h}$$

or $$h = \frac{2.00 \text{ cm}}{\cos 19.5°} = 2.12 \text{ cm}$$

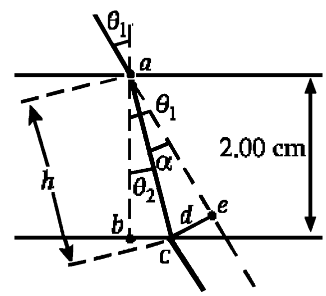


FIG. P35.19

The angle of deviation upon entry is $\alpha = \theta_1 - \theta_2 = 30.0° - 19.5° = 10.5°$

The offset distance comes from $\sin\alpha = \frac{d}{h}$: $d = (2.21 \text{ cm})\sin 10.5° = \boxed{0.388 \text{ cm}}$

P35.20 The distance h traveled by the light is $h = \frac{2.00 \text{ cm}}{\cos 19.5°} = 2.12 \text{ cm}$

The speed of light in the material is $v = \frac{c}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.50} = 2.00 \times 10^8 \text{ m/s}$

Therefore, $t = \frac{h}{v} = \frac{2.12 \times 10^{-2} \text{ m}}{2.00 \times 10^8 \text{ m/s}} = 1.06 \times 10^{-10} \text{ s} = \boxed{106 \text{ ps}}$

P35.21 Applying Snell's law at the air-oil interface,

$$n_{\text{air}} \sin\theta = n_{\text{oil}} \sin 20.0°$$

yields $\boxed{\theta = 30.4°}$

Applying Snell's law at the oil-water interface

$$n_w \sin\theta' = n_{\text{oil}} \sin 20.0°$$

yields $\boxed{\theta' = 22.3°}$

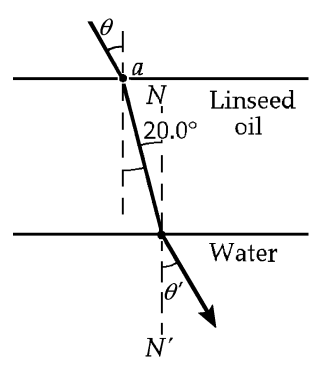


FIG. P35.21

P35.22 For sheets 1 and 2 as described,

$n_1 \sin 26.5° = n_2 \sin 31.7°$

$0.849 n_1 = n_2$

For the trial with sheets 3 and 2,

$n_3 \sin 26.5° = n_2 \sin 36.7°$

$0.747 n_3 = n_2$

Now

$0.747 n_3 = 0.849 n_1$

$n_3 = 1.14 n_1$

For the third trial,

$n_1 \sin 26.5° = n_3 \sin\theta_3 = 1.14 n_1 \sin\theta_3$

$\theta_3 = \boxed{23.1°}$

***P35.23** Refraction proceeds according to $(1.00)\sin\theta_1 = (1.66)\sin\theta_2$ (1)

(a) For the normal component of velocity to be constant, $v_1 \cos\theta_1 = v_2 \cos\theta_2$

or $(c)\cos\theta_1 = \left(\dfrac{c}{1.66}\right)\cos\theta_2$ (2)

We multiply Equations (1) and (2), obtaining: $\sin\theta_1 \cos\theta_1 = \sin\theta_2 \cos\theta_2$

or $\sin 2\theta_1 = \sin 2\theta_2$

The solution $\theta_1 = \theta_2 = 0$ does not satisfy Equation (2) and must be rejected. The physical solution is $2\theta_1 = 180° - 2\theta_2$ or $\theta_2 = 90.0° - \theta_1$. Then Equation (1) becomes:

$\sin\theta_1 = 1.66\cos\theta_1$, or $\tan\theta_1 = 1.66$

which yields $\boxed{\theta_1 = 58.9°}$

In this case, $\boxed{\text{yes}}$, the perpendicular velocity component does remain constant.

(b) Light entering the glass slows down and makes a smaller angle with the normal. Both effects reduce the velocity component parallel to the surface of the glass.
Then

$\boxed{\text{no, the parallel velocity component cannot remain constant, or will remain constant only in the trivial case } \theta_1 = \theta_2 = 0}$.

P35.24 Consider glass with an index of refraction of 1.5, which is 3 mm thick. The speed of light in the glass is

$$\frac{3\times 10^{8}\ \text{m/s}}{1.5} = 2\times 10^{8}\ \text{m/s}$$

The extra travel time is $\dfrac{3\times 10^{-3}\ \text{m}}{2\times 10^{8}\ \text{m/s}} - \dfrac{3\times 10^{-3}\ \text{m}}{3\times 10^{8}\ \text{m/s}}\ \boxed{\sim 10^{-11}\ \text{s}}$

For light of wavelength 600 nm in vacuum and wavelength $\dfrac{600\ \text{nm}}{1.5} = 400$ nm in glass,

the extra optical path, in wavelengths, is $\dfrac{3\times 10^{-3}\ \text{m}}{4\times 10^{-7}\ \text{m}} - \dfrac{3\times 10^{-3}\ \text{m}}{6\times 10^{-7}\ \text{m}}\ \boxed{\sim 10^{3}\ \text{wavelengths}}$

P35.25 Taking Φ to be the apex angle and δ_{min} to be the angle of minimum deviation, from Equation 35.9, the index of refraction of the prism material is

$$n = \frac{\sin\left[\left(\Phi + \delta_{\text{min}}\right)/2\right]}{\sin\left(\Phi/2\right)}$$

Solving for δ_{min}, $\quad \delta_{\text{min}} = 2\sin^{-1}\left(n\sin\dfrac{\Phi}{2}\right) - \Phi = 2\sin^{-1}\left[(2.20)\sin(25.0°)\right] - 50.0° = \boxed{86.8°}$

P35.26 $n(700\ \text{nm}) = 1.458$

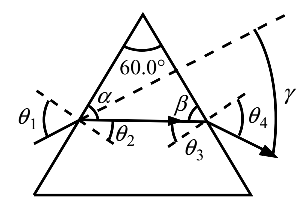


FIG. P35.26

(a) $(1.00)\sin 75.0° = 1.458\sin\theta_2;\ \theta_2 = \boxed{41.5°}$

(b) Let $\quad \theta_3 + \beta = 90.0°,\ \theta_2 + \alpha = 90.0°$ then $\alpha + \beta + 60.0° = 180°$

So $\quad 60.0° - \theta_2 - \theta_3 = 0 \Rightarrow 60.0° - 41.5° = \theta_3 = \boxed{18.5°}$

(c) $1.458\sin 18.5° = 1.00\sin\theta_4 \qquad \theta_4 = \boxed{27.6°}$

(d) $\gamma = (\theta_1 - \theta_2) + \left[\beta - (90.0° - \theta_4)\right]$

$\gamma = 75.0° - 41.5° + (90.0° - 18.5°) - (90.0° - 27.6°) = \boxed{42.6°}$

P35.27 At the first refraction, $\quad 1.00\sin\theta_1 = n\sin\theta_2$

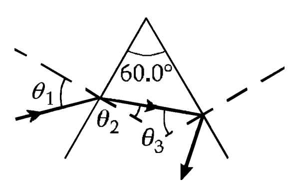


FIG. P35.27

The critical angle at the second surface is given by $n\sin\theta_3 = 1.00$:

or $\quad \theta_3 = \sin^{-1}\left(\dfrac{1.00}{1.50}\right) = 41.8°$

But, $\quad \theta_2 = 60.0° - \theta_3$

Thus, to avoid total internal reflection at the second surface (i.e., have $\theta_3 < 41.8°$)

it is necessary that $\quad \theta_2 > 18.2°$

Since $\sin\theta_1 = n\sin\theta_2$, this becomes $\quad \sin\theta_1 > 1.50\sin 18.2° = 0.468$

or $\quad \theta_1 > \boxed{27.9°}$

P35.28 At the first refraction, $1.00\sin\theta_1 = n\sin\theta_2$

The critical angle at the second surface is given by

$n\sin\theta_3 = 1.00$, or $\theta_3 = \sin^{-1}\left(\dfrac{1.00}{n}\right)$

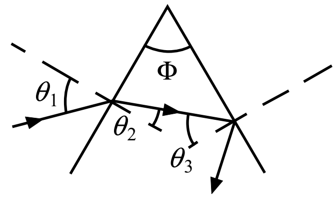


FIG. P35.28

But $(90.0° - \theta_2) + (90.0° - \theta_3) + \Phi = 180°$

which gives $\theta_2 = \Phi - \theta_3$

Thus, to have $\theta_3 < \sin^{-1}\left(\dfrac{1.00}{n}\right)$ and avoid total internal reflection at the second surface,

it is necessary that $\theta_2 > \Phi - \sin^{-1}\left(\dfrac{1.00}{n}\right)$

Since $\sin\theta_1 = n\sin\theta_2$, this requirement becomes $\sin\theta_1 > n\sin\left[\Phi - \sin^{-1}\left(\dfrac{1.00}{n}\right)\right]$

or $\theta_1 > \boxed{\sin^{-1}\left(n\sin\left[\Phi - \sin^{-1}\left(\dfrac{1.00}{n}\right)\right]\right)}$

Through the application of trigonometric identities, $\theta_1 > \boxed{\sin^{-1}\left(\sqrt{n^2 - 1}\sin\Phi - \cos\Phi\right)}$

P35.29 Note for use in every part: $\Phi + (90.0° - \theta_2) + (90.0° - \theta_3) = 180°$

so $\theta_3 = \Phi - \theta_2$

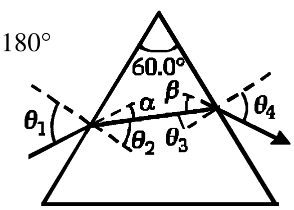


FIG. P35.29

At the first surface the deviation is $\alpha = \theta_1 - \theta_2$

At exit, the deviation is $\beta = \theta_4 - \theta_3$

The total deviation is therefore $\delta = \alpha + \beta = \theta_1 + \theta_4 - \theta_2 - \theta_3 = \theta_1 + \theta_4 - \Phi$

(a) At entry: $n_1\sin\theta_1 = n_2\sin\theta_2$ or $\theta_2 = \sin^{-1}\left(\dfrac{\sin 48.6°}{1.50}\right) = 30.0°$

Thus, $\theta_3 = 60.0° - 30.0° = 30.0°$

At exit: $1.50\sin 30.0° = 1.00\sin\theta_4$ or $\theta_4 = \sin^{-1}\left[1.50\sin(30.0°)\right] = 48.6°$

so the path through the prism is symmetric when $\theta_1 = 48.6°$.

(b) $\delta = 48.6° + 48.6° - 60.0° = \boxed{37.2°}$

(c) At entry: $\sin\theta_2 = \dfrac{\sin 45.6°}{1.50} \Rightarrow \theta_2 = 28.4°$ $\theta_3 = 60.0° - 28.4° = 31.6°$

At exit: $\sin\theta_4 = 1.50\sin(31.6°) \Rightarrow \theta_4 = 51.7°$ $\delta = 45.6° + 51.7° - 60.0° = \boxed{37.3°}$

continued on next page

(d) At entry: $\sin\theta_2 = \dfrac{\sin 51.6°}{1.50} \Rightarrow \theta_2 = 31.5°$ $\theta_3 = 60.0° - 31.5° = 28.5°$

At exit: $\sin\theta_4 = 1.50\sin(28.5°) \Rightarrow \theta_4 = 45.7°$ $\delta = 51.6° + 45.7° - 60.0° = \boxed{37.3°}$

Section 35.6 **Huygens's Principle**

P35.30 (a) For the diagrams of contour lines and wave fronts and rays, see Figures (a) and (b) below. As the waves move to shallower water, the wave fronts bend to become more nearly parallel to the contour lines.

(b) For the diagrams of contour lines and wave fronts and rays, see Figures (c) and (d) below. We suppose that the headlands are steep underwater, as they are above water. The rays are everywhere perpendicular to the wave fronts of the incoming refracting waves. As shown, the rays bend toward the headlands and deliver more energy per length at the headlands.

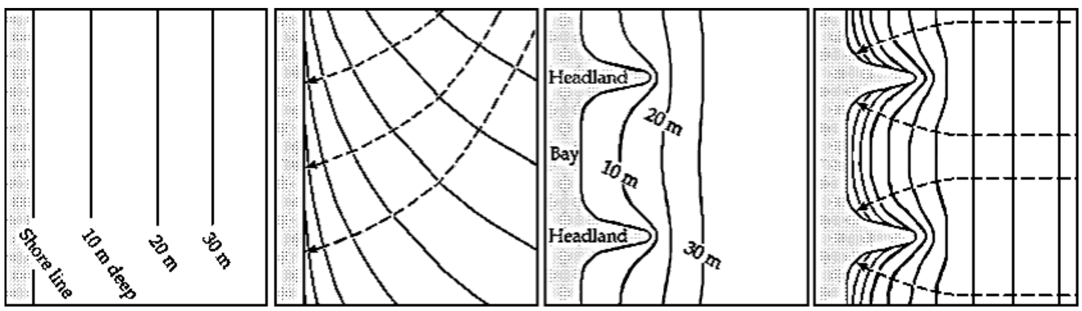


FIG. P35.30

Section 35.7 **Dispersion**

P35.31 For the incoming ray, $\sin\theta_2 = \dfrac{\sin\theta_1}{n}$

Using the figure to the right,

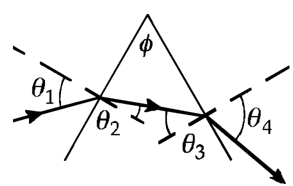


FIG. P35.31

$(\theta_2)_{\text{violet}} = \sin^{-1}\left(\dfrac{\sin 50.0°}{1.66}\right) = 27.48°$

$(\theta_2)_{\text{red}} = \sin^{-1}\left(\dfrac{\sin 50.0°}{1.62}\right) = 28.22°$

For the outgoing ray, $\theta_3 = 60.0° - \theta_2$

and $\sin\theta_4 = n\sin\theta_3$: $(\theta_4)_{\text{violet}} = \sin^{-1}[1.66\sin 32.52°] = 63.17°$

$(\theta_4)_{\text{red}} = \sin^{-1}[1.62\sin 31.78°] = 58.56°$

The angular dispersion is the difference $\Delta\theta_4 = (\theta_4)_{\text{violet}} - (\theta_4)_{\text{red}} = 63.17° - 58.56° = \boxed{4.61°}$

P35.32 From Fig 35.21 $n_v = 1.470$ at 400 nm and $n_r = 1.458$ at 700 nm

Then $1.00\sin\theta = 1.470\sin\theta_v$ and $1.00\sin\theta = 1.458\sin\theta_r$

$$\delta_r - \delta_v = \theta_r - \theta_v = \sin^{-1}\left(\frac{\sin\theta}{1.458}\right) - \sin^{-1}\left(\frac{\sin\theta}{1.470}\right)$$

$$\Delta\delta = \sin^{-1}\left(\frac{\sin 30.0°}{1.458}\right) - \sin^{-1}\left(\frac{\sin 30.0°}{1.470}\right) = \boxed{0.171°}$$

Section 35.8 **Total Internal Reflection**

P35.33 $n\sin\theta = 1$. From Table 35.1,

(a) $\theta = \sin^{-1}\left(\frac{1}{2.419}\right) = \boxed{24.4°}$

(b) $\theta = \sin^{-1}\left(\frac{1}{1.66}\right) = \boxed{37.0°}$

(c) $\theta = \sin^{-1}\left(\frac{1}{1.309}\right) = \boxed{49.8°}$

***P35.34** For total internal reflection, $n_1 \sin\theta_1 = n_2 \sin 90.0°$

$1.50 \sin\theta_1 = 1.33(1.00)$ or $\theta_1 = \boxed{62.5°}$

P35.35 $\sin\theta_c = \frac{n_2}{n_1}$

$n_2 = n_1 \sin 88.8° = (1.000\ 3)(0.999\ 8) = \boxed{1.000\ 08}$

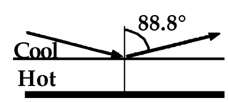


FIG. P35.35

P35.36 $\sin\theta_c = \frac{n_{\text{air}}}{n_{\text{pipe}}} = \frac{1.00}{1.36} = 0.735$ $\theta_c = 47.3°$

Geometry shows that the angle of refraction at the end is

$\phi = 90.0° - \theta_c = 90.0° - 47.3° = 42.7°$

Then, Snell's law at the end, $1.00\sin\theta = 1.36\sin 42.7°$

gives $\boxed{\theta = 67.2°}$

The 2-μm diameter is unnecessary information.

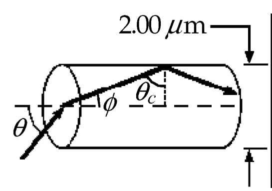


FIG. P35.36

P35.37 (a) A ray along the inner edge will escape if any ray escapes. Its angle of incidence is described by $\sin\theta = \frac{R-d}{R}$ and by $n\sin\theta > 1\sin 90°$. Then

$$\frac{n(R-d)}{R} > 1 \qquad nR - nd > R \qquad nR - R > nd \qquad R > \boxed{\frac{nd}{n-1}}$$

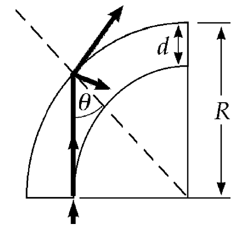


FIG. P35.37

(b) As $d \to 0$, $R_{\min} \to 0$. This is reasonable.

As n increases, $R_{\min}$ decreases. This is reasonable.

As n decreases toward 1, $R_{\min}$ increases. This is reasonable.

(c) $$R_{\min} = \frac{1.40(100\times10^{-6}\text{ m})}{0.40} = \boxed{350\times10^{-6}\text{ m}}$$

P35.38 (a) $\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$ and $\theta_2 = 90.0°$ at the critical angle

$$\frac{\sin 90.0°}{\sin\theta_c} = \frac{1\,850\text{ m/s}}{343\text{ m/s}} \quad \text{so} \quad \theta_c = \sin^{-1}(0.185) = \boxed{10.7°}$$

(b) Sound can be totally reflected if it is traveling in the medium where it travels slower: $\boxed{\text{air}}$.

(c) $\boxed{\text{Sound in air falling on the wall from most directions is 100\% reflected}}$, so the wall is a good mirror.

P35.39 For plastic with index of refraction $\boxed{n \ge 1.42}$ surrounded by air, the critical angle for total internal reflection is given by

$$\theta_c = \sin^{-1}\left(\frac{1}{n}\right) \le \sin^{-1}\left(\frac{1}{1.42}\right) = 44.8°$$

In the gasoline gauge, skylight from above travels down the plastic. The rays close to the vertical are totally reflected from the sides of the slab and from both facets at the lower end of the plastic, where it is not immersed in gasoline. This light returns up inside the plastic and makes it look bright. Where the plastic is immersed in gasoline, with index of refraction about 1.50, total internal reflection should not happen. The light passes out of the lower end of the plastic with little reflected, making this part of the gauge look dark. To frustrate total internal reflection in the gasoline, the index of refraction of the plastic should be $\boxed{n < 2.12}$.

since $$\theta_c = \sin^{-1}\left(\frac{1.50}{2.12}\right) = 45.0°$$

Additional Problems

***P35.40** From the textbook figure we have $w = 2b + a$

so $$b = \frac{w-a}{2} = \frac{700\ \mu\text{m} - 1\ \mu\text{m}}{2} = 349.5\ \mu\text{m}$$

$$\tan\theta_2 = \frac{b}{t} = \frac{349.5\ \mu\text{m}}{1\,200\ \mu\text{m}} = 0.291 \qquad \theta_2 = 16.2°$$

For refraction at entry, $$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$\theta_1 = \sin^{-1}\frac{n_2\sin\theta_2}{n_1} = \sin^{-1}\frac{1.55\sin 16.2°}{1.00} = \sin^{-1}0.433 = \boxed{25.7°}$$

P35.41 Scattered light leaves the center of the photograph (a) in all horizontal directions between $\theta_1 = 0°$ and $90°$ from the normal. When it immediately enters the water (b), it is gathered into a fan between $0°$ and $\theta_{2\,\max}$ given by

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$1.00 \sin 90 = 1.333 \sin\theta_{2\,\max}$$
$$\theta_{2\,\max} = 48.6°$$

The light leaves the cylinder without deviation, so the viewer only receives light from the center of the photograph when he has turned by an angle less than 48.6°. When the paperweight is turned farther, light at the back surface undergoes total internal reflection (c). The viewer sees things outside the globe on the far side.

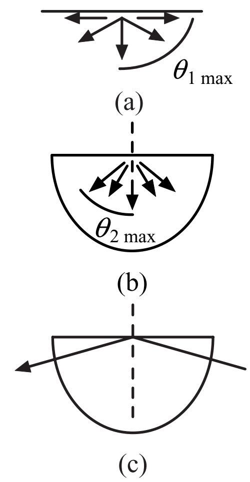


FIG. P35.41

P35.42 Let $n(x)$ be the index of refraction at distance x below the top of the atmosphere and $n(x = h) = n$ be its value at the planet surface.

Then, $$n(x) = 1.000 + \left(\frac{n - 1.000}{h}\right)x$$

(a) The total time interval required to traverse the atmosphere is

$$\Delta t = \int_0^h \frac{dx}{v} = \int_0^h \frac{n(x)}{c}dx: \qquad \Delta t = \frac{1}{c}\int_0^h \left[1.000 + \left(\frac{n-1.000}{h}\right)x\right]dx$$

$$\Delta t = \frac{h}{c} + \frac{(n-1.000)}{ch}\left(\frac{h^2}{2}\right) = \boxed{\frac{h}{c}\left(\frac{n+1.000}{2}\right)}$$

(b) The travel time in the absence of an atmosphere would be $\frac{h}{c}$.

Thus, the time in the presence of an atmosphere is $\boxed{\left(\frac{n+1.000}{2}\right) \text{ times larger}}$.

P35.43 Let the air and glass be medium 1 and 2, respectively. By Snell's law, $n_2 \sin\theta_2 = n_1 \sin\theta_1$

or $1.56 \sin\theta_2 = \sin\theta_1$

But the conditions of the problem are such that $\theta_1 = 2\theta_2$. $1.56 \sin\theta_2 = \sin 2\theta_2$

We now use the double-angle trig identity suggested. $1.56 \sin\theta_2 = 2\sin\theta_2 \cos\theta_2$

or $\cos\theta_2 = \frac{1.56}{2} = 0.780$

Thus, $\theta_2 = 38.7°$ and $\theta_1 = 2\theta_2 = \boxed{77.5°}$.

P35.44 (a) $\theta_1' = \theta_1 = \boxed{30.0°}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1.00 \sin 30.0° = 1.55 \sin\theta_2$

$\theta_2 = \boxed{18.8°}$

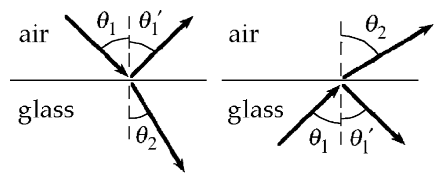


FIG. P35.44

(b) $\theta_1' = \theta_1 = \boxed{30.0°}$

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right)$$

$$= \sin^{-1}\left(\frac{1.55 \sin 30.0°}{1}\right) = \boxed{50.8°}$$

(c), (d) The other entries are computed similarly, and are shown in the table below.

(c) air into glass, angles in degrees			(d) glass into air, angles in degrees		
incidence	reflection	refraction	incidence	reflection	refraction
0	0	0	0	0	0
10.0	10.0	6.43	10.0	10.0	15.6
20.0	20.0	12.7	20.0	20.0	32.0
30.0	30.0	18.8	30.0	30.0	50.8
40.0	40.0	24.5	40.0	40.0	85.1
50.0	50.0	29.6	50.0	50.0	none*
60.0	60.0	34.0	60.0	60.0	none*
70.0	70.0	37.3	70.0	70.0	none*
80.0	80.0	39.4	80.0	80.0	none*
90.0	90.0	40.2	90.0	90.0	none*

*total internal reflection

P35.45 For water, $\sin\theta_c = \dfrac{1}{4/3} = \dfrac{3}{4}$

Thus $\theta_c = \sin^{-1}(0.750) = 48.6°$

and $d = 2\left[(1.00\text{ m})\tan\theta_c\right]$

$d = (2.00\text{ m})\tan 48.6° = \boxed{2.27\text{ m}}$

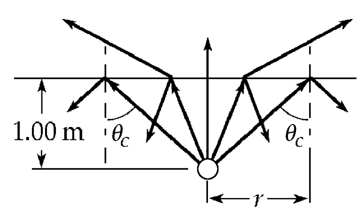


FIG. P35.45

P35.46 (a) We see the Sun moving from east to west across the sky. Its angular speed is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\text{ rad}}{86\,400\text{ s}} = 7.27\times10^{-5}\text{ rad/s}$$

The direction of sunlight crossing the cell from the window changes at this rate, moving on the opposite wall at speed

$$v = r\omega = (2.37\text{ m})(7.27\times10^{-5}\text{ rad/s}) = 1.72\times10^{-4}\text{ m/s} = \boxed{0.172\text{ mm/s}}$$

(b) The mirror folds into the cell the motion that would occur in a room twice as wide:

$$v = r\omega = 2(0.174\text{ mm/s}) = \boxed{0.345\text{ mm/s}}$$

(c), (d) As the Sun moves southward and upward at 50.0°, we may regard the corner of the window as fixed, and both patches of light move $\boxed{\text{northward and downward at 50.0°}}$.

P35.47 Horizontal light rays from the setting Sun pass above the hiker. The light rays are twice refracted and once reflected, as in Figure (b). The most intense light reaching the hiker, that which represents the visible rainbow, is located between angles of 40° and 42° from the hiker's shadow. The hiker sees a greater percentage of the violet inner edge, so we consider the red outer edge. The radius R of the circle of droplets is

$$R = (8.00\text{ km})\sin 42.0° = 5.35\text{ km}$$

Then the angle ϕ, between the vertical and the radius where the bow touches the ground, is given by

$$\cos\phi = \frac{2.00\text{ km}}{R} = \frac{2.00\text{ km}}{5.35\text{ km}} = 0.374$$

or $\phi = 68.1°$

The angle filled by the visible bow is $360° - (2 \times 68.1°) = 224°$

so the visible bow is $\frac{224°}{360°} = \boxed{62.2\%\text{ of a circle}}$.

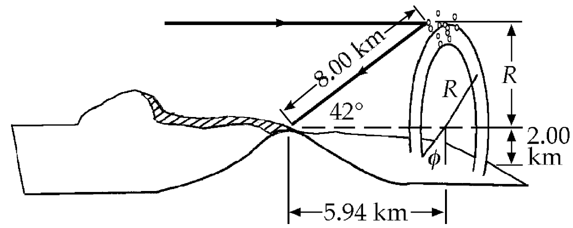


Figure (a)

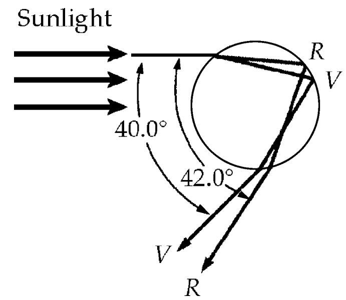


Figure (b)

FIG. P35.47

P35.48 (a) $\boxed{45.0°}$ as shown in the first figure to the right.

(b) $\boxed{\text{Yes}}$

If grazing angle is halved, the number of reflections from the side faces is doubled.

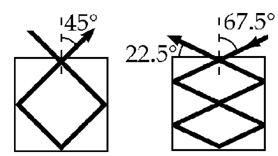


FIG. P35.48

P35.49 As the beam enters the slab,

$$1.00\sin 50.0° = 1.48\sin\theta_2$$

giving $\theta_2 = 31.2°$

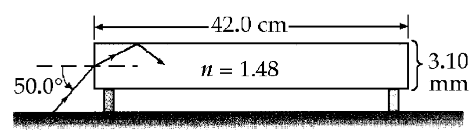


FIG. P35.49

The beam then strikes the top of the slab at $x_1 = \frac{1.55\text{ mm}}{\tan 31.2°}$ from the left end. Thereafter, the beam strikes a face each time it has traveled a distance of $2x_1$ along the length of the slab. Since the slab is 420 mm long, the beam has an additional 420 mm $- x_1$ to travel after the first reflection. The number of additional reflections is

$$\frac{420\text{ mm} - x_1}{2x_1} = \frac{420\text{ mm} - 1.55\text{ mm}/\tan 31.2°}{3.10\text{ mm}/\tan 31.2°} = 81.5 \quad \text{or 81 reflections}$$

since the answer must be an integer. The total number of reflections made in the slab is then $\boxed{82}$.

P35.50 Light passing the top of the pole makes an angle of incidence $\phi_1 = 90.0° - \theta$. It falls on the water surface at distance from the pole

$$s_1 = \frac{L-d}{\tan\theta}$$

and has an angle of refraction ϕ_2 from $1.00\sin\phi_1 = n\sin\phi_2$

Then $s_2 = d\tan\phi_2$

and the whole shadow length is

$$s_1 + s_2 = \frac{L-d}{\tan\theta} + d\tan\left(\sin^{-1}\left(\frac{\sin\phi_1}{n}\right)\right)$$

$$s_1 + s_2 = \frac{L-d}{\tan\theta} + d\tan\left(\sin^{-1}\left(\frac{\cos\theta}{n}\right)\right)$$

$$= \frac{2.00\text{ m}}{\tan 40.0°} + (2.00\text{ m})\tan\left(\sin^{-1}\left(\frac{\cos 40.0°}{1.33}\right)\right) = \boxed{3.79\text{ m}}$$

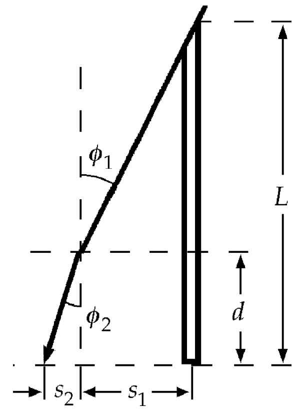


FIG. P35.50

P35.51 Define n_1 to be the index of refraction of the surrounding medium and n_2 to be that for the prism material. We can use the critical angle of 42.0° to find the ratio $\frac{n_2}{n_1}$:

$$n_2 \sin 42.0° = n_1 \sin 90.0°$$

So, $$\frac{n_2}{n_1} = \frac{1}{\sin 42.0°} = 1.49$$

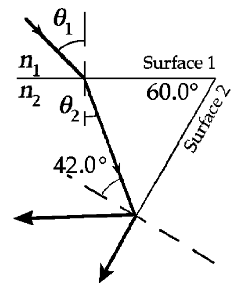


FIG. P35.51

Call the angle of refraction θ_2 at the surface 1. The ray inside the prism forms a triangle with surfaces 1 and 2, so the sum of the interior angles of this triangle must be 180°.

Thus, $(90.0° - \theta_2) + 60.0° + (90.0° - 42.0°) = 180°$

Therefore, $\theta_2 = 18.0°$

Applying Snell's law at surface 1, $n_1 \sin\theta_1 = n_2 \sin 18.0°$

$$\sin\theta_1 = \left(\frac{n_2}{n_1}\right)\sin\theta_2 = 1.49\sin 18.0° \qquad \boxed{\theta_1 = 27.5°}$$

***P35.52** (a) As the mirror turns through angle θ, the angle of incidence increases by θ and so does the angle of reflection. The incident ray is stationary, so the reflected ray turns through angle 2θ. The angular speed of the reflected ray is $2\omega_m$. The speed of the dot of light on the circular wall is $2\omega_m R = 2(35\text{ rad/s})(3\text{ m}) = \boxed{210\text{ m/s}}$.

(b) The two angles marked θ in the figure to the right are equal because their sides are perpendicular, right side to right side and left side to left side.

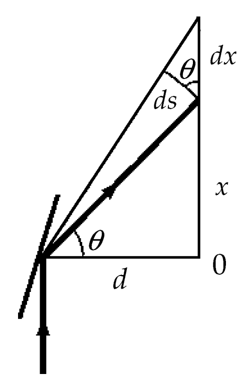


FIG. P35.52

We have $$\cos\theta = \frac{d}{\sqrt{x^2+d^2}} = \frac{ds}{dx}$$

and $$\frac{ds}{dt} = 2\omega_m\sqrt{x^2+d^2}.$$ So

$$\frac{dx}{dt} = \frac{ds}{dt}\frac{\sqrt{x^2+d^2}}{d} = 2\omega_m\frac{x^2+d^2}{d} = 2(35\text{ rad/s})\frac{x^2+(3\text{ m})^2}{(3\text{ m})} = \boxed{23.3\frac{x^2+9\text{ m}^2}{\text{m}\cdot\text{s}}}$$

continued on next page

(c) The minimum value is $\boxed{210\text{ m/s}}$ for $x = 0$, just as for the circular wall.

(d) The maximum speed goes to infinity as x goes to $\boxed{\text{infinity}}$, which happens when the mirror turns by 45°.

(e) To turn by 45° takes the time interval $(\pi/4\text{ rad})/(35\text{ rad/s}) = \boxed{22.4\text{ ms}}$.

***P35.53** (a) For polystyrene *surrounded by air,* internal reflection requires

$$\theta_3 = \sin^{-1}\left(\frac{1.00}{1.49}\right) = 42.2°$$

Then from geometry, $\theta_2 = 90.0° - \theta_3 = 47.8°$

From Snell's law, $\sin\theta_1 = 1.49\sin 47.8° = 1.10$

This has no solution.

Therefore, total internal reflection $\boxed{\text{always happens}}$ or the greatest angle is 90°.

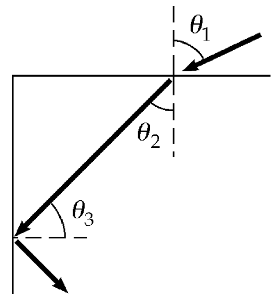


FIG. P35.53

(b) For polystyrene *surrounded by water,* $\theta_3 = \sin^{-1}\left(\frac{1.33}{1.49}\right) = 63.2°$

and $\theta_2 = 26.8°$

From Snell's law, $\theta_1 = \boxed{30.3°}$

(c) $\boxed{\text{No internal refraction is possible}}$ since the beam is initially traveling in a medium of lower index of refraction. No angle exists.

***P35.54** (a) The optical day is longer. Incoming sunlight is refracted downward at the top of the atmosphere, so an observer can see the rising Sun when it is still geometrically below the horizon. Light from the setting Sun reaches her after the Sun is below the horizon geometrically.

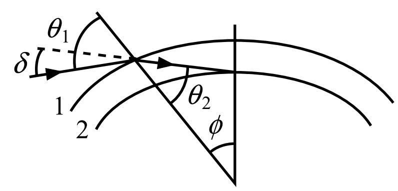


FIG. P35.54

(b) The picture illustrates optical sunrise. At the center of the Earth,

$$\cos\phi = \frac{6.37\times10^6\text{ m}}{6.37\times10^6\text{ m} + 8\,614}$$

$\phi = 2.98°$

$\theta_2 = 90 - 2.98° = 87.0°$

At the top of the atmosphere

$n_1\sin\theta_1 = n_2\sin\theta_2$

$1\sin\theta_1 = 1.000\,293\sin 87.0°$

$\theta_1 = 87.4°$

Deviation upon entry is

$\delta = |\theta_1 - \theta_2|$

$\delta = 87.364° - 87.022° = 0.342°$

Sunrise of the optical day is before geometric sunrise by $0.342°\left(\frac{86\,400\text{ s}}{360°}\right) = 82.2\text{ s}.$

Optical sunset occurs later too, so the optical day is longer by $\boxed{164\text{ s}}$.

P35.55 $\tan\theta_1 = \dfrac{4.00\text{ cm}}{h}$

and $\tan\theta_2 = \dfrac{2.00\text{ cm}}{h}$

$$\tan^2\theta_1 = (2.00\tan\theta_2)^2 = 4.00\tan^2\theta_2$$

$$\frac{\sin^2\theta_1}{1-\sin^2\theta_1} = 4.00\left(\frac{\sin^2\theta_2}{1-\sin^2\theta_2}\right) \quad (1)$$

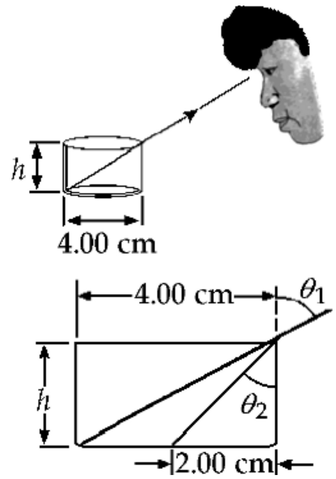


FIG. P35.55

Snell's law in this case is: $n_1\sin\theta_1 = n_2\sin\theta_2$

$$\sin\theta_1 = 1.333\sin\theta_2$$

Squaring both sides, $\sin^2\theta_1 = 1.777\sin^2\theta_2 \quad (2)$

Substituting (2) into (1), $\dfrac{1.777\sin^2\theta_2}{1-1.777\sin^2\theta_2} = 4.00\left(\dfrac{\sin^2\theta_2}{1-\sin^2\theta_2}\right)$

Defining $x = \sin^2\theta$, $\dfrac{0.444}{1-1.777x} = \dfrac{1}{1-x}$

Solving for x, $0.444 - 0.444x = 1 - 1.777x$ and $x = 0.417$

From x we can solve for θ_2: $\theta_2 = \sin^{-1}\sqrt{0.417} = 40.2°$

Thus, the height is $h = \dfrac{2.00\text{ cm}}{\tan\theta_2} = \dfrac{2.00\text{ cm}}{\tan 40.2°} = \boxed{2.36\text{ cm}}$

P35.56 $\delta = \theta_1 - \theta_2 = 10.0°$ and $n_1\sin\theta_1 = n_2\sin\theta_2$

with $n_1 = 1,\ n_2 = \dfrac{4}{3}$

Thus, $\theta_1 = \sin^{-1}(n_2\sin\theta_2) = \sin^{-1}\left[n_2\sin(\theta_1 - 10.0°)\right]$

(You can use a calculator to home in on an approximate solution to this equation, testing different values of θ_1 until you find that $\theta_1 = \boxed{36.5°}$. Alternatively, you can solve for θ_1 exactly, as shown below.)

We are given that $\sin\theta_1 = \dfrac{4}{3}\sin(\theta_1 - 10.0°)$

This is the sine of a difference, so $\dfrac{3}{4}\sin\theta_1 = \sin\theta_1\cos 10.0° - \cos\theta_1\sin 10.0°$

Rearranging, $\sin 10.0°\cos\theta_1 = \left(\cos 10.0° - \dfrac{3}{4}\right)\sin\theta_1$

$\dfrac{\sin 10.0°}{\cos 10.0° - 0.750} = \tan\theta_1$ and $\theta_1 = \tan^{-1}(0.740) = \boxed{36.5°}$

P35.57 Observe in the sketch that the angle of incidence at point P is γ, and using triangle OPQ:

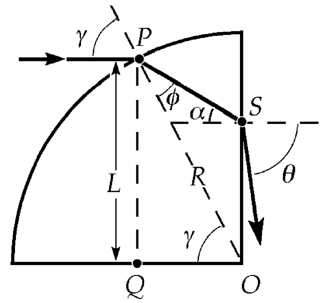


FIG. P35.57

$$\sin\gamma = \frac{L}{R}$$

Also, $$\cos\gamma = \sqrt{1-\sin^2\gamma} = \frac{\sqrt{R^2-L^2}}{R}$$

Applying Snell's law at point P, $1.00\sin\gamma = n\sin\phi$.

Thus, $$\sin\phi = \frac{\sin\gamma}{n} = \frac{L}{nR}$$

and $$\cos\phi = \sqrt{1-\sin^2\phi} = \frac{\sqrt{n^2R^2-L^2}}{nR}$$

From triangle OPS, $\phi + (\alpha + 90.0°) + (90.0° - \gamma) = 180°$ or the angle of incidence at point S is $\alpha = \gamma - \phi$. Then, applying Snell's law at point S

gives $$1.00\sin\theta = n\sin\alpha = n\sin(\gamma-\phi)$$

or $$\sin\theta = n[\sin\gamma\cos\phi - \cos\gamma\sin\phi] = n\left[\left(\frac{L}{R}\right)\frac{\sqrt{n^2R^2-L^2}}{nR} - \frac{\sqrt{R^2-L^2}}{R}\left(\frac{L}{nR}\right)\right]$$

$$\sin\theta = \frac{L}{R^2}\left(\sqrt{n^2R^2-L^2} - \sqrt{R^2-L^2}\right)$$

and $$\theta = \boxed{\sin^{-1}\left[\frac{L}{R^2}\left(\sqrt{n^2R^2-L^2} - \sqrt{R^2-L^2}\right)\right]}$$

***P35.58** (a) In the textbook figure we have $r_1 = \sqrt{a^2+x^2}$ and $r_2 = \sqrt{b^2+(d-x)^2}$. The speeds in the two media are $v_1 = c/n_1$ and $v_2 = c/n_2$ so the travel time for the light from P to Q is indeed

$$t = \frac{r_1}{v_1} + \frac{r_2}{v_2} = \frac{n_1\sqrt{a^2+x^2}}{c} + \frac{n_2\sqrt{b^2+(d-x)^2}}{c}$$

(b) Now $\frac{dt}{dx} = \frac{n_1}{2c}\left(a^2+x^2\right)^{-1/2}2x + \frac{n_2}{2c}\left(b^2+(d-x)^2\right)^{-1/2}2(d-x)(-1) = 0$ is the requirement

for minimal travel time, which simplifies to $\frac{n_1x}{\sqrt{a^2+x^2}} = \frac{n_2(d-x)}{\sqrt{b^2+(d-x)^2}}$.

(c) Now $\sin\theta_1 = \frac{x}{\sqrt{a^2+x^2}}$ and $\sin\theta_2 = \frac{d-x}{\sqrt{b^2+(d-x)^2}}$ so we have directly $n_1\sin\theta_1 = n_2\sin\theta_2$.

P35.59 To derive the law of *reflection,* locate point O so that the time of travel from point A to point B will be minimum.

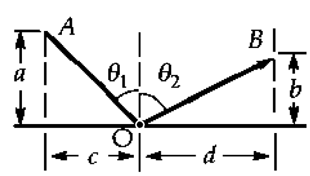


FIG. P35.59

The *total* light path is $L = a\sec\theta_1 + b\sec\theta_2$.

The time of travel is $t = \left(\frac{1}{v}\right)(a\sec\theta_1 + b\sec\theta_2)$.

If point O is displaced by dx, then

$$dt = \left(\frac{1}{v}\right)(a\sec\theta_1\tan\theta_1 d\theta_1 + b\sec\theta_2\tan\theta_2 d\theta_2) = 0 \quad (1)$$

(since for minimum time $dt = 0$).

continued on next page

Also, $c + d = a\tan\theta_1 + b\tan\theta_2 = \text{constant}$

so, $a\sec^2\theta_1 d\theta_1 + b\sec^2\theta_2 d\theta_2 = 0$ (2)

Divide equations (1) and (2) to find $\boxed{\theta_1 = \theta_2}$.

P35.60 As shown in the sketch, the angle of incidence at point A is:

$$\theta = \sin^{-1}\left(\frac{d/2}{R}\right) = \sin^{-1}\left(\frac{1.00\text{ m}}{2.00\text{ m}}\right) = 30.0°$$

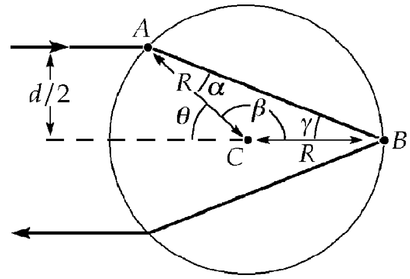


FIG. P35.60

If the emerging ray is to be parallel to the incident ray, the path must be symmetric about the centerline CB of the cylinder. In the isosceles triangle ABC,

$$\gamma = \alpha \qquad \text{and} \qquad \beta = 180° - \theta$$

Therefore, $\alpha + \beta + \gamma = 180°$

becomes $2\alpha + 180° - \theta = 180°$

or $\alpha = \dfrac{\theta}{2} = 15.0°$

Then, applying Snell's law at point A,

$$n\sin\alpha = 1.00\sin\theta$$

or $n = \dfrac{\sin\theta}{\sin\alpha} = \dfrac{\sin 30.0°}{\sin 15.0°} = \boxed{1.93}$

P35.61 (a) The apparent radius of the glowing sphere is R_3 as shown. For it

$$\sin\theta_1 = \frac{R_1}{R_2}$$

$$\sin\theta_2 = \frac{R_3}{R_2}$$

$$n\sin\theta_1 = 1\sin\theta_2$$

$$n\frac{R_1}{R_2} = \frac{R_3}{R_2} \qquad \boxed{R_3 = nR_1}$$

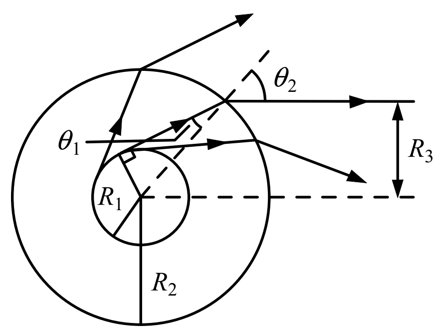


FIG. P35.61 (a)

(b) If $nR_1 > R_2$, then $\sin\theta_2$ cannot be equal to $\dfrac{nR_1}{R_2}$. The ray considered in part (a) undergoes total internal reflection. In this case a ray escaping the atmosphere as shown here is responsible for the apparent radius of the glowing sphere and $\boxed{R_3 = R_2}$.

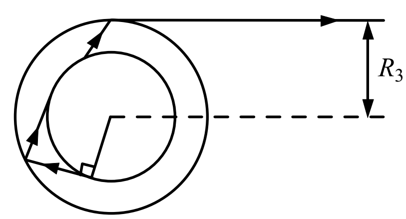


FIG. P35.61 (b)

P35.62 (a) At the boundary of the air and glass, the critical angle is given by $\sin\theta_c = \dfrac{1}{n}$

Consider the critical ray PBB': $\tan\theta_c = \dfrac{d/4}{t}$ or $\dfrac{\sin\theta_c}{\cos\theta_c} = \dfrac{d}{4t}$.

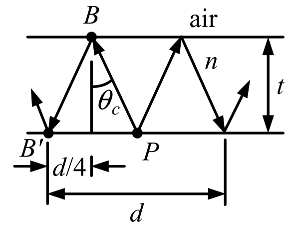


FIG. P35.62

Squaring the last equation gives: $\dfrac{\sin^2\theta_c}{\cos^2\theta_c} = \dfrac{\sin^2\theta_c}{1-\sin^2\theta_c} = \left(\dfrac{d}{4t}\right)^2$.

Since $\sin\theta_c = \dfrac{1}{n}$, this becomes $\dfrac{1}{n^2-1} = \left(\dfrac{d}{4t}\right)^2$ or $\boxed{n = \sqrt{1+\left(\dfrac{4t}{d}\right)^2}}$

(b) Solving for d, $d = \dfrac{4t}{\sqrt{n^2-1}}$.

Thus, if $n = 1.52$ and $t = 0.600$ cm, $d = \dfrac{4(0.600\text{ cm})}{\sqrt{(1.52)^2-1}} = \boxed{2.10\text{ cm}}$

(c) Since violet light has a larger index of refraction, it will lead to a smaller critical angle and the inner edge of the white halo will be tinged with $\boxed{\text{violet}}$ light.

P35.63 (a) Given that $\theta_1 = 45.0°$ and $\theta_2 = 76.0°$

Snell's law at the first surface gives

$$n\sin\alpha = 1.00\sin 45.0° \qquad (1)$$

Observe that the angle of incidence at the second surface is

$$\beta = 90.0° - \alpha.$$

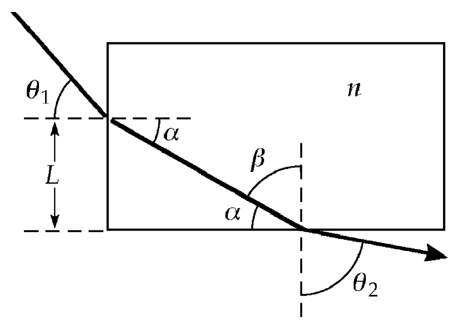


FIG. P35.63

Thus, Snell's law at the second surface yields

$n\sin\beta = n\sin(90.0° - \alpha) = 1.00\sin 76.0°$

or $n\cos\alpha = \sin 76.0°$. (2)

Dividing Equation (1) by Equation (2), $\tan\alpha = \dfrac{\sin 45.0°}{\sin 76.0°} = 0.729$

or $\alpha = 36.1°$

Then, from Equation (1), $n = \dfrac{\sin 45.0°}{\sin\alpha} = \dfrac{\sin 45.0°}{\sin 36.1°} = \boxed{1.20}$

(b) From the sketch, observe that the distance the light travels in the plastic is $d = \dfrac{L}{\sin\alpha}$. Also, the speed of light in the plastic is $v = \dfrac{c}{n}$, so the time required to travel through the plastic is

$$\Delta t = \frac{d}{v} = \frac{nL}{c\sin\alpha} = \frac{1.20(0.500\text{ m})}{(3.00\times10^8\text{ m/s})\sin 36.1°} = 3.40\times10^{-9}\text{ s} = \boxed{3.40\text{ ns}}$$

***P35.64**

$\sin\theta_1$	$\sin\theta_2$	$\dfrac{\sin\theta_1}{\sin\theta_2}$
0.174	0.131	1.330 4
0.342	0.261	1.312 9
0.500	0.379	1.317 7
0.643	0.480	1.338 5
0.766	0.576	1.328 9
0.866	0.647	1.339 0
0.940	0.711	1.322 0
0.985	0.740	1.331 5

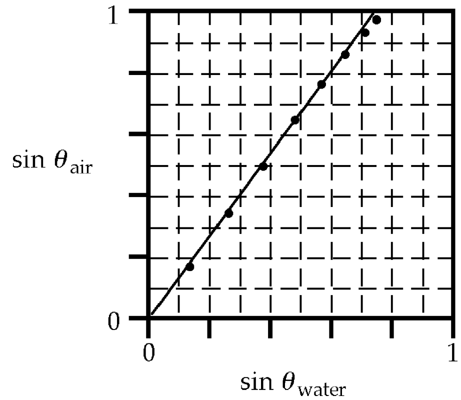


FIG. P35.64

The straightness of the graph line demonstrates Snell's proportionality of the sine of the angle of refraction to the sine of the angle of incidence.

The slope of the line is $\bar{n} = 1.327\,6 \pm 0.01$

The equation $\sin\theta_1 = n\sin\theta_2$ shows that this slope is the index of refraction,

$$n = \boxed{1.328 \pm 0.8\%}$$

***P35.65** Consider an insulated box with the imagined one-way mirror forming one face, installed so that 90% of the electromagnetic radiation incident from the outside is transmitted to the inside and only a lower percentage of the electromagnetic waves from the inside make it through to the outside. Suppose the interior and exterior of the box are originally at the same temperature. Objects within and without are radiating and absorbing electromagnetic waves. They would all maintain constant temperature if the box had an open window. With the glass letting more energy in than out, the interior of the box will rise in temperature. But this is impossible, according to Clausius's statement of the second law. This reduction to a contradiction proves that it is impossible for the one-way mirror to exist.

ANSWERS TO EVEN PROBLEMS

P35.2 227 Mm/s

P35.4 (a) and (b) See the solution. (c) 0.055 7°

P35.6 See the solution.

P35.8 27.1 ns

P35.10 15.4°, 2.56 m. The light wave slows down as it moves from air to water but the sound wave speeds up by a large factor. The light wave bends toward the normal and its wavelength shortens, but the sound wave bends away from the normal and its wavelength increases.

P35.12 (a) 474 THz (b) 422 nm (c) 200 Mm/s

P35.14 (a) 2.0×10^8 m/s (b) 474 THz (c) 4.2×10^{-7} m

P35.16 6.30 cm

P35.18 3.39 m

P35.20 106 ps

P35.22 23.1°

P35.24 ~10^{-11} s; between 10^3 and 10^4 wavelengths

P35.26 (a) 41.5° (b) 18.5° (c) 27.6° (d) 42.6°

P35.28 $\sin^{-1}\left(\sqrt{n^2-1}\sin\Phi - \cos\Phi\right)$

P35.30 See the solution.

P35.32 0.171°

P35.34 62.5°

P35.36 67.2°

P35.38 (a) 10.7° (b) air (c) Sound falling on the wall from most directions is 100% reflected.

P35.40 25.7°

P35.42 (a) $\frac{h}{c}\left(\frac{n+1}{2}\right)$ (b) larger by $\frac{n+1}{2}$ times

P35.44 (a) See the solution; the angles are $\theta_1' = 30.0°$ and $\theta_2 = 18.8°$ (b) See the solution; the angles are $\theta_1' = 30.0°$ and $\theta_2 = 50.8°$. (c) and (d) See the solution.

P35.46 (a) 0.172 mm/s (b) 0.345 mm/s (c) Northward at 50.0° below the horizontal (d) Northward at 50.0° below the horizontal

P35.48 (a) 45.0° (b) Yes; see the solution.

P35.50 3.79 m

P35.52 (a) 210 m/s (b) 23.3 $(x^2 + 9\text{ m}^2)$/m·s (c) 210 m/s for $x = 0$; this is the same as the speed on the circular wall. (d) The speed goes to infinity as x goes to infinity, when the mirror turns through 45° from the $x = 0$ point. (e) 22.4 ms

P35.54 (a) The optical day is longer. Incoming sunlight is refracted downward at the top of the atmosphere, so an observer can see the rising Sun when it is still geometrically below the horizon. Light from the setting Sun reaches her after the Sun is already below the horizon geometrically.
(b) 164 s

P35.56 36.5°

P35.58 See the solution.

P35.60 1.93

P35.62 (a) $n = [1 + (4t/d)^2]^{1/2}$ (b) 2.10 cm (c) violet

P35.64 See the solution; $n = 1.328 \pm 0.8\%$.

36

Image Formation

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q36.1 With a concave spherical mirror, for objects beyond the focal length the image will be real and inverted. For objects inside the focal length, the image will be virtual, upright, and magnified. Try a shaving or makeup mirror as an example.

Q36.2 With a convex spherical mirror, all images of real objects are upright, virtual and smaller than the object. As seen in Question 36.1, you only get a change of orientation when you pass the focal point—but the focal point of a convex mirror is on the non-reflecting side!

***Q36.3** (i) When we flatten a curved mirror we move its center of curvature out to infinity. The focal length is still half the radius of curvature and is infinite. Answer (d).

(ii) The image is actual size and right side up. The magnification is 1. Answer (b).

Q36.4 The mirror equation and the magnification equation apply to plane mirrors. A curved mirror is made flat by increasing its radius of curvature without bound, so that its focal length goes to infinity. From $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=0$ we have $\frac{1}{p}=-\frac{1}{q}$; therefore, $p=-q$. The virtual image is as far behind the mirror as the object is in front. The magnification is $M=-\frac{q}{p}=\frac{p}{p}=1$. The image is right side up and actual size.

***Q36.5** (i) Answer (c). (ii) Answer (c). When the object is at the focal point the image can be thought of as a right side up image behind the mirror at infinity, or as an inverted image in front of the mirror.

Q36.6 In the diagram, only two of the three principal rays have been used to locate images to reduce the amount of visual clutter. The upright shaded arrows are the objects, and the correspondingly numbered inverted arrows are the images. As you can see, object 2 is closer to the focal point than object 1, and image 2 is farther to the left than image 1.

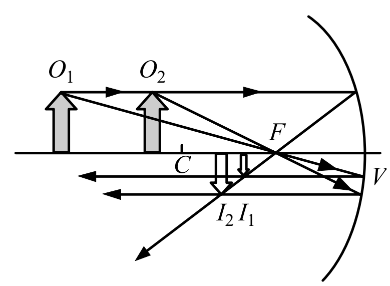


FIG. Q36.6

***Q36.7** (i) (a) positive (b) negative (c) negative (d) negative (e) positive (f) positive (g) positive

(ii) (a) positive (b) positive (c) positive (d) positive (e) negative (f) negative

(iii) (a) positive (b) negative (c) positive (d) negative (e) negative (f) positive

***Q36.8** Answer (c). The angle of refraction for the light coming from fish to person is 60°. The angle of incidence is smaller, so the fish is deeper than it appears.

***Q36.9** The ranking is e > d > g > a > b > f > c. In case e, the object is at infinite distance. In d the object distance is very large but not infinite. In g the object distance is several times the focal length. In a, the object distance is a little larger than the focal length. In b the object distance is very slightly larger than the focal length. In f it is equal to the focal length. In c the object distance is less than the focal length.

Q36.10 An infinite number. In general, an infinite number of rays leave each point of any object and travel in all directions. Note that the three principal rays that we use for imaging are just a subset of the infinite number of rays. All three principal rays can be drawn in a ray diagram, provided that we extend the plane of the lens as shown in Figure Q36.10.

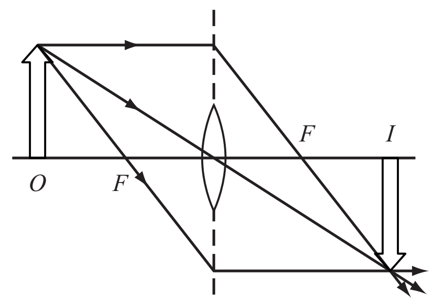


FIG. Q36.10

***Q36.11** Answer (d). The entire image is visible, but only at half the intensity. Each point on the object is a source of rays that travel in all directions. Thus, light from all parts of the object goes through all unblocked parts of the lens and forms an image. If you block part of the lens, you are blocking some of the rays, but the remaining ones still come from all parts of the object.

***Q36.12** Answer (e). If the object distance is $2f$ the image distance is also $2f$ and the distance between object and real image is minimal.

***Q36.13** The focal point is defined as the location of the image formed by rays originally parallel to the axis. An object at a large but finite distance will radiate rays nearly but not exactly parallel. Infinite object distance describes the definite limiting case in which these rays become parallel. To measure the focal length of a converging lens, set it up to form an image of the farthest object you can see outside a window. The image distance will be equal to the focal length within one percent or better if the object distance is a hundred times larger or more.

***Q36.14** Use a converging lens as the projection lens in a slide projector. Place the brightly illuminated slide slightly farther than its focal length away from it, so that the lens will produce a real, inverted, enlarged image on the screen.

***Q36.15** Answer (e). The water drop functions as a lens of short focal length, forming a real image of the distant object in space, outside the drop on the side where the light exits the drop. The camera lens is focused on the real image.

Q36.16 Chromatic aberration arises because a material medium's refractive index can be frequency dependent. A mirror changes the direction of light by reflection, not refraction. Light of all wavelengths follows the same path according to the law of reflection, so no chromatic aberration happens.

Q36.17 If the converging lens is immersed in a liquid with an index of refraction significantly greater than that of the lens itself, it will make light from a distant source diverge. This is not the case with a converging (concave) mirror, as the law of reflection has nothing to do with the indices of refraction.

Q36.18 As in the diagram, let the center of curvature C of the fishbowl and the bottom of the fish define the optical axis, intersecting the fishbowl at vertex V. A ray from the top of the fish that reaches the bowl surface along a radial line through C has angle of incidence zero and angle of refraction zero. This ray exits from the bowl unchanged in direction. A ray from the top of the fish to V is refracted to bend away from the normal. Its extension back inside the fishbowl determines the location of the image and the characteristics of the image. The image is upright, virtual, and enlarged.

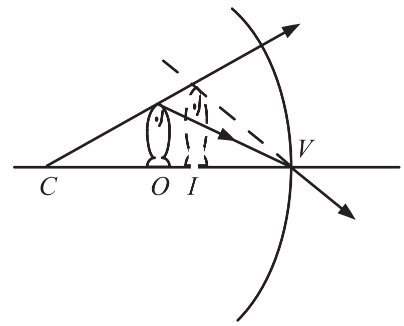


FIG. Q36.18

Q36.19 Because when you look at the ƎƆИA⅃UᙠMA in your rear view mirror, the apparent left-right inversion clearly displays the name of the AMBULANCE behind you. Do not jam on your brakes when a MIAMI city bus is right behind you.

Q36.20 With the meniscus design, when you direct your gaze near the outer circumference of the lens you receive a ray that has passed through glass with more nearly parallel surfaces of entry and exit. Thus, the lens minimally distorts the direction to the object you are looking at. If you wear glasses, turn them around and look through them the wrong way to maximize this distortion.

Q36.21 Answer (b). The outer surface should be flat so that it will not produce a fuzzy or distorted image for the diver when the mask is used either in air or in water.

Q36.22 The eyeglasses on the left are diverging lenses that correct for nearsightedness. If you look carefully at the edge of the person's face through the lens, you will see that everything viewed through these glasses is reduced in size. The eyeglasses on the right are converging lenses, which correct for farsightedness. These lenses make everything that is viewed through them look larger.

Q36.23 The eyeglass wearer's eye is at an object distance from the lens that is quite small—the eye is on the order of 10^{-2} meter from the lens. The focal length of an eyeglass lens is several decimeters, positive or negative. Therefore the image distance will be similar in magnitude to the object distance. The onlooker sees a sharp image of the eye behind the lens. Look closely at the left side of Figure Q36.22 and notice that the wearer's eyes seem not only to be smaller, but also positioned a bit behind the plane of his face—namely where they would be if he were not wearing glasses. Similarly, in the right half of Figure Q36.22, his eyes seem to be in front of the plane of his face and magnified. We as observers take this light information coming from the object through the lens and perceive or photograph the image as if it were an object.

Q36.24 Absolutely. Only absorbed light, not transmitted light, contributes internal energy to a transparent object. A clear lens can stay ice-cold and solid as megajoules of light energy pass through it.

Q36.25 Make the mirror an efficient reflector (shiny). Make it reflect to the image even rays far from the axis, by giving it a parabolic shape. Most important, make it large in diameter to intercept a lot of solar power. And you get higher temperature if the image is smaller, as you get with shorter focal length; and if the furnace enclosure is an efficient absorber (black).

Q36.26 The artist's statements are accurate, perceptive, and eloquent. The image you see is "almost one's whole surroundings," including things behind you and things farther in front of you than the globe is, but nothing eclipsed by the opaque globe or by your head. For example, we cannot see Escher's index and middle fingers or their reflections in the globe.

The point halfway between your eyes is indeed the focus in a figurative sense, but it is not an optical focus. The principal axis will always lie in a line that runs through the center of the sphere and the bridge of your nose. Outside the globe, you are at the center of your observable universe. If you wink at the ball, the center of the looking-glass world hops over to the location of the image of your open eye.

Q36.27 You have likely seen a Fresnel mirror for sound. The diagram represents first a side view of a band shell. It is a concave mirror for sound, designed to channel sound into a beam toward the audience in front of the band shell. Sections of its surface can be kept at the right orientations as they are pushed around inside a rectangular box to form an auditorium with good diffusion of sound from stage to audience, with a floor plan suggested by the second part of the diagram.

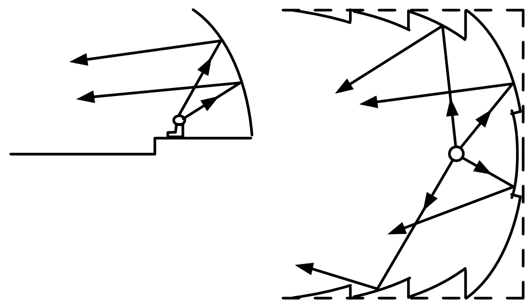


FIG. Q36.27

SOLUTIONS TO PROBLEMS

Section 36.1 Images Formed by Flat Mirrors

P36.1 I stand 40 cm from my bathroom mirror. I scatter light, which travels to the mirror and back to me in time

$$\frac{0.8\text{ m}}{3\times10^{8}\text{ m/s}}\ \boxed{\sim10^{-9}\text{ s}}$$

showing me a view of myself as I was at that look-back time. I'm no Dorian Gray!

P36.2 The virtual image is as far behind the mirror as the choir is in front of the mirror. Thus, the image is 5.30 m behind the mirror. The image of the choir is $0.800\text{ m}+5.30\text{ m}=6.10\text{ m}$ from the organist. Using similar triangles:

$$\frac{h'}{0.600\text{ m}}=\frac{6.10\text{ m}}{0.800\text{ m}}$$

or $$h'=(0.600\text{ m})\left(\frac{6.10\text{ m}}{0.800\text{ m}}\right)=\boxed{4.58\text{ m}}$$

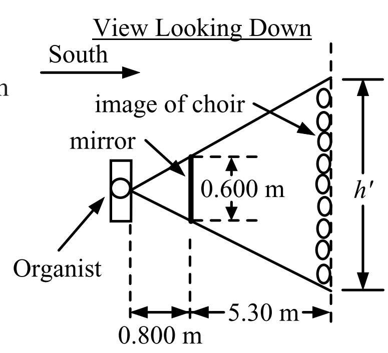


FIG. P36.2

P36.3 The flatness of the mirror is described

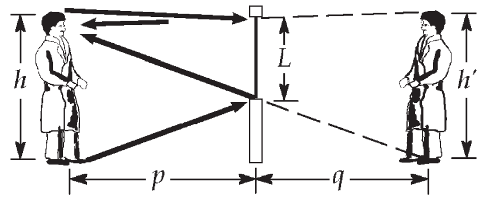


FIG. P36.3

by $R = \infty\, ,\ f = \infty$

and $\frac{1}{f} = 0$

By our general mirror equation,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = 0$$

or $q = -p$

Thus, the image is as far behind the mirror as the person is in front. The magnification is then

$$M = \frac{-q}{p} = 1 = \frac{h'}{h}$$

so $h' = h = 70.0$ inches

The required height of the mirror is defined by the triangle from the person's eyes to the top and bottom of his image, as shown. From the geometry of the triangle, we see that the mirror height must be:

$$h'\left(\frac{p}{p-q}\right) = h'\left(\frac{p}{2p}\right) = \frac{h'}{2}$$

Thus, the mirror must be $\boxed{\text{at least 35.0 inches high}}$.

P36.4 (1) The first image in the left mirror is 5.00 ft behind the mirror, or $\boxed{10.0 \text{ ft}}$ from the position of the person.

(2) The first image in the right mirror is located 10.0 ft behind the right mirror, but this location is 25.0 ft from the left mirror. Thus, the second image in the left mirror is 25.0 ft behind the mirror, or $\boxed{30.0 \text{ ft}}$ from the person.

(3) The first image in the left mirror forms an image in the right mirror. This first image is 20.0 ft from the right mirror, and, thus, an image 20.0 ft behind the right mirror is formed. This image in the right mirror also forms an image in the left mirror. The distance from this image in the right mirror to the left mirror is 35.0 ft. The third image in the left mirror is, thus, 35.0 ft behind the mirror, or $\boxed{40.0 \text{ ft}}$ from the person.

P36.5 (a) The flat mirrors have

$$R \to \infty$$

and $f \to \infty$

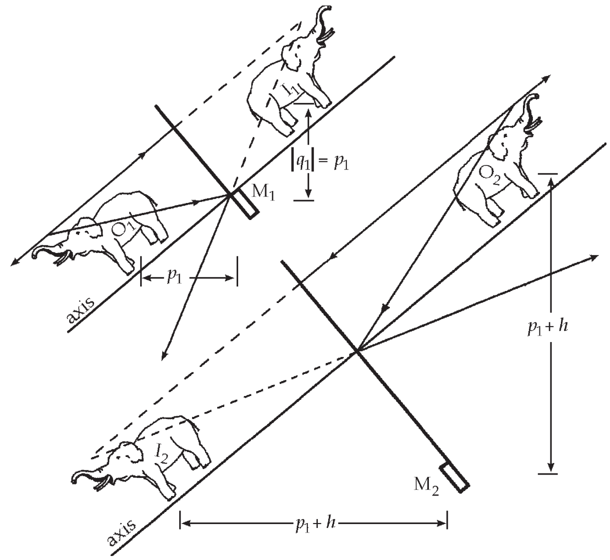


FIG. P36.5

The upper mirror M_1 produces a virtual, actual sized image I_1 according to

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f} = \frac{1}{\infty} = 0$$

$$q_1 = -p_1$$

with $M_1 = -\frac{q_1}{p_1} = +1$

As shown, this image is above the upper mirror. It is the object for mirror M_2, at object distance

$$p_2 = p_1 + h$$

The lower mirror produces a virtual, actual-size, right-side-up image according to

$$\frac{1}{p_2} + \frac{1}{q_2} = 0$$

$$q_2 = -p_2 = -(p_1 + h)$$

with $M_2 = -\frac{q_2}{p_2} = +1$ and $M_{\text{overall}} = M_1 M_2 = 1.$

Thus the final image is at distance $\boxed{p_1 + h}$ behind the lower mirror.

(b) It is $\boxed{\text{virtual}}$.

(c) $\boxed{\text{Upright}}$

(d) With magnification $\boxed{+1}$.

(e) It $\boxed{\text{does not appear to be reversed}}$ left and right. In a top view of the periscope, parallel rays from the right and left sides of the object stay parallel and on the right and left.

Section 36.2 **Images Formed by Spherical Mirrors**

P36.6 For a concave mirror, both R and f are positive.

We also know that $$f = \frac{R}{2} = 10.0 \text{ cm}$$

(a) $$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0 \text{ cm}} - \frac{1}{40.0 \text{ cm}} = \frac{3}{40.0 \text{ cm}}$$

and $$\boxed{q = 13.3 \text{ cm}}$$

$$M = \frac{q}{p} = -\frac{13.3 \text{ cm}}{40.0 \text{ cm}} = \boxed{-0.333}$$

The image is 13.3 cm in front of the mirror, $\boxed{\text{real, and inverted}}$.

(b) $$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0 \text{ cm}} - \frac{1}{20.0 \text{ cm}} = \frac{1}{20.0 \text{ cm}}$$

and $$\boxed{q = 20.0 \text{ cm}}$$

$$M = \frac{q}{p} = -\frac{20.0 \text{ cm}}{20.0 \text{ cm}} = \boxed{-1.00}$$

The image is 20.0 cm in front of the mirror, $\boxed{\text{real, and inverted}}$.

(c) $$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0 \text{ cm}} - \frac{1}{10.0 \text{ cm}} = 0$$

Thus, $$q = \text{infinity}$$

$\boxed{\text{No image is formed}}$. The rays are reflected parallel to each other.

P36.7 (a) $$\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$$ gives $$\frac{1}{30.0 \text{ cm}} + \frac{1}{q} = \frac{2}{(-40.0 \text{ cm})}$$

$$\frac{1}{q} = -\frac{2}{40.0 \text{ cm}} - \frac{1}{30.0 \text{ cm}} = -0.083\,3 \text{ cm}^{-1}$$ so $$q = \boxed{-12.0 \text{ cm}}$$

$$M = \frac{-q}{p} = -\frac{(-12.0 \text{ cm})}{30.0 \text{ cm}} = \boxed{0.400}$$

(b) $$\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$$ gives $$\frac{1}{60.0 \text{ cm}} + \frac{1}{q} = \frac{2}{(-40.0 \text{ cm})}$$

$$\frac{1}{q} = -\frac{2}{40.0 \text{ cm}} - \frac{1}{60.0 \text{ cm}} = -0.066\,6 \text{ cm}^{-1}$$ so $$q = \boxed{-15.0 \text{ cm}}$$

$$M = \frac{-q}{p} = -\frac{(-15.0 \text{ cm})}{60.0 \text{ cm}} = \boxed{0.250}$$

(c) Since $M > 0$, the images are $\boxed{\text{upright}}$.

P36.8 $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=-\frac{1}{0.275\text{ m}}-\frac{1}{10.0\text{ m}}$ gives $\boxed{q=-0.267\text{ m}}$

Thus, the image is $\boxed{\text{virtual}}$.

$$M=\frac{-q}{p}=-\frac{-0.267}{10.0\text{ m}}=\boxed{0.026\,7}$$

Thus, the image is $\boxed{\text{upright}}\,(+M)$ and $\boxed{\text{diminished}}\,(|M|<1)$

P36.9 (a) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ becomes $\frac{1}{q}=\frac{2}{60.0\text{ cm}}-\frac{1}{90.0\text{ cm}}$

$q=\boxed{45.0\text{ cm}}$ and $M=\frac{-q}{p}=-\frac{45.0\text{ cm}}{90.0\text{ cm}}=\boxed{-0.500}$

(b) $\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$ becomes $\frac{1}{q}=\frac{2}{60.0\text{ cm}}-\frac{1}{20.0\text{ cm}}$

$q=\boxed{-60.0\text{ cm}}$ and $M=\frac{-q}{p}=-\frac{(-60.0\text{ cm})}{(20.0\text{ cm})}=\boxed{3.00}$

(c) The image (a) is real, inverted, and diminished. That of (b) is virtual, upright, and enlarged. The ray diagrams are similar to Figures 36.13(a) and 36.13(b) in the text, respectively.

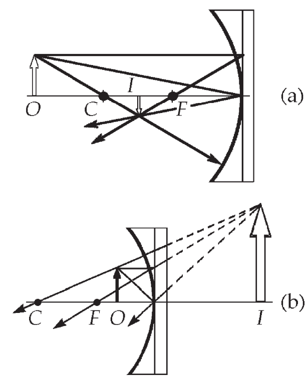


FIG. P36.9

P36.10 With radius 2.50 m, the cylindrical wall is a highly efficient mirror for sound, with focal length

$$f=\frac{R}{2}=1.25\text{ m}$$

In a vertical plane the sound disperses as usual, but that radiated in a horizontal plane is concentrated in a sound image at distance q from the back of the niche, where

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}\quad\text{so}\quad\frac{1}{2.00\text{ m}}+\frac{1}{q}=\frac{1}{1.25\text{ m}}$$

$$q=\boxed{3.33\text{ m}}$$

***P36.11** (a) Since the object is in front of the mirror, $p>0$. With the image behind the mirror, $q<0$. The mirror equation gives the radius of curvature as $\frac{2}{R}=\frac{1}{p}+\frac{1}{q}=\frac{1}{1.00\text{ cm}}-\frac{1}{10.0\text{ cm}}=\frac{10-1}{10.0\text{ cm}}$,

or $R=2\left(\frac{10.0\text{ cm}}{9}\right)=\boxed{+2.22\text{ cm}}$.

(b) The magnification is $M=-\frac{q}{p}=-\frac{(-10.0\text{ cm})}{1.00\text{ cm}}=\boxed{+10.0}$.

P36.12 The ball is a convex mirror with $R = -4.25$ cm and $f = \frac{R}{2} = -2.125$ cm. We have

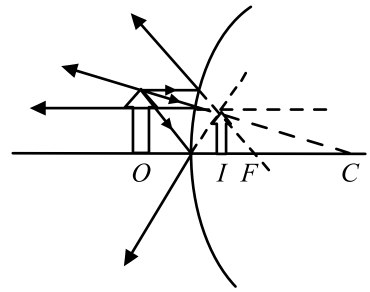


FIG. P36.12

$$M = \frac{3}{4} = -\frac{q}{p}$$

$$q = -\frac{3}{4}p$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{p} + \frac{1}{-(3/4)p} = \frac{1}{-2.125 \text{ cm}}$$

$$\frac{3}{3p} - \frac{4}{3p} = \frac{1}{-2.125 \text{ cm}}$$

$$3p = 2.125 \text{ cm}$$

$\boxed{p = 0.708 \text{ cm}}$ in front of the sphere.

The image is upright, virtual, and diminished.

P36.13 (a) $M = -4 = -\frac{q}{p}$ $\quad q = 4p$

$q - p = 0.60 \text{ m} = 4p - p$ $\quad p = 0.2$ m $\quad q = 0.8$ m

$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{0.2 \text{ m}} + \frac{1}{0.8 \text{ m}}$ $\quad f = \boxed{160 \text{ mm}}$

(b) $M = +\frac{1}{2} = -\frac{q}{p}$ $\quad p = -2q$

$|q| + p = 0.20 \text{ m} = -q + p = -q - 2q$

$q = -66.7$ mm $\quad p = 133$ mm

$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} = \frac{1}{0.133 \text{ m}} + \frac{1}{-0.066\,7 \text{ m}}$ $\quad R = \boxed{-267 \text{ mm}}$

P36.14 $M = -\frac{q}{p}$

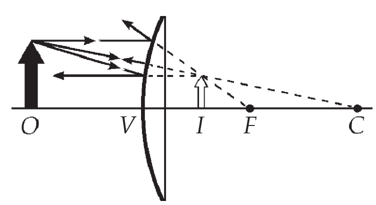


FIG. P36.14

$$q = -Mp = -0.013(30 \text{ cm}) = -0.39 \text{ cm}$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R}$$

$$\frac{1}{30 \text{ cm}} + \frac{1}{-0.39 \text{ cm}} = \frac{2}{R}$$

$$R = \frac{2}{-2.53 \text{ m}^{-1}} = -0.790 \text{ cm}$$

The cornea is convex, with radius of curvature $\boxed{0.790 \text{ cm}}$.

P36.15 With

$$M=\frac{h'}{h}=\frac{+4.00\text{ cm}}{10.0\text{ cm}}=+0.400=-\frac{q}{p}$$

$$q=-0.400p$$

the image must be virtual.

(a) It is a $\boxed{\text{convex}}$ mirror that produces a diminished upright virtual image.

(b) We must have

$$p+|q|=42.0\text{ cm}=p-q$$

$$p=42.0\text{ cm}+q$$

$$p=42.0\text{ cm}-0.400p$$

$$p=\frac{42.0\text{ cm}}{1.40}=30.0\text{ cm}$$

The mirror is $\boxed{\text{at the 30.0 cm mark}}$.

(c) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{1}{30\text{ cm}}+\frac{1}{-0.4(30\text{ cm})}=\frac{1}{f}=-0.050\ 0/\text{cm}$ $\boxed{f=-20.0\text{ cm}}$

The ray diagram looks like Figure 36.13(c) in the text.

P36.16 Assume that the object distance is the same in both cases (i.e., her face is the same distance from the hubcap regardless of which way it is turned). Also realize that the near image ($q=-10.0$ cm) occurs when using the convex side of the hubcap. Applying the mirror equation to both cases gives:

(concave side: $R=|R|$, $q=-30.0$ cm)

$$\frac{1}{p}-\frac{1}{30.0}=\frac{2}{|R|}$$

or

$$\frac{2}{|R|}=\frac{30.0\text{ cm}-p}{(30.0\text{ cm})p} \quad (1)$$

(convex side: $R=-|R|$, $q=-10.0$ cm)

$$\frac{1}{p}-\frac{1}{10.0}=-\frac{2}{|R|}$$

or

$$\frac{2}{|R|}=\frac{p-10.0\text{ cm}}{(10.0\text{ cm})p} \quad (2)$$

(a) Equating Equations (1) and (2) gives:

$$\frac{30.0\text{ cm}-p}{3.00}=p-10.0\text{ cm}$$

or

$$p=15.0\text{ cm}$$

Thus, her face is $\boxed{15.0\text{ cm}}$ from the hubcap.

continued on next page

(b) Using the above result ($p = 15.0$ cm) in Equation (1) gives:

$$\frac{2}{|R|} = \frac{30.0\text{ cm} - 15.0\text{ cm}}{(30.0\text{ cm})(15.0\text{ cm})}$$

or $$\frac{2}{|R|} = \frac{1}{30.0\text{ cm}}$$

and $$|R| = 60.0\text{ cm}$$

The radius of the hubcap is $\boxed{60.0\text{ cm}}$.

P36.17 (a) $q = (p + 5.00\text{ m})$ and, since the image must be real,

$$M = -\frac{q}{p} = -5 \quad \text{or} \quad q = 5p$$

Therefore, $$p + 5.00\text{ m} = 5p$$

or $p = 1.25$ m and $q = 6.25$ m

From $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$, $$R = \frac{2pq}{p+q} = \frac{2(1.25)(6.25)}{1.25 + 6.25}$$

$$= \boxed{2.08\text{ m (concave)}}$$

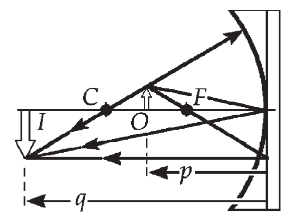


FIG. P36.17

(b) From part (a), $p = 1.25$ m; the mirror should be $\boxed{1.25\text{ m}}$ in front of the object.

P36.18 (a) The flat mirror produces an image according to

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R} \qquad \frac{1}{24\text{ cm}} + \frac{1}{q} = \frac{1}{\infty} = 0 \qquad q = -24.0\text{ m}$$

The image is 24.0 m behind the mirror, distant from your eyes by

$$1.55\text{ m} + 24.0\text{ m} = \boxed{25.6\text{ m}}$$

(b) The image is the same size as the object, so $\theta = \frac{h}{d} = \frac{1.50\text{ m}}{25.6\text{ m}} = \boxed{0.058\,7\text{ rad}}$

(c) $$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} \qquad \frac{1}{24\text{ m}} + \frac{1}{q} = \frac{2}{(-2\text{ m})} \qquad q = \frac{1}{-(1/1\text{ m}) - (1/24\text{ m})} = -0.960\text{ m}$$

This image is distant from your eyes by $1.55\text{ m} + 0.960\text{ m} = \boxed{2.51\text{ m}}$

(d) The image size is given by $M = \frac{h'}{h} = -\frac{q}{p}$ $$h' = -h\frac{q}{p} = -1.50\text{ m}\left(\frac{-0.960\text{ m}}{24\text{ m}}\right)$$

$$= 0.060\,0\text{ m.}$$

So its angular size at your eye is $\theta' = \frac{h'}{d} = \frac{0.06\text{ m}}{2.51\text{ m}} = \boxed{0.023\,9\text{ rad}}$

(e) Your brain assumes that the car is 1.50 m high and calculates its distance as

$$d' = \frac{h}{\theta'} = \frac{1.50\text{ m}}{0.023\,9} = \boxed{62.8\text{ m}}$$

P36.19 (a) The image starts from a point whose height above the mirror vertex is given by

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{2}{R} \qquad \frac{1}{3.00\text{ m}}+\frac{1}{q}=\frac{1}{0.500\text{ m}} \qquad \text{Therefore,} \qquad q=0.600\text{ m}$$

As the ball falls, p decreases and q increases. Ball and image pass when $q_1 = p_1$. When this is true,

$$\frac{1}{p_1}+\frac{1}{p_1}=\frac{1}{0.500\text{ m}}=\frac{2}{p_1} \qquad \text{or} \qquad p_1=1.00\text{ m}$$

As the ball passes the focal point, the image switches from infinitely far above the mirror to infinitely far below the mirror. As the ball approaches the mirror from above, the virtual image approaches the mirror from below, reaching it together when $p_2 = q_2 = 0$.

(b) The falling ball passes its real image when it has fallen

$$3.00\text{ m}-1.00\text{ m}=2.00\text{ m}=\frac{1}{2}gt^2,\text{ or when } t=\sqrt{\frac{2(2.00\text{ m})}{9.80\text{ m/s}^2}}=\boxed{0.639\text{ s}}$$

The ball reaches its virtual image when it has traversed

$$3.00\text{ m}-0=3.00\text{ m}=\frac{1}{2}gt^2,\text{ or at } t=\sqrt{\frac{2(3.00\text{ m})}{9.80\text{ m/s}^2}}=\boxed{0.782\text{ s}}$$

Section 36.3 **Images Formed by Refraction**

P36.20 When $R\to\infty$, the equation describing image formation at a single refracting surface becomes $q=-p\left(\frac{n_2}{n_1}\right)$. We use this to locate the final images of the two surfaces of the glass plate. First, find the image the glass forms of the *bottom* of the plate.

$$q_{B1}=-\left(\frac{1.33}{1.66}\right)(8.00\text{ cm})=-6.41\text{ cm}$$

This virtual image is 6.41 cm below the top surface of the glass of 18.41 cm below the water surface. Next, use this image as an object and locate the image the water forms of the bottom of the plate.

$$q_{B2}=-\left(\frac{1.00}{1.33}\right)(18.41\text{ cm})=-13.84\text{ cm} \qquad \text{or} \qquad 13.84\text{ cm below the water surface}$$

Now find image the water forms of the *top* surface of the glass.

$$q_3=-\left(\frac{1}{1.33}\right)(12.0\text{ cm})=-9.02\text{ cm} \qquad \text{or} \qquad 9.02\text{ cm below the water surface}$$

Therefore, the apparent thickness of the glass is $\Delta t=13.84\text{ cm}-9.02\text{ cm}=\boxed{4.82\text{ cm}}$.

P36.21 $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}=0$ and $R\to\infty$

$$q=-\frac{n_2}{n_1}p=-\frac{1}{1.309}(50.0\text{ cm})=-38.2\text{ cm}$$

Thus, the virtual image of the dust speck is $\boxed{\text{38.2 cm below the top surface}}$ of the ice.

P36.22 $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$ becomes $\frac{1.00}{p}+\frac{1.50}{q}=\frac{1.50-1.00}{6.00\text{ cm}}=\frac{1}{12.0\text{ cm}}$

(a) $\frac{1.00}{20.0\text{ cm}}+\frac{1.50}{q}=\frac{1}{12.0\text{ cm}}$ or $q=\frac{1.50}{\left[(1.00/12.0\text{ cm})-(1.00/20.0\text{ cm})\right]}=\boxed{45.0\text{ cm}}$

(b) $\frac{1.00}{10.0\text{ cm}}+\frac{1.50}{q}=\frac{1}{12.0\text{ cm}}$ or $q=\frac{1.50}{\left[(1.00/12.0\text{ cm})-(1.00/10.0\text{ cm})\right]}=\boxed{-90.0\text{ cm}}$

(c) $\frac{1.00}{3.0\text{ cm}}+\frac{1.50}{q}=\frac{1}{12.0\text{ cm}}$ or $q=\frac{1.50}{\left[(1.00/12.0\text{ cm})-(1.00/3.0\text{ cm})\right]}=\boxed{-6.00\text{ cm}}$

P36.23 From Equation 36.8 $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$

Solve for q to find $q=\frac{n_2Rp}{p(n_2-n_1)-n_1R}$

In this case, $n_1=1.50,\ n_2=1.00,\ R=-15.0\text{ cm}$

and $p=10.0\text{ cm}$

So $q=\frac{(1.00)(-15.0\text{ cm})(10.0\text{ cm})}{(10.0\text{ cm})(1.00-1.50)-(1.50)(-15.0\text{ cm})}=-8.57\text{ cm}$

Therefore, the $\boxed{\text{apparent depth is 8.57 cm}}$

***P36.24** In the right triangle lying between O and the center of the curved surface, $\tan\theta_1=h/p$. In the right triangle lying between I and the center of the surface, $\tan\theta_2=-h'/q$. We need the negative sign because the image height is counted as negative while the angle is not. We substitute into the given $n_1\tan\theta_1=n_2\tan\theta_2$ to obtain $n_1h/p=-n_2h'/q$. Then the magnification, defined by $M=h'/h$, is given by $M=h'/h=-n_1q/n_2p$.

***P36.25** (a) The center of curvature is on the object side, so the radius of curvature is negative.

$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$ becomes $\frac{1.33}{30\text{ cm}}+\frac{1.00}{q}=\frac{1.00-1.33}{-80\text{ cm}}$ $q=-24.9\text{ cm}$

So $\boxed{\text{the image is inside the tank, 24.9 cm behind the front wall; virtual, right side up, enlarged.}}$

(b) Now we have $\frac{1.33}{90\text{ cm}}+\frac{1.00}{q}=\frac{1.00-1.33}{-80\text{ cm}}$ $q=-93.9\text{ cm}$

So $\boxed{\text{the image is inside the tank, 93.9 cm behind the front wall; virtual, right side up, enlarged.}}$

(c) In case (a) the result of problem 24 gives $M=-\frac{n_1q}{n_2p}=-\frac{1.33(-24.9)}{1.00(30)}=\boxed{+1.10}$

In case (b) we have $M=-\frac{1.33(-93.9)}{1.00(90)}=\boxed{+1.39}$

(d) In case (a) $h'=Mh=1.10(9.00\text{ cm})=\boxed{9.92\text{ cm}}$. In case (b), the farther lobster looms larger:

$h'=Mh=1.30(9.00\text{ cm})=\boxed{12.5\text{ cm}}$

(e) The plastic has uniform thickness, so the surfaces of entry and exit for any particular ray are very nearly parallel. The ray is slightly displaced, but it would not be changed in direction by going through the plastic wall with air on both sides. Only the difference between the air and water is responsible for the refraction of the light.

P36.26 For a plane surface, $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$ becomes $q=-\frac{n_2 p}{n_1}$

Thus, the magnitudes of the rate of change in the image and object positions are related by

$$\left|\frac{dq}{dt}\right|=\frac{n_2}{n_1}\left|\frac{dp}{dt}\right|$$

If the fish swims toward the wall with a speed of 2.00 cm/s, the speed of the image is given by

$$v_{\text{image}}=\left|\frac{dq}{dt}\right|=\frac{1.00}{1.33}(2.00\ \text{cm/s})=\boxed{1.50\ \text{cm/s}}$$

Section 36.4 Thin Lenses

P36.27 (a) $\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=(0.440)\left[\frac{1}{12.0\ \text{cm}}-\frac{1}{(-18.0\ \text{cm})}\right]$

$f=\boxed{16.4\ \text{cm}}$

(b) $\frac{1}{f}=(0.440)\left[\frac{1}{18.0\ \text{cm}}-\frac{1}{(-12.0\ \text{cm})}\right]$

$f=\boxed{16.4\ \text{cm}}$

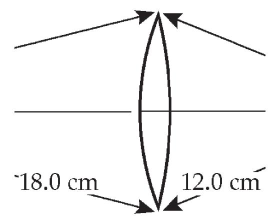


FIG. P36.27

P36.28 Let R_1 = outer radius and R_2 = inner radius

$$\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=(1.50-1)\left[\frac{1}{2.00\ \text{m}}-\frac{1}{2.50\ \text{cm}}\right]=0.050\,0\ \text{cm}^{-1}$$

so $f=\boxed{20.0\ \text{cm}}$

P36.29 For a converging lens, f is positive. We use $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$.

(a) $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{20.0\ \text{cm}}-\frac{1}{40.0\ \text{cm}}=\frac{1}{40.0\ \text{cm}}$ $\boxed{q=40.0\ \text{cm}}$

$M=-\frac{q}{p}=-\frac{40.0}{40.0}=\boxed{-1.00}$

The image is $\boxed{\text{real, inverted}}$, and located 40.0 cm past the lens.

(b) $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{20.0\ \text{cm}}-\frac{1}{20.0\ \text{cm}}=0$ $\boxed{q=\text{infinity}}$

$\boxed{\text{No image}}$ is formed. The rays emerging from the lens are parallel to each other.

(c) $\frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{1}{20.0\ \text{cm}}-\frac{1}{10.0\ \text{cm}}=-\frac{1}{20.0\ \text{cm}}$ $\boxed{q=-20.0\ \text{cm}}$

$M=-\frac{q}{p}=-\frac{(-20.0)}{10.0}=\boxed{2.00}$

The image is $\boxed{\text{upright, virtual}}$ and 20.0 cm in front of the lens.

P36.30 (a) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$: $\quad \frac{1}{32.0\text{ cm}}+\frac{1}{8.00\text{ cm}}=\frac{1}{f}$

so $\boxed{f=6.40\text{ cm}}$

(b) $M=-\frac{q}{p}=-\frac{8.00\text{ cm}}{32.0\text{ cm}}=\boxed{-0.250}$

(c) Since $f>0$, the lens is $\boxed{\text{converging}}$.

P36.31 We are looking at an enlarged, upright, virtual image:

$M=\frac{h'}{h}=2=-\frac{q}{p}$ so $p=-\frac{q}{2}=-\frac{(-2.84\text{ cm})}{2}=+1.42\text{ cm}$

$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ gives $\frac{1}{1.42\text{ cm}}+\frac{1}{(-2.84\text{ cm})}=\frac{1}{f}$

$\boxed{f=2.84\text{ cm}}$

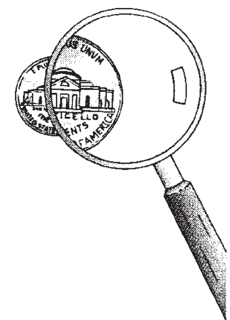


FIG. P36.31

P36.32 $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$: $\quad p^{-1}+q^{-1}=\text{constant}$

We may differentiate through with respect to p: $\quad -1p^{-2}-1q^{-2}\frac{dq}{dp}=0$

$$\frac{dq}{dp}=-\frac{q^2}{p^2}=-M^2$$

P36.33 (a) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ $\quad \frac{1}{20.0\text{ cm}}+\frac{1}{q}=\frac{1}{(-32.0\text{ cm})}$

so $q=-\left(\frac{1}{20.0}+\frac{1}{32.0}\right)^{-1}=\boxed{-12.3\text{ cm}}$

The image is 12.3 cm to the left of the lens.

(b) $M=-\frac{q}{p}=-\frac{(-12.3\text{ cm})}{20.0\text{ cm}}=\boxed{0.615}$

(c) See the ray diagram to the right.

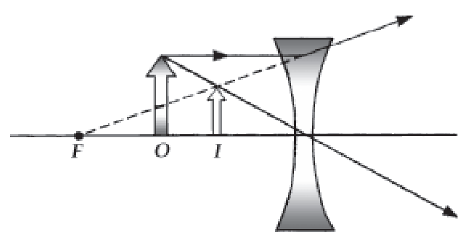


FIG. P36.33

P36.34 The image is inverted: $\quad M=\frac{h'}{h}=\frac{-1.80\text{ m}}{0.024\,0\text{ m}}=-75.0=\frac{-q}{p}$ $\quad q=75.0p$

(a) $q+p=3.00\text{ m}=75.0p+p$ $\quad p=\boxed{39.5\text{ mm}}$

(b) $q=2.96\text{ m}$ $\quad \frac{1}{f}=\frac{1}{p}+\frac{1}{q}=\frac{1}{0.039\,5\text{ m}}+\frac{1}{2.96\text{ m}}$ $\quad f=\boxed{39.0\text{ mm}}$

***P36.35** Comparing $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ with $\frac{1}{p}+\frac{1}{-3.5p}=\frac{1}{7.5\text{ cm}}$ we see $q=-3.5p$ and $f=7.50$ cm for a converging lens.

(a) To solve, we add the fractions:

$$\frac{-3.5+1}{-3.5p}=\frac{1}{7.5\text{ cm}}$$

$$\frac{3.5p}{2.5}=7.5\text{ cm}$$

$$p=\boxed{5.36\text{ cm}}$$

(b) $q=-3.5(5.36\text{ cm})=\boxed{-18.8\text{ cm}}$ $\qquad M=-\frac{q}{p}=-\frac{-18.8\text{ cm}}{5.36\text{ cm}}=+3.50$

(c)

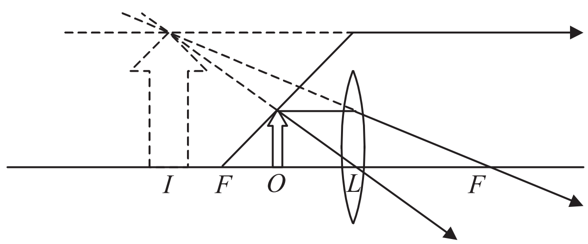


FIG. P36.35(c)

The image is $\boxed{\text{enlarged, upright, and virtual.}}$

(d) The lens is being used as a magnifying glass. Statement: A magnifying glass with focal length 7.50 cm is used to form an image of a stamp, enlarged 3.50 times. Find the object distance. Locate and describe the image.

P36.36 In $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ or $p^{-1}+q^{-1}=$ constant, we differentiate with respect to time:

$$-1\left(p^{-2}\right)\frac{dp}{dt}-1\left(q^{-2}\right)\frac{dq}{dt}=0$$

$$\frac{dq}{dt}=\frac{-q^2}{p^2}\frac{dp}{dt}$$

We must find the momentary image location q:

$$\frac{1}{20\text{ m}}+\frac{1}{q}=\frac{1}{0.3\text{ m}}$$

$$q=0.305\text{ m}$$

Now $\frac{dq}{dt}=-\frac{(0.305\text{ m})^2}{(20\text{ m})^2}5\text{ m/s}=-0.00116\text{ m/s}=\boxed{1.16\text{ mm/s toward the lens}}$.

P36.37 (a) The image distance is: $q = d - p$

Thus, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ becomes $\frac{1}{p} + \frac{1}{d-p} = \frac{1}{f}$

This reduces to a quadratic equation: $p^2 + (-d)p + fd = 0$

which yields: $\boxed{p = \frac{d \pm \sqrt{d^2 - 4fd}}{2} = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} - fd}}$

Since $f < \frac{d}{4}$, both solutions are meaningful and the two solutions are not equal to each other. Thus, there are two distinct lens positions that form an image on the screen.

(b) The smaller solution for p gives a larger value for q, with a $\boxed{\text{real, enlarged, inverted image}}$.

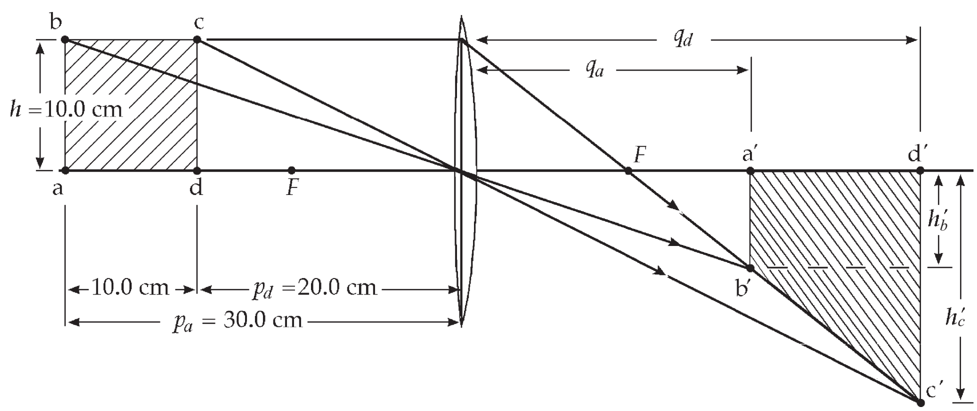
The larger solution for p describes a $\boxed{\text{real, diminished, inverted image}}$.

***P36.38** (a) $\frac{1}{p_a} + \frac{1}{q_a} = \frac{1}{f}$ becomes $\frac{1}{30.0 \text{ cm}} + \frac{1}{q_1} = \frac{1}{14.0 \text{ cm}}$ or $\boxed{q_a = 26.2 \text{ cm}}$

$$h'_b = hM_a = h\left(\frac{-q_a}{p_a}\right) = (10.0 \text{ cm})(-0.875) = \boxed{-8.75 \text{ cm}}$$

$\frac{1}{20.0 \text{ cm}} + \frac{1}{q_d} = \frac{1}{14.0 \text{ cm}}$ or $\boxed{q_d = 46.7 \text{ cm}}$

$$h'_c = hM_d = (10.0 \text{ cm})(-2.33) = \boxed{-23.3 \text{ cm}}$$



The square is imaged as a trapezoid.

FIG. P36.38(b)

(b) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ becomes $\frac{1}{p} + \frac{1}{q} = \frac{1}{14 \text{ cm}}$ or $1/p = 1/14 \text{ cm} - 1/q$

$$|h'| = |hM| = \left| h\left(\frac{-q}{p}\right) \right| = (10.0 \text{ cm})q\left(\frac{1}{14 \text{ cm}} - \frac{1}{q}\right)$$

(c) The quantity $\int_{q_a}^{q_d} |h'| dq$ adds up the geometrical (positive) areas of thin vertical ribbons comprising the whole area of the image. We have

$$\int_{q_a}^{q_d} |h'| dq = \int_{q_a}^{q_d} (10.0 \text{ cm})\left(\frac{q}{14 \text{ cm}} - 1\right) dq = (10.0 \text{ cm})\left(\frac{q^2}{28 \text{ cm}} - q\right)_{26.2 \text{ cm}}^{46.7 \text{ cm}}$$

$$\text{Area} = (10.0 \text{ cm})\left(\frac{46.7^2 - 26.2^2}{28} - 46.7 + 26.2\right) \text{ cm} = \boxed{328 \text{ cm}^2}$$

P36.39 To properly focus the image of a distant object, the lens must be at a distance equal to the focal length from the film ($q_1 = 65.0$ mm). For the closer object:

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f}$$

becomes $$\frac{1}{2\,000 \text{ mm}} + \frac{1}{q_2} = \frac{1}{65.0 \text{ mm}}$$

and $$q_2 = (65.0 \text{ mm})\left(\frac{2\,000}{2\,000 - 65.0}\right)$$

The lens must be moved $\boxed{\text{away from the film}}$ by a distance

$$D = q_2 - q_1 = (65.0 \text{ mm})\left(\frac{2\,000}{2\,000 - 65.0}\right) - 65.0 \text{ mm} = \boxed{2.18 \text{ mm}}$$

Section 36.5 **Lens Aberrations**

P36.40 (a) The focal length of the lens is given by

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.53 - 1.00)\left(\frac{1}{-32.5 \text{ cm}} - \frac{1}{42.5 \text{ cm}}\right)$$

$f = -34.7$ cm

Note that R_1 is negative because the center of curvature of the first surface is on the virtual image side.

When $p = \infty$

the thin lens equation gives $q = f$

Thus, the violet image of a very distant object is formed

at $\boxed{q = -34.7 \text{ cm}}$

The image is $\boxed{\text{virtual, upright and diminshed}}$.

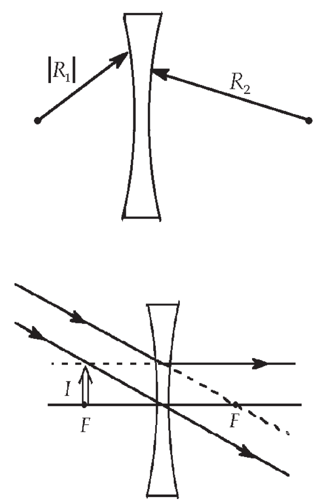


FIG. P36.40

(b) The same ray diagram and image characteristics apply for red light.

Again, $q = f$

and now $$\frac{1}{f} = (1.51 - 1.00)\left(\frac{1}{-32.5 \text{ cm}} - \frac{1}{42.5 \text{ cm}}\right)$$

giving $f = \boxed{-36.1 \text{ cm}}$

P36.41 Ray h_1 is undeviated at the plane surface and strikes the second surface at angle of incidence given by

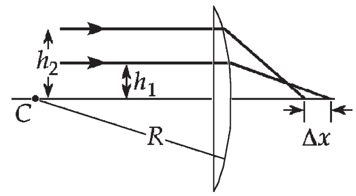


FIG. P36.41

$$\theta_1 = \sin^{-1}\left(\frac{h_1}{R}\right) = \sin^{-1}\left(\frac{0.500 \text{ cm}}{20.0 \text{ cm}}\right) = 1.43°$$

Then, $$1.00 \sin\theta_2 = 1.60 \sin\theta_1 = (1.60)\left(\frac{0.500}{20.0 \text{ cm}}\right)$$

so $$\theta_2 = 2.29°$$

The angle this emerging ray makes with the horizontal is $\theta_2 - \theta_1 = 0.860°$

It crosses the axis at a point farther out by f_1

where $$f_1 = \frac{h_1}{\tan(\theta_2 - \theta_1)} = \frac{0.500 \text{ cm}}{\tan(0.860°)} = 33.3 \text{ cm}$$

The point of exit for this ray is distant axially from the lens vertex by

$$20.0 \text{ cm} - \sqrt{(20.0 \text{ cm})^2 - (0.500 \text{ cm})^2} = 0.006\ 25 \text{ cm}$$

so ray h_1 crosses the axis at this distance from the vertex:

$$x_1 = 33.3 \text{ cm} - 0.006\ 25 \text{ cm} = 33.3 \text{ cm}$$

Now we repeat this calculation for ray h_2:

$$\theta = \sin^{-1}\left(\frac{12.0 \text{ cm}}{20.0 \text{ cm}}\right) = 36.9°$$

$$1.00 \sin\theta_2 = 1.60 \sin\theta_1 = (1.60)\left(\frac{12.00}{20.0}\right) \qquad \theta_2 = 73.7°$$

$$f_2 = \frac{h_2}{\tan(\theta_1 - \theta_2)} = \frac{12.0 \text{ cm}}{\tan 36.8°} = 16.0 \text{ cm}$$

$$x_2 = (16.0 \text{ cm})\left(20.0 \text{ cm} - \sqrt{(20.0 \text{ cm})^2 - (12.0 \text{ cm})^2}\right) = 12.0 \text{ cm}$$

Now $$\Delta x = 33.3 \text{ cm} - 12.0 \text{ cm} = \boxed{21.3 \text{ cm}}$$

Section 36.6 **The Camera**

P36.42 The same light intensity is received from the subject, and the same light energy on the film is required:

$$IA_1\Delta t_1 = IA_2\Delta t_2$$

$$\frac{\pi d_1^2}{4}\Delta t_1 = \frac{\pi d_2^2}{4}\Delta t_2$$

$$\left(\frac{f}{4}\right)^2\left(\frac{1}{16} \text{ s}\right) = d_2^2\left(\frac{1}{128} \text{ s}\right)$$

$$d_2 = \sqrt{\frac{128}{16}}\frac{f}{4} = \boxed{\frac{f}{1.41}}$$

Section 36.7 The Eye

P36.43 $P = \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{\infty} - \frac{1}{0.250 \text{ m}} = -4.00 \text{ diopters} = \boxed{-4.00 \text{ diopters, a diverging lens}}$

P36.44 For starlight going through Nick's glasses, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$$\frac{1}{\infty} + \frac{1}{(-0.800 \text{ m})} = \frac{1}{f} = -1.25 \text{ diopters}$$

For a nearby object, $\frac{1}{p} + \frac{1}{(-0.180 \text{ m})} = -1.25 \text{ m}^{-1}$, so $p = \boxed{23.2 \text{ cm}}$

Section 36.8 The Simple Magnifier

Section 36.9 The Compound Microscope

Section 36.10 The Telescope

P36.45 (a) From the thin lens equation: $\frac{1}{p} + \frac{1}{(-25.0 \text{ cm})} = \frac{1}{5.00 \text{ cm}}$ or $p = \boxed{4.17 \text{ cm}}$

(b) $M = -\frac{q}{p} = 1 + \frac{25.0 \text{ cm}}{f} = 1 + \frac{25.0 \text{ cm}}{5.00 \text{ cm}} = \boxed{6.00}$

P36.46 Using Equation 36.26, $M \approx -\left(\frac{L}{f_o}\right)\left(\frac{25.0 \text{ cm}}{f_e}\right) = -\left(\frac{23.0 \text{ cm}}{0.400 \text{ cm}}\right)\left(\frac{25.0 \text{ cm}}{2.50 \text{ cm}}\right) = \boxed{-575}$

P36.47 $f_o = 20.0 \text{ m}$ $\quad f_e = 0.025\,0 \text{ m}$

(a) The angular magnification produced by this telescope is $m = -\frac{f_o}{f_e} = \boxed{-800}$.

(b) Since $m < 0$, the image is $\boxed{\text{inverted}}$.

P36.48 (a) The mirror-and-lens equation $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

gives $q = \frac{1}{1/f - 1/p} = \frac{1}{(p-f)/fp} = \frac{fp}{p-f}$

Then, $M = \frac{h'}{h} = -\frac{q}{p} = -\frac{f}{p-f}$

gives $\boxed{h' = \frac{hf}{f-p}}$

(b) For $p >> f$, $f - p \approx -p$. Then, $h' = \boxed{-\frac{hf}{p}}$

(c) Suppose the telescope observes the space station at the zenith:

$$h' = -\frac{hf}{p} = -\frac{(108.6 \text{ m})(4.00 \text{ m})}{407 \times 10^3 \text{ m}} = \boxed{-1.07 \text{ mm}}$$

P36.49 Let I_0 represent the intensity of the light from the nebula and θ_0 its angular diameter. With the first telescope, the image diameter h' on the film is given by $\theta_o = -\frac{h'}{f_o}$ as $h' = -\theta_o(2\,000\text{ mm})$.

The light power captured by the telescope aperture is $\mathcal{P}_1 = I_0 A_1 = I_0\left[\frac{\pi(200\text{ mm})^2}{4}\right]$, and the light energy focused on the film during the exposure is $E_1 = \mathcal{P}_1 \Delta t_1 = I_0\left[\frac{\pi(200\text{ mm})^2}{4}\right](1.50\text{ min})$.

Likewise, the light power captured by the aperture of the second telescope is

$\mathcal{P}_2 = I_0 A_2 = I_0\left[\frac{\pi(60.0\text{ mm})^2}{4}\right]$ and the light energy is $E_2 = I_0\left[\frac{\pi(60.0\text{ mm})^2}{4}\right]\Delta t_2$. Therefore, to have the same light energy per unit area, it is necessary that

$$\frac{I_0\left[\pi(60.0\text{ mm})^2/4\right]\Delta t_2}{\pi\left[\theta_o(900\text{ mm})^2/4\right]} = \frac{I_0\left[\pi(200\text{ mm})^2/4\right](1.50\text{ min})}{\pi\left[\theta_o(2\,000\text{ mm})^2/4\right]}$$

The required exposure time with the second telescope is

$$\Delta t_2 = \frac{(200\text{ mm})^2(900\text{ mm})^2}{(60.0\text{ mm})^2(2\,000\text{ mm})^2}(1.50\text{ min}) = \boxed{3.38\text{ min}}$$

Additional Problems

P36.50 (a) $\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{5\text{ cm}} - \frac{1}{7.5\text{ cm}}$ $\quad\therefore q_1 = 15\text{ cm}$

$$M_1 = -\frac{q_1}{p_1} = -\frac{15\text{ cm}}{7.5\text{ cm}} = -2$$

$$M = M_1 M_2 \qquad \therefore 1 = (-2)M_2$$

$$\therefore M_2 = -\frac{1}{2} = -\frac{q_2}{p_2} \qquad \therefore p_2 = 2q_2$$

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2} \qquad \therefore \frac{1}{2q_2} + \frac{1}{q_2} = \frac{1}{10\text{ cm}} \qquad \therefore q_2 = 15\text{ cm},\ p_2 = 30\text{ cm}$$

$$p_1 + q_1 + p_2 + q_2 = 7.5\text{ cm} + 15\text{ cm} + 30\text{ cm} + 15\text{ cm} = \boxed{67.5\text{ cm}}$$

(b) $\frac{1}{p_1'} + \frac{1}{q_1'} = \frac{1}{f_1} = \frac{1}{5\text{ cm}}$

Solve for q_1' in terms of p_1': $q_1' = \frac{5p_1'}{p_1' - 5}$ (1)

$M_1' = -\frac{q_1'}{p_1'} = -\frac{5}{p_1' - 5}$, using (1).

$$M' = M_1' M_2' \qquad \therefore M_2' = \frac{M'}{M_1'} = -\frac{3}{5}(p_1' - 5) = -\frac{q_2'}{p_2'}$$

$\therefore q_2' = \frac{3}{5}p_2'(p_1' - 5)$ (2)

Substitute (2) into the lens equation $\frac{1}{p_2'} + \frac{1}{q_2'} = \frac{1}{f_2} = \frac{1}{10\text{ cm}}$ and obtain p_2' in terms of p_1':

continued on next page

$$p_2' = \frac{10(3p_1' - 10)}{3(p_1' - 5)} \qquad (3)$$

Substituting (3) in (2), obtain q_2' in terms of p_1':

$$q_2' = 2(3p_1' - 10) \qquad (4)$$

Now, $p_1' + q_1' + p_2' + q_2' =$ a constant.

Using (1), (3), and (4), and the value obtained in (a):

$$p_1' + \frac{5p_1'}{p_1' - 5} + \frac{10(3p_1' - 10)}{3(p' - 5)} + 2(3p_1' - 10) = 67.5$$

This reduces to the quadratic equation

$$21p_1'^2 - 322.5p_1' + 1\,212.5 = 0$$

which has solutions $p_1' = 8.784$ cm and 6.573 cm.

Case 1: $p_1' = 8.784$ cm

$\therefore\ p_1' - p_1 = 8.784 \text{ cm} - 7.5 \text{ cm} = 1.28 \text{ cm}$

From (4): $q_2' = 32.7$ cm

$\therefore\ q_2' - q_2 = 32.7 \text{ cm} - 15 \text{ cm} = 17.7 \text{ cm}$

Case 2: $p_1' = 6.573$ cm

$\therefore\ p_1' - p_1 = 6.573 \text{ cm} - 7.5 \text{ cm} = -0.927 \text{ cm}$

From (4): $q_2' = 19.44$ cm

$\therefore\ q_2' = q_2 = 19.44 \text{ cm} - 15 \text{ cm} = 4.44 \text{ cm}$

From these results it is concluded that:

The lenses can be displaced in two ways. The first lens can be moved 1.28 cm farther from the object and the second lens 17.7 cm toward the object. Alternatively, the first lens can be moved 0.927 cm toward the object and the second lens 4.44 cm toward the object.

P36.51 Only a diverging lens gives an upright diminished image. The image is virtual and

$d = p - |q| = p + q$: $\quad M = -\frac{q}{p}$ so $q = -Mp$ and $d = p - Mp$

$p = \frac{d}{1 - M}$: $\quad \frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{p} + \frac{1}{-Mp} = \frac{-M + 1}{-Mp} = \frac{(1 - M)^2}{-Md}$

$$f = \frac{-Md}{(1 - M)^2} = \frac{-(0.500)(20.0 \text{ cm})}{(1 - 0.500)^2} = \boxed{-40.0 \text{ cm}}$$

P36.52 If $M < 1$, the lens is diverging and the image is virtual. $d = p - |q| = p + q$

$M = -\frac{q}{p}$ so $q = -Mp$ and $d = p - Mp$

$p = \frac{d}{1 - M}$: $\quad \frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{p} + \frac{1}{(-Mp)} = \frac{(-M + 1)}{-Mp} = \frac{(1 - M)^2}{-Md} \quad \boxed{f = \frac{-Md}{(1 - M)^2}}$

If $M > 1$, the lens is converging and the image is still virtual.

Now $\quad d = -q - p$

We obtain in this case $\boxed{f = \frac{Md}{(M - 1)^2}}$.

***P36.53** The real image formed by the concave mirror serves as a real object for the convex mirror with $p = 50$ cm and $q = -10$ cm. Therefore,

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \qquad \frac{1}{f} = \frac{1}{50\text{ cm}} + \frac{1}{(-10\text{ cm})}$$

gives $f = -12.5$ cm and $R = 2f = \boxed{-25.0\text{ cm}}$.

***P36.54** Start with the first pass through the lens.

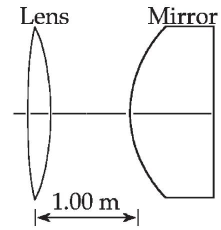


FIG. P36.54

$$\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{80.0\text{ cm}} - \frac{1}{100\text{ cm}} \qquad q_1 = 400\text{ cm to right of lens}$$

For the mirror, $p_2 = -300$ cm

$$\frac{1}{q_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{(-50.0\text{ cm})} - \frac{1}{(-300\text{ cm})} \qquad q_2 = -60.0\text{ cm}$$

For the second pass through the lens, $p_3 = 160$ cm

$$\frac{1}{q_3} = \frac{1}{f_1} - \frac{1}{p_3} = \frac{1}{80.0\text{ cm}} - \frac{1}{160\text{ cm}} \qquad q_3 = \boxed{160\text{ cm to the left of lens}}$$

$$M_1 = -\frac{q_1}{p_1} = -\frac{400\text{ cm}}{100\text{ cm}} = -4.00 \qquad M_2 = -\frac{q_2}{p_2} = -\frac{(-60.0\text{ cm})}{(-300\text{ cm})} = -\frac{1}{5}$$

$$M_3 = -\frac{q_3}{p_3} = -\frac{160\text{ cm}}{160\text{ cm}} = -1 \qquad M = M_1M_2M_3 = \boxed{-0.800}$$

Since $M < 0$ the final image is $\boxed{\text{inverted}}$.

***P36.55** When the meterstick coordinate of the object is 0, its object distance is $p_i = 32$ cm. When the meterstick coordinate of the object is x, its object distance is $p = 32\text{ cm} - x$. The image distance from the lens is given by

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad \frac{1}{32 - x} + \frac{1}{q} = \frac{1}{26} \qquad \frac{1}{q} = \frac{32 - x - 26}{26(32 - x)} \qquad q = \frac{832 - 26x}{6 - x}$$

The image meterstick coordinate is

$x' = 32 + q = (32(6 - x) + 832 - 26x)/(6 - x) = \boxed{(1\,024\text{ cm} - 58x)\text{ cm}/(6\text{ cm} - x)}$. The image starts at the position $x_i' = 171$ cm and moves in the positive x direction, faster and faster, until it is out at infinity when the object is at the position $x = 6$ cm. At this instant the rays from the top of the object are parallel as they leave the lens. Their intersection point can be described as at $x' = \infty$ to the right or equally well at $x' = -\infty$ on the left. From $x' = -\infty$ the image continues moving to the right, now slowing down. It reaches, for example, –280 cm when the object is at 8 cm, and –55 cm when the object is finally at 12 cm. The image has traveled always to the right, to infinity and beyond.

P36.56 $\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{10.0 \text{ cm}} - \frac{1}{12.5 \text{ cm}}$

so $q_1 = 50.0$ cm (to left of mirror)

This serves as an object for the lens (a virtual object), so

$$\frac{1}{q_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{(-16.7 \text{ cm})} - \frac{1}{(-25.0 \text{ cm})} \text{ and } q_2 = -50.3 \text{ cm}$$

meaning 50.3 cm to the right of the lens. Thus, the final image is located $\boxed{\text{25.3 cm to right of mirror}}$.

$$M_1 = -\frac{q_1}{p_1} = -\frac{50.0 \text{ cm}}{12.5 \text{ cm}} = -4.00$$

$$M_2 = -\frac{q_2}{p_2} = -\frac{(-50.3 \text{ cm})}{(-25.0 \text{ cm})} = -2.01$$

$$M = M_1 M_2 = \boxed{8.05}$$

Thus, the final image is $\boxed{\text{virtual, upright}}$, 8.05 times the size of object, and 25.3 cm to right of the mirror.

P36.57 A telescope with an eyepiece decreases the diameter of a beam of parallel rays. When light is sent through the same device in the opposite direction, the beam expands. Send the light first through the diverging lens. It will then be diverging from a virtual image found like this:

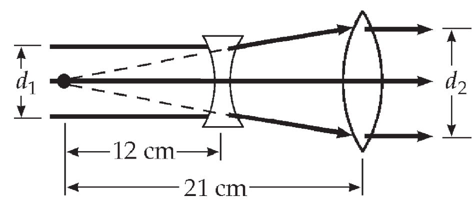


FIG. P36.57

$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ $\qquad \frac{1}{\infty} + \frac{1}{q} = \frac{1}{-12 \text{ cm}}$

$q = -12$ cm

Use this image as a real object for the converging lens, placing it at the focal point on the object side of the lens, at $p = 21$cm. Then

$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ $\qquad \frac{1}{21 \text{ cm}} + \frac{1}{q} = \frac{1}{21 \text{ cm}}$

$q = \infty$

The exiting rays will be parallel. The lenses must be 21.0 cm − 12.0 cm = 9.00 cm apart.

By similar triangles, $\frac{d_2}{d_1} = \frac{21 \text{ cm}}{12 \text{ cm}} = \boxed{\text{1.75 times}}$

***P36.58** (a) For the lens in air,

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{79\text{ cm}} = (1.55-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For the same lens in water

$$\frac{1}{f'} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f'} = \left(\frac{1.55}{1.33} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

By division,

$$\frac{1/79\text{ cm}}{1/f'} = \frac{0.55}{0.165} = \frac{f'}{79\text{ cm}}$$

$$f' = 79\text{ cm}(3.32) = \boxed{263\text{ cm}}$$

(b) The path of a reflected ray does not depend on the refractive index of the medium which the reflecting surface bounds. Therefore the focal length of a mirror does not change when it is put into a different medium: $f' = \frac{R}{2} = f = \boxed{79.0\text{ cm}}$.

P36.59 A hemisphere is too thick to be described as a thin lens. The light is undeviated on entry into the flat face. We next consider the light's exit from the second surface, for which $R = -6.00$ cm.

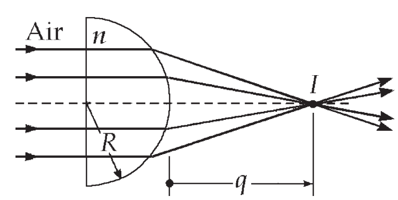


FIG. P36.59

The incident rays are parallel, so $p = \infty$

Then, $$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

becomes $$0 + \frac{1}{q} = \frac{1.00 - 1.56}{-6.00\text{ cm}}$$

and $$\boxed{q = 10.7\text{ cm}}$$

P36.60 (a) $$I = \frac{\mathcal{P}}{4\pi r^2} = \frac{4.50\text{ W}}{4\pi\left(1.60\times10^{-2}\text{ m}\right)^2} = \boxed{1.40\text{ kW/m}^2}$$

(b) $$I = \frac{\mathcal{P}}{4\pi r^2} = \frac{4.50\text{ W}}{4\pi(7.20\text{ m})^2} = \boxed{6.91\text{ mW/m}^2}$$

(c) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$: $$\frac{1}{7.20\text{ m}} + \frac{1}{q} = \frac{1}{0.350\text{ m}}$$

so $$q = 0.368\text{ m}$$

and $$M = \frac{h'}{3.20\text{ cm}} = -\frac{q}{p} = -\frac{0.368\text{ m}}{7.20\text{ m}}$$

$$h' = \boxed{0.164\text{ cm}}$$

(d) The lens intercepts power given by $$\mathcal{P} = IA = \left(6.91\times10^{-3}\text{ W/m}^2\right)\left[\frac{\pi}{4}(0.150\text{ m})^2\right]$$

and puts it all onto the image where $$I = \frac{\mathcal{P}}{A} = \frac{\left(6.91\times10^{-3}\text{ W/m}^2\right)\left[\pi(15.0\text{ cm})^2/4\right]}{\pi(0.164\text{ cm})^2/4}$$

$$I = \boxed{58.1\text{ W/m}^2}$$

P36.61 From the thin lens equation, $q_1 = \frac{f_1 p_1}{p_1 - f_1} = \frac{(-6.00\text{ cm})(12.0\text{ cm})}{12.0\text{ cm} - (-6.00\text{ cm})} = -4.00\text{ cm}$

When we require that $q_2 \to \infty$, the thin lens equation becomes $p_2 = f_2$.

In this case, $p_2 = d - (-4.00\text{ cm})$

Therefore, $d + 4.00\text{ cm} = f_2 = 12.0\text{ cm}$ and $\boxed{d = 8.00\text{ cm}}$

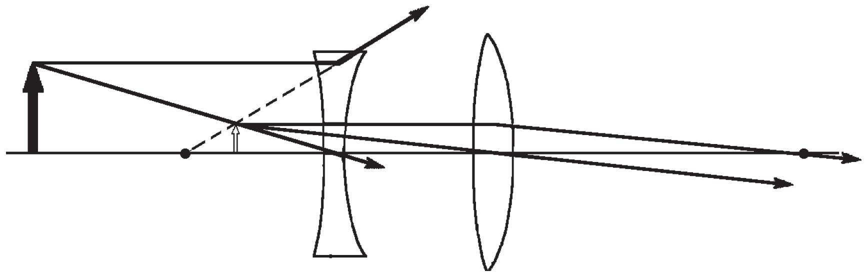


FIG. P36.61

P36.62 (a) For the light the mirror intercepts,

$$\mathcal{P} = I_0 A = I_0 \pi R_a^2$$

$$350\text{ W} = \left(1\,000\text{ W/m}^2\right)\pi R_a^2$$

and $R_a = \boxed{0.334\text{ m or larger}}$

(b) In $\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R}$

we have $p \to \infty$

so $q = \frac{R}{2}$

$$M = \frac{h'}{h} = -\frac{q}{p}$$

so $h' = -q\left(\frac{h}{p}\right) = -\left(\frac{R}{2}\right)\left[0.533°\left(\frac{\pi\text{ rad}}{180°}\right)\right] = -\left(\frac{R}{2}\right)(9.30\text{ m rad})$

where $\frac{h}{p}$ is the angle the Sun subtends. The intensity at the image is

then $I = \frac{\mathcal{P}}{\pi h'^2/4} = \frac{4I_0 \pi R_a^2}{\pi h'^2} = \frac{4I_0 R_a^2}{(R/2)^2\left(9.30\times10^{-3}\text{ rad}\right)^2}$

$$120\times10^3\text{ W/m}^2 = \frac{16\left(1\,000\text{ W/m}^2\right)R_a^2}{R^2\left(9.30\times10^{-3}\text{ rad}\right)^2}$$

so $\boxed{\frac{R_a}{R} = 0.025\,5\text{ or larger}}$

P36.63 For the mirror, $f = \frac{R}{2} = +1.50$ m. In addition, because the distance to the Sun is so much larger than any other distances, we can take $p = \infty$.

The mirror equation, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, then gives $q = f = \boxed{1.50 \text{ m}}$

Now, in $M = -\frac{q}{p} = \frac{h'}{h}$

the magnification is nearly zero, but we can be more precise: $\frac{h}{p}$ is the angular diameter of the object. Thus, the image diameter is

$$h' = -\frac{hq}{p} = (-0.533°)\left(\frac{\pi \text{ rad}}{180°}\right)(1.50 \text{ m}) = -0.014\,0 \text{ m} = \boxed{-1.40 \text{ cm}}$$

P36.64 (a) The lens makers' equation, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

becomes: $\frac{1}{5.00 \text{ cm}} = (n-1)\left[\frac{1}{9.00 \text{ cm}} - \frac{1}{(-11.0 \text{ cm})}\right]$ giving $n = \boxed{1.99}$.

(b) As the light passes through the lens for the first time, the thin lens equation

$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f}$ becomes: $\frac{1}{8.00 \text{ cm}} + \frac{1}{q_1} = \frac{1}{5.00 \text{ cm}}$

or $q_1 = 13.3$ cm, and $M_1 = -\frac{q_1}{p_1} = -\frac{13.3 \text{ cm}}{8.00 \text{ cm}} = -1.67$

This image becomes the object for the concave mirror with:

$p_m = 20.0 \text{ cm} - q_1 = 20.0 \text{ cm} - 13.3 \text{ cm} = 6.67 \text{ cm}$ and $f = \frac{R}{2} = +4.00 \text{ cm}$

The mirror equation becomes: $\frac{1}{6.67 \text{ cm}} + \frac{1}{q_m} = \frac{1}{4.00 \text{ cm}}$

giving $q_m = 10.0$ cm and $M_2 = -\frac{q_m}{p_m} = -\frac{10.0 \text{ cm}}{6.67 \text{ cm}} = -1.50$

The image formed by the mirror serves as a real object for the lens on the second pass of the light through the lens with:

$$p_3 = 20.0 \text{ cm} - q_m = +10.0 \text{ cm}$$

The thin lens equation yields: $\frac{1}{10.0 \text{ cm}} + \frac{1}{q_3} = \frac{1}{5.00 \text{ cm}}$

or $q_3 = 10.0$ cm and $M_3 = -\frac{q_3}{p_3} = -\frac{10.0 \text{ cm}}{10.0 \text{ cm}} = -1.00$

The final image is a real image located $\boxed{\text{10.0 cm to the left of the lens}}$.

The overall magnification is $M_{\text{total}} = M_1 M_2 M_3 = \boxed{-2.50}$

(c) Since the total magnification is negative, this final image is $\boxed{\text{inverted}}$.

P36.65 In the original situation, $p_1 + q_1 = 1.50\text{ m}$

In the final situation, $p_2 = p_1 + 0.900\text{ m}$

and $q_2 = q_1 - 0.900\text{ m} = 0.600\text{ m} - p_1$

Our lens equation is $\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f} = \frac{1}{p_2} + \frac{1}{q_2}$

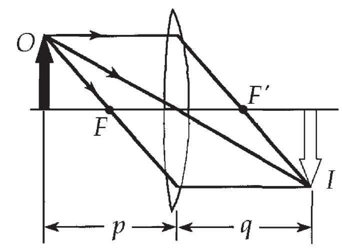


FIG. P36.65

Substituting, we have $\frac{1}{p_1} + \frac{1}{1.50\text{ m} - p_1} = \frac{1}{p_1 + 0.900} + \frac{1}{0.600 - p_1}$

Adding the fractions, $\frac{1.50\text{ m} - p_1 + p_1}{p_1(1.50\text{ m} - p_1)} = \frac{0.600 - p_1 + p_1 + 0.900}{(p_1 + 0.900)(0.600 - p_1)}$

Simplified, this becomes $p_1(1.50\text{ m} - p_1) = (p_1 + 0.900)(0.600 - p_1)$

(a) Thus, $p_1 = \frac{0.540}{1.80}\text{ m} = \boxed{0.300\text{ m}}$ $p_2 = p_1 + 0.900 = \boxed{1.20\text{ m}}$

(b) $\frac{1}{f} = \frac{1}{0.300\text{ m}} + \frac{1}{1.50\text{ m} - 0.300\text{ m}}$ and $f = \boxed{0.240\text{ m}}$

(c) The second image is $\boxed{\text{real, inverted, and diminished}}$

with $M = -\frac{q_2}{p_2} = \boxed{-0.250}$

P36.66 The object is located at the focal point of the upper mirror. Thus, the upper mirror creates an image at infinity (i.e., parallel rays leave this mirror).

The lower mirror focuses these parallel rays at its focal point, located at the hole in the upper mirror.

Thus, the $\boxed{\text{image is real, inverted, and actual size}}$.

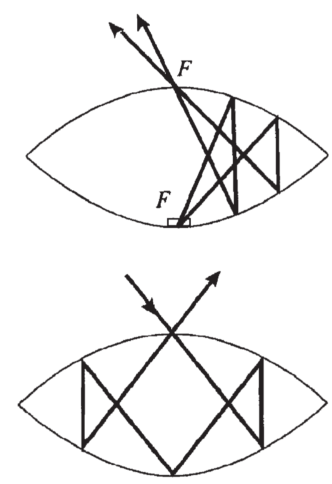


FIG. P36.66

For the upper mirror:

$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$: $\frac{1}{7.50\text{ cm}} + \frac{1}{q_1} = \frac{1}{7.50\text{ cm}}$ $q_1 = \infty$

For the lower mirror:

$$\frac{1}{\infty} + \frac{1}{q_2} = \frac{1}{7.50\text{ cm}} \qquad q_2 = 7.50\text{ cm}$$

Light directed into the hole in the upper mirror reflects as shown, to behave as if it were reflecting from the hole.

P36.67 (a) For lens one, as shown in the first figure,

$$\frac{1}{40.0\text{ cm}}+\frac{1}{q_1}=\frac{1}{30.0\text{ cm}}$$

$$q_1=120\text{ cm}$$

$$M_1=-\frac{q_1}{p_1}=-\frac{120\text{ cm}}{40.0\text{ cm}}=-3.00$$

This real image $I_1=O_2$ is a virtual object for the second lens. That is, it is *behind* the lens, as shown in the second figure. The object distance is

$$p_2=110\text{ cm}-120\text{ cm}=-10.0\text{ cm}$$

$$\frac{1}{-10.0\text{ cm}}+\frac{1}{q_2}=\frac{1}{-20.0\text{ cm}}:$$

$$q_2=\boxed{20.0\text{ cm}}$$

$$M_2=-\frac{q_2}{p_2}=-\frac{20.0\text{ cm}}{(-10.0\text{ cm})}=+2.00$$

$$M_{\text{overall}}=M_1M_2=\boxed{-6.00}$$

(b) $M_{\text{overall}}<0$, so final image is $\boxed{\text{inverted}}$.

(c) If lens two is a converging lens (third figure):

$$\frac{1}{-10.0\text{ cm}}+\frac{1}{q_2}=\frac{1}{20.0\text{ cm}}$$

$$q_2=\boxed{6.67\text{ cm}}$$

$$M_2=-\frac{6.67\text{ cm}}{(-10.0\text{ cm})}=+0.667$$

$$M_{\text{overall}}=M_1M_2=\boxed{-2.00}$$

Again, $M_{\text{overall}}<0$ and the final image is $\boxed{\text{inverted}}$.

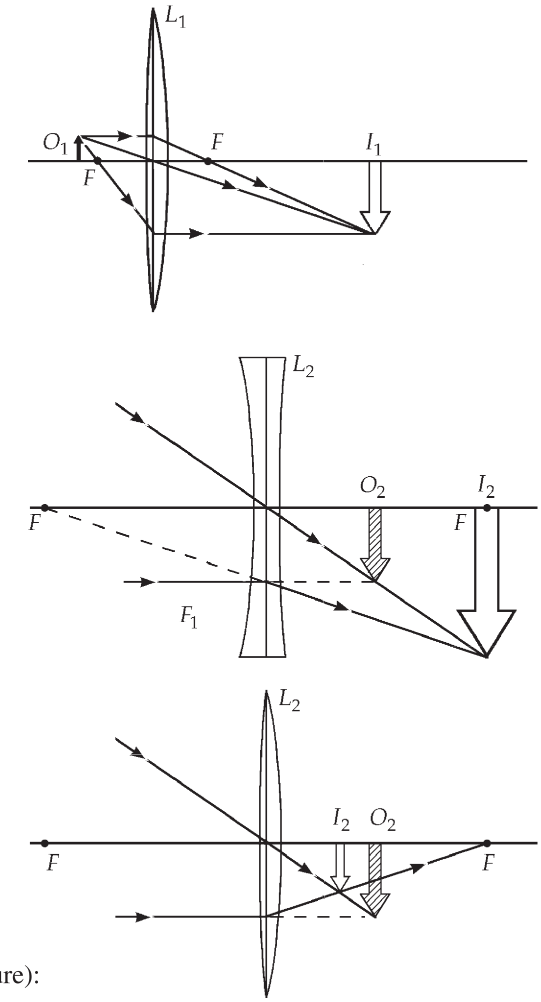


FIG. P36.67

P36.68 The first lens has focal length described by

$$\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{R_{11}} - \frac{1}{R_{12}}\right) = (n_1 - 1)\left(\frac{1}{\infty} - \frac{1}{R}\right) = -\frac{n_1 - 1}{R}$$

For the second lens

$$\frac{1}{f_2} = (n_2 - 1)\left(\frac{1}{R_{21}} - \frac{1}{R_{22}}\right) = (n_2 - 1)\left(\frac{1}{+R} - \frac{1}{-R}\right) = +\frac{2(n_2 - 1)}{R}$$

Let an object be placed at any distance p_1 large compared to the thickness of the doublet. The first lens forms an image according to

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1}$$

$$\frac{1}{q_1} = \frac{-n_1 + 1}{R} - \frac{1}{p_1}$$

This virtual $(q_1 < 0)$ image is a real object for the second lens at distance $p_2 = -q_1$. For the second lens

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}$$

$$\frac{1}{q_2} = \frac{2n_2 - 2}{R} - \frac{1}{p_2} = \frac{2n_2 - 2}{R} + \frac{1}{q_1} = \frac{2n_2 - 2}{R} + \frac{-n_1 + 1}{R} - \frac{1}{p_1} = \frac{2n_2 - n_1 - 1}{R} - \frac{1}{p_1}$$

Then $\frac{1}{p_1} + \frac{1}{q_2} = \frac{2n_2 - n_1 - 1}{R}$ so the doublet behaves like a single lens with $\frac{1}{f} = \frac{2n_2 - n_1 - 1}{R}$.

ANSWERS TO EVEN PROBLEMS

P36.2 4.58 m

P36.4 10.0 ft, 30.0 ft, 40.0 ft

P36.6 (a) 13.3 cm, –0.333, real and inverted (b) 20.0 cm, –1.00, real and inverted (c) No image is formed.

P36.8 at $q = -0.267$ m virtual, upright, and diminished with $M = 0.026\,7$

P36.10 at 3.33 m from the deepest point of the niche

P36.12 At 0.708 cm in front of the reflecting surface. Image is virtual, upright, and diminished.

P36.14 7.90 mm

P36.16 (a) 15.0 cm (b) 60.0 cm

P36.18 (a) 25.6 m (b) 0.058 7 rad (c) 2.51 m (d) 0.023 9 rad (e) 62.8 m from your eyes

P36.20 4.82 cm

P36.22 (a) 45.0 cm (b) –90.0 cm (c) –6.00 cm

P36.24 See the solution.

P36.26 1.50 cm/s

P36.28 20.0 cm

P36.30 (a) 6.40 cm (b) –0.250 (c) converging

P36.32 See the solution.

P36.34 (a) 39.0 mm (b) 39.5 mm

P36.36 1.16 mm/s toward the lens

P36.38 (a) $q_a = 26.2$ cm $q_d = 46.7$ cm $h_b' = -8.75$ cm $h_c' = -23.3$ cm. See the solution. (b) The equation follows from $h'/h = -q/p$ and $1/p + 1/q = 1/f$. (c) The integral stated adds up the areas of ribbons covering the whole image, each with vertical dimension $|h'|$ and horizontal width dq. 328 cm^2

P36.40 (a) at $q = -34.7$ cm virtual, upright, and diminished (b) at $q = -36.1$ cm virtual, upright, and diminished

P36.42 $\dfrac{f}{1.41}$

P36.44 23.2 cm

P36.46 –575

P36.48 (a) See the solution. (b) $h' = -\dfrac{hf}{p}$ (c) -1.07 mm

P36.50 (a) 67.5 cm (b) The lenses can be displaced in two ways. The first lens can be displaced 1.28 cm farther away from the object, and the second lens 17.7 cm toward the object. Alternatively, the first lens can be displaced 0.927 cm toward the object and the second lens 4.44 cm toward the object.

P36.52 if $M < 1$, $f = \dfrac{-Md}{(1-M)^2}$, if $M > 1$, $f = \dfrac{Md}{(M-1)^2}$

P36.54 160 cm to the left of the lens, inverted, $M = -0.800$

P36.56 25.3 cm to right of mirror, virtual, upright, enlarged 8.05 times

P36.58 (a) 263 cm (b) 79.0 cm

P36.60 (a) $1.40\ \text{kW/m}^2$ (b) $6.91\ \text{mW/m}^2$ (c) 0.164 cm (d) $58.1\ \text{W/m}^2$

P36.62 (a) 0.334 m or larger (b) $\dfrac{R_a}{R} = 0.025\,5$ or larger

P36.64 (a) 1.99 (b) 10.0 cm to the left of the lens; -2.50 (c) inverted

P36.66 See the solution; real, inverted, and actual size.

P36.68 See the solution.

37

Interference of Light Waves

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q37.1 (a) Two waves interfere constructively if their path difference is zero, or an integral multiple of the wavelength, according to $\delta = m\lambda$, with $m = 0, 1, 2, 3, \ldots$.

(b) Two waves interfere destructively if their path difference is a half wavelength, or an odd multiple of $\frac{\lambda}{2}$, described by $\delta = \left(m + \frac{1}{2}\right)\lambda$, with $m = 0, 1, 2, 3, \ldots$.

Q37.2 The light from the flashlights consists of many different wavelengths (that's why it's white) with random time differences between the light waves. There is no *coherence* between the two sources. The light from the two flashlights does not maintain a constant phase relationship over time. These three equivalent statements mean no possibility of an interference pattern.

***Q37.3** (i) The angles in the interference pattern are controlled by λ/d, which we estimate in each case: (a) $0.45\ \mu\text{m}/400\ \mu\text{m} \approx 1.1 \times 10^{-3}$ (b) $0.7\ \mu\text{m}/400\ \mu\text{m} \approx 1.6 \times 10^{-3}$ (c) and (d) $0.7\ \mu\text{m}/800\ \mu\text{m} \approx 0.9 \times 10^{-3}$. The ranking is b > a > c = d.

(ii) Now we consider $L\lambda/d$: (a) $4\text{ m}(0.45\ \mu\text{m}/400\ \mu\text{m}) \approx 4.4\text{ mm}$ (b) $4\text{ m}(0.7\ \mu\text{m}/400\ \mu\text{m}) \approx 7\text{ mm}$ (c) $4\text{ m}(0.7\ \mu\text{m}/800\ \mu\text{m}) \approx 3\text{ mm}$ (d) $8\text{ m}(0.7\ \mu\text{m}/800\ \mu\text{m}) \approx 7\text{ mm}$. The ranking is b = d > a > c.

***Q37.4** Yes. A single beam of laser light going into the slits divides up into several fuzzy-edged beams diverging from the point halfway between the slits.

***Q37.5** Answer (c). Underwater, the wavelength of the light decreases according to $\lambda_{\text{water}} = \frac{\lambda_{\text{air}}}{n_{\text{water}}}$. Since the angles between positions of light and dark bands are proportional to λ, the underwater fringe separations decrease.

Q37.6 Every color produces its own pattern, with a spacing between the maxima that is characteristic of the wavelength. With white light, the central maximum is white. The first side maximum is a full spectrum with violet on the inner edge and red on the outer edge on each side. Each side maximum farther out is in principle a full spectrum, but they overlap one another and are hard to distinguish. Using monochromatic light can eliminate this problem.

***Q37.7** With two fine slits separated by a distance d slightly less than λ, the equation $d \sin\theta = 0$ has the usual solution $\theta = 0$. But $d \sin\theta = 1\lambda$ has no solution. There is no first side maximum. $d \sin\theta = (1/2)\lambda$ has a solution. Minima flank the central maximum on each side. Answer (b).

Q37.8 As water evaporates from the "soap" bubble, the thickness of the bubble wall approaches zero. Since light reflecting from the front of the water surface is phase-shifted 180° and light reflecting from the back of the soap film is phase-shifted 0°, the reflected light meets the conditions for a minimum. Thus the soap film appears black, as in the textbook illustration accompanying this question.

***Q37.9** Answer (b). If the thickness of the oil film were smaller than half of the wavelengths of visible light, no colors would appear. If the thickness of the oil film were much larger, the colors would overlap to mix to white or gray.

***Q37.10** (i) Answer (b). If the oil film is brightest where it is thinnest, then $n_{air} < n_{oil} < n_{glass}$. With this condition, light reflecting from both the top and the bottom surface of the oil film will undergo phase reversal. Then these two beams will be in phase with each other where the film is very thin. This is the condition for constructive interference as the thickness of the oil film decreases toward zero. If the oil film is dark where it is thinnest, then $n_{air} < n_{oil} > n_{glass}$. In this case, reflecting light undergoes phase reversal upon reflection from the front surface but no phase reversal upon reflection from the back surface. The two reflected beams are 180° out of phase and interfere destructively as the oil film thickness goes to zero.

(ii) Yes. It should be lower in index than both kinds of glass.

(iii) Yes. It should be higher in refractive index than both kinds of glass.

(iv) No.

Q37.11 If R is large, light reflecting from the lower surface of the lens can interfere with light reflecting from the upper surface of the flat. The latter undergoes phase reversal on reflection while the former does not. Where there is negligible distance between the surfaces, at the center of the pattern you will see a dark spot because of the destructive interference associated with the 180° phase shift. Colored rings surround the dark spot. If the lens is a perfect sphere the rings are perfect circles. Distorted rings reveal bumps or hollows on the fine scale of the wavelength of visible light.

Q37.12 A camera lens will have more than one element, to correct (at least) for chromatic aberration. It will have several surfaces, each of which would reflect some fraction of the incident light. To maximize light throughput the surfaces need antireflective coatings. The coating thickness is chosen to produce destructive interference for reflected light of some wavelength.

***Q37.13** (i) Answer (c). The distance between nodes is one-half the wavelength.

(ii) Answer (d). Moving one mirror by 125 nm lengthens the path of light reflecting from it by 250 nm. Since this is half a wavelength, the action reverses constructive into destructive interference.

(iii) Answer (e). The wavelength of the light in the film is 500 nm/2 = 250 nm. If the film is made 62.5 nm thicker, the light reflecting inside the film has a path length 125 nm greater. This is half a wavelength, to reverse constructive into destructive interference.

***Q37.14** Answer (a). If the mirrors do not move the character of the interference stays the same. The light does not get tired before entering the interferometer or undergo any change on the way from the source to the half-silvered mirror.

SOLUTIONS TO PROBLEMS

Section 37.1 **Conditions for Interference**

Section 37.2 **Young's Double-Slit Experiment**

Section 37.3 **Light Waves in Interference**

P37.1 $\Delta y_{\text{bright}} = \dfrac{\lambda L}{d} = \dfrac{(632.8\times10^{-9})(5.00)}{2.00\times10^{-4}}\ \text{m} = \boxed{1.58\ \text{cm}}$

P37.2 $y_{\text{bright}} = \dfrac{\lambda L}{d} m$

For $m = 1$, $\lambda = \dfrac{yd}{L} = \dfrac{(3.40\times10^{-3}\ \text{m})(5.00\times10^{-4}\ \text{m})}{3.30\ \text{m}} = \boxed{515\ \text{nm}}$

P37.3

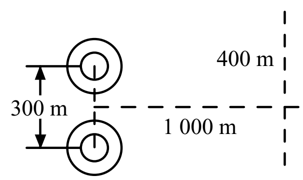


FIG. P37.3

Note, with the conditions given, the small angle approximation **does not work well**. That is, $\sin\theta$, $\tan\theta$, and θ are significantly different. We treat the interference as a Fraunhofer pattern.

(a) At the $m = 2$ maximum, $\tan\theta = \dfrac{400\ \text{m}}{1\,000\ \text{m}} = 0.400$

$\theta = 21.8°$

So $\lambda = \dfrac{d\sin\theta}{m} = \dfrac{(300\ \text{m})\sin 21.8°}{2} = \boxed{55.7\ \text{m}}$

(b) The next minimum encountered is the $m = 2$ minimum, and at that point,

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$$

which becomes $d\sin\theta = \dfrac{5}{2}\lambda$

or $\sin\theta = \dfrac{5}{2}\dfrac{\lambda}{d} = \dfrac{5}{2}\left(\dfrac{55.7\ \text{m}}{300\ \text{m}}\right) = 0.464$

and $\theta = 27.7°$

so $y = (1\,000\ \text{m})\tan 27.7° = 524\ \text{m}$

Therefore, the car must travel an additional $\boxed{124\ \text{m}}$.

If we considered Fresnel interference, we would more precisely find

(a) $\lambda = \dfrac{1}{2}\left(\sqrt{550^2 + 1\,000^2} - \sqrt{250^2 + 1\,000^2}\right) = 55.2$ m and (b) 123 m

P37.4 $\lambda = \dfrac{v}{f} = \dfrac{354\ \text{m/s}}{2\,000\ \text{s}^{-1}} = 0.177\ \text{m}$

(a) $d\sin\theta = m\lambda$ so $(0.300\ \text{m})\sin\theta = 1(0.177\ \text{m})$ and $\theta = \boxed{36.2°}$

(b) $d\sin\theta = m\lambda$ so $d\sin 36.2° = 1(0.030\,0\ \text{m})$ and $d = \boxed{5.08\ \text{cm}}$

(c) $(1.00\times10^{-6}\ \text{m})\sin 36.2° = (1)\lambda$ so $\lambda = 590\ \text{nm}$

$$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\ \text{m/s}}{5.90\times10^{-7}\ \text{m}} = \boxed{508\ \text{THz}}$$

P37.5 In the equation $d\sin\theta = \left(m+\dfrac{1}{2}\right)\lambda$

The first minimum is described by $m = 0$

and the tenth by $m = 9$: $\sin\theta = \dfrac{\lambda}{d}\left(9+\dfrac{1}{2}\right)$

Also, $\tan\theta = \dfrac{y}{L}$

but for small θ, $\sin\theta \approx \tan\theta$

Thus, $d = \dfrac{9.5\lambda}{\sin\theta} = \dfrac{9.5\lambda L}{y}$

$$d = \frac{9.5(5\,890\times10^{-10}\ \text{m})(2.00\ \text{m})}{7.26\times10^{-3}\ \text{m}} = 1.54\times10^{-3}\ \text{m} = \boxed{1.54\ \text{mm}}$$

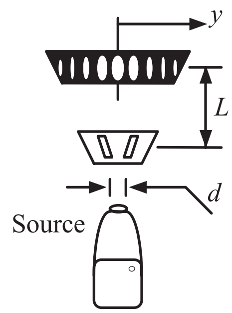


FIG. P37.5

***P37.6** Problem statement: A single oscillator makes the two speakers of a boom box, 35.0 cm apart, vibrate in phase at 1.62 kHz. At what angles, measured from the perpendicular bisector of the line joining the speakers, will a distant observer hear maximum sound intensity? Minimum sound intensity? The ambient temperature is 20°C.

We solve the first equation for λ, substitute into the others, and solve for each angle to find this answer: The wavelength of the sound is 21.2 cm. Interference maxima occur at angles of 0° and 37.2° to the left and right. Minima occur at angles of 17.6° and 65.1°. No second-order or higher-order maximum exists. No angle exists, smaller or larger than 90°, for which $\sin\theta_{2\ \text{loud}} = 1.21$. No location exists in the Universe that is two wavelengths farther from one speaker than from the other.

P37.7 (a) For the bright fringe,

$$y_{\text{bright}} = \frac{m\lambda L}{d} \text{ where } m = 1$$

$$y = \frac{(546.1\times10^{-9}\ \text{m})(1.20\ \text{m})}{0.250\times10^{-3}\ \text{m}} = 2.62\times10^{-3}\ \text{m} = \boxed{2.62\ \text{mm}}$$

(b) For the dark bands, $y_{\text{dark}} = \dfrac{\lambda L}{d}\left(m+\dfrac{1}{2}\right)$; $m = 0, 1, 2, 3, \ldots$

$$y_2 - y_1 = \frac{\lambda L}{d}\left[\left(1+\frac{1}{2}\right)-\left(0+\frac{1}{2}\right)\right] = \frac{\lambda L}{d}(1)$$

$$= \frac{(546.1\times10^{-9}\ \text{m})(1.20\ \text{m})}{0.250\times10^{-3}\ \text{m}}$$

$$\Delta y = \boxed{2.62\ \text{mm}}$$

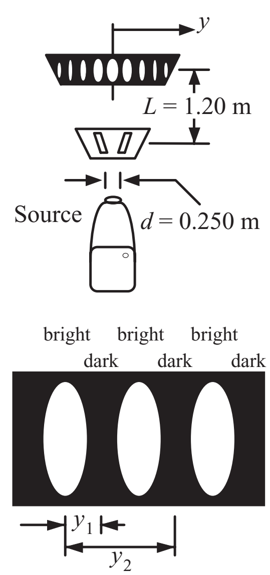


FIG. P37.7

P37.8 Location of A = central maximum,

Location of B = first minimum.

So, $\Delta y = [y_{\min} - y_{\max}] = \dfrac{\lambda L}{d}\left(0 + \dfrac{1}{2}\right) - 0 = \dfrac{1}{2}\dfrac{\lambda L}{d} = 20.0 \text{ m}$

Thus, $d = \dfrac{\lambda L}{2(20.0 \text{ m})} = \dfrac{(3.00 \text{ m})(150 \text{ m})}{40.0 \text{ m}} = \boxed{11.3 \text{ m}}$

P37.9 Taking $m = 0$ and $y = 0.200$ mm in Equations 37.3 and 37.4 gives

$$L \approx \frac{2dy}{\lambda} = \frac{2(0.400 \times 10^{-3} \text{ m})(0.200 \times 10^{-3} \text{ m})}{442 \times 10^{-9} \text{ m}} = 0.362 \text{ m}$$

$L \approx \boxed{36.2 \text{ cm}}$

Geometric optics or a particle theory of light would incorrectly predict bright regions opposite the slits and darkness in between. But, as this example shows, interference can produce just the opposite.

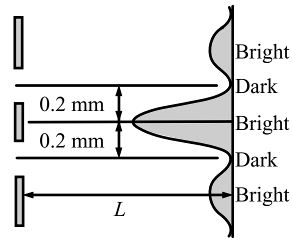


FIG. P37.9

P37.10 At 30.0°, $d \sin\theta = m\lambda$

$(3.20 \times 10^{-4} \text{ m})\sin 30.0° = m(500 \times 10^{-9} \text{ m})$ so $m = 320$

There are 320 maxima to the right, 320 to the left, and one for $m = 0$ straight ahead.

There are $\boxed{641 \text{ maxima}}$.

P37.11 Observe that the pilot must not only home in on the airport, but must be headed in the right direction when she arrives at the end of the runway.

(a) $\lambda = \dfrac{c}{f} = \dfrac{3 \times 10^{8} \text{ m/s}}{30 \times 10^{6} \text{ s}^{-1}} = \boxed{10.0 \text{ m}}$

(b) The first side maximum is at an angle given by $d \sin\theta = (1)\lambda$.

$(40 \text{ m})\sin\theta = 10 \text{ m}$ $\theta = 14.5°$ $\tan\theta = \dfrac{y}{L}$

$y = L\tan\theta = (2\,000 \text{ m})\tan 14.5° = \boxed{516 \text{ m}}$

(c) The signal of 10-m wavelength in parts (a) and (b) would show maxima at 0°, 14.5°, 30.0°, 48.6°, and 90°. A signal of wavelength 11.23 m would show maxima at 0°, 16.3°, 34.2°, and 57.3°. The only value in common is 0°. If λ_1 and λ_2 were related by a ratio of small integers (a just musical consonance!) in $\dfrac{\lambda_1}{\lambda_2} = \dfrac{n_1}{n_2}$, then the equations $d \sin\theta = n_2\lambda_1$ and $d \sin\theta = n_1\lambda_2$ would both be satisfied for the same nonzero angle. The pilot could come flying in with that inappropriate bearing, and run off the runway immediately after touchdown.

P37.12 In $d \sin\theta = m\lambda$ $d\dfrac{y}{L} = m\lambda$ $y = \dfrac{m\lambda L}{d}$

$$\frac{dy}{dt} = \frac{m\lambda}{d}\frac{dL}{dt} = \frac{1(633 \times 10^{-9} \text{ m})}{(0.3 \times 10^{-3} \text{ m})} 3 \text{ m/s} = \boxed{6.33 \text{ mm/s}}$$

P37.13 $\phi = \frac{2\pi}{\lambda} d \sin\theta = \frac{2\pi}{\lambda} d\left(\frac{y}{L}\right)$

(a) $\phi = \frac{2\pi}{\left(5.00\times10^{-7}\text{ m}\right)}\left(1.20\times10^{-4}\text{ m}\right)\sin(0.500°) = \boxed{13.2\text{ rad}}$

(b) $\phi = \frac{2\pi}{\left(5.00\times10^{-7}\text{ m}\right)}\left(1.20\times10^{-4}\text{ m}\right)\left(\frac{5.00\times10^{-3}\text{ m}}{1.20\text{ m}}\right) = \boxed{6.28\text{ rad}}$

(c) If $\phi = 0.333\text{ rad} = \frac{2\pi d\sin\theta}{\lambda}$ $\quad\theta = \sin^{-1}\left(\frac{\lambda\phi}{2\pi d}\right) = \sin^{-1}\left[\frac{\left(5.00\times10^{-7}\text{ m}\right)(0.333\text{ rad})}{2\pi\left(1.20\times10^{-4}\text{ m}\right)}\right]$

$\theta = \boxed{1.27\times10^{-2}\text{ deg}}$

(d) If $d\sin\theta = \frac{\lambda}{4}$ $\quad\theta = \sin^{-1}\left(\frac{\lambda}{4d}\right) = \sin^{-1}\left[\frac{5\times10^{-7}\text{ m}}{4\left(1.20\times10^{-4}\text{ m}\right)}\right]$

$\theta = \boxed{5.97\times10^{-2}\text{ deg}}$

P37.14 The path difference between rays 1 and 2 is: $\delta = d\sin\theta_1 - d\sin\theta_2$
For constructive interference, this path difference must be equal to an integral number of wavelengths: $d\sin\theta_1 - d\sin\theta_2 = m\lambda$, or

$\boxed{d\left(\sin\theta_1 - \sin\theta_2\right) = m\lambda}$

***P37.15** (a) The path difference $\delta = d\sin\theta$ and when $L >> y$

$\delta = \frac{yd}{L} = \frac{\left(1.80\times10^{-2}\text{ m}\right)\left(1.50\times10^{-4}\text{ m}\right)}{1.40\text{ m}} = 1.93\times10^{-6}\text{ m} = \boxed{1.93\ \mu\text{m}}$

(b) $\frac{\delta}{\lambda} = \frac{1.93\times10^{-6}\text{ m}}{6.43\times10^{-7}\text{ m}} = 3.00$, or $\boxed{\delta = 3.00\lambda}$

(c) Point P will be a $\boxed{\text{maximum}}$ because the path difference is an integer multiple of the wavelength.

Section 37.4 Intensity Distribution of the Double-Slit Interference Pattern

P37.16 (a) $\frac{I}{I_{max}} = \cos^2\left(\frac{\phi}{2}\right)$ (Equation 37.12)

Therefore, $\phi = 2\cos^{-1}\sqrt{\frac{I}{I_{max}}} = 2\cos^{-1}\sqrt{0.640} = \boxed{1.29\text{ rad}}$

(b) $\delta = \frac{\lambda\phi}{2\pi} = \frac{(486.1\text{ nm})(1.29\text{ rad})}{2\pi} = \boxed{99.6\text{ nm}}$

P37.17 $I_{av} = I_{max}\cos^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right)$

For small θ, $\sin\theta = \dfrac{y}{L}$

and $I_{av} = 0.750 I_{max}$

$$y = \frac{\lambda L}{\pi d}\cos^{-1}\sqrt{\frac{I_{av}}{I_{max}}}$$

$$y = \frac{(6.00\times10^{-7})(1.20\text{ m})}{\pi(2.50\times10^{-3}\text{ m})}\cos^{-1}\sqrt{\frac{0.750 I_{max}}{I_{max}}} = \boxed{48.0\ \mu\text{m}}$$

P37.18 $I = I_{max}\cos^2\left(\dfrac{\pi yd}{\lambda L}\right)$

$$\frac{I}{I_{max}} = \cos^2\left[\frac{\pi(6.00\times10^{-3}\text{ m})(1.80\times10^{-4}\text{ m})}{(656.3\times10^{-9}\text{ m})(0.800\text{ m})}\right] = \boxed{0.968}$$

***P37.19** We use trigonometric identities to write

$6\sin(100\pi t) + 8\sin(100\pi t + \pi/2) = 6\sin(100\pi t) + 8\sin(100\pi t)\cos(\pi/2) + 8\cos(100\pi t)\sin(\pi/2)$

$E_1 + E_2 = 6\sin(100\pi t) + 8\cos(100\pi t)$

and $E_R\sin(100\pi t + \phi) = E_R\sin(100\pi t)\cos\phi + E_R\cos(100\pi t)\sin\phi$

The equation $E_1 + E_2 = E_R\sin(100\pi t + \phi)$ is satisfied if we require just

$6 = E_R\cos\phi$ and $8 = E_R\sin\phi$

or $6^2 + 8^2 = E_R^2(\cos^2\phi + \sin^2\phi)$ $\boxed{E_R = 10}$

and $\tan\phi = \sin\phi/\cos\phi = 8/6 = 1.33$ $\boxed{\phi = 53.1°}$

***P37.20** In $I_{av} = I_{max}\cos^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right)$ for angles between $-0.3°$ and $+0.3°$ we take $\sin\theta = \theta$ to find

$I = I_{max}\cos^2\left(\dfrac{\pi\, 250\ \mu\text{m}\ \theta}{0.5461\ \mu\text{m}}\right)$ $I/I_{max} = \cos^2(1438\ \theta)$

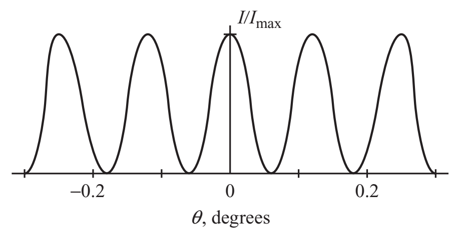


FIG. P37.20

This equation is correct assuming θ is in radians; but we can then equally well substitute in values for θ in degrees and interpret the argument of the cosine function as a number of degrees. We get the same answers for θ negative and for θ positive. We evaluate

θ, degrees	–0.3	–0.25	–0.2	–0.15	–0.1	–0.05	0	0.05	0.1	0.15	0.2	0.25	0.3
I/I_{max}	0.101	1.00	0.092	0.659	0.652	0.096	1	0.096	0.652	0.659	0.092	1.00	0.101

The cosine-squared function has maximum values of 1 at $\theta = 0$, at $1438\,\theta = 180°$ with $\theta = 0.125°$, and at $1438\,\theta = 360°$ with $\theta = 0.250°$. It has minimum values of zero halfway between the maximum values. The graph then has the appearance shown.

P37.21 (a) From Equation 37.9,

$$\phi = \frac{2\pi d}{\lambda}\sin\theta = \frac{2\pi d}{\lambda}\cdot\frac{y}{\sqrt{y^2+D^2}}$$

$$\phi \approx \frac{2\pi yd}{\lambda D} = \frac{2\pi\left(0.850\times10^{-3}\text{ m}\right)\left(2.50\times10^{-3}\text{ m}\right)}{\left(600\times10^{-9}\text{ m}\right)(2.80\text{ m})} = \boxed{7.95\text{ rad}}$$

(b) $$\frac{I}{I_{max}} = \frac{\cos^2\left[(\pi d/\lambda)\sin\theta\right]}{\cos^2\left[(\pi d/\lambda)\sin\theta_{max}\right]} = \frac{\cos^2(\phi/2)}{\cos^2 m\pi}$$

$$\frac{I}{I_{max}} = \cos^2\frac{\phi}{2} = \cos^2\left(\frac{7.95\text{ rad}}{2}\right) = \boxed{0.453}$$

***P37.22** (a) The resultant amplitude is

$$E_r = E_0\sin\omega t + E_0\sin(\omega t+\phi) + E_0\sin(\omega t+2\phi),\text{ where }\phi = \frac{2\pi}{\lambda}d\sin\theta$$

$$E_r = E_0\left(\sin\omega t + \sin\omega t\cos\phi + \cos\omega t\sin\phi + \sin\omega t\cos2\phi + \cos\omega t\sin2\phi\right)$$

$$E_r = E_0(\sin\omega t)\left(1+\cos\phi+2\cos^2\phi-1\right) + E_0(\cos\omega t)(\sin\phi + 2\sin\phi\cos\phi)$$

$$E_r = E_0(1+2\cos\phi)(\sin\omega t\cos\phi + \cos\omega t\sin\phi) = E_0(1+2\cos\phi)\sin(\omega t+\phi)$$

Then the intensity is $\quad I \propto E_r^2 = E_0^2(1+2\cos\phi)^2\left(\frac{1}{2}\right)$

where the time average of $\sin^2(\omega t+\phi)$ is $\frac{1}{2}$.

From one slit alone we would get intensity $I_{max} \propto E_0^2\left(\frac{1}{2}\right)$ so

$$\boxed{I = I_{max}\left[1+2\cos\left(\frac{2\pi d\sin\theta}{\lambda}\right)\right]^2}$$

(b) Look at the $N = 3$ graph in the textbook Figure 37.8. Minimum intensity is zero, attained where $\cos\phi = -\frac{1}{2}$. One relative maximum occurs at $\cos\phi = -1.00$, where

$I = I_{max}[1-2]^2 = I_{max}$

The larger local maximum happens where $\cos\phi = +1.00$, giving $I = I_{max}[1+2]^2 = 9.00I_{max}$.

The ratio of intensities at primary versus secondary maxima is $\boxed{9.00}$.

Section 37.5 **Change of Phase Due to Reflection**

Section 37.6 **Interference in Thin Films**

P37.23 (a) The light reflected from the top of the oil film undergoes phase reversal. Since $1.45 > 1.33$, the light reflected from the bottom undergoes no reversal. For constructive interference of reflected light, we then have

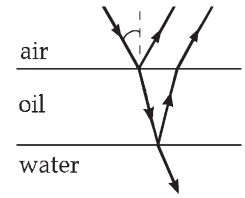


FIG. P37.23

$$2nt = \left(m + \frac{1}{2}\right)\lambda$$

or $$\lambda_m = \frac{2nt}{m + (1/2)} = \frac{2(1.45)(280 \text{ nm})}{m + (1/2)}$$

Substituting for m gives: $m = 0$, $\lambda_0 = 1\,620$ nm (infrared)

$m = 1$, $\lambda_1 = 541$ nm (green)

$m = 2$, $\lambda_2 = 325$ nm (ultraviolet)

Both infrared and ultraviolet light are invisible to the human eye, so the dominant color in reflected light is $\boxed{\text{green}}$.

(b) The dominant wavelengths in the transmitted light are those that produce destructive interference in the reflected light. The condition for destructive interference upon reflection is

$$2nt = m\lambda$$

or $$\lambda_m = \frac{2nt}{m} = \frac{812 \text{ nm}}{m}$$

Substituting for m gives: $m = 1$, $\lambda_1 = 812$ nm (near infrared)

$m = 2$, $\lambda_2 = 406$ nm (violet)

$m = 3$, $\lambda_3 = 271$ nm (ultraviolet)

Of these, the only wavelength visible to the human eye (and hence the dominant wavelength observed in the transmitted light) is 406 nm. Thus, the dominant color in the transmitted light is $\boxed{\text{violet}}$.

P37.24 Light reflecting from the first surface suffers phase reversal. Light reflecting from the second surface does not, but passes twice through the thickness t of the film. So, for constructive interference, we require

$$\frac{\lambda_n}{2} + 2t = \lambda_n$$

where $\lambda_n = \dfrac{\lambda}{n}$ is the wavelength in the material.

Then $$2t = \frac{\lambda_n}{2} = \frac{\lambda}{2n}$$

$$\lambda = 4nt = 4(1.33)(115 \text{ nm}) = \boxed{612 \text{ nm}}$$

P37.25 Since $1 < 1.25 < 1.33$, light reflected both from the top and from the bottom surface of the oil suffers phase reversal.

For constructive interference we require $2t = \frac{m\lambda_{cons}}{n}$

and for destructive interference, $2t = \frac{[m+(1/2)]\lambda_{des}}{n}$

Then $\frac{\lambda_{cons}}{\lambda_{dest}} = 1 + \frac{1}{2m} = \frac{640 \text{ nm}}{512 \text{ nm}} = 1.25$ and $m = 2$

Therefore, $t = \frac{2(640 \text{ nm})}{2(1.25)} = \boxed{512 \text{ nm}}$

P37.26 Treating the anti-reflectance coating like a camera-lens coating,

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n}$$

Let $m = 0$: $t = \frac{\lambda}{4n} = \frac{3.00 \text{ cm}}{4(1.50)} = \boxed{0.500 \text{ cm}}$

This anti-reflectance coating could be easily countered by changing the wavelength of the radar to 1.50 cm. Then the coating would exhibit maximum reflection!

P37.27 $2nt = \left(m + \frac{1}{2}\right)\lambda$ so $t = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n}$

Minimum $t = \left(\frac{1}{2}\right)\frac{(500 \text{ nm})}{2(1.30)} = \boxed{96.2 \text{ nm}}$

P37.28 Since the light undergoes a 180° phase change at each surface of the film, the condition for *constructive* interference is $2nt = m\lambda$, or $\lambda = \frac{2nt}{m}$. The film thickness is $t = 1.00 \times 10^{-5} \text{ cm} = 1.00 \times 10^{-7} \text{ m} = 100 \text{ nm}$. Therefore, the wavelengths intensified in the reflected light are

$$\lambda = \frac{2(1.38)(100 \text{ nm})}{m} = \frac{276 \text{ nm}}{m} \text{ where } m = 1, 2, 3, \ldots$$

or $\lambda_1 = 276$ nm, $\lambda_2 = 138$ nm, ... All reflection maxima are in the ultraviolet and beyond.

$\boxed{\text{No visible wavelengths are intensified.}}$

P37.29 (a) For maximum transmission, we want destructive interference in the light reflected from the front and back surfaces of the film.

If the surrounding glass has refractive index greater than 1.378, light reflected from the front surface suffers no phase reversal and light reflected from the back does undergo phase reversal. This effect by itself would produce destructive interference, so we want the distance down and back to be one whole wavelength in the film: $2t = \frac{\lambda}{n}$.

$$t = \frac{\lambda}{2n} = \frac{656.3 \text{ nm}}{2(1.378)} = \boxed{238 \text{ nm}}$$

(b) The filter will undergo thermal expansion. As t increases in $2nt = \lambda$, so does $\boxed{\lambda \text{ increase}}$.

(c) Destructive interference for reflected light happens also for λ in $2nt = 2\lambda$,

or $\lambda = 1.378(238 \text{ nm}) = \boxed{328 \text{ nm}}$ (near ultraviolet)

P37.30 If the path length difference $\Delta = \lambda$, the transmitted light will be bright. Since $\Delta = 2d = \lambda$,

$$d_{\min} = \frac{\lambda}{2} = \frac{580\text{ nm}}{2} = \boxed{290\text{ nm}}$$

P37.31 For destructive interference in the air,

$$2t = m\lambda$$

For 30 dark fringes, including the one where the plates meet,

$$t = \frac{29(600\text{ nm})}{2} = 8.70\times10^{-6}\text{ m}$$

Therefore, the *radius* of the wire is

$$r = \frac{t}{2} = \frac{8.70\ \mu\text{m}}{2} = \boxed{4.35\ \mu\text{m}}$$

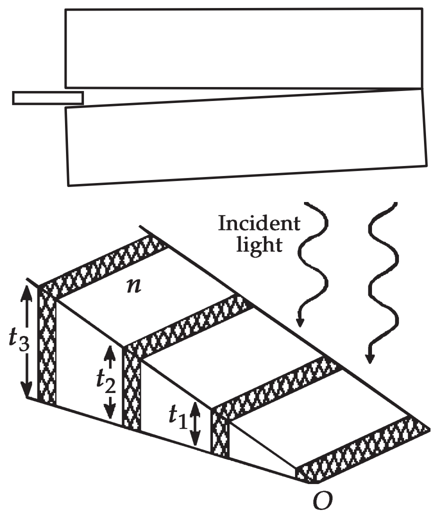


FIG. P37.31

P37.32 The condition for bright fringes is

$$2t + \frac{\lambda}{2n} = m\frac{\lambda}{n} \qquad m = 1, 2, 3, \ldots$$

From the sketch, observe that

$$t = R(1-\cos\theta) \approx R\left(1 - 1 + \frac{\theta^2}{2}\right) = \frac{R}{2}\left(\frac{r}{R}\right)^2 = \frac{r^2}{2R}$$

The condition for a bright fringe becomes $\quad \dfrac{r^2}{R} = \left(m - \dfrac{1}{2}\right)\dfrac{\lambda}{n}$

Thus, for fixed m and λ, $\quad nr^2 = \text{constant}$

Therefore, $n_{\text{liquid}} r_f^2 = n_{\text{air}} r_i^2$ and $\quad n_{\text{liquid}} = (1.00)\dfrac{(1.50\text{ cm})^2}{(1.31\text{ cm})^2} = \boxed{1.31}$

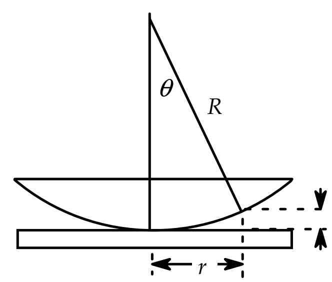


FIG. P37.32

P37.33 For total darkness, we want destructive interference for reflected light for both 400 nm and 600 nm. With phase reversal at just one reflecting surface, the condition for destructive interference is

$$2n_{\text{air}}t = m\lambda \qquad m = 0, 1, 2, \ldots$$

The least common multiple of these two wavelengths is 1 200 nm, so we get no reflected light at $2(1.00)t = 3(400\text{ nm}) = 2(600\text{ nm}) = 1\,200\text{ nm}$, so $t = 600$ nm at this second dark fringe.

By similar triangles, $\quad \dfrac{600\text{ nm}}{x} = \dfrac{0.050\,0\text{ mm}}{10.0\text{ cm}}$

or the distance from the contact point is $\quad x = \left(600\times10^{-9}\text{ m}\right)\left(\dfrac{0.100\text{ m}}{5.00\times10^{-5}\text{ m}}\right) = \boxed{1.20\text{ mm}}$

Section 37.7 **The Michelson Interferometer**

P37.34 Distance $= 2\left(3.82\times10^{-4}\text{ m}\right) = 1\,700\lambda \qquad \lambda = 4.49\times10^{-7}\text{ m} = \boxed{449\text{ nm}}$

The light is $\boxed{\text{blue}}$.

P37.35 When the mirror on one arm is displaced by $\Delta\ell$, the path difference changes by $2\Delta\ell$. A shift resulting in the reversal between dark and bright fringes requires a path length change of one-half wavelength. Therefore, $2\Delta\ell = \dfrac{m\lambda}{2}$, where in this case, $m = 250$.

$$\Delta\ell = m\frac{\lambda}{4} = \frac{(250)\left(6.328\times10^{-7}\text{ m}\right)}{4} = \boxed{39.6\ \mu\text{m}}$$

P37.36 Counting light going both directions, the number of wavelengths originally in the cylinder is $m_1 = \dfrac{2L}{\lambda}$. It changes to $m_2 = \dfrac{2L}{\lambda/n} = \dfrac{2nL}{\lambda}$ as the cylinder is filled with gas. If N is the number of bright fringes passing, $N = m_2 - m_1 = \dfrac{2L}{\lambda}(n-1)$, or the index of refraction of the gas is

$$n = \boxed{1+\frac{N\lambda}{2L}}$$

Additional Problems

***P37.37** The same light source will radiate into the syrup light with wavelength 560 nm/1.38 = 406 nm. The first side bright fringe is separated from the central bright fringe by distance y described by

$d\sin\theta = 1\lambda \quad dy/L \approx \lambda \qquad y = \lambda L/d = 406\times10^{-9}\text{m}(1.20\text{ m})/(30\times10^{-6}\text{m}) = \boxed{1.62\text{ cm}}$

P37.38 (a) Where fringes of the two colors coincide we have $d\sin\theta = m\lambda = m'\lambda'$, requiring $\dfrac{\lambda}{\lambda'} = \dfrac{m'}{m}$.

(b) $\lambda = 430$ nm, $\lambda' = 510$ nm

$\therefore \dfrac{m'}{m} = \dfrac{430\text{ nm}}{510\text{ nm}} = \dfrac{43}{51}$, which cannot be reduced any further. Then $m = 51$, $m' = 43$.

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{(51)\left(430\times10^{-9}\text{ m}\right)}{0.025\times10^{-3}\text{ m}}\right] = 61.3^\circ$$

$$y_m = L\tan\theta_m = (1.5\text{ m})\tan 61.3^\circ = \boxed{2.74\text{ m}}$$

P37.39 The wavelength is $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\text{ m/s}}{60.0\times10^{6}\text{ s}^{-1}} = 5.00$ m.

Along the line AB the two traveling waves going in opposite directions add to give a standing wave. The two transmitters are exactly 2.00 wavelengths apart and the signal from B, when it arrives at A, will always be in phase with transmitter B. Since B is 180° out of phase with A, the two signals always interfere destructively at the position of A.

The first antinode (point of constructive interference) is located at distance

$\dfrac{\lambda}{4} = \dfrac{5.00\text{ m}}{4} = \boxed{1.25\text{ m}}$ from the node at A.

P37.40 Along the line of length d joining the source, two identical waves moving in opposite directions add to give a standing wave.

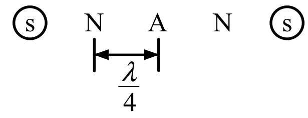


FIG. P37.40

An antinode is halfway between the sources. If $\frac{d}{2} > \frac{\lambda}{2}$, there is space for two more antinodes for a total of three. If $\frac{d}{2} > \lambda$, there will be at least five antinodes, and so on. To repeat, if $\frac{d}{\lambda} > 0$, the number of antinodes is 1 or more. If $\frac{d}{\lambda} > 1$, the number of antinodes is 3 or more. If $\frac{d}{\lambda} > 2$, the number of antinodes is 5 or more. In general,

the number of antinodes is 1 plus 2 times the greatest integer less than or equal to $\frac{d}{\lambda}$.

If $\frac{d}{2} < \frac{\lambda}{4}$, there will be no nodes. If $\frac{d}{2} > \frac{\lambda}{4}$, there will be space for at least two nodes, as shown in the picture. If $\frac{d}{2} > \frac{3\lambda}{4}$, there will be at least four nodes. If $\frac{d}{2} > \frac{5\lambda}{4}$, six or more nodes will fit in, and so on. To repeat, if $2d < \lambda$, the number of nodes is 0. If $2d > \lambda$, the number of nodes is 2 or more. If $2d > 3\lambda$, the number of nodes is 4 or more. If $2d > 5\lambda$, the number of nodes is 6 or more. Again, if $\left(\frac{d}{\lambda} + \frac{1}{2}\right) > 1$, the number of nodes is at least 2. If $\left(\frac{d}{\lambda} + \frac{1}{2}\right) > 2$, the number of nodes is at least 4. If $\left(\frac{d}{\lambda} + \frac{1}{2}\right) > 3$, the number of nodes is at least 6. In general,

the number of nodes is 2 times the greatest nonzero integer less than $\left(\frac{d}{\lambda} + \frac{1}{2}\right)$.

Next, we enumerate the zones of constructive interference. They are described by $d\sin\theta = m\lambda$, $m = 0, 1, 2, \ldots$ with θ counted as positive both left and right of the maximum at $\theta = 0$ in the center. The number of side maxima on each side is the greatest integer satisfying $\sin\theta \le 1$, $d1 \ge m\lambda$, $m \le \frac{d}{\lambda}$. So the total

number of bright fringes is one plus 2 times the greatest integer less than or equal to $\frac{d}{\lambda}$.

It is equal to the number of antinodes on the line joining the sources.

The interference minima are to the left and right at angles described by $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$, $m = 0, 1, 2, \ldots$. With $\sin\theta < 1$, $d1 > \left(m_{max} + \frac{1}{2}\right)\lambda$, $m_{max} < \frac{d}{\lambda} - \frac{1}{2}$ or $m_{max} + 1 < \frac{d}{\lambda} + \frac{1}{2}$. Let $n = 1, 2, 3, \ldots$. Then the number of side minima is the greatest integer n less than $\frac{d}{\lambda} + \frac{1}{2}$. Counting both left and right,

the number of dark fringes is two times the greatest positive integer less than $\left(\frac{d}{\lambda} + \frac{1}{2}\right)$.

It is equal to the number of nodes in the standing wave between the sources.

P37.41 My middle finger has width $d = 2$ cm.

(a) Two adjacent directions of constructive interference for 600-nm light are described by

$$d\sin\theta = m\lambda$$

$$\theta_0 = 0$$

$$\left(2\times10^{-2}\text{ m}\right)\sin\theta_1 = 1\left(6\times10^{-7}\text{ m}\right)$$

Thus, $\theta_1 = 2\times10^{-3}$ degree

and $\theta_1 - \theta_0$ $\boxed{\sim10^{-3}\text{ degree}}$

(b) Choose $\theta_1 = 20°$

$$\left(2\times10^{-2}\text{ m}\right)\sin20° = (1)\lambda$$

$$\lambda = 7\text{ mm}$$

Millimeter waves are $\boxed{\text{microwaves}}$.

$f = \dfrac{c}{\lambda}$: $\quad f = \dfrac{3\times10^{8}\text{ m/s}}{7\times10^{-3}\text{ m}}\ \boxed{\sim10^{11}\text{ Hz}}$

***P37.42** Constructive interference occurs where $m = 0, 1, 2, 3, \ldots, -1, -2, -3, \ldots$, for

$$\left(\frac{2\pi x_1}{\lambda} - 2\pi ft + \frac{\pi}{6}\right) - \left(\frac{2\pi x_2}{\lambda} - 2\pi ft + \frac{\pi}{8}\right) = 2\pi m \qquad \frac{2\pi\left(x_1 - x_2\right)}{\lambda} + \left(\frac{\pi}{6} - \frac{\pi}{8}\right) = 2\pi m$$

$$\frac{\left(x_1 - x_2\right)}{650\text{ nm}} + \frac{1}{12} - \frac{1}{16} = m \quad \boxed{x_1 - x_2 = \left(m - \frac{1}{48}\right)650\text{ nm with } m = 0, 1, 2, 3, \ldots, -1, -2, -3, \ldots}$$

***P37.43** A bright line for the green light requires $dy/L = m_1\lambda_1$. A blue interference maximum requires $dy/L = m_2\lambda_2$ for integers m_1 and m_2. Then m_1 540 nm $= m_2$ 450 nm. The smallest integers satisfying the equation are $m_1 = 5$ and $m_2 = 6$. Then for both

$dy/L = 2\,700$ nm $\qquad y = (1.4\text{ m})\ 2.7\ \mu m/\ 150\mu m = \boxed{2.52\text{ cm}}$

P37.44 If the center point on the screen is to be a dark spot rather than bright, passage through the plastic must delay the light by one-half wavelength. Calling the thickness of the plastic t,

$$\frac{t}{\lambda} + \frac{1}{2} = \frac{t}{\lambda/n} = \frac{nt}{\lambda} \quad \text{or} \quad t = \boxed{\frac{\lambda}{2(n-1)}}$$ where n is the index of refraction for the plastic.

P37.45 No phase shift upon reflection from the upper surface (glass to air) of the film, but there will be a shift of $\frac{\lambda}{2}$ due to the reflection at the lower surface of the film (air to metal). The total phase difference in the two reflected beams is

then $\delta = 2nt + \frac{\lambda}{2}$

For constructive interference, $\delta = m\lambda$

or $2(1.00)t + \frac{\lambda}{2} = m\lambda$

Thus, the film thickness for the mth order bright fringe is

$$t_m = \left(m - \frac{1}{2}\right)\frac{\lambda}{2} = m\left(\frac{\lambda}{2}\right) - \frac{\lambda}{4}$$

and the thickness for the $m-1$ bright fringe is:

$$t_{m-1} = (m-1)\left(\frac{\lambda}{2}\right) - \frac{\lambda}{4}$$

Therefore, the change in thickness required to go from one bright fringe to the next is

$$\Delta t = t_m - t_{m-1} = \frac{\lambda}{2}$$

To go through 200 bright fringes, the change in thickness of the air film must be:

$$200\left(\frac{\lambda}{2}\right) = 100\lambda$$

Thus, the increase in the length of the rod is

$$\Delta L = 100\lambda = 100\left(5.00\times10^{-7}\text{ m}\right) = 5.00\times10^{-5}\text{ m}$$

From $\Delta L = L_i\alpha\,\Delta T$

we have: $\alpha = \frac{\Delta L}{L_i\Delta T} = \frac{5.00\times10^{-5}\text{ m}}{(0.100\text{ m})(25.0\,°\text{C})} = \boxed{20.0\times10^{-6}\,°\text{C}^{-1}}$

P37.46 Since $1 < 1.25 < 1.34$, light reflected from top and bottom surfaces of the oil undergoes phase reversal. The path difference is then $2t$, which must be equal to

$$m\lambda_n = \frac{m\lambda}{n}$$

for maximum reflection, with $m = 1$ for the given first-order condition and $n = 1.25$. So

$$t = \frac{m\lambda}{2n} = \frac{1(500\text{ nm})}{2(1.25)} = 200\text{ nm}$$

The volume we assume to be constant: $1.00\text{ m}^3 = (200\text{ nm})A$

$$A = \frac{1.00\text{ m}^3}{200\left(10^{-9}\text{ m}\right)} = 5.00\times10^6\text{ m}^2 = \boxed{5.00\text{ km}^2}$$

P37.47 One radio wave reaches the receiver R directly from the distant source at an angle θ above the horizontal. The other wave undergoes phase reversal as it reflects from the water at P.

Constructive interference first occurs for a path difference of

$$d = \frac{\lambda}{2} \qquad (1)$$

It is equally far from P to R as from P to R', the mirror image of the telescope.

The angles θ in the figure are equal because they each form part of a right triangle with a shared angle at R'.

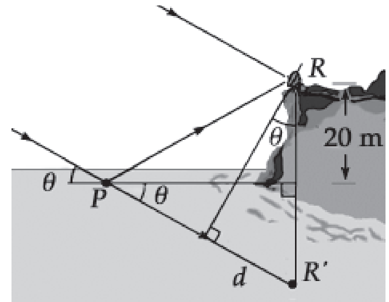


FIG. P37.47

So the path difference is $d = 2(20.0\text{ m})\sin\theta = (40.0\text{ m})\sin\theta$

The wavelength is $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^8\text{ m/s}}{60.0\times10^6\text{ Hz}} = 5.00\text{ m}$

Substituting for d and λ in Equation (1), $(40.0\text{ m})\sin\theta = \dfrac{5.00\text{ m}}{2}$

Solving for the angle θ, $\sin\theta = \dfrac{5.00\text{ m}}{80.0\text{ m}}$ and $\boxed{\theta = 3.58°}$.

P37.48 For destructive interference, the path length must differ by $m\lambda$. We may treat this problem as a double slit experiment if we remember the light undergoes a $\frac{\pi}{2}$-phase shift at the mirror. The second slit is the mirror image of the source, 1.00 cm below the mirror plane. Modifying Equation 37.7,

$$y_{\text{dark}} = \frac{m\lambda L}{d} = \frac{1(5.00\times10^{-7}\text{ m})(100\text{ m})}{(2.00\times10^{-2}\text{ m})} = \boxed{2.50\text{ mm}}$$

P37.49 $2\sqrt{(15.0\text{ km})^2 + h^2} = 30.175\text{ km}$

$(15.0\text{ km})^2 + h^2 = 227.63$

$h = \boxed{1.62\text{ km}}$

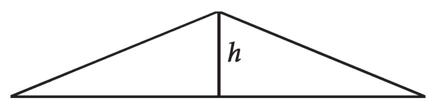


FIG. P37.49

P37.50 For dark fringes, $2nt = m\lambda$

and at the edge of the wedge, $t = \dfrac{84(500\text{ nm})}{2}$

When submerged in water, $2nt = m\lambda$

$$m = \frac{2(1.33)(42)(500\text{ nm})}{500\text{ nm}}$$

so $m + 1 = \boxed{113\text{ dark fringes}}$

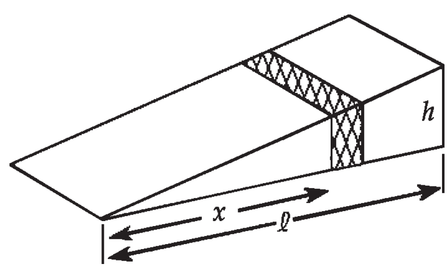


FIG. P37.50

P37.51 From Equation 37.14, $$\frac{I}{I_{\max}} = \cos^2\left(\frac{\pi yd}{\lambda L}\right)$$

Let λ_2 equal the wavelength for which $$\frac{I}{I_{\max}} \to \frac{I_2}{I_{\max}} = 0.640$$

Then $$\lambda_2 = \frac{\pi yd/L}{\cos^{-1}\left(I_2/I_{\max}\right)^{1/2}}$$

But $$\frac{\pi yd}{L} = \lambda_1 \cos^{-1}\left(\frac{I_1}{I_{\max}}\right)^{1/2} = (600 \text{ nm})\cos^{-1}(0.900) = 271 \text{ nm}$$

Substituting this value into the expression for λ_2, $$\lambda_2 = \frac{271 \text{ nm}}{\cos^{-1}\left(0.640^{1/2}\right)} = \boxed{421 \text{ nm}}$$

Note that in this problem, $\cos^{-1}\left(\frac{I}{I_{\max}}\right)^{1/2}$ must be expressed in radians.

P37.52 At entrance, $1.00 \sin 30.0° = 1.38 \sin\theta_2$ $\qquad \theta_2 = 21.2°$
Call t the unknown thickness. Then

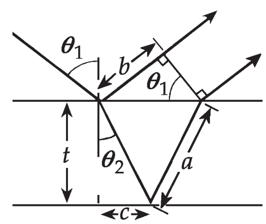


FIG. P37.52

$$\cos 21.2° = \frac{t}{a} \qquad a = \frac{t}{\cos 21.2°}$$

$$\tan 21.2° = \frac{c}{t} \qquad c = t\tan 21.2°$$

$$\sin\theta_1 = \frac{b}{2c} \qquad b = 2t\tan 21.2° \sin 30.0°$$

The net shift for the second ray, including the phase reversal on reflection of the first, is

$$2an - b - \frac{\lambda}{2}$$

where the factor n accounts for the shorter wavelength in the film. For constructive interference, we require

$$2an - b - \frac{\lambda}{2} = m\lambda$$

The minimum thickness will be given by $$2an - b - \frac{\lambda}{2} = 0$$

$$\frac{\lambda}{2} = 2an - b = 2\frac{nt}{\cos 21.2°} - 2t\,(\tan 21.2°)\sin 30.0°$$

$$\frac{590 \text{ nm}}{2} = \left(\frac{2\times 1.38}{\cos 21.2°} - 2\tan 21.2° \ \sin 30.0°\right)t = 2.57t \qquad t = \boxed{115 \text{ nm}}$$

P37.53 The shift between the two reflected waves is $\delta = 2na - b - \frac{\lambda}{2}$ where a and b are as shown in the ray diagram, n is the index of refraction, and the term $\frac{\lambda}{2}$ is due to phase reversal at the top surface. For constructive interference, $\delta = m\lambda$ where m has integer values. This condition becomes

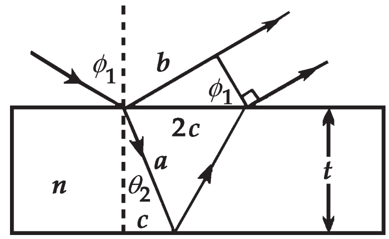


FIG. P37.53

$$2na - b = \left(m + \frac{1}{2}\right)\lambda \qquad (1)$$

From the figure's geometry, $a = \dfrac{t}{\cos\theta_2}$

$$c = a\sin\theta_2 = \frac{t\sin\theta_2}{\cos\theta_2}$$

$$b = 2c\sin\phi_1 = \frac{2t\sin\theta_2}{\cos\theta_2}\sin\phi_1$$

Also, from Snell's law, $\sin\phi_1 = n\sin\theta_2$

Thus, $b = \dfrac{2nt\sin^2\theta_2}{\cos\theta_2}$

With these results, the condition for constructive interference given in Equation (1) becomes:

$$2n\left(\frac{t}{\cos\theta_2}\right) - \frac{2nt\sin^2\theta_2}{\cos\theta_2} = \frac{2nt}{\cos\theta_2}\left(1 - \sin^2\theta_2\right) = \left(m + \frac{1}{2}\right)\lambda$$

or $\boxed{2nt\cos\theta_2 = \left(m + \frac{1}{2}\right)\lambda}$

***P37.54** (a) For a linear function taking the value 1.90 at $y = 0$ and 1.33 at $y = 20$ cm we write $n(y) = 1.90 + (1.33 - 1.90)y/20\text{ cm}$ or $\boxed{n(y) = 1.90 - 0.0285\ y/cm}$

(b) $$\int_0^{20\text{ cm}} n(y)dy = \int_0^{20\text{ cm}} [1.90 - 0.0285y/\text{cm}]dy = 1.90y - \frac{0.0285y^2}{2}\Bigg|_0^{20\text{ cm}}$$

$$= 38.0\text{ cm} - 5.7\text{ cm}$$

$$= \boxed{32.3\text{ cm}}$$

(c) The beam will continuously curve downward.

P37.55 (a) Minimum: $2nt = m\lambda_2$ for $m = 0, 1, 2, \ldots$

Maximum: $2nt = \left(m' + \frac{1}{2}\right)\lambda_1$ for $m' = 0, 1, 2, \ldots$

for $\lambda_1 > \lambda_2$, $\left(m' + \frac{1}{2}\right) < m$

so $m' = m - 1$

Then $2nt = m\lambda_2 = \left(m - \frac{1}{2}\right)\lambda_1$

$2m\lambda_2 = 2m\lambda_1 - \lambda_1$

so $\boxed{m = \dfrac{\lambda_1}{2(\lambda_1 - \lambda_2)}}$

(b) $m = \dfrac{500}{2(500 - 370)} = 1.92 \to 2$ (wavelengths measured to ± 5 nm)

Minimum: $2nt = m\lambda_2$

$2(1.40)t = 2(370 \text{ nm})$ $t = 264$ nm

Maximum: $2nt = \left(m - 1 + \frac{1}{2}\right)\lambda = 1.5\lambda$

$2(1.40)t = 1.5(500 \text{ nm})$ $t = 268$ nm

Film thickness = $\boxed{266 \text{ nm}}$

P37.56 From the sketch, observe that

$$x = \sqrt{h^2 + \left(\frac{d}{2}\right)^2} = \frac{\sqrt{4h^2 + d^2}}{2}$$

Including the phase reversal due to reflection from the ground, the total shift between the two waves is $\delta = 2x - d - \dfrac{\lambda}{2}$.

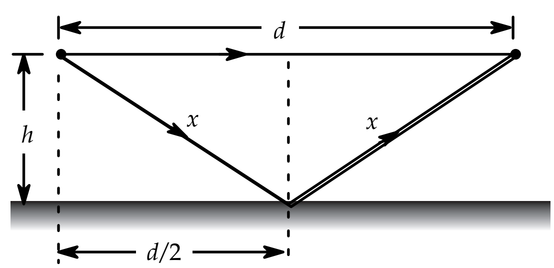


FIG. P37.56

(a) For constructive interference, the total shift must be an integral number of wavelengths, or $\delta = m\lambda$ where $m = 0, 1, 2, 3, \ldots$

Thus, $2x - d = \left(m + \frac{1}{2}\right)\lambda$ or $\lambda = \dfrac{4x - 2d}{2m + 1}$

For the longest wavelength, $m = 0$, giving $\lambda = 4x - 2d = \boxed{2\sqrt{4h^2 + d^2} - 2d}$

(b) For destructive interference, $\delta = \left(m - \frac{1}{2}\right)\lambda$ where $m = 0, 1, 2, 3, \ldots$.

Thus, $2x - d = m\lambda$ or $\lambda = \dfrac{2x - d}{m}$

For the longest wavelength, $m = 1$ giving $\lambda = 2x - d = \boxed{\sqrt{4h^2 + d^2} - d}$

P37.57 Call t the thickness of the film. The central maximum corresponds to zero phase difference. Thus, the added distance Δr traveled by the light from the lower slit must introduce a phase difference equal to that introduced by the plastic film. The phase difference ϕ is

$$\phi = 2\pi\left(\frac{t}{\lambda_a}\right)(n-1)$$

The corresponding difference in **path length** Δr is

$$\Delta r = \phi\left(\frac{\lambda_a}{2\pi}\right) = 2\pi\left(\frac{t}{\lambda_a}\right)(n-1)\left(\frac{\lambda_a}{2\pi}\right) = t(n-1)$$

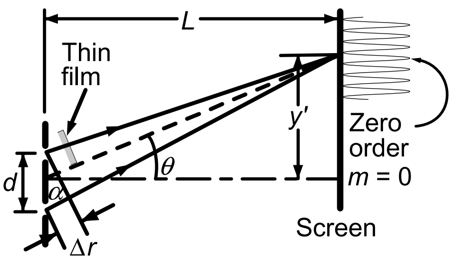


FIG. P37.57

Note that the wavelength of the light does not appear in this equation. In the figure, the two rays from the slits are essentially parallel.

Thus the angle θ may be expressed as $\tan\theta = \frac{\Delta r}{d} = \frac{y'}{L}$

Eliminating Δr by substitution, $\frac{y'}{L} = \frac{t(n-1)}{d}$ gives $\boxed{y' = \frac{t(n-1)L}{d}}$

P37.58 Bright fringes occur when $2t = \frac{\lambda}{n}\left(m+\frac{1}{2}\right)$

and dark fringes occur when $2t = \left(\frac{\lambda}{n}\right)m$

The thickness of the film at x is $t = \left(\frac{h}{\ell}\right)x$

Therefore, $\boxed{x_{\text{bright}} = \frac{\lambda\ell}{2hn}\left(m+\frac{1}{2}\right)}$ and $\boxed{x_{\text{dark}} = \frac{\lambda\ell m}{2hn}}$.

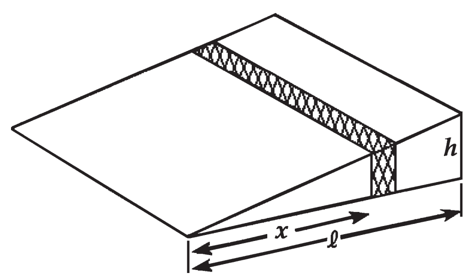


FIG. P37.58

P37.59 (a) Constructive interference in the reflected light requires $2t = \left(m+\frac{1}{2}\right)\lambda$. The first bright ring has $m = 0$ and the 55th has $m = 54$, so at the edge of the lens

$$t = \frac{54.5\left(650\times10^{-9}\text{ m}\right)}{2} = 17.7\ \mu\text{m}$$

Now from the geometry in textbook Figure 37.12, we can find the distance t from the curved surface down to the flat plate by considering distances down from the center of curvature:

$$\sqrt{R^2 - r^2} = R - t \text{ or } R^2 - r^2 = R^2 - 2Rt + t^2$$

$$R = \frac{r^2+t^2}{2t} = \frac{\left(5.00\times10^{-2}\text{ m}\right)^2 + \left(1.77\times10^{-5}\text{ m}\right)^2}{2\left(1.77\times10^{-5}\text{ m}\right)} = \boxed{70.6\text{ m}}$$

(b) $\frac{1}{f} = (n-1)\left(\frac{1}{R_2} - \frac{1}{R_2}\right) = 0.520\left(\frac{1}{\infty} - \frac{1}{-70.6\text{ m}}\right)$ so $f = \boxed{136\text{ m}}$

P37.60 The shift between the waves reflecting from the top and bottom surfaces of the film at the point where the film has thickness t is $\delta = 2tn_{\text{film}} + \frac{\lambda}{2}$, with the factor of $\frac{\lambda}{2}$ being due to a phase reversal at *one* of the surfaces.

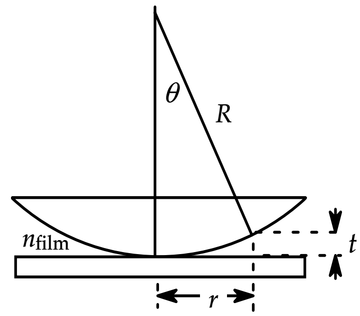


FIG. P37.60

For the dark rings (destructive interference), the total shift should be $\delta = \left(m + \frac{1}{2}\right)\lambda$ with $m = 0, 1, 2, 3, \ldots$.

This requires that $t = \frac{m\lambda}{2n_{\text{film}}}$.

To find t in terms of r and R, $\quad R^2 = r^2 + (R - t)^2 \quad$ so $\quad r^2 = 2Rt + t^2$

Since t is much smaller than R, $\quad t^2 << 2Rt \quad$ and $\quad r^2 \approx 2Rt = 2R\left(\frac{m\lambda}{2n_{\text{film}}}\right)$

Thus, where m is an integer, $\quad \boxed{r \approx \sqrt{\frac{m\lambda R}{n_{\text{film}}}}}$

P37.61 Light reflecting from the upper interface of the air layer suffers no phase change, while light reflecting from the lower interface is reversed 180°. Then there is indeed a dark fringe at the outer circumference of the lens, and a dark fringe wherever the air thickness t satisfies $2t = m\lambda$, $m = 0, 1, 2, \ldots$.

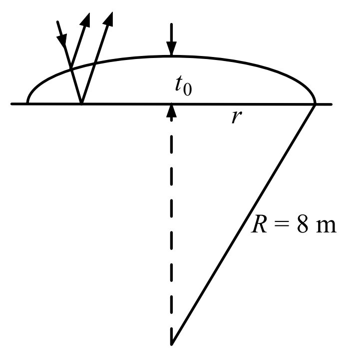


FIG. P37.61

(a) At the central dark spot $m = 50$ and

$$t_0 = \frac{50\lambda}{2} = 25\left(589 \times 10^{-9} \text{ m}\right) = \boxed{1.47 \times 10^{-5} \text{ m}}$$

(b) In the right triangle,

$$(8 \text{ m})^2 = r^2 + \left(8 \text{ m} - 1.47 \times 10^{-5} \text{ m}\right)^2 = r^2 + (8 \text{ m})^2 - 2(8 \text{ m})\left(1.47 \times 10^{-5} \text{ m}\right) + 2 \times 10^{-10} \text{ m}^2$$

The last term is negligible. $r = \sqrt{2(8 \text{ m})\left(1.47 \times 10^{-5} \text{ m}\right)} = \boxed{1.53 \times 10^{-2} \text{ m}}$

(c) $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.50 - 1)\left(\frac{1}{\infty} - \frac{1}{8.00 \text{ m}}\right)$

$\boxed{f = -16.0 \text{ m}}$

P37.62 For bright rings the gap t between surfaces is given by $2t = \left(m + \frac{1}{2}\right)\lambda$. The first bright ring has $m = 0$ and the hundredth has $m = 99$.

So, $t = \frac{1}{2}(99.5)\left(500 \times 10^{-9}\text{ m}\right) = 24.9\ \mu\text{m}$.

Call r_b the ring radius. From the geometry of the figure at the right,

$$t = r - \sqrt{r^2 - r_b^2} - \left(R - \sqrt{R^2 - r_b^2}\right)$$

Since $r_b << r$, we can expand in series:

$$t = r - r\left(1 - \frac{1}{2}\frac{r_b^2}{r^2}\right) - R + R\left(1 - \frac{1}{2}\frac{r_b^2}{R^2}\right) = \frac{1}{2}\frac{r_b^2}{r} - \frac{1}{2}\frac{r_b^2}{R}$$

$$r_b = \left[\frac{2t}{1/r - 1/R}\right]^{1/2} = \left[\frac{2\left(24.9 \times 10^{-6}\text{ m}\right)}{1/4.00\text{ m} - 1/12.0\text{ m}}\right]^{1/2} = \boxed{1.73\text{ cm}}$$

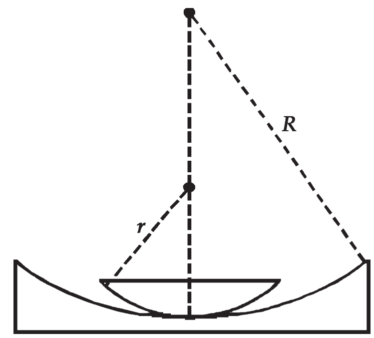


FIG. P37.62

P37.63 (a) Bright bands are observed when $2nt = \left(m + \frac{1}{2}\right)\lambda$.

Hence, the first bright band ($m = 0$) corresponds to $nt = \frac{\lambda}{4}$.

Since $\frac{x_1}{x_2} = \frac{t_1}{t_2}$

we have $x_2 = x_1\left(\frac{t_2}{t_1}\right) = x_1\left(\frac{\lambda_2}{\lambda_1}\right) = (3.00\text{ cm})\left(\frac{680\text{ nm}}{420\text{ nm}}\right) = \boxed{4.86\text{ cm}}$

(b) $t_1 = \frac{\lambda_1}{4n} = \frac{420\text{ nm}}{4(1.33)} = \boxed{78.9\text{ nm}}$

$t_2 = \frac{\lambda_2}{4n} = \frac{680\text{ nm}}{4(1.33)} = \boxed{128\text{ nm}}$

(c) $\theta \approx \tan\theta = \frac{t_1}{x_1} = \frac{78.9\text{ nm}}{3.00\text{ cm}} = \boxed{2.63 \times 10^{-6}\text{ rad}}$

P37.64 Depth = one-quarter of the wavelength in plastic.

$$t = \frac{\lambda}{4n} = \frac{780\text{ nm}}{4(1.50)} = \boxed{130\text{ nm}}$$

P37.65 $2h\sin\theta = \left(m + \frac{1}{2}\right)\lambda$ bright

$2h\left(\frac{\Delta y}{2L}\right) = \frac{1}{2}\lambda$ so $h = \frac{L\lambda}{2\Delta y} = \frac{(2.00\text{ m})\left(606 \times 10^{-9}\text{ m}\right)}{2\left(1.2 \times 10^{-3}\text{ m}\right)} = \boxed{0.505\text{ mm}}$

P37.66 Superposing the two vectors, $E_R = \left|\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2\right|$

$$E_R = \left|\vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2\right| = \sqrt{\left(E_0 + \frac{E_0}{3}\cos\phi\right)^2 + \left(\frac{E_0}{3}\sin\phi\right)^2} = \sqrt{E_0^2 + \frac{2}{3}E_0^2\cos\phi + \frac{E_0^2}{9}\cos^2\phi + \frac{E_0^2}{9}\sin^2\phi}$$

$$E_R = \sqrt{\frac{10}{9}E_0^2 + \frac{2}{3}E_0^2\cos\phi}$$

Since intensity is proportional to the square of the amplitude,

$$I = \frac{10}{9}I_{\max} + \frac{2}{3}I_{\max}\cos\phi$$

Using the trigonometric identity $\cos\phi = 2\cos^2\frac{\phi}{2} - 1$, this becomes

$$I = \frac{10}{9}I_{\max} + \frac{2}{3}I_{\max}\left(2\cos^2\frac{\phi}{2} - 1\right) = \frac{4}{9}I_{\max} + \frac{4}{3}I_{\max}\cos^2\frac{\phi}{2}$$

or $$\boxed{I = \frac{4}{9}I_{\max}\left(1 + 3\cos^2\frac{\phi}{2}\right)}$$

P37.67 Represent the light radiated from each slit to point P as a phasor. The two have equal amplitudes E. Since intensity is proportional to amplitude squared, they add to amplitude $\sqrt{3}E$.

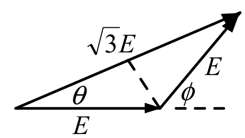


FIG. P37.67

Then $\cos\theta = \frac{\sqrt{3}E/2}{E}$, $\theta = 30°$. Next, the obtuse angle between the two phasors is $180 - 30 - 30 = 120°$, and

$\phi = 180 - 120° = 60°$. The phase difference between the

two phasors is caused by the path difference $\delta = \overline{SS_2} - \overline{SS_1}$

according to $\frac{\delta}{\lambda} = \frac{\phi}{360°}$, $\delta = \lambda\frac{60°}{360°} = \frac{\lambda}{6}$. Then

$$\sqrt{L^2 + d^2} - L = \frac{\lambda}{6}$$

$$L^2 + d^2 = L^2 + \frac{2L\lambda}{6} + \frac{\lambda^2}{36}$$

The last term is negligible, so

$$d = \left(\frac{2L\lambda}{6}\right)^{1/2} = \sqrt{\frac{2(1.2\text{ m})620\times10^{-9}\text{ m}}{6}} = \boxed{0.498\text{ mm}}$$

ANSWERS TO EVEN PROBLEMS

P37.2 515 nm

P37.4 (a) 36.2° (b) 5.08 cm (c) 508 THz

P37.6 Question: A single oscillator makes the two speakers of a boom box, 35.0 cm apart, vibrate in phase at 1.62 kHz. At what angles, measured from the perpendicular bisector of the line joining the speakers, will a distant observer hear maximum sound intensity? Minimum sound intensity? The ambient temperature is 20°C. Answer: The wavelength of the sound is 21.2 cm. Interference maxima occur at angles of 0° and 37.2° to the left and right. Minima occur at angles of 17.6° and 65.1°. No second-order or higher-order maximum exists. No angle exists, smaller or larger than 90°, for which $\sin\theta_{2\ \text{loud}} = 1.21$. No location exists in the Universe that is two wavelengths farther from one speaker than from the other.

P37.8 11.3 m

P37.10 641

P37.12 6.33 mm/s

P37.14 See the solution.

P37.16 (a) 1.29 rad (b) 99.6 nm

P37.18 0.968

P37.20 See the solution.

P37.22 (a) See the solution. (b) The cosine function takes on the extreme value −1 to describe the secondary maxima. The cosine function takes on the extreme value +1 to describe the primary maxima. The ratio is 9.00.

P37.24 612 nm

P37.26 0.500 cm

P37.28 No reflection maxima in the visible spectrum

P37.30 290 nm

P37.32 1.31

P37.34 449 nm; blue

P37.36 $1 + N\lambda/2L$

P37.38 (a) See the solution. (b) 2.74 m

P37.40 number of antinodes = number of constructive interference zones

$$= 1 \text{ plus 2 times the greatest positive integer} \le \frac{d}{\lambda}$$

number of nodes = number of destructive interference zones

$$= 2 \text{ times the greatest positive integer} < \left(\frac{d}{\lambda} + \frac{1}{2}\right)$$

P37.42 $x_1 - x_2 = \left(m - \frac{1}{48}\right)650 \text{ nm}$ with $m = 0, 1, 2, 3, \ldots, -1, -2, -3, \ldots$

P37.44 $\dfrac{\lambda}{2(n-1)}$

P37.46 5.00 km^2

P37.48 2.50 mm

P37.50 113

P37.52 115 nm

P37.54 (a) $n(y) = 1.90 - 0.0285\ y/\text{cm}$ (b) 32.3 cm (c) The beam will continuously curve downward.

P37.56 (a) $2(4h^2 + d^2)^{1/2} - 2d$ (b) $(4h^2 + d^2)^{1/2} - d$

P37.58 See the solution.

P37.60 See the solution.

P37.62 1.73 cm

P37.64 130 nm

P37.66 See the solution.

38

Diffraction Patterns and Polarization

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q38.1 Audible sound has wavelengths on the order of meters or centimeters, while visible light has a wavelength on the order of half a micrometer. In this world of breadbox-sized objects, $\frac{\lambda}{a}$ is large for sound, and sound diffracts around behind walls with doorways. But $\frac{\lambda}{a}$ is a tiny fraction for visible light passing ordinary-size objects or apertures, so light changes its direction by only very small angles when it diffracts.

Another way of phrasing the answer: We can see by a small angle around a small obstacle or around the edge of a small opening. The side fringes in Figure 38.1 and the Arago spot in the center of Figure 38.3 show this diffraction. We cannot always hear around corners. Out-of-doors, away from reflecting surfaces, have someone a few meters distant face away from you and whisper. The high-frequency, short-wavelength, information-carrying components of the sound do not diffract around his head enough for you to understand his words.

Suppose an opera singer loses the tempo and cannot immediately get it from the orchestra conductor. Then the prompter may make rhythmic kissing noises with her lips and teeth. Try it—you will sound like a birdwatcher trying to lure out a curious bird. This sound is clear on the stage but does not diffract around the prompter's box enough for the audience to hear it.

Q38.2 The wavelength of visible light is extremely small in comparison to the dimensions of your hand, so the diffraction of light around an obstacle the size of your hand is totally negligible. However, sound waves have wavelengths that are comparable to the dimensions of the hand or even larger. Therefore, significant diffraction of sound waves occurs around hand-sized obstacles.

***Q38.3** Answer (d). The power of the light coming through the slit decreases, as you would expect. The central maximum increases in width as the width of the slit decreases. In the condition $\sin\theta = \frac{\lambda}{a}$ for destructive interference on each side of the central maximum, θ increases as a decreases.

***Q38.4** We consider the fraction $\lambda L/a$. In case (a) and in case (f) it is $\lambda_0 L/a$. In case (b) it is $2\lambda_0 L/3a$. In case (c) it is $3\lambda_0 L/2a$. In case (d) it is $\lambda_0 L/2a$. In case (e) it is $\lambda_0 2L/a$. The ranking is e > c > a = f > b > d.

***Q38.5** Answer (b). The wavelength will be much smaller than with visible light.

***Q38.6** Answer (c). The ability to resolve light sources depends on diffraction, not on intensity.

Q38.7 Consider incident light nearly parallel to the horizontal ruler. Suppose it scatters from bumps at distance d apart to produce a diffraction pattern on a vertical wall a distance L away. At a point of height y, where $\theta = \frac{y}{L}$ gives the scattering angle θ, the character of the interference is determined by the shift δ between beams scattered by adjacent bumps, where $\delta = \frac{d}{\cos\theta} - d$.

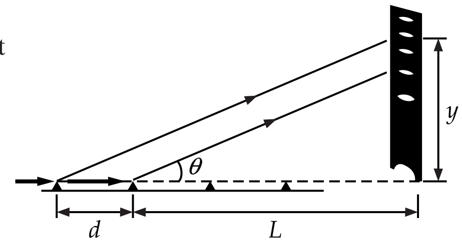


FIG. Q38.7

Bright spots appear for $\delta = m\lambda$, where $m = 0, 1, 2, 3, \ldots$. For small θ, these equations combine and reduce to $m\lambda = \frac{y_m^2 d}{2L^2}$. Measurement of the heights y_m of bright spots allows calculation of the wavelength of the light.

***Q38.8** Answer (b). No diffraction effects are observed because the separation distance between adjacent ribs is so much greater than the wavelength of x-rays. Diffraction does not limit the resolution of an x-ray image. Diffraction might sometimes limit the resolution of an ultrasonogram.

***Q38.9** Answer (a). Glare, as usually encountered when driving or boating, is horizontally polarized. Reflected light is polarized in the same plane as the reflecting surface. As unpolarized light hits a shiny horizontal surface, the atoms on the surface absorb and then reemit the light energy as a reflection. We can model the surface as containing conduction electrons free to vibrate easily along the surface, but not to move easily out of surface. The light emitted from a vibrating electron is partially or completely polarized along the plane of vibration, thus horizontally.

Q38.10 Light from the sky is partially polarized. Light from the blue sky that is polarized at 90° to the polarization axis of the glasses will be blocked, making the sky look darker as compared to the clouds.

***Q38.11** Answer (a). The grooves in a diffraction grating are not electrically conducting. Sending light through a diffraction grating is not like sending a vibration on a rope through a picket fence. The electric field in light does not have an amplitude in real space. Its amplitude is in newtons per coulomb, not in millimeters.

Q38.12 First think about the glass without a coin and about one particular point P on the screen. We can divide up the area of the glass into ring-shaped zones centered on the line joining P and the light source, with successive zones contributing alternately in-phase and out-of-phase with the light that takes the straight-line path to P. These Fresnel zones have nearly equal areas. An outer zone contributes only slightly less to the total wave disturbance at P than does the central circular zone. Now insert the coin. If P is in line with its center, the coin will block off the light from some particular number of zones. The first unblocked zone around its circumference will send light to P with significant amplitude. Zones farther out will predominantly interfere destructively with each other, and the Arago spot is bright. Slightly off the axis there is nearly complete destructive interference, so most of the geometrical shadow is dark. A bug on the screen crawling out past the edge of the geometrical shadow would in effect see the central few zones coming out of eclipse. As the light from them interferes alternately constructively and destructively, the bug moves through bright and dark fringes on the screen. The diffraction pattern is shown in Figure 38.3 in the text.

Q38.13 Since the obsidian is opaque, a standard method of measuring incidence and refraction angles and using Snell's Law is ineffective. Reflect unpolarized light from the horizontal surface of the obsidian through a vertically polarized filter. Change the angle of incidence until you observe that none of the reflected light is transmitted through the filter. This means that the reflected light is completely horizontally polarized, and that the incidence and reflection angles are the polarization angle. The tangent of the polarization angle is the index of refraction of the obsidian.

Q38.14 The fine hair blocks off light that would otherwise go through a fine slit and produce a diffraction pattern on a distant screen. The width of the central maximum in the pattern is inversely proportional to the distance across the slit. When the hair is in place, it subtracts the same diffraction pattern from the projected disk of laser light. The hair produces a diffraction minimum that crosses the bright circle on the screen. The width of the minimum is inversely proportional to the diameter of the hair. The central minimum is flanked by narrower maxima and minima. Measure the width $2y$ of the central minimum between the maxima bracketing it, and use Equation 38.1 in the form $\frac{y}{L} = \frac{\lambda}{a}$ to find the width a of the hair.

Q38.15 The condition for constructive interference is that the three radio signals arrive at the city in phase. We know the speed of the waves (it is the speed of light c), the angular bearing θ of the city east of north from the broadcast site, and the distance d between adjacent towers. The wave from the westernmost tower must travel an extra distance $2d\sin\theta$ to reach the city, compared to the signal from the eastern tower. For each cycle of the carrier wave, the western antenna would transmit first, the center antenna after a time delay $\frac{d\sin\theta}{c}$, and the eastern antenna after an additional equal time delay.

Q38.16 It is shown in the correct orientation. If the horizontal width of the opening is equal to or less than the wavelength of the sound, then the equation $a\sin\theta = (1)\lambda$ has the solution $\theta = 90°$, or has no solution. The central diffraction maximum covers the whole seaward side. If the vertical height of the opening is large compared to the wavelength, then the angle in $a\sin\theta = (1)\lambda$ will be small, and the central diffraction maximum will form a thin horizontal sheet.

Featured in the motion picture *M*A*S*H* (20th Century Fox, Aspen Productions, 1970) is a loudspeaker mounted on an exterior wall of an Army barracks. It has an approximately rectangular aperture, and it is installed incorrectly. The longer side is horizontal, to maximize sound spreading in a vertical plane and to minimize sound radiated in different horizontal directions.

SOLUTIONS TO PROBLEMS

Section 38.1 **Introduction to Diffraction Patterns**

Section 38.2 **Diffraction Patterns from Narrow Slits**

P38.1 $$\sin\theta = \frac{\lambda}{a} = \frac{6.328\times10^{-7}}{3.00\times10^{-4}} = 2.11\times10^{-3}$$

$$\frac{y}{1.00\text{ m}} = \tan\theta \approx \sin\theta \approx \theta \text{ (for small } \theta\text{)}$$

$$2y = \boxed{4.22\text{ mm}}$$

P38.2 The positions of the first-order minima are $\frac{y}{L} \approx \sin\theta = \pm\frac{\lambda}{a}$. Thus, the spacing between these two minima is $\Delta y = 2\left(\frac{\lambda}{a}\right)L$ and the wavelength is

$$\lambda = \left(\frac{\Delta y}{2}\right)\left(\frac{a}{L}\right) = \left(\frac{4.10\times 10^{-3}\ \text{m}}{2}\right)\left(\frac{0.550\times 10^{-3}\ \text{m}}{2.06\ \text{m}}\right) = \boxed{547\ \text{nm}}$$

P38.3 $\frac{y}{L} \approx \sin\theta = \frac{m\lambda}{a}$ $\qquad$ $\Delta y = 3.00\times 10^{-3}\ \text{nm}$

$\Delta m = 3 - 1 = 2$ $\qquad$ and $\qquad$ $a = \frac{\Delta m \lambda L}{\Delta y}$

$$a = \frac{2\left(690\times 10^{-9}\ \text{m}\right)(0.500\ \text{m})}{\left(3.00\times 10^{-3}\ \text{m}\right)} = \boxed{2.30\times 10^{-4}\ \text{m}}$$

P38.4 For destructive interference,

$$\sin\theta = m\frac{\lambda}{a} = \frac{\lambda}{a} = \frac{5.00\ \text{cm}}{36.0\ \text{cm}} = 0.139$$

and $\qquad \theta = 7.98°$

$$\frac{y}{L} = \tan\theta$$

gives $\qquad y = L\tan\theta = (6.50\ \text{m})\tan 7.98° = 0.912\ \text{m}$

$$y = \boxed{91.2\ \text{cm}}$$

P38.5 If the speed of sound is 340 m/s, $\qquad \lambda = \frac{v}{f} = \frac{340\ \text{m/s}}{650\ \text{s}^{-1}} = 0.523\ \text{m}$

Diffraction minima occur at angles described by $\quad a\sin\theta = m\lambda$

$(1.10\ \text{m})\sin\theta_1 = 1(0.523\ \text{m})$ $\qquad \theta_1 = 28.4°$

$(1.10\ \text{m})\sin\theta_2 = 2(0.523\ \text{m})$ $\qquad \theta_2 = 72.0°$

$(1.10\ \text{m})\sin\theta_3 = 3(0.523\ \text{m})$ $\qquad \theta_3$ nonexistent

Maxima appear straight ahead at $\boxed{0°}$ and left and right at an angle given approximately by

$(1.10\ \text{m})\sin\theta_x = 1.5(0.523\ \text{m})$ $\qquad \theta_x\ \boxed{\approx 46°}$

There is no solution to $a\sin\theta = 2.5\lambda$, so our answer is already complete, with $\boxed{\text{three}}$ sound maxima.

***P38.6** (a) The rectangular patch on the wall is wider than it is tall. The aperture will be taller than it is wide. For horizontal spreading we have

$$\tan\theta_{\text{width}} = \frac{y_{\text{width}}}{L} = \frac{0.110\text{ m}/2}{4.5\text{ m}} = 0.012\,2$$

$$a_{\text{width}}\sin\theta_{\text{width}} = 1\lambda$$

$$a_{\text{width}} = \frac{632.8\times10^{-9}\text{ m}}{0.012\,2} = \boxed{5.18\times10^{-5}\text{ m}}$$

For vertical spreading, similarly

$$\tan\theta_{\text{height}} = \frac{0.006\text{ m}/2}{4.5\text{ m}} = 0.000\,667$$

$$a_{\text{height}} = \frac{1\lambda}{\sin\theta_h} = \frac{632.8\times10^{-9}\text{ m}}{0.000\,667}$$

$$= \boxed{9.49\times10^{-4}\text{ m}}$$

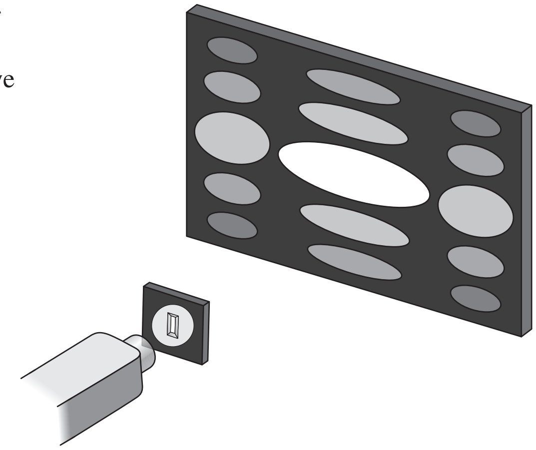


FIG. P38.6

(b) The central bright patch is horizontal. The aperture is vertical.

A smaller distance between aperture edges causes a wider diffraction angle. The longer dimension of each rectangle is 18.3 times larger than the smaller dimension.

P38.7 $\sin\theta \approx \frac{y}{L} = \frac{4.10\times10^{-3}\text{ m}}{1.20\text{ m}}$

We define $\phi = \frac{\pi a\sin\theta}{\lambda} = \frac{\pi\left(4.00\times10^{-4}\text{ m}\right)}{546.1\times10^{-9}\text{ m}}\left(\frac{4.10\times10^{-3}\text{ m}}{1.20\text{ m}}\right) = 7.86\text{ rad}$

$$\frac{I}{I_{\max}} = \left[\frac{\sin(\phi)}{\phi}\right]^2 = \left[\frac{\sin(7.86)}{7.86}\right]^2 = \boxed{1.62\times10^{-2}}$$

P38.8 Equation 38.1 states that $\sin\theta = \frac{m\lambda}{a}$, where $m = \pm1, \pm2, \pm3, \ldots$.

The requirement for $m = 1$ is from an analysis of the extra path distance traveled by ray 1 compared to ray 3 in the textbook Figure 38.5. This extra distance must be equal to $\frac{\lambda}{2}$ for destructive interference. When the source rays approach the slit at an angle β, there is a distance added to the path difference (of ray 1 compared to ray 3) of $\frac{a}{2}\sin\beta$. Then, for destructive interference,

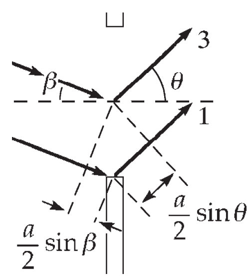


FIG. P38.8

$$\frac{a}{2}\sin\beta + \frac{a}{2}\sin\theta = \frac{\lambda}{2}\ \text{ so }\ \sin\theta = \frac{\lambda}{a}0 - \sin\beta.$$

Dividing the slit into 4 parts leads to the 2nd order minimum: $\sin\theta = \frac{2\lambda}{a} - \sin\beta$

Dividing the slit into 6 parts gives the third order minimum: $\sin\theta = \frac{3\lambda}{a} - \sin\beta$

Generalizing, we obtain the condition for the *m*th order minimum: $\sin\theta = \frac{m\lambda}{a} - \sin\beta$

***P38.9** The diffraction envelope shows a broad central maximum flanked by zeros at $a\sin\theta = 1\lambda$ and $a\sin\theta = 2\lambda$. That is, the zeros are at $(\pi a\sin\theta)/\lambda = \pi, -\pi, 2\pi, -2\pi, \ldots$. Noting that the distance between slits is $d = 9\,\mu\text{m} = 3a$, we say that within the diffraction envelope the interference pattern shows closely spaced maxima at $d\sin\theta = m\lambda$, giving $(\pi\, 3a\sin\theta)/\lambda = m\pi$ or $(\pi a\sin\theta)/\lambda = 0, \pi/3, -\pi/3, 2\pi/3, -2\pi/3$. The third-order interference maxima are missing because they fall at the same directions as diffraction minima, but the fourth order can be visible at $(\pi a\sin\theta)/\lambda = 4\pi/3$ and $-4\pi/3$ as diagrammed.

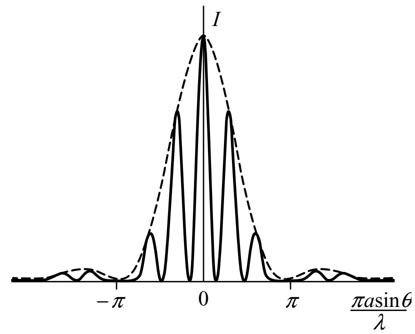


FIG. P38.9

P38.10 (a) Double-slit interference maxima are at angles given by $d\sin\theta = m\lambda$.

For $m = 0$, $\theta_0 = \boxed{0°}$

For $m = 1$, $(2.80\ \mu\text{m})\sin\theta = 1(0.501\,5\ \mu\text{m})$: $\theta_1 = \sin^{-1}(0.179) = \boxed{10.3°}$

Similarly, for $m = 2, 3, 4, 5$ and 6, $\theta_2 = \boxed{21.0°}$, $\theta_3 = \boxed{32.5°}$, $\theta_4 = \boxed{45.8°}$

$\theta_5 = \boxed{63.6°}$, and $\theta_6 = \sin^{-1}(1.07) =$ nonexistent

Thus, there are $5 + 5 + 1 = 11$ directions for interference maxima.

(b) We check for missing orders by looking for single-slit diffraction minima, at $a\sin\theta = m\lambda$.

For $m = 1$, $(0.700\ \mu\text{m})\sin\theta = 1(0.501\,5\ \mu\text{m})$ and $\theta_1 = 45.8°$

Thus, there is no bright fringe at this angle. There are only $\boxed{\text{nine bright fringes}}$, at $\boxed{\theta = 0°, \pm 10.3°, \pm 21.0°, \pm 32.5°, \text{ and } \pm 63.6°}$.

(c) $$I = I_{max}\left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi\sin\theta/\lambda}\right]^2$$

At $\theta = 0°$, $\dfrac{\sin\theta}{\theta} \to 1$ and $\dfrac{I}{I_{max}} \to \boxed{1.00}$

At $\theta = 10.3°$, $$\frac{\pi a\sin\theta}{\lambda} = \frac{\pi(0.700\ \mu\text{m})\sin 10.3°}{0.501\,5\ \mu\text{m}} = 0.785 \text{ rad} = 45.0°$$

$$\frac{I}{I_{max}} = \left[\frac{\sin 45.0°}{0.785}\right]^2 = \boxed{0.811}$$

Similarly, at $\theta = 21.0°$, $\dfrac{\pi a\sin\theta}{\lambda} = 1.57 \text{ rad} = 90.0°$ and $\dfrac{I}{I_{max}} = \boxed{0.405}$

At $\theta = 32.5°$, $\dfrac{\pi a\sin\theta}{\lambda} = 2.36 \text{ rad} = 135°$ and $\dfrac{I}{I_{max}} = \boxed{0.090\,1}$

At $\theta = 63.6°$, $\dfrac{\pi a\sin\theta}{\lambda} = 3.93 \text{ rad} = 225°$ and $\dfrac{I}{I_{max}} = \boxed{0.032\,4}$

Section 38.3 **Resolution of Single-Slit and Circular Apertures**

P38.11 We assume Rayleigh's criterion applies to the predator's eye with pupil narrowed. (It is a few times too optimistic for a normal human eye with pupil dilated.)

$$\sin\theta = \frac{\lambda}{a} = \frac{5.00\times10^{-7}\text{ m}}{5.00\times10^{-4}} = \boxed{1.00\times10^{-3}\text{ rad}}$$

***P38.12** (a) $1.22\lambda/D = 1.22(589\text{ nm})/(9\,000\,000\text{ nm}) = \boxed{79.8\ \mu\text{rad}}$

(b) For a smaller angle of diffraction we choose the smallest visible wavelength, $\boxed{\text{violet}}$ at 400 nm, to obtain $1.22\lambda/D = 1.22(400\text{ nm})/(9\,000\,000\text{ nm}) = \boxed{54.2\ \mu\text{rad}}$.

(c) The wavelength in water is shortened to its vacuum value divided by the index of refraction. $\boxed{\text{The resolving power is improved, with the minimum resolvable angle becoming}}$

$1.22\lambda/D = 1.22(589\text{ nm}/1.33)/(9\,000\,000\text{ nm}) = \boxed{60.0\ \mu\text{rad}}$

Better than water for many purposes is oil immersion.

P38.13 Undergoing diffraction from a circular opening, the beam spreads into a cone of half-angle

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{632.8\times10^{-9}\text{ m}}{0.005\,00\text{ m}}\right) = 1.54\times10^{-4}\text{ rad}$$

The radius of the beam ten kilometers away is, from the definition of radian measure,

$$r_{\text{beam}} = \theta_{\min}\left(1.00\times10^{4}\text{ m}\right) = 1.544\text{ m}$$

and its diameter is $d_{\text{beam}} = 2r_{\text{beam}} = \boxed{3.09\text{ m}}$.

***P38.14** When the pupil is open wide, it appears that the resolving power of human vision is limited by the coarseness of light sensors on the retina. But we use Rayleigh's criterion as a handy indicator of how good our vision might be. We take $\theta_{\min} = \frac{d}{L} = 1.22\frac{\lambda}{D}$, where $\theta_{\min}$ is the smallest angular separation of two objects for which they are resolved by an aperture of diameter D, d is the separation of the two objects, and L is the maximum distance of the aperture from the two objects at which they can be resolved.

Two objects can be resolved if their angular separation is greater than $\theta_{\min}$. Thus, $\theta_{\min}$ should be as small as possible. Therefore, light with the smaller of the two given wavelengths is easier to resolve, i.e., $\boxed{\text{blue}}$.

$$L = \frac{Dd}{1.22\lambda} = \frac{\left(5.20\times10^{-3}\text{ m}\right)\left(2.80\times10^{-2}\text{ m}\right)}{1.22\lambda} = \frac{1.193\times10^{-4}\text{ m}^2}{\lambda}$$

Thus $L = 186$ m for $\lambda = 640$ nm, and $L = 271$ m for $\lambda = 440$ nm. The viewer with the assumed diffraction-limited vision could resolve adjacent tubes of blue in the range $\boxed{186\text{ m to }271\text{ m}}$, but cannot resolve adjacent tubes of red in this range.

P38.15 When the pupil is open wide, it appears that the resolving power of human vision is limited by the coarseness of light sensors on the retina. But we use Rayleigh's criterion as a handy indicator of how good our vision might be. According to this criterion, two dots separated center-to-center by 2.00 mm would overlap

when $$\theta_{\min} = \frac{d}{L} = 1.22\frac{\lambda}{D}$$

Thus, $$L = \frac{dD}{1.22\lambda} = \frac{\left(2.00\times10^{-3}\text{ m}\right)\left(4.00\times10^{-3}\text{ m}\right)}{1.22\left(500\times10^{-9}\text{ m}\right)} = \boxed{13.1\text{ m}}$$

P38.16 When the pupil is open wide, it appears that the resolving power of human vision is limited by the coarseness of light sensors on the retina. But we use Rayleigh's criterion as a handy indicator of how good our vision might be.

$D = 250 \times 10^3$ m
$\lambda = 5.00 \times 10^{-7}$ m
$d = 5.00 \times 10^{-3}$ m

$$x = 1.22\frac{\lambda}{d}D = 1.22\left(\frac{5.00\times10^{-7}\text{ m}}{5.00\times10^{-3}\text{ m}}\right)\left(250\times10^3\text{ m}\right) = \boxed{30.5\text{ m}}$$

P38.17 The concave mirror of the spy satellite is probably about 2 m in diameter, and is surely not more than 5 m in diameter. That is the size of the largest piece of glass successfully cast to a precise shape, for the mirror of the Hale telescope on Mount Palomar. If the spy satellite had a larger mirror, its manufacture could not be kept secret, and it would be visible from the ground. Outer space is probably closer than your state capitol, but the satellite is surely above 200-km altitude, for reasonably low air friction. We find the distance between barely resolvable objects at a distance of 200 km, seen in yellow light through a 5-m aperture:

$$\frac{y}{L} = 1.22\frac{\lambda}{D}$$

$$y = (200\ 000\text{ m})(1.22)\left(\frac{6\times10^{-7}\text{ m}}{5\text{ m}}\right) = 3\text{ cm}$$

(Considering atmospheric seeing caused by variations in air density and temperature, the distance between barely resolvable objects is more like $(200\ 000\text{ m})(1\text{ s})\left(\frac{1°}{3\ 600\text{ s}}\right)\left(\frac{\pi\text{ rad}}{180°}\right) = 97$ cm.)

Thus the snooping spy satellite cannot see the difference between III and II or IV on a license plate. It cannot count coins spilled on a sidewalk, much less read the dates on them.

P38.18 $1.22\frac{\lambda}{D} = \frac{d}{L}$ $\quad \lambda = \frac{c}{f} = 0.020\ 0$ m

$D = 2.10$ m $\quad L = 9\ 000$ m

$$d = 1.22\frac{(0.020\ 0\text{ m})(9\ 000\text{ m})}{2.10\text{ m}} = \boxed{105\text{ m}}$$

Section 38.4 **The Diffraction Grating**

P38.19 $d = \frac{1.00\text{ cm}}{2\ 000} = \frac{1.00\times10^{-2}\text{ m}}{2\ 000} = 5.00\ \mu\text{m}$

$$\sin\theta = \frac{m\lambda}{d} = \frac{1\left(640\times10^{-9}\text{ m}\right)}{5.00\times10^{-6}\text{ m}} = 0.128 \quad \theta = \boxed{7.35°}$$

P38.20 The principal maxima are defined by

$d\sin\theta = m\lambda \qquad m = 0, 1, 2, \ldots$

For $m = 1$, $\qquad \lambda = d\sin\theta$

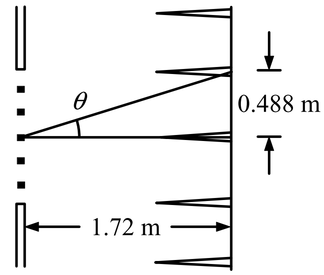


FIG. P38.20

where θ is the angle between the central ($m = 0$) and the first order ($m = 1$) maxima. The value of θ can be determined from the information given about the distance between maxima and the grating-to-screen distance. From the figure,

$$\tan\theta = \frac{0.488 \text{ m}}{1.72 \text{ m}} = 0.284$$

so $\qquad \theta = 15.8°$

and $\qquad \sin\theta = 0.273$

The distance between grating "slits" equals the reciprocal of the number of grating lines per centimeter

$$d = \frac{1}{5\,310 \text{ cm}^{-1}} = 1.88\times10^{-4}\text{ cm} = 1.88\times10^{3}\text{ nm}$$

The wavelength is $\qquad \lambda = d\sin\theta = \left(1.88\times10^{3}\text{ nm}\right)(0.273) = \boxed{514 \text{ nm}}$

P38.21 The grating spacing is $\qquad d = \dfrac{1.00\times10^{-2}\text{ m}}{4\,500} = 2.22\times10^{-6}\text{ m}$

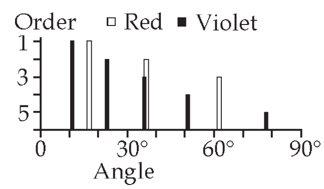


FIG. P38.21

In the 1st-order spectrum, diffraction angles are given by

$\sin\theta = \dfrac{\lambda}{d}$: $\qquad \sin\theta_1 = \dfrac{656\times10^{-9}\text{ m}}{2.22\times10^{-6}\text{ m}} = 0.295$

so that for red $\qquad \theta_1 = 17.17°$

and for blue $\qquad \sin\theta_2 = \dfrac{434\times10^{-9}\text{ m}}{2.22\times10^{-6}\text{ m}} = 0.195$

so that $\qquad \theta_2 = 11.26°$

The angular separation is in first-order, $\qquad \Delta\theta = 17.17° - 11.26° = \boxed{5.91°}$

In the second-order spectrum, $\qquad \Delta\theta = \sin^{-1}\left(\dfrac{2\lambda_1}{d}\right) - \sin^{-1}\left(\dfrac{2\lambda_2}{d}\right) = \boxed{13.2°}$

Again, in the third order, $\qquad \Delta\theta = \sin^{-1}\left(\dfrac{3\lambda_1}{d}\right) - \sin^{-1}\left(\dfrac{3\lambda_2}{d}\right) = \boxed{26.5°}$

Since the red does not appear in the fourth-order spectrum, the answer is complete.

P38.22 $\sin\theta = 0.350$: $d = \dfrac{\lambda}{\sin\theta} = \dfrac{632.8\text{ nm}}{0.350} = 1.81\times10^3\text{ nm}$

Line spacing = $\boxed{1.81\ \mu\text{m}}$

P38.23 (a) $d = \dfrac{1}{3\,660\ \text{lines/cm}} = 2.732\times10^{-4}\ \text{cm} = 2.732\times10^{-6}\ \text{m} = 2\,732\ \text{nm}$

$\lambda = \dfrac{d\sin\theta}{m}$: At $\theta = 10.09°$, $\lambda = \boxed{478.7\text{ nm}}$

At $\theta = 13.71°$, $\lambda = \boxed{647.6\text{ nm}}$

At $\theta = 14.77°$, $\lambda = \boxed{696.6\text{ nm}}$

(b) $d = \dfrac{\lambda}{\sin\theta_1}$ and $2\lambda = d\sin\theta_2$ so $\sin\theta_2 = \dfrac{2\lambda}{d} = \dfrac{2\lambda}{\lambda/\sin\theta_1} = 2\sin\theta_1$

Therefore, if $\theta_1 = 10.09°$ then $\sin\theta_2 = 2\sin(10.09°)$ gives $\theta_2 = \boxed{20.51°}$.

Similarly, for $\theta_1 = 13.71°$, $\theta_2 = \boxed{28.30°}$ and for $\theta_1 = 14.77°$, $\theta_2 = \boxed{30.66°}$.

P38.24 $\sin\theta = \dfrac{m\lambda}{d}$

Therefore, taking the ends of the visible spectrum to be $\lambda_v = 400$ nm and $\lambda_r = 750$ nm, the ends of the different order spectra are defined by:

End of second order: $\sin\theta_{2r} = \dfrac{2\lambda_r}{d} = \dfrac{1\,500\text{ nm}}{d}$

Start of third order: $\sin\theta_{3v} = \dfrac{3\lambda_v}{d} = \dfrac{1\,200\text{ nm}}{d}$

Thus, it is seen that $\boxed{\theta_{2r} > \theta_{3v}\text{ and these orders must overlap}}$ regardless of the value of the grating spacing *d*.

***P38.25** The sound has wavelength $\lambda = \dfrac{v}{f} = \dfrac{343\ \text{m/s}}{37.2\times10^3/\text{s}} = 9.22\times10^{-3}\text{ m}$. Each diffracted beam is described

by $d\ \sin\theta = m\lambda,\quad m = 0,\ 1,\ 2,\ \ldots$

The zero-order beam is at $m = 0$, $\theta = 0$. The beams in the first order of interference are to the left and right at $\theta = \sin^{-1}\dfrac{1\lambda}{d} = \sin^{-1}\dfrac{9.22\times10^{-3}\ \text{m}}{1.3\times10^{-2}\ \text{m}} = \sin^{-1}0.709 = 45.2°$. For a second-order beam we would need $\theta = \sin^{-1}\dfrac{2\lambda}{d} = \sin^{-1}2(0.709) = \sin^{-1}1.42$. No angle, smaller or larger than 90°, has a sine greater than 1. Then a diffracted beam does not exist for the second order or any higher order. The whole answer is then, $\boxed{\text{three beams, at } 0°\text{ and at } 45.2°\text{ to the right and to the left}}$.

P38.26 For a side maximum, $\tan\theta = \dfrac{y}{L} = \dfrac{0.4\ \mu\text{m}}{6.9\ \mu\text{m}}$

$$\theta = 3.32°$$

$d\sin\theta = m\lambda$ $\qquad d = \dfrac{(1)(780\times10^{-9}\ \text{m})}{\sin 3.32°} = 13.5\ \mu\text{m}$

The number of grooves per millimeter $= \dfrac{1\times10^{-3}\ \text{m}}{13.5\times10^{-6}\ \text{m}} = \boxed{74.2}$

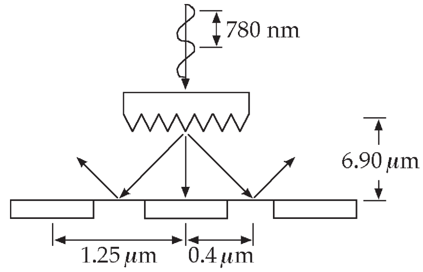


FIG. P38.26

P38.27 $d = \dfrac{1.00\times10^{-3}\ \text{m/mm}}{250\ \text{lines/mm}} = 4.00\times10^{-6}\ \text{m} = 4\ 000\ \text{nm}$ $\qquad d\sin\theta = m\lambda \Rightarrow m = \dfrac{d\sin\theta}{\lambda}$

(a) The number of times a complete order is seen is the same as the number of orders in which the long wavelength limit is visible.

$$m_{\max} = \frac{d\sin\theta_{\max}}{\lambda} = \frac{(4\ 000\ \text{nm})\sin 90.0°}{700\ \text{nm}} = 5.71$$

or $\boxed{\text{5 orders is the maximum}}$

(b) The highest order in which the violet end of the spectrum can be seen is:

$$m_{\max} = \frac{d\sin\theta_{\max}}{\lambda} = \frac{(4\ 000\ \text{nm})\sin 90.0°}{400\ \text{nm}} = 10.0$$

or $\boxed{\text{10 orders in the short-wavelength region}}$

P38.28 (a) The several narrow parallel slits make a diffraction grating. The zeroth- and first-order maxima are separated according to

$d\sin\theta = (1)\lambda$ $\qquad \sin\theta = \dfrac{\lambda}{d} = \dfrac{632.8\times10^{-9}\,\text{m}}{1.2\times10^{-3}\text{m}}$

$$\theta = \sin^{-1}(0.000\ 527) = 0.000\ 527\ \text{rad}$$

$$y = L\tan\theta = (1.40\ \text{m})(0.000\ 527) = \boxed{0.738\ \text{mm}}$$

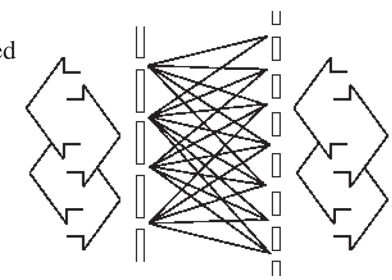


FIG. P38.28

(b) Many equally spaced transparent lines appear on the film. It is itself a diffraction grating. When the same light is sent through the film, it produces interference maxima separated according to

$d\sin\theta = (1)\lambda$ $\qquad \sin\theta = \dfrac{\lambda}{d} = \dfrac{632.8\times10^{-9}\ \text{m}}{0.738\times10^{-3}\ \text{m}} = 0.000\ 857$

$$y = L\tan\theta = (1.40\ \text{m})(0.000\ 857) = 1.20\ \text{mm}$$

An image of the original set of slits appears on the screen. If the screen is removed, light diverges from the real images with the same wave fronts reconstructed as the original slits produced. Reasoning from the mathematics of Fourier transforms, Gabor showed that light diverging from any object, not just a set of slits, could be used. In the picture, the slits or maxima on the left are separated by 1.20 mm. The slits or maxima on the right are separated by 0.738 mm. The length difference between any pair of lines is an integer number of wavelengths. Light can be sent through equally well toward the right or toward the left. Soccer players shift smoothly between offensive and defensive tactics.

P38.29 $d = \dfrac{1}{4\,200/\text{cm}} = 2.38 \times 10^{-6}\text{ m} = 2\,380\text{ nm}$

$d\sin\theta = m\lambda$ or $\theta = \sin^{-1}\left(\dfrac{m\lambda}{d}\right)$ and $y = L\tan\theta = L\tan\left[\sin^{-1}\left(\dfrac{m\lambda}{d}\right)\right]$

Thus, $$\Delta y = L\left\{\tan\left[\sin^{-1}\left(\frac{m\lambda_2}{d}\right)\right] - \tan\left[\sin^{-1}\left(\frac{m\lambda_1}{d}\right)\right]\right\}$$

For $m = 1$, $$\Delta y = (2.00\text{ m})\left\{\tan\left[\sin^{-1}\left(\frac{589.6}{2\,380}\right)\right] - \tan\left[\sin^{-1}\left(\frac{589}{2\,380}\right)\right]\right\} = 0.554\text{ mm}$$

For $m = 2$, $$\Delta y = (2.00\text{ m})\left\{\tan\left[\sin^{-1}\left(\frac{2(589.6)}{2\,380}\right)\right] - \tan\left[\sin^{-1}\left(\frac{2(589)}{2\,380}\right)\right]\right\} = 1.54\text{ mm}$$

For $m = 3$, $$\Delta y = (2.00\text{ m})\left\{\tan\left[\sin^{-1}\left(\frac{3(589.6)}{2\,380}\right)\right] - \tan\left[\sin^{-1}\left(\frac{3(589)}{2\,380}\right)\right]\right\} = 5.04\text{ mm}$$

Thus, the observed order must be $\boxed{m = 2}$.

Section 38.5 **Diffraction of X-Rays by Crystals**

P38.30 $2d\sin\theta = m\lambda$: $$\lambda = \frac{2d\sin\theta}{m} = \frac{2\left(0.353 \times 10^{-9}\text{ m}\right)\sin 7.60°}{1} = 9.34 \times 10^{-11}\text{ m} = \boxed{0.093\,4\text{ nm}}$$

P38.31 $2d\sin\theta = m\lambda$: $$\sin\theta = \frac{m\lambda}{2d} = \frac{1\left(0.140 \times 10^{-9}\text{ m}\right)}{2\left(0.281 \times 10^{-9}\text{ m}\right)} = 0.249$$

and $\boxed{\theta = 14.4°}$

P38.32 Figure 38.23 of the text shows the situation.

$2d\sin\theta = m\lambda$ or $\lambda = \dfrac{2d\sin\theta}{m}$

$m = 1$: $$\lambda_1 = \frac{2(2.80\text{ m})\sin 80.0°}{1} = \boxed{5.51\text{ m}}$$

$m = 2$: $$\lambda_2 = \frac{2(2.80\text{ m})\sin 80.0°}{2} = \boxed{2.76\text{ m}}$$

$m = 3$: $$\lambda_3 = \frac{2(2.80\text{ m})\sin 80.0°}{3} = \boxed{1.84\text{ m}}$$

***P38.33** The crystal cannot produce diffracted beams of visible light. The wavelengths of visible light are some hundreds of nanometers. There is no angle whose sine is greater than 1. Bragg's law $2d\sin\theta = m\lambda$ cannot be satisfied for a wavelength much larger than the distance between atomic planes in the crystal.

Section 38.6 **Polarization of Light Waves**

P38.34 The average value of the cosine-squared function is one-half, so the first polarizer transmits $\frac{1}{2}$ the light. The second transmits $\cos^2 30.0° = \frac{3}{4}$.

$$I_f = \frac{1}{2} \times \frac{3}{4} I_i = \boxed{\frac{3}{8} I_i}$$

P38.35 $I = I_{max} \cos^2 \theta \quad \Rightarrow \quad \theta = \cos^{-1} \sqrt{\frac{I}{I_{max}}}$

(a) $\frac{I}{I_{max}} = \frac{1}{3.00} \quad \Rightarrow \quad \theta = \cos^{-1} \sqrt{\frac{1}{3.00}} = \boxed{54.7°}$

(b) $\frac{I}{I_{max}} = \frac{1}{5.00} \quad \Rightarrow \quad \theta = \cos^{-1} \sqrt{\frac{1}{5.00}} = \boxed{63.4°}$

(c) $\frac{I}{I_{max}} = \frac{1}{10.0} \quad \Rightarrow \quad \theta = \cos^{-1} \sqrt{\frac{1}{10.0}} = \boxed{71.6°}$

P38.36 By Brewster's law, $n = \tan \theta_p = \tan(48.0°) = \boxed{1.11}$

P38.37 $\sin \theta_c = \frac{1}{n}$ or $n = \frac{1}{\sin \theta_c} = \frac{1}{\sin 34.4°} = 1.77$

Also, $\tan \theta_p = n$. Thus, $\theta_p = \tan^{-1}(n) = \tan^{-1}(1.77) = \boxed{60.5°}$

P38.38 $\sin \theta_c = \frac{1}{n}$ and $\tan \theta_p = n$

Thus, $\sin \theta_c = \frac{1}{\tan \theta_p}$ or $\boxed{\cot \theta_p = \sin \theta_c}$.

P38.39 (a) At incidence, $n_1 \sin \theta_1 = n_2 \sin \theta_2$ and $\theta_1' = \theta_1$. For complete polarization of the reflected light,

$$(90 - \theta_1') + (90 - \theta_2) = 90°$$

$$\theta_1' + \theta_2 = 90 = \theta_1 + \theta_2$$

Then $n_1 \sin \theta_1 = n_2 \sin(90 - \theta_1) = n_2 \cos \theta_1$

$$\frac{\sin \theta_1}{\cos \theta_1} = \frac{n_2}{n_1} = \tan \theta_1$$

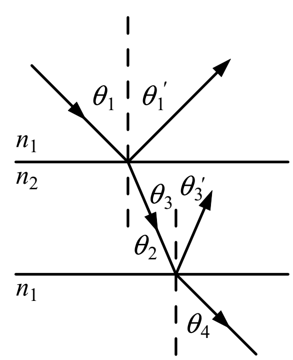


FIG. P38.39(a)

At the bottom surface, $\theta_3 = \theta_2$ because the normals to the surfaces of entry and exit are parallel.

Then $n_2 \sin \theta_3 = n_1 \sin \theta_4$ and $\theta_3' = \theta_3$

$n_2 \sin \theta_2 = n_1 \sin \theta_4$ and $\theta_4 = \theta_1$

The condition for complete polarization of the reflected light is

$90 - \theta_3' + 90 - \theta_4 = 90° \qquad \theta_2 + \theta_1 = 90°$

This is the same as the condition for θ_1 to be Brewster's angle at the top surface.

continued on next page

(b) We consider light moving in a plane perpendicular to the line where the surfaces of the prism meet at the unknown angle Φ. We require

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$\theta_1 + \theta_2 = 90°$$

So $n_1 \sin(90 - \theta_2) = n_2 \sin\theta_2 \qquad \dfrac{n_1}{n_2} = \tan\theta_2$

And $\quad n_2 \sin\theta_3 = n_3 \sin\theta_4 \qquad \theta_3 + \theta_4 = 90°$

$$n_2 \sin\theta_3 = n_3 \cos\theta_3 \qquad \tan\theta_3 = \frac{n_3}{n_2}$$

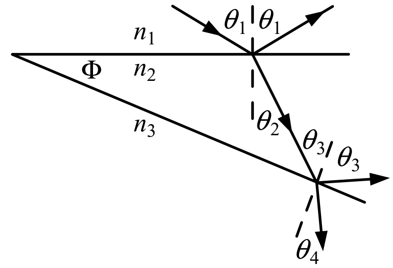


FIG. P38.39(b)

In the triangle made by the faces of the prism and the ray in the prism,

$$\Phi + 90 + \theta_2 + (90 - \theta_3) = 180$$

So one particular apex angle is required, and it is

$$\Phi = \theta_3 - \theta_2 = \boxed{\tan^{-1}\left(\frac{n_3}{n_2}\right) - \tan^{-1}\left(\frac{n_1}{n_2}\right)}$$

Here a negative result is to be interpreted as meaning the same as a positive result.

P38.40 For incident unpolarized light of intensity $I_{\max}$:

The average value of the cosine-squared function is one half, so the intensity after transmission by the first disk is

$$I = \frac{1}{2} I_{\max}$$

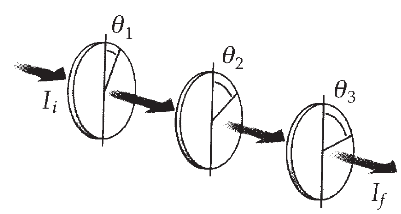


FIG. P38.40

After transmitting 2nd disk: $I = \dfrac{1}{2} I_{\max} \cos^2\theta$

After transmitting 3rd disk: $I = \dfrac{1}{2} I_{\max} \cos^2\theta \cos^2(90° - \theta)$

where the angle between the first and second disk is $\theta = \omega t$.

Using trigonometric identities $\cos^2\theta = \dfrac{1}{2}(1 + \cos 2\theta)$

and $\qquad \cos^2(90° - \theta) = \sin^2\theta = \dfrac{1}{2}(1 - \cos 2\theta)$

we have $\qquad I = \dfrac{1}{2} I_{\max} \left[\dfrac{(1+\cos 2\theta)}{2}\right]\left[\dfrac{(1-\cos 2\theta)}{2}\right]$

$$I = \frac{1}{8} I_{\max}\left(1 - \cos^2 2\theta\right) = \frac{1}{8} I_{\max}\left(\frac{1}{2}\right)(1 - \cos 4\theta)$$

Since $\theta = \omega t$, the intensity of the emerging beam is given by $\boxed{I = \dfrac{1}{16} I_{\max}(1 - 4\omega t)}$.

***P38.41** (a) Let I_0 represent the intensity of unpolarized light incident on the first polarizer. In Malus's law the average value of the cosine-squared function is 1/2, so the first filter lets through 1/2 of the incident intensity. Of the light reaching them, the second filter passes $\cos^2 45° = 1/2$ and the third filter also $\cos^2 45° = 1/2$. The transmitted intensity is then $I_0(1/2)(1/2)(1/2) = 0.125\, I_0$. The reduction in intensity is by a factor of $\boxed{0.875}$ of the incident intensity.

(b) By the same logic as in part (a) we have transmitted $I_0(1/2)(\cos^2 30°)(\cos^2 30°)(\cos^2 30°) = I_0(1/2)(\cos^2 30°)^3 = 0.211\, I_0$. Then the fraction absorbed is $\boxed{0.789}$.

(c) Yet again we compute transmission $I_0(1/2)(\cos^2 15°)^6 = 0.330\, I_0$. And the fraction absorbed is $\boxed{0.670}$.

(d) We can get more and more of the incident light through the stack of ideal filters, approaching 50%, by reducing the angle between the transmission axes of each one and the next.

Additional Problems

***P38.42** The central bright fringe is wider than the side bright fringes, so the light must have been diffracted by a $\boxed{\text{single slit}}$. For precision, we measure from the second minimum on one side of the center to the second minimum on the other side:

$$2y = (11.7 - 6.3)\text{ cm} = 5.4\text{ cm} \qquad y = 2.7\text{ cm}$$

$$\tan\theta = \frac{y}{L} = \frac{0.027\text{ m}}{2.6\text{ m}} = 0.010\ 4$$

$$\theta = 0.595° = 0.010\ 4\text{ rad}$$

$$a\sin\theta = m\lambda$$

$$a = \frac{m\lambda}{\sin\theta} = \frac{2\left(632.8\times10^{-9}\text{ m}\right)}{\sin 0.595°} = \frac{2\left(632.8\times10^{-9}\text{ m}\right)}{0.010\ 4} = \boxed{1.22\times10^{-4}\text{ m}}$$

P38.43 Let the first sheet have its axis at angle θ to the original plane of polarization, and let each further sheet have its axis turned by the same angle.

The first sheet passes intensity $\quad I_{\max}\cos^2\theta$

The second sheet passes $\quad I_{\max}\cos^4\theta$

and the nth sheet lets through $\quad I_{\max}\cos^{2n}\theta \ge 0.90 I_{\max} \quad$ where $\theta = \dfrac{45°}{n}$

Try different integers to find $\quad \cos^{2\times5}\left(\dfrac{45°}{5}\right) = 0.885 \qquad \cos^{2\times6}\left(\dfrac{45°}{6}\right) = 0.902$

(a) So $n = \boxed{6}$

(b) $\theta = \boxed{7.50°}$

P38.44 Consider vocal sound moving at 340 m/s and of frequency 3 000 Hz. Its wavelength is

$$\lambda = \frac{v}{f} = \frac{340 \text{ m/s}}{3\,000 \text{ Hz}} = 0.113 \text{ m}$$

If your mouth, for horizontal dispersion, behaves similarly to a slit 6.00 cm wide, then $a\sin\theta = m\lambda$ predicts no diffraction minima. You are a nearly isotropic source of this sound. It spreads out from you nearly equally in all directions. On the other hand, if you use a megaphone with width 60.0 cm at its wide end, then $a\sin\theta = m\lambda$ predicts the first diffraction minimum at

$$\theta = \sin^{-1}\left(\frac{m\lambda}{a}\right) = \sin^{-1}\left(\frac{0.113 \text{ m}}{0.600 \text{ m}}\right) = 10.9°$$

This suggests that the sound is radiated mostly toward the front into a diverging beam of angular diameter only about 20°. With less sound energy wasted in other directions, more is available for your intended auditors. We could check that a distant observer to the side or behind you receives less sound when a megaphone is used.

***P38.45** (a) We assume the first side maximum is at $a\sin\theta = 1.5\lambda$. (Its location is determined more precisely in problem 66.) Then the required fractional intensity is

$$\frac{I}{I_{\max}} = \left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right]^2 = \left[\frac{\sin(1.5\pi)}{1.5\pi}\right]^2 = \frac{1}{2.25\pi^2} = \boxed{0.0450}$$

(b) Proceeding as in part (a) we have $a\sin\theta/\lambda = 2.5$ and

$$\frac{I}{I_{\max}} = \left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right]^2 = \left[\frac{\sin(2.5\pi)}{2.5\pi}\right]^2 = \frac{1}{6.25\pi^2} = \boxed{0.0162}$$

***P38.46** The energy in the central maximum we can estimate in Figure 38.6 as proportional to

$$(\text{height})(\text{width}) = I_{\max}(2\pi)$$

As in problem 45, the maximum height of the first side maximum is approximately

$$I_{\max}[\sin(3\pi/2)/(3\pi/2)]^2 = 4I_{\max}/9\pi^2$$

Then the energy in one side maximum is proportional to $\pi(4I_{\max}/9\pi^2)$,

and that in both of the first side maxima together is proportional to $2\pi(4I_{\max}/9\pi^2)$.

Similarly and more precisely, and always with the same proportionality constant, the energy in both of the second side maxima is proportional to $2\pi(4I_{\max}/25\pi^2)$.

The energy in all of the side maxima together is proportional to

$$2\pi(4I_{\max}/\pi^2)[1/3^2 + 1/5^2 + 1/7^2 + \cdots] = 2\pi(4I_{\max}/\pi^2)[\pi^2/8 - 1] = (8I_{\max}/\pi)[\pi^2/8 - 1]$$

$$= I_{\max}(\pi - 8/\pi) = 0.595I_{\max}$$

The ratio of the energy in the central maximum to the total energy is then

$$I_{\max}(2\pi)/\{I_{\max}(2\pi) + 0.595I_{\max}\} = 2\pi/6.88 = 0.913 = 91.3\%$$

Our calculation is only a rough estimate, because the shape of the central maximum in particular is not just a vertically-stretched cycle of a cosine curve. It is slimmer than that.

P38.47 The first minimum is at $a\sin\theta = (1)\lambda$

This has no solution if $\dfrac{\lambda}{a} > 1$

or if $a < \lambda = \boxed{632.8\text{ nm}}$

***P38.48** With light in effect moving through vacuum, Rayleigh's criterion limits the resolution according to

$$d/L = 1.22\,\lambda/D \qquad D = 1.22\,\lambda L/d = 1.22\,(885\times10^{-9}\text{ m})\ 12\ 000\text{ m}/2.3\text{ m} = \boxed{5.63\text{ mm}}$$

The assumption is absurd. Over a horizontal path of 12 km in air, density variations associated with convection ("heat waves" or what an astronomer calls "seeing") would make the motor-cycles completely unresolvable with any optical device.

P38.49 $d = \dfrac{1}{400\text{ mm}^{-1}} = 2.50\times10^{-6}\text{ m}$

(a) $d\sin\theta = m\lambda$ $\qquad \theta_a = \sin^{-1}\left(\dfrac{2\times541\times10^{-9}\text{ m}}{2.50\times10^{-6}\text{ m}}\right) = \boxed{25.6°}$

(b) $\lambda = \dfrac{541\times10^{-9}\text{ m}}{1.33} = 4.07\times10^{-7}\text{ m}$ $\qquad \theta_b = \sin^{-1}\left(\dfrac{2\times4.07\times10^{-7}\text{ m}}{2.50\times10^{-6}\text{ m}}\right) = \boxed{19.0°}$

(c) $d\sin\theta_a = 2\lambda$ and $d\sin\theta_b = \dfrac{2\lambda}{n}$

combine by substitution to give $\qquad n\sin\theta_b = (1)\sin\theta_a$

P38.50 (a) $\lambda = \dfrac{v}{f}$: $\qquad \lambda = \dfrac{3.00\times10^{8}\text{ m/s}}{1.40\times10^{9}\text{ s}^{-1}} = 0.214\text{ m}$

$\theta_{\min} = 1.22\dfrac{\lambda}{D}$: $\qquad \theta_{\min} = 1.22\left(\dfrac{0.214\text{ m}}{3.60\times10^{4}\text{ m}}\right) = \boxed{7.26\ \mu\text{rad}}$

$\theta_{\min} = 7.26\ \mu\text{rad}\left(\dfrac{180\times60\times60\text{ s}}{\pi}\right) = \boxed{1.50\text{ arc seconds}}$

(b) $\theta_{\min} = \dfrac{d}{L}$: $\qquad d = \theta_{\min}L = (7.26\times10^{-6}\text{ rad})(26\ 000\text{ ly}) = \boxed{0.189\text{ ly}}$

(c) It is not true for humans, but we assume the hawk's visual acuity is limited only by Rayleigh's criterion.

$\theta_{\min} = 1.22\dfrac{\lambda}{D}$ $\quad \theta_{\min} = 1.22\left(\dfrac{500\times10^{-9}\text{ m}}{12.0\times10^{-3}\text{ m}}\right) = \boxed{50.8\ \mu\text{rad}}$ (10.5 seconds of arc)

(d) $d = \theta_{\min}L = (50.8\times10^{-6}\text{ rad})(30.0\text{ m}) = 1.52\times10^{-3}\text{ m} = \boxed{1.52\text{ mm}}$

P38.51 With a grazing angle of 36.0°, the angle of incidence is 54.0°

$\tan\theta_p = n = \tan 54.0° = 1.38$

In the liquid, $\lambda_n = \frac{\lambda}{n} = \frac{750 \text{ nm}}{1.38} = \boxed{545 \text{ nm}}$.

P38.52 (a) Bragg's law applies to the space lattice of melanin rods. Consider the planes $d = 0.25\ \mu\text{m}$ apart. For light at near-normal incidence, strong reflection happens for the wavelength given by $2d\sin\theta = m\lambda$. The longest wavelength reflected strongly corresponds to $m = 1$:

$2(0.25\times10^{-6}\text{ m})\sin 90° = 1\lambda$ $\lambda = 500$ nm. This is the blue-green color.

(b) For light incident at grazing angle 60°, $2d\sin\theta = m\lambda$ gives $1\lambda = 2(0.25\times10^{-6}\text{ m})\sin 60° = 433$ nm. This is violet.

(c) Your two eyes receive light reflected from the feather at different angles, so they receive light incident at different angles and containing different colors reinforced by constructive interference.

(d) The longest wavelength that can be reflected with extra strength by these melanin rods is the one we computed first, 500 nm blue-green.

(e) If the melanin rods were farther apart (say $0.32\ \mu\text{m}$) they could reflect red with constructive interference.

P38.53 (a) $d\sin\theta = m\lambda$

or $d = \frac{m\lambda}{\sin\theta} = \frac{3(500\times10^{-9}\text{ m})}{\sin 32.0°} = 2.83\ \mu\text{m}$

Therefore, lines per unit length $= \frac{1}{d} = \frac{1}{2.83\times10^{-6}\text{ m}}$

or lines per unit length $= 3.53\times10^{5}\text{ m}^{-1} = \boxed{3.53\times10^{3}\text{ cm}^{-1}}$.

(b) $\sin\theta = \frac{m\lambda}{d} = \frac{m(500\times10^{-9}\text{ m})}{2.83\times10^{-6}\text{ m}} = m(0.177)$

For $\sin\theta \le 1.00$, we must have $m(0.177) \le 1.00$

or $m \le 5.66$

Therefore, the highest order observed is $m = 5$

Total number of primary maxima observed is $2m + 1 = \boxed{11}$

***P38.54** (a) For the air-to-water interface,

$$\tan\theta_p = \frac{n_{\text{water}}}{n_{\text{air}}} = \frac{1.33}{1.00} \qquad \theta_p = 53.1°$$

and $(1.00)\sin\theta_p = (1.33)\sin\theta_2$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 53.1°}{1.33}\right) = 36.9°$$

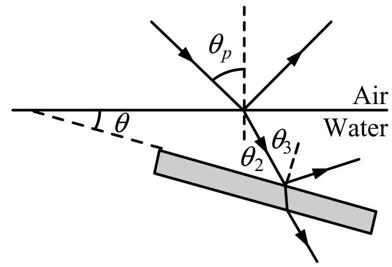


FIG. P38.54

For the water-to-slab interface, $\tan\theta_p = \tan\theta_3 = \frac{n_{\text{slab}}}{n_{\text{water}}} = \frac{n}{1.33}$

so $\theta_3 = \tan^{-1}(n/1.33)$.

The angle between surfaces is $\theta = \theta_3 - \theta_2 = \tan^{-1}(n/1.33) - 36.9°$.

(b) If we imagine $n \to \infty$, then $\theta \to 53.1°$. The material of the slab in this case is higher in optical density than any gem. The light in the water skims along the upper surface of the slab.

(c) If we imagine $n = 1$, then $\theta = 0$. The slab is so low in optical density that it is like air. The light strikes parallel surfaces as it enters and exits the water, both at the polarizing angle.

P38.55 A central maximum and side maxima in seven orders of interference appear. If the seventh order is just at 90°,

$$d\sin\theta = m\lambda \qquad d(1) = 7\left(654\times10^{-9}\text{ m}\right) \qquad d = 4.58\ \mu\text{m}$$

If the seventh order is at less than 90°, the eighth order might be nearly ready to appear according to

$$d(1) = 8\left(654\times10^{-9}\text{ m}\right) \qquad d = 5.23\ \mu\text{m}$$

Thus $\boxed{4.58\ \mu\text{m} < d < 5.23\ \mu\text{m}}$.

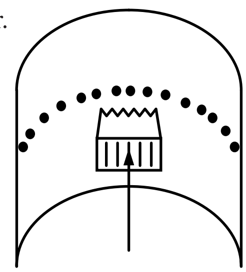


FIG. P38.55

P38.56 (a) We require $\theta_{\min} = 1.22\frac{\lambda}{D} = \frac{\text{radius of diffraction disk}}{L} = \frac{D}{2L}$

Then $\boxed{D^2 = 2.44\lambda L}$

(b) $D = \sqrt{2.44\left(500\times10^{-9}\text{ m}\right)(0.150\text{ m})} = \boxed{428\ \mu\text{m}}$

P38.57 For the limiting angle of resolution between lines we assume $\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\frac{\left(550\times10^{-9}\text{ m}\right)}{\left(5.00\times10^{-3}\text{ m}\right)} = 1.34\times10^{-4}\text{ rad}$.

Assuming a picture screen with vertical dimension ℓ, the minimum viewing distance for no visible lines is found from $\theta_{\min} = \frac{\ell/485}{L}$. The desired ratio is then

$$\frac{L}{\ell} = \frac{1}{485\theta_{\min}} = \frac{1}{485\left(1.34\times10^{-4}\text{ rad}\right)} = \boxed{15.4}$$

When the pupil of a human eye is wide open, its actual resolving power is significantly poorer than Rayleigh's criterion suggests.

P38.58 (a) Applying Snell's law gives $n_2 \sin\phi = n_1 \sin\theta$. From the sketch, we also see that:

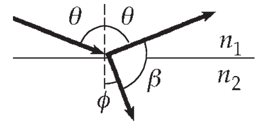


FIG. P38.58(a)

$$\theta + \phi + \beta = \pi, \text{ or } \phi = \pi - (\theta + \beta)$$

Using the given identity:

$$\sin\phi = \sin\pi\cos(\theta+\beta) - \cos\pi\sin(\theta+\beta)$$

which reduces to: $\sin\phi = \sin(\theta+\beta)$

Applying the identity again: $\sin\phi = \sin\theta\cos\beta + \cos\theta\sin\beta$

Snell's law then becomes: $n_2(\sin\theta\cos\beta + \cos\theta\sin\beta) = n_1 \sin\theta$

or (after dividing by $\cos\theta$): $n_2(\tan\theta\cos\beta + \sin\beta) = n_1 \tan\theta$

Solving for $\tan\theta$ gives: $\boxed{\tan\theta = \dfrac{n_2 \sin\beta}{n_1 - n_2\cos\beta}}$

(b) If $\beta = 90.0°$, $n_1 = 1.00$, and $n_2 = n$, the above result becomes:

$\tan\theta = \dfrac{n(1.00)}{1.00 - 0}$, or $n = \tan\theta$, which is Brewster's law.

P38.59 (a) From Equation 38.1, $\theta = \sin^{-1}\left(\dfrac{m\lambda}{a}\right)$

In this case $m = 1$ and $\lambda = \dfrac{c}{f} = \dfrac{3.00\times 10^{8}\ \text{m/s}}{7.50\times 10^{9}\ \text{Hz}} = 4.00\times 10^{-2}\ \text{m}$

Thus, $\theta = \sin^{-1}\left(\dfrac{4.00\times 10^{-2}\ \text{m}}{6.00\times 10^{-2}\ \text{m}}\right) = \boxed{41.8°}$

(b) From Equation 38.2, $\dfrac{I}{I_{\max}} = \left[\dfrac{\sin(\phi)}{\phi}\right]^2$ where $\phi = \dfrac{\pi a \sin\theta}{\lambda}$

When $\theta = 15.0°$, $\phi = \dfrac{\pi(0.060\ 0\ \text{m})\sin 15.0°}{0.040\ 0\ \text{m}} = 1.22\ \text{rad}$

and $\dfrac{I}{I_{\max}} = \left[\dfrac{\sin(1.22\ \text{rad})}{1.22\ \text{rad}}\right]^2 = \boxed{0.593}$

(c) $\sin\theta = \dfrac{\lambda}{a}$ so $\theta = 41.8°$:

This is the minimum angle subtended by the two sources at the slit. Let α be the half angle between the sources, each a distance $\ell = 0.100$ m from the center line and a distance L from the slit plane. Then,

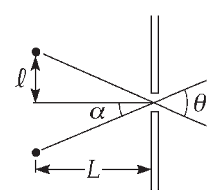


FIG. P38.59(c)

$$L = \ell\cot\alpha = (0.100\ \text{m})\cot\left(\frac{41.8°}{2}\right) = \boxed{0.262\ \text{m}}$$

***P38.60** (a) The first sheet transmits one-half the intensity of the originally unpolarized light, because the average value of the cosine-squared function in Malus's law is one-half. Then

$$\frac{I}{I_{max}} = \frac{1}{2}\left(\cos^2 45.0°\right) \quad \left(\cos^2 45.0°\right) = \boxed{\frac{1}{8}}$$

(b) No recipes remain. The two experiments follow precisely analogous steps, but the results are different. The middle filter in part (a) changes the polarization state of the light that passes through it, but the recipe selections do not change individual recipes. The result for light gives us a glimpse of how quantum-mechanical measurements differ from classical measurements.

P38.61 (a) Constructive interference of light of wavelength λ on the screen is described by

$d\sin\theta = m\lambda$ where $\tan\theta = \frac{y}{L}$ so $\sin\theta = \frac{y}{\sqrt{L^2+y^2}}$. Then $(d)y\left(L^2+y^2\right)^{-1/2} = m\lambda$.

Differentiating with respect to y gives

$$d1\left(L^2+y^2\right)^{-1/2} + (d)y\left(-\frac{1}{2}\right)\left(L^2+y^2\right)^{-3/2}(0+2y) = m\frac{d\lambda}{dy}$$

$$\frac{d}{\left(L^2+y^2\right)^{1/2}} - \frac{(d)y^2}{\left(L^2+y^2\right)^{3/2}} = m\frac{d\lambda}{dy} = \frac{(d)L^2+(d)y^2-(d)y^2}{\left(L^2+y^2\right)^{3/2}}$$

$$\frac{d\lambda}{dy} = \frac{(d)L^2}{m\left(L^2+y^2\right)^{3/2}}$$

(b) Here $d\sin\theta = m\lambda$ gives $\frac{10^{-2}\text{ m}}{8\,000}\sin\theta = 1\left(550\times10^{-9}\text{ m}\right)$, $\theta = \sin^{-1}\left(\frac{0.55\times10^{-6}\text{ m}}{1.25\times10^{-6}\text{ m}}\right) = 26.1°$

$$y = L\tan\theta = 2.40\text{ m}\tan 26.1° = 1.18\text{ m}$$

Now $\frac{d\lambda}{dy} = \frac{dL^2}{m\left(L^2+y^2\right)^{3/2}} = \frac{1.25\times10^{-6}\text{ m}(2.40\text{ m})^2}{1\left((2.4\text{ m})^2+(1.18\text{ m})^2\right)^{3/2}} = 3.77\times10^{-7} = \boxed{3.77\text{ nm/cm}}$.

P38.62 (a) The angles of bright beams diffracted from the grating are given by $(d)\sin\theta = m\lambda$. The angular dispersion is defined as the derivative $\frac{d\theta}{d\lambda}$: $\quad (d)\cos\theta\frac{d\theta}{d\lambda} = m \quad \frac{d\theta}{d\lambda} = \frac{m}{d\cos\theta}$

(b) For the average wavelength 578 nm,

$$d\sin\theta = m\lambda \qquad \frac{0.02\text{ m}}{8\,000}\sin\theta = 2\left(578\times10^{-9}\text{ m}\right)$$

$$\theta = \sin^{-1}\frac{2\times578\times10^{-9}\text{ m}}{2.5\times10^{-6}\text{ m}} = 27.5°$$

The separation angle between the lines is

$$\Delta\theta = \frac{d\theta}{d\lambda}\Delta\lambda = \frac{m}{d\cos\theta}\Delta\lambda = \frac{2}{2.5\times10^{-6}\text{ m}\cos 27.5°}2.11\times10^{-9}\text{ m}$$

$$= 0.001\,90 = 0.001\,90\text{ rad} = 0.001\,90\text{ rad}\left(\frac{180°}{\pi\text{ rad}}\right) = \boxed{0.109°}$$

P38.63 (a) The E and O rays, in phase at the surface of the plate, will have a phase difference

$$\theta = \left(\frac{2\pi}{\lambda}\right)\delta$$

after traveling distance d through the plate. Here δ is the difference in the *optical path* lengths of these rays. The optical path length between two points is the product of the actual path length d and the index of refraction. Therefore,

$$\delta = \left|dn_O - dn_E\right|$$

The absolute value is used since $\frac{n_O}{n_E}$ may be more or less than unity. Therefore,

$$\theta = \left(\frac{2\pi}{\lambda}\right)\left|dn_O - dn_E\right| = \boxed{\left(\frac{2\pi}{\lambda}\right)d\left|n_O - n_E\right|}$$

(b) $$d = \frac{\lambda\theta}{2\pi\left|n_O - n_E\right|} = \frac{\left(550\times10^{-9}\text{ m}\right)(\pi/2)}{2\pi\left|1.544 - 1.553\right|} = 1.53\times10^{-5}\text{ m} = \boxed{15.3\ \mu\text{m}}$$

P38.64 (a) From Equation 38.2, $\frac{I}{I_{\max}} = \left[\frac{\sin(\phi)}{\phi}\right]^2$

where we define $\phi \equiv \frac{\pi a\sin\theta}{\lambda}$

Therefore, when $\frac{I}{I_{\max}} = \frac{1}{2}$ we must have $\frac{\sin\phi}{\phi} = \frac{1}{\sqrt{2}}$, or $\boxed{\sin\phi = \frac{\phi}{\sqrt{2}}}$

(b) Let $y_1 = \sin\phi$ and $y_2 = \frac{\phi}{\sqrt{2}}$.

A plot of y_1 and y_2 in the range $1.00 \le \phi \le \frac{\pi}{2}$ is shown to the right.

The solution to the transcendental equation is found to be $\boxed{\phi = 1.39\text{ rad}}$.

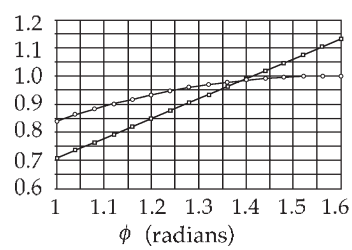


FIG. P38.64(b)

(c) $\frac{\pi a\sin\theta}{\lambda} = \phi$ gives $\sin\theta = \left(\frac{\phi}{\pi}\right)\frac{\lambda}{a} = 0.443\frac{\lambda}{a}$.

If $\frac{\lambda}{a}$ is small, then $\theta \approx 0.443\frac{\lambda}{a}$.

This gives the half-width, measured away from the maximum at $\theta = 0$. The pattern is symmetric, so the full width is given by

$$\Delta\theta = 0.443\frac{\lambda}{a} - \left(-0.443\frac{\lambda}{a}\right) = \boxed{\frac{0.886\lambda}{a}}$$

P38.65

ϕ	$\sqrt{2}\sin\phi$	
1	1.19	bigger than ϕ
2	1.29	smaller than ϕ
1.5	1.41	smaller
1.4	1.394	
1.39	1.391	bigger
1.395	1.392	
1.392	1.391 7	smaller
1.391 5	1.391 54	bigger
1.391 52	1.391 55	bigger
1.391 6	1.391 568	smaller
1.391 58	1.391 563	
1.391 57	1.391 561	
1.391 56	1.391 558	
1.391 559	1.391 557 8	
1.391 558	1.391 557 5	
1.391 557	1.391 557 3	
1.391 557 4	1.391 557 4	

We get the answer as 1.391 557 4 to seven digits after 17 steps. Clever guessing, like using the value of $\sqrt{2}\sin\phi$ as the next guess for ϕ, could reduce this to around 13 steps.

P38.66 In $I = I_{\max}\left[\frac{\sin(\phi)}{\phi}\right]^2$ we find $\frac{dI}{d\phi} = I_{\max}\left(\frac{2\sin(\phi)}{\phi}\right)\left[\frac{(\phi)\cos(\phi)-\sin(\phi)}{(\phi)^2}\right]$

and require that it be zero. The possibility $\sin(\phi) = 0$ locates all of the minima and the central maximum, according to

$\phi = 0,\ \pi,\ 2\pi,\ \ldots;$ $\quad\phi = \frac{\pi a\sin\theta}{\lambda} = 0,\ \pi,\ 2\pi,\ \ldots;\ a\sin\theta = 0,\ \lambda,\ 2\lambda,\ \ldots$

The side maxima are found from $\quad\phi\cos(\phi) - \sin(\phi) = 0$, or $\tan(\phi) = \phi$

This has solutions $\quad\boxed{\phi = 4.493\,4}$, $\boxed{\phi = 7.725\,3}$, and others, giving

(a) $\quad\pi a\sin\theta = 4.493\,4\lambda \qquad \boxed{a\sin\theta = 1.430\,3\lambda}$

(b) $\quad\pi a\sin\theta = 7.725\,3\lambda \qquad \boxed{a\sin\theta = 2.459\,0\lambda}$

P38.67 The first minimum in the single-slit diffraction pattern occurs at

$$\sin\theta = \frac{\lambda}{a} \approx \frac{y_{\min}}{L}$$

Thus, the slit width is given by

$$a = \frac{\lambda L}{y_{\min}}$$

For a minimum located at $y_{\min} = 6.36\text{ mm} \pm 0.08\text{ mm}$, the width is

$$a = \frac{(632.8\times10^{-9}\text{ m})(1.00\text{ m})}{6.36\times10^{-3}\text{ m}} = \boxed{99.5\ \mu\text{m} \pm 1\%}$$

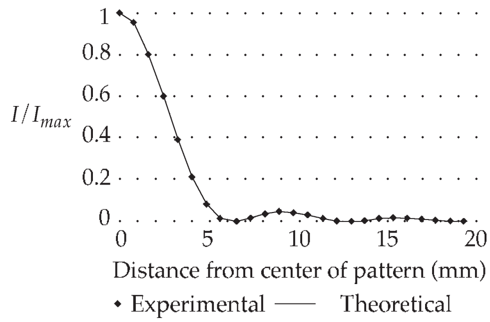


FIG. P38.67

ANSWERS TO EVEN PROBLEMS

P38.2 547 nm

P38.4 91.2 cm

P38.6 (a) 51.8 μm wide and 949 μm high (b) horizontal; vertical. See the solution. A smaller distance between aperture edges causes a wider diffraction angle. The longer dimension of each rectangle is 18.3 times larger than the smaller dimension.

P38.8 See the solution.

P38.10 (a) 0°, 10.3°, 21.0°, 32.5°, 45.8°, 63.6° (b) nine bright fringes, at 0° and on either side at 10.3°, 21.0°, 32.5°, and 63.6° (c) 1.00, 0.811, 0.405, 0.090 1, 0.032 4

P38.12 (a) 79.8 μrad (b) violet, 54.2 μrad (c) The resolving power is improved, with the minimum resolvable angle becoming 60.0 μrad.

P38.14 Between 186 m and 271 m; blue is resolvable at larger distances because it has shorter wavelength.

P38.16 30.5 m

P38.18 105 m

P38.20 514 nm

P38.22 1.81 μm

P38.24 See the solution.

P38.26 74.2 grooves/mm

P38.28 (a) 0.738 mm (b) See the solution.

P38.30 93.4 pm

P38.32 5.51 m, 2.76 m, 1.84 m

P38.34 $\frac{3}{8}$

P38.36 1.11

P38.38 See the solution.

P38.40 See the solution.

P38.42 One slit 0.122 mm wide. The central maximum is twice as wide as the other maxima.

P38.44 See the solution.

P38.46 See the solution.

P38.48 5.63 mm. The assumption is absurd. Over a horizontal path of 12 km in air, density variations associated with convection ("heat waves" or what an astronomer calls "seeing") would make the motorcycles completely unresolvable with any optical device.

P38.50 (a) 1.50 sec (b) 0.189 ly (c) 10.5 sec (d) 1.52 mm

P38.52 See the solution.

P38.54 (a) $\theta = \tan^{-1}(n/1.33) - 36.9°$ (b) If we imagine $n \to \infty$, then $\theta \to 53.1°$. The material of the slab in this case is higher in optical density than any gem. The light in the water skims along the upper surface of the slab. (c) If we imagine $n = 1$, then $\theta = 0$. The slab is so low in optical density that it is like air. The light strikes parallel surfaces as it enters and exits the water, both at the polarizing angle.

P38.56 (a) See the solution. (b) 428 μm

P38.58 See the solution.

P38.60 (a) 1/8 (b) No recipes remain. The two experiments follow precisely analogous steps, but the results are different. The middle filter in part (a) changes the polarization state of the light that passes through it, but the recipe selection processes do not change individual recipes.

P38.62 (a) See the solution. (b) 0.109°

P38.64 (a) See the solution. (b) and (c) See the solution.

P38.66 (a) $a \sin\theta = 1.430\,3\lambda$ (b) $a \sin\theta = 2.459\,0\lambda$

39

Relativity

Note: In chapters 39, 40, and 41 we use u to represent the speed of a particle with mass, reserving v for the speeds associated with reference frames, wave functions, and photons.

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q39.1 No. The principle of relativity implies that nothing can travel faster than the speed of light in a vacuum, which is 300 Mm/s. The electron would emit light in a conical shock wave of Cerenkov radiation.

***Q39.2** Answer (c). The dimension parallel to the direction of motion is reduced by the factor γ and the other dimensions are unchanged.

***Q39.3** Answer (c). An oblate spheroid. The dimension in the direction of motion would be measured to be scrunched in.

***Q39.4** Answer (e). The relativistic time dilation effect is symmetric between the observers.

Q39.5 Suppose a railroad train is moving past you. One way to measure its length is this: You mark the tracks at the cowcatcher forming the front of the moving engine at 9:00:00 AM, while your assistant marks the tracks at the back of the caboose at the same time. Then you find the distance between the marks on the tracks with a tape measure. You and your assistant must make the marks simultaneously in your frame of reference, for otherwise the motion of the train would make its length different from the distance between marks.

***Q39.6** (i) Answer (c). The Earth observer measures the clock in orbit to run slower.

(ii) Answer (b). They are not synchronized. They both tick at the same rate after return, but a time difference has developed between the two clocks.

Q39.7 (a) Yours does.

(b) His does.

(c) If the velocity of relative motion is constant, both observers have equally valid views.

Q39.8 Get a *Mr. Tompkins* book by George Gamow for a wonderful fictional exploration of this question. Driving home in a hurry, you push on the gas pedal not to increase your speed by very much, but rather to make the blocks get shorter. Big Doppler shifts in wave frequencies make red lights look green as you approach them and make car horns and car radios useless. High-speed transportation is very expensive, requiring huge fuel purchases. And it is dangerous, as a speeding car can knock down a building. Having had breakfast at home, you return hungry for lunch, but you find you have missed dinner. There is a five-day delay in transmission when you watch the Olympics in Australia on live television. It takes ninety-five years for sunlight to reach Earth. We cannot see the Milky Way; the fireball of the Big Bang surrounds us at the distance of Rigel or Deneb.

Q39.9 By a curved line. This can be seen in the middle of Speedo's world-line in Figure 39.11, where he turns around and begins his trip home.

Q39.10 A microwave pulse is reflected from a moving object. The waves that are reflected back are Doppler shifted in frequency according to the speed of the target. The receiver in the radar gun detects the reflected wave and compares its frequency to that of the emitted pulse. Using the frequency shift, the speed can be calculated to high precision. Be forewarned: this technique works if you are either traveling toward or away from your local law enforcement agent!

Q39.11 This system would be seen as a star moving in an elliptical path. Just like the light from a star in a binary star system, the spectrum of light from the star would undergo a series of Doppler shifts depending on the star's speed and direction of motion relative to the observer. The repetition rate of the Doppler shift pattern is the period of the orbit. Information about the orbit size can be calculated from the size of the Doppler shifts.

Q39.12 According to $\vec{\mathbf{p}} = \gamma m\vec{\mathbf{u}}$, doubling the speed u will make the momentum of an object increase by the factor $2\left[\dfrac{c^2 - u^2}{c^2 - 4u^2}\right]^{1/2}$.

***Q39.13** From $E^2 = p^2c^2 + m^2c^4 = (mc^2 + K)^2$ we consider $pc = \sqrt{2Kmc^2 + K^2}$. For a photon, $pc = E$. For particles with mass, the greater the mass the greater the momentum if K is always 1 MeV. The ranking is d > b > c > a.

Q39.14 As the object approaches the speed of light, its kinetic energy grows without limit. It would take an infinite investment of work to accelerate the object to the speed of light.

***Q39.15** (i) Answer (a).

(ii) (c) and (iii) (d). There is no upper limit on the momentum or energy of an electron. As more energy E is fed into the object without limit, its speed approaches the speed of light and its momentum approaches $\dfrac{E}{c}$.

***Q39.16** Answer (b). Quasar light moves at three hundred million meters per second, just like the light from a firefly at rest.

Q39.17 Any physical theory must agree with experimental measurements within some domain. Newtonian mechanics agrees with experiment for objects moving slowly compared to the speed of light. Relativistic mechanics agrees with experiment for objects at all speeds. Thus the two theories must and do agree with each other for ordinary nonrelativistic objects. Both statements given in the question are formally correct, but the first is clumsily phrased. It seems to suggest that relativistic mechanics applies only to fast-moving objects.

Q39.18 The point of intersection moves to the right. To state the problem precisely, let us assume that each of the two cards moves toward the other parallel to the long dimension of the picture, with velocity of magnitude u. The point of intersection moves to the right at speed $\frac{2u}{\tan\phi} = 2u\cot\phi$, where ϕ is the small angle between the cards. As ϕ approaches zero, $\cot\phi$ approaches infinity. Thus the point of intersection can move with a speed faster than c if v is sufficiently large and ϕ sufficiently small. For example, take $u = 500\ \text{m/s}$ and $\phi = 0.000\,19°$. If you are worried about holding the cards steady enough to be sure of the angle, cut the edge of one card along a curve so that the angle will necessarily be sufficiently small at some place along the edge.

Let us assume the spinning flashlight is at the center of a grain elevator, forming a circular screen of radius R. The linear speed of the spot on the screen is given by $v = \omega R$, where ω is the angular speed of rotation of the flashlight. With sufficiently large ω and R, the speed of the spot moving on the screen can exceed c.

Neither of these examples violates the principle of relativity. Both cases are describing a point of intersection: in the first case, the intersection of two cards and in the second case, the intersection of a light beam with a screen. A point of intersection is not made of matter so it has no mass, and hence no energy. A bug momentarily at the intersection point could yelp, take a bite out of one card, or reflect the light. None of these actions would result in communication reaching another bug so soon as the intersection point reaches him. The second bug would have to wait for sound or light to travel across the distance between the first bug and himself, to get the message.

As a child, the author used an Erector set to build a superluminal speed generator using the intersecting-cards method. Can you get a visible dot to run across a computer screen faster than light? Want'a see it again?

Q39.19 Special relativity describes inertial reference frames: that is, reference frames that are not accelerating. General relativity describes all reference frames.

Q39.20 The downstairs clock runs more slowly because it is closer to the Earth and hence in a stronger gravitational field than the upstairs clock.

SOLUTIONS TO PROBLEMS

Section 39.1 **The Principle of Galilean Relativity**

P39.1 The first observer watches some object accelerate under applied forces. Call the instantaneous velocity of the object $\vec{\mathbf{v}}_1$. The second observer has constant velocity $\vec{\mathbf{v}}_{21}$ relative to the first, and measures the object to have velocity $\vec{\mathbf{v}}_2 = \vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_{21}$.

The second observer measures an acceleration of $\quad \vec{\mathbf{a}}_2 = \frac{d\vec{\mathbf{v}}_2}{dt} = \frac{d\vec{\mathbf{v}}_1}{dt}$.

This is the same as that measured by the first observer. In this nonrelativistic case, they measure the same forces as well. Thus, the second observer also confirms that $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$.

P39.2 The laboratory observer notes Newton's second law to hold: $\vec{\mathbf{F}}_1 = m\vec{\mathbf{a}}_1$
(where the subscript 1 denotes the measurement was made in the laboratory frame of reference). The observer in the accelerating frame measures the acceleration of the mass as $\vec{\mathbf{a}}_2 = \vec{\mathbf{a}}_1 - \vec{\mathbf{a}}'$ (where the subscript 2 implies the measurement was made in the accelerating frame of reference, and the primed acceleration term is the acceleration of the accelerated frame with respect to the laboratory frame of reference). If Newton's second law held for the accelerating frame, that observer would then find valid the relation $\vec{\mathbf{F}}_2 = m\vec{\mathbf{a}}_2$ or $\vec{\mathbf{F}}_1 = m\vec{\mathbf{a}}_2$

(since $\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_2$ and the mass is unchanged in each). But, instead, the accelerating frame observer will find that $\vec{\mathbf{F}}_2 = m\vec{\mathbf{a}}_2 - m\vec{\mathbf{a}}'$, which is *not* Newton's second law.

P39.3 In the rest frame,

$$p_i = m_1 v_{1i} + m_2 v_{2i} = (2\,000\text{ kg})(20.0\text{ m/s}) + (1\,500\text{ kg})(0\text{ m/s}) = 4.00 \times 10^4\text{ kg}\cdot\text{m/s}$$

$$p_f = (m_1 + m_2) v_f = (2\,000\text{ kg} + 1\,500\text{ kg}) v_f$$

Since $p_i = p_f$, $\quad v_f = \dfrac{4.00 \times 10^4\text{ kg}\cdot\text{m/s}}{2\,000\text{ kg} + 1\,500\text{ kg}} = 11.429\text{ m/s}$

In the moving frame, these velocities are all reduced by +10.0 m/s.

$$v'_{1i} = v_{1i} - v' = 20.0\text{ m/s} - (+10.0\text{ m/s}) = 10.0\text{ m/s}$$

$$v'_{2i} = v_{2i} - v' = 0\text{ m/s} - (+10.0\text{ m/s}) = -10.0\text{ m/s}$$

$$v'_f = 11.429\text{ m/s} - (+10.0\text{ m/s}) = 1.429\text{ m/s}$$

Our initial momentum is then

$$p'_i = m_1 v'_{1i} + m_2 v'_{2i} = (2\,000\text{ kg})(10.0\text{ m/s}) + (1\,500\text{ kg})(-10.0\text{ m/s}) = 5\,000\text{ kg}\cdot\text{m/s}$$

and our final momentum has the same value:

$$p'_f = (2\,000\text{ kg} + 1\,500\text{ kg}) v'_f = (3\,500\text{ kg})(1.429\text{ m/s}) = 5\,000\text{ kg}\cdot\text{m/s}$$

Section 39.2 **The Michelson-Morley Experiment**

Section 39.3 **Einstein's Principle of Relativity**

Section 39.4 **Consequences of the Special Theory of Relativity**

P39.4 $L = L_p\sqrt{1 - \dfrac{v^2}{c^2}}$ $\qquad v = c\sqrt{1 - \left(\dfrac{L}{L_p}\right)^2}$

Taking $L = \dfrac{L_p}{2}$ where $L_p = 1.00\text{ m}$ gives $v = c\sqrt{1 - \left(\dfrac{L_p/2}{L_p}\right)^2} = c\sqrt{1 - \dfrac{1}{4}} = \boxed{0.866c}$

P39.5 $\Delta t = \dfrac{\Delta t_p}{\left[1 - (v/c)^2\right]^{1/2}}$ $\qquad$ so $\qquad v = c\left[1 - \left(\dfrac{\Delta t_p}{\Delta t}\right)^2\right]^{1/2}$

For $\Delta t = 2\Delta t_p$ $\qquad v = c\left[1 - \left(\dfrac{\Delta t_p}{2\Delta t_p}\right)^2\right]^{1/2} = c\left[1 - \dfrac{1}{4}\right]^{1/2} = \boxed{0.866c}$

P39.6 (a) $\gamma = \dfrac{1}{\sqrt{1 - (v/c)^2}} = \dfrac{1}{\sqrt{1 - (0.500)^2}} = \dfrac{2}{\sqrt{3}}$

The time interval between pulses as measured by the Earth observer is

$$\Delta t = \gamma \Delta t_p = \frac{2}{\sqrt{3}}\left(\frac{60.0\text{ s}}{75.0}\right) = 0.924\text{ s}$$

Thus, the Earth observer records a pulse rate of $\dfrac{60.0\text{ s/min}}{0.924\text{ s}} = \boxed{64.9/\text{min}}$.

(b) At a relative speed $v = 0.990c$, the relativistic factor γ increases to 7.09 and the pulse rate recorded by the Earth observer decreases to $\boxed{10.6/\text{min}}$. That is, the life span of the astronaut (reckoned by the duration of the total number of his heartbeats) is much longer as measured by an Earth clock than by a clock aboard the space vehicle.

P39.7 $\Delta t = \gamma \Delta t_p = \dfrac{\Delta t_p}{\sqrt{1 - v^2/c^2}}$ so $\Delta t_p = \left(\sqrt{1 - \dfrac{v^2}{c^2}}\right)\Delta t \cong \left(1 - \dfrac{v^2}{2c^2}\right)\Delta t$

and $\Delta t - \Delta t_p = \left(\dfrac{v^2}{2c^2}\right)\Delta t$

If $v = 1\,000 \text{ km/h} = \dfrac{1.00 \times 10^6 \text{ m}}{3\,600 \text{ s}} = 277.8 \text{ m/s}$

then $\dfrac{v}{c} = 9.26 \times 10^{-7}$

and $(\Delta t - \Delta t_p) = (4.28 \times 10^{-13})(3\,600 \text{ s}) = 1.54 \times 10^{-9} \text{ s} = \boxed{1.54 \text{ ns}}$

P39.8 For $\dfrac{v}{c} = 0.990$, $\gamma = 7.09$

(a) The muon's lifetime as measured in the Earth's rest frame is

$$\Delta t = \frac{4.60 \text{ km}}{0.990c}$$

and the lifetime measured in the muon's rest frame is

$$\Delta t_p = \frac{\Delta t}{\gamma} = \frac{1}{7.09}\left[\frac{4.60 \times 10^3 \text{ m}}{0.990(3.00 \times 10^8 \text{ m/s})}\right] = \boxed{2.18\ \mu\text{s}}$$

(b) $L = L_p\sqrt{1 - \left(\dfrac{v}{c}\right)^2} = \dfrac{L_p}{\gamma} = \dfrac{4.60 \times 10^3 \text{ m}}{7.09} = \boxed{649 \text{ m}}$

P39.9 The spaceship is measured by the Earth observer to be length-contracted to

$L = L_p\sqrt{1 - \dfrac{v^2}{c^2}}$ or $L^2 = L_p^2\left(1 - \dfrac{v^2}{c^2}\right)$

Also, the contracted length is related to the time required to pass overhead by:

$L = vt$ or $L^2 = v^2t^2 = \dfrac{v^2}{c^2}(ct)^2$

Equating these two expressions gives $L_p^2 - L_p^2\dfrac{v^2}{c^2} = (ct)^2\dfrac{v^2}{c^2}$

or $\left[L_p^2 + (ct)^2\right]\dfrac{v^2}{c^2} = L_p^2$

Using the given values: $L_p = 300 \text{ m}$ and $t = 7.50 \times 10^{-7} \text{ s}$

this becomes $(1.41 \times 10^5 \text{ m}^2)\dfrac{v^2}{c^2} = 9.00 \times 10^4 \text{ m}^2$

giving $v = \boxed{0.800c}$

P39.10 (a) The spaceship is measured by Earth observers to be of length L, where

$$L = L_p\sqrt{1-\frac{v^2}{c^2}} \qquad \text{and} \qquad L = v\Delta t$$

$$v\Delta t = L_p\sqrt{1-\frac{v^2}{c^2}} \qquad \text{and} \qquad v^2\Delta t^2 = L_p^2\left(1-\frac{v^2}{c^2}\right)$$

Solving for v, $v^2\left(\Delta t^2 + \frac{L_p^2}{c^2}\right) = L_p^2$ $\quad \boxed{v = \frac{cL_p}{\sqrt{c^2\Delta t^2 + L_p^2}}}$

(b) The tanks move nonrelativistically, so we have $v = \frac{300 \text{ m}}{75 \text{ s}} = \boxed{4.00 \text{ m/s}}$.

(c) For the data in problem 9,

$$v = \frac{c(300 \text{ m})}{\sqrt{(3\times10^8 \text{ m/s})^2(0.75\times10^{-6} \text{ s})^2 + (300 \text{ m})^2}} = \frac{c(300 \text{ m})}{\sqrt{225^2+300^2} \text{ m}} = 0.800c$$

in agreement with problem 9. For the data in part (b),

$$v = \frac{c(300 \text{ m})}{\sqrt{(3\times10^8 \text{ m/s})^2(75 \text{ s})^2 + (300 \text{ m})^2}} = \frac{c(300 \text{ m})}{\sqrt{(2.25\times10^{10})^2+300^2} \text{ m}}$$

$$= 1.33\times10^{-8}c = 4.00 \text{ m/s}$$

in agreement with part (b).

P39.11 We find Cooper's speed: $\frac{GMm}{r^2} = \frac{mv^2}{r}$

Solving, $v = \left[\frac{GM}{(R+h)}\right]^{1/2} = \left[\frac{(6.67\times10^{-11})(5.98\times10^{24})}{(6.37\times10^6 + 0.160\times10^6)}\right]^{1/2} = 7.82 \text{ km/s}$

Then the time period of one orbit is $T = \frac{2\pi(R+h)}{v} = \frac{2\pi(6.53\times10^6)}{7.82\times10^3} = 5.25\times10^3 \text{ s}$

(a) The time difference for 22 orbits is $\Delta t - \Delta t_p = (\gamma-1)\Delta t_p = \left[\left(1-\frac{v^2}{c^2}\right)^{-1/2} - 1\right](22T)$

$$\Delta t - \Delta t_p \approx \left(1+\frac{1}{2}\frac{v^2}{c^2}-1\right)(22T) = \frac{1}{2}\left(\frac{7.82\times10^3 \text{ m/s}}{3\times10^8 \text{ m/s}}\right)^2 22(5.25\times10^3 \text{ s}) = \boxed{39.2\ \mu\text{s}}$$

(b) For each one orbit Cooper aged less by $\Delta t - \Delta t_p = \frac{39.2\ \mu\text{s}}{22} = 1.78\ \mu\text{s}$. The press report is $\boxed{\text{accurate to one digit}}$.

P39.12 $\gamma = \frac{1}{\sqrt{1-(v^2/c^2)}} = 1.01$ $\qquad$ so $\qquad \boxed{v = 0.140c}$

P39.13 (a) Since your ship is identical to his, and you are at rest with respect to your own ship, its length is $\boxed{20.0\text{ m}}$.

(b) His ship is in motion relative to you, so you measure its length contracted to $\boxed{19.0\text{ m}}$.

(c) We have $$L = L_p\sqrt{1-\frac{v^2}{c^2}}$$

from which $$\frac{L}{L_p} = \frac{19.0\text{ m}}{20.0\text{ m}} = 0.950 = \sqrt{1-\frac{v^2}{c^2}} \text{ and } \boxed{v = 0.312c}$$

P39.14 In the Earth frame, Speedo's trip lasts for a time

$$\Delta t = \frac{\Delta x}{v} = \frac{20.0\text{ ly}}{0.950\text{ ly/yr}} = 21.05\text{ yr}$$

Speedo's age advances only by the proper time interval

$$\Delta t_p = \frac{\Delta t}{\gamma} = 21.05\text{ yr}\sqrt{1-0.95^2} = 6.574\text{ yr during his trip}$$

Similarly for Goslo,

$$\Delta t_p = \frac{\Delta x}{v}\sqrt{1-\frac{v^2}{c^2}} = \frac{20.0\text{ ly}}{0.750\text{ ly/yr}}\sqrt{1-0.75^2} = 17.64\text{ yr}$$

While Speedo has landed on Planet X and is waiting for his brother, he ages by

$$\frac{20.0\text{ ly}}{0.750\text{ ly/yr}} - \frac{20.0\text{ ly}}{0.950\text{ ly/yr}} = 5.614\text{ yr}$$

Then $\boxed{\text{Goslo}}$ ends up older by $17.64\text{ yr} - (6.574\text{ yr} + 5.614\text{ yr}) = \boxed{5.45\text{ yr}}$.

P39.15 The orbital speed of the Earth is as described by $\Sigma F = ma$: $\frac{Gm_S m_E}{r^2} = \frac{m_E v^2}{r}$

$$v = \sqrt{\frac{Gm_S}{r}} = \sqrt{\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})}{1.496\times10^{11}\text{ m}}} = 2.98\times10^4\text{ m/s}$$

The maximum frequency received by the extraterrestrials is

$$f_{\text{obs}} = f_{\text{source}}\sqrt{\frac{1+v/c}{1-v/c}} = (57.0\times10^6\text{ Hz})\sqrt{\frac{1+(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}{1-(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}}$$
$$= 57.005\,66\times10^6\text{ Hz}$$

The minimum frequency received is

$$f_{\text{obs}} = f_{\text{source}}\sqrt{\frac{1-v/c}{1+v/c}} = (57.0\times10^6\text{ Hz})\sqrt{\frac{1-(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}{1+(2.98\times10^4\text{ m/s})/(3.00\times10^8\text{ m/s})}}$$
$$= 56.994\,34\times10^6\text{ Hz}$$

The difference, which lets them figure out the speed of our planet, is

$$(57.005\,66 - 56.994\,34)\times10^6\text{ Hz} = \boxed{1.13\times10^4\text{ Hz}}$$

P39.16 (a) Let f_c be the frequency as seen by the car. Thus, $f_c = f_{\text{source}}\sqrt{\dfrac{c+v}{c-v}}$

and, if f is the frequency of the reflected wave, $f = f_c\sqrt{\dfrac{c+v}{c-v}}$

Combining gives $\boxed{f = f_{\text{source}}\dfrac{(c+v)}{(c-v)}}$

(b) Using the above result, $f(c-v) = f_{\text{source}}(c+v)$

which gives $(f - f_{\text{source}})c = (f + f_{\text{source}})v \approx 2f_{\text{source}}v$

The beat frequency is then $f_{\text{beat}} = f - f_{\text{source}} = \dfrac{2f_{\text{source}}v}{c} = \boxed{\dfrac{2v}{\lambda}}$

(c) $$f_{\text{beat}} = \frac{(2)(30.0\ \text{m/s})(10.0\times10^9\ \text{Hz})}{3.00\times10^8\ \text{m/s}} = \frac{(2)(30.0\ \text{m/s})}{(0.030\ 0\ \text{m})} = 2\ 000\ \text{Hz} = \boxed{2.00\ \text{kHz}}$$

$$\lambda = \frac{c}{f_{\text{source}}} = \frac{3.00\times10^8\ \text{m/s}}{10.0\times10^9\ \text{Hz}} = 3.00\ \text{cm}$$

(d) $$v = \frac{f_{\text{beat}}\lambda}{2} \quad \text{so} \quad \Delta v = \frac{\Delta f_{\text{beat}}\lambda}{2} = \frac{(5\ \text{Hz})(0.030\ 0\ \text{m})}{2} = \boxed{0.075\ 0\ \text{m/s} \approx 0.2\ \text{mi/h}}$$

P39.17 (a) When the source moves away from an observer, the observed frequency is

$$f_{\text{obs}} = f_{\text{source}}\left(\frac{c - v_s}{c + v_s}\right)^{1/2} \text{ where } v_s = v_{\text{source}}$$

When $v_s << c$, the binomial expansion gives

$$\left(\frac{c-v_s}{c+v_s}\right)^{1/2} = \left[1 - \left(\frac{v_s}{c}\right)\right]^{1/2}\left[1 + \left(\frac{v_s}{c}\right)\right]^{-1/2} \approx \left(1 - \frac{v_s}{2c}\right)\left(1 - \frac{v_s}{2c}\right) \approx \left(1 - \frac{v_s}{c}\right)$$

So, $$f_{\text{obs}} \approx f_{\text{source}}\left(1 - \frac{v_s}{c}\right)$$

The observed wavelength is found from $c = \lambda_{\text{obs}}f_{\text{obs}} = \lambda f_{\text{source}}$:

$$\lambda_{\text{obs}} = \frac{\lambda f_{\text{source}}}{f_{\text{obs}}} \approx \frac{\lambda f_{\text{source}}}{f_{\text{source}}(1 - v_s/c)} = \frac{\lambda}{1 - v_s/c}$$

$$\Delta\lambda = \lambda_{\text{obs}} - \lambda = \lambda\left(\frac{1}{1 - v_s/c} - 1\right) = \lambda\left(\frac{v_s/c}{1 - v_s/c}\right)$$

Since $1 - \dfrac{v_s}{c} \approx 1$, $\boxed{\dfrac{\Delta\lambda}{\lambda} \approx \dfrac{v_{\text{source}}}{c}}$

(b) $$v_{\text{source}} = c\left(\frac{\Delta\lambda}{\lambda}\right) = c\left(\frac{20.0\ \text{nm}}{397\ \text{nm}}\right) = \boxed{0.050\ 4c}$$

P39.18 For the light as observed

$$f_{\text{obs}} = \frac{c}{\lambda_{\text{obs}}} = \sqrt{\frac{1+v/c}{1-v/c}}\, f_{\text{source}} = \sqrt{\frac{1+v/c}{1-v/c}}\, \frac{c}{\lambda_{\text{source}}}$$

$$\sqrt{\frac{1+v/c}{1-v/c}} = \frac{\lambda_{\text{source}}}{\lambda_{\text{obs}}} = \frac{650\text{ nm}}{520\text{ nm}} \qquad \frac{1+v/c}{1-v/c} = 1.25^2 = 1.562$$

$$1+\frac{v}{c} = 1.562 - 1.562\frac{v}{c} \qquad \frac{v}{c} = \frac{0.562}{2.562} = 0.220$$

$$v = \boxed{0.220c} = 6.59\times 10^7\text{ m/s}$$

Section 39.5 **The Lorentz Transformation Equations**

P39.19 Let Suzanne be fixed in reference from S and see the two light-emission events with coordinates $x_1 = 0,\ t_1 = 0,\ x_2 = 0,\ t_2 = 3\ \mu\text{s}$. Let Mark be fixed in reference frame S′ and give the events coordinate $x_1' = 0$, $t_1' = 0$, $t_2' = 9\ \mu\text{s}$.

(a) Then we have

$$t_2' = \gamma\left(t_2 - \frac{v}{c^2}x_2\right)$$

$$9\ \mu\text{s} = \frac{1}{\sqrt{1-v^2/c^2}}(3\ \mu\text{s} - 0) \qquad \sqrt{1-\frac{v^2}{c^2}} = \frac{1}{3}$$

$$\frac{v^2}{c^2} = \frac{8}{9} \qquad \boxed{v = 0.943c}$$

(b) $x_2' = \gamma(x_2 - vt_2) = 3\left(0 - 0.943c\times 3\times 10^{-6}\text{ s}\right)\left(\frac{3\times 10^8\text{ m/s}}{c}\right) = \boxed{2.55\times 10^3\text{ m}}$

P39.20 $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}} = \dfrac{1}{\sqrt{1-0.995^2}} = 10.0$

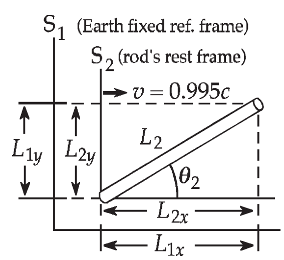


FIG. P39.20

We are also given: $L_1 = 2.00$ m, and $\theta = 30.0°$ (both measured in a reference frame moving relative to the rod).

Thus, $L_{1x} = L_1\cos\theta_1 = (2.00\text{ m})(0.867) = 1.73\text{ m}$

and $L_{1y} = L_1\sin\theta_1 = (2.00\text{ m})(0.500) = 1.00\text{ m}$

L_{2x} is a proper length, related to L_{1x} by $\quad L_{1x} = \dfrac{L_{2x}}{\gamma}$

Therefore, $\quad L_{2x} = 10.0L_{1x} = 17.3\text{ m}$

and $\quad L_{2y} = L_{1y} = 1.00\text{ m}$

(Lengths perpendicular to the motion are unchanged.)

(a) $L_2 = \sqrt{(L_{2x})^2 + (L_{2y})^2}$ gives $\boxed{L_2 = 17.4\text{ m}}$

(b) $\theta_2 = \tan^{-1}\dfrac{L_{2y}}{L_{2x}}$ gives $\boxed{\theta_2 = 3.30°}$

P39.21 Einstein's reasoning about lightning striking the ends of a train shows that the moving observer sees the event toward which she is moving, event B, as occurring first. The S-frame coordinates of the events we may take as ($x = 0,\ y = 0,\ z = 0,\ t = 0$) and ($x = 100$ m, $y = 0,\ z = 0,\ t = 0$). Then the coordinates in S' are given by the Lorentz transformation. Event A is at ($x' = 0$, $y' = 0,\ z' = 0,\ t' = 0$). The time of event B is

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) = \frac{1}{\sqrt{1-0.8^2}}\left(0 - \frac{0.8c}{c^2}(100\text{ m})\right) = 1.667\left(-\frac{80\text{ m}}{3\times10^8\text{ m/s}}\right) = -4.44\times10^{-7}\text{ s}$$

The time elapsing before A occurs is $\boxed{444\text{ ns}}$.

P39.22 (a) From the Lorentz transformation, the separations between the blue-light and red-light events are described by

$$\Delta x' = \gamma(\Delta x - v\Delta t) \qquad 0 = \gamma\left[2.00\text{ m} - v\left(8.00\times10^{-9}\text{ s}\right)\right]$$

$$v = \frac{2.00\text{ m}}{8.00\times10^{-9}\text{ s}} = \boxed{2.50\times10^8\text{ m/s}} \qquad \gamma = \frac{1}{\sqrt{1-\left(2.50\times10^8\text{ m/s}\right)^2/\left(3.00\times10^8\text{ m/s}\right)^2}} = 1.81$$

(b) Again from the Lorentz transformation,

$$x' = \gamma(x - vt):$$

$$x' = 1.81\left[3.00\text{ m} - \left(2.50\times10^8\text{ m/s}\right)\left(1.00\times10^{-9}\text{ s}\right)\right]$$

$$x' = \boxed{4.97\text{ m}}$$

(c) $t' = \gamma\left(t - \dfrac{v}{c^2}x\right)$:

$$t' = 1.81\left[1.00\times10^{-9}\text{ s} - \frac{\left(2.50\times10^8\text{ m/s}\right)}{\left(3.00\times10^8\text{ m/s}\right)^2}(3.00\text{ m})\right]$$

$$t' = \boxed{-1.33\times10^{-8}\text{ s}}$$

Section 39.6 **The Lorentz Velocity Transformation Equations**

P39.23 $$u_x' = \frac{u_x - v}{1 - u_x v/c^2} = \frac{-0.750c - 0.750c}{1-(-0.750)(0.750)} = -0.960c$$

speed = $\boxed{0.960\text{ c}}$

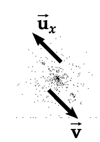


FIG. P39.23

P39.24 u_x = Enterprise velocity

v = Klingon velocity

From Equation 39.16

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2} = \frac{0.900c - 0.800c}{1-(0.900)(0.800)} = \boxed{0.357c}$$

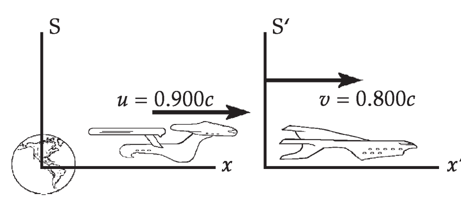


FIG. P39.24

Section 39.7 **Relativistic Linear Momentum**

P39.25 (a) $p = \gamma mu$; for an electron moving at $0.010\,0c$,

$$\gamma = \frac{1}{\sqrt{1-(u/c)^2}} = \frac{1}{\sqrt{1-(0.010\,0)^2}} = 1.000\,05 \approx 1.00$$

Thus, $$p = 1.00\left(9.11\times10^{-31}\text{ kg}\right)(0.010\,0)\left(3.00\times10^{8}\text{ m/s}\right)$$

$$p = \boxed{2.73\times10^{-24}\text{ kg}\cdot\text{m/s}}$$

(b) Following the same steps as used in part (a),

we find at $0.500c$, $\gamma = 1.15$ and $p = \boxed{1.58\times10^{-22}\text{ kg}\cdot\text{m/s}}$

(c) At $0.900c$, $\gamma = 2.29$ and $p = \boxed{5.64\times10^{-22}\text{ kg}\cdot\text{m/s}}$

P39.26 Using the relativistic form, $$p = \frac{mu}{\sqrt{1-(u/c)^2}} = \gamma mu$$

we find the difference Δ from the classical momentum, mu: $\Delta p = \gamma mu - mu = (\gamma-1)mu$

(a) The difference is 1.00% when $(\gamma-1)mu = 0.010\,0\gamma mu$: $\gamma = \frac{1}{0.990} = \frac{1}{\sqrt{1-(u/c)^2}}$

thus $1-\left(\frac{u}{c}\right)^2 = (0.990)^2$, and $u = \boxed{0.141c}$

(b) The difference is 10.0% when $(\gamma-1)mu = 0.100\gamma mu$: $\gamma = \frac{1}{0.900} = \frac{1}{\sqrt{1-(u/c)^2}}$

thus $1-\left(\frac{u}{c}\right)^2 = (0.900)^2$ and $u = \boxed{0.436c}$

P39.27 $\frac{p-mu}{mu} = \frac{\gamma mu - mu}{mu} = \gamma - 1$: $$\gamma - 1 = \frac{1}{\sqrt{1-(u/c)^2}} - 1 \approx 1 + \frac{1}{2}\left(\frac{u}{c}\right)^2 - 1 = \frac{1}{2}\left(\frac{u}{c}\right)^2$$

$$\frac{p-mu}{mu} = \frac{1}{2}\left(\frac{90.0\text{ m/s}}{3.00\times10^{8}\text{ m/s}}\right)^2 = \boxed{4.50\times10^{-14}}$$

***P39.28** We can express the proportionality of the speeding fine to the excess momentum as $F = d(p - p_\ell)$ where F is the fine, d is a proportionality constant, p is the magnitude of the vehicle's momentum, and p_ℓ is the magnitude of its momentum as it travels at 90 km/h.

(a) METHOD ONE. For low speeds, the classical expression for momentum is accurate. Our equation describes the two cases

$$\$80 = d\left[m(190\ \text{km/h}) - m(90\ \text{km/h})\right] \text{ and } F_a = d\left[m(1\,090\ \text{km/h}) - m(90\ \text{km/h})\right]$$

Dividing gives $\dfrac{F_a}{\$80} = \dfrac{m(1\,000\ \text{km/h})}{m(100\ \text{km/h})} \Rightarrow \boxed{F_a = \$800}$.

METHOD TWO. The relativistic momentum expression is always accurate:

$$\$80 = d\left(\frac{m(190\ \text{km/h})}{\sqrt{1-(190\ \text{km/h})^2/c^2}} - \frac{m(90\ \text{km/h})}{\sqrt{1-(90\ \text{km/h})^2/c^2}}\right)$$

$$F_a = d\left(\frac{m(1\,090\ \text{km/h})}{\sqrt{1-(1\,090\ \text{km/h})^2/c^2}} - \frac{m(190\ \text{km/h})}{\sqrt{1-(190\ \text{km/h})^2/c^2}}\right)$$

To three or even to six digits, the answer is the same: $F_a = \$80(10) = \$800$.

(b) Now the high-speed case must be described relativistically. The speed of light is $3\times 10^8\ \text{m/s} = 1.08\times 10^9\ \text{km/h}$.

$$\$80 = dm(100\ \text{km/h})$$

$$F_b = d\left(\frac{m(1.000\,000\,09\times 10^9\ \text{km/h})}{\sqrt{1-\left(1.000\,000\,09\times 10^9/1.08\times 10^9\right)^2}} - m(90\ \text{km/h})\right)$$

$$F_b = dm(2.65\times 10^9\ \text{km/h})$$

again dividing, $\dfrac{F_b}{\$80} = \dfrac{2.65\times 10^9}{10^2} \Rightarrow F_b = \boxed{\$2.12\times 10^9}$.

P39.29 Relativistic momentum of the system of fragments must be conserved. For total momentum to be zero after as it was before, we must have, with subscript 2 referring to the heavier fragment, and subscript 1 to the lighter, $p_2 = p_1$

or $$\gamma_2 m_2 u_2 = \gamma_1 m_1 u_1 = \frac{2.50\times10^{-28}\ \text{kg}}{\sqrt{1-(0.893)^2}}\times(0.893c)$$

or $$\frac{\left(1.67\times10^{-27}\ \text{kg}\right)u_2}{\sqrt{1-\left(u_2/c\right)^2}} = \left(4.960\times10^{-28}\ \text{kg}\right)c$$

Proceeding to solve, we find $$\left(\frac{1.67\times10^{-27}}{4.960\times10^{-28}}\frac{u_2}{c}\right)^2 = 1-\frac{u_2^2}{c^2}$$

$$12.3\frac{u_2^2}{c^2} = 1 \text{ and } u_2 = \boxed{0.285c}$$

Section 39.8 **Relativistic Energy**

***P39.30** (a) $K = E - E_R = 5E_R$

$E = 6E_R = 6\left(9.11\times10^{-31}\ \text{kg}\right)\left(3.00\times10^{8}\ \text{m/s}\right)^2 = 4.92\times10^{-13}\ \text{J} = \boxed{3.07\ \text{MeV}}$

(b) $E = \gamma mc^2 = \gamma E_R$

Thus $\gamma = \dfrac{E}{E_R} = 6 = \dfrac{1}{\sqrt{1-u^2/c^2}}$ which yields $\boxed{u = 0.986c}$

P39.31 $$\Sigma W = K_f - K_i = \left(\frac{1}{\sqrt{1-\left(v_f/c\right)^2}}-1\right)mc^2 - \left(\frac{1}{\sqrt{1-\left(v_i/c\right)^2}}\right)mc^2$$

or $$\Sigma W = \left(\frac{1}{\sqrt{1-\left(v_f/c\right)^2}} - \frac{1}{\sqrt{1-\left(v_i/c\right)^2}}\right)mc^2$$

(a) $$\Sigma W = \left(\frac{1}{\sqrt{1-(0.750)^2}} - \frac{1}{\sqrt{1-(0.500)^2}}\right)\left(1.673\times10^{-27}\ \text{kg}\right)\left(2.998\times10^{8}\ \text{m/s}\right)^2$$

$$\Sigma W = \boxed{5.37\times10^{-11}\ \text{J}}$$

(b) $$\Sigma W = \left(\frac{1}{\sqrt{1-(0.995)^2}} - \frac{1}{\sqrt{1-(0.500)^2}}\right)\left(1.673\times10^{-27}\ \text{kg}\right)\left(2.998\times10^{8}\ \text{m/s}\right)^2$$

$$\Sigma W = \boxed{1.33\times10^{-9}\ \text{J}}$$

P39.32 The relativistic kinetic energy of an object of mass m and speed u is $K_r = \left(\dfrac{1}{\sqrt{1-u^2/c^2}} - 1\right)mc^2$.

For $u = 0.100c$, $$K_r = \left(\frac{1}{\sqrt{1-0.010\,0}} - 1\right)mc^2 = 0.005\,038mc^2$$

The classical equation $K_c = \frac{1}{2}mu^2$ gives $$K_c = \frac{1}{2}m(0.100c)^2 = 0.005\,000mc^2$$

different by $$\frac{0.005\,038 - 0.005\,000}{0.005\,038} = 0.751\%$$

For still smaller speeds the agreement will be still better.

P39.33 $E = \gamma mc^2 = 2mc^2$ or $\gamma = 2$

Thus, $\dfrac{u}{c} = \sqrt{1-\left(\dfrac{1}{\gamma}\right)^2} = \dfrac{\sqrt{3}}{2}$ or $u = \dfrac{c\sqrt{3}}{2}$

The momentum is then $$p = \gamma mu = 2m\left(\frac{c\sqrt{3}}{2}\right) = \left(\frac{mc^2}{c}\right)\sqrt{3}$$

$$p = \left(\frac{938.3\text{ MeV}}{c}\right)\sqrt{3} = \boxed{1.63\times10^3\ \frac{\text{MeV}}{c}}$$

***P39.34** (a) Using the classical equation, $$K = \frac{1}{2}mu^2 = \frac{1}{2}(78.0\text{ kg})(1.06\times10^5\text{ m/s})^2 = \boxed{4.38\times10^{11}\text{ J}}$$

(b) Using the relativistic equation, $$K = \left(\frac{1}{\sqrt{1-(u/c)^2}} - 1\right)mc^2$$

$$K = \left[\frac{1}{\sqrt{1-\left(1.06\times10^5/2.998\times10^8\right)^2}} - 1\right](78.0\text{ kg})(2.998\times10^8\text{ m/s})^2 = \boxed{4.38\times10^{11}\text{ J}}$$

(c) When $\dfrac{u}{c} \ll 1$, the binomial series expansion gives $$\left[1-\left(\frac{u}{c}\right)^2\right]^{-1/2} \approx 1 + \frac{1}{2}\left(\frac{u}{c}\right)^2$$

Thus, $$\left[1-\left(\frac{u}{c}\right)^2\right]^{-1/2} - 1 \approx \frac{1}{2}\left(\frac{u}{c}\right)^2$$

and the relativistic expression for kinetic energy becomes $K \approx \dfrac{1}{2}\left(\dfrac{u}{c}\right)^2 mc^2 = \dfrac{1}{2}mu^2$.

That is, in the limit of speeds much smaller than the speed of light, the relativistic and classical expressions yield the same results. In this situation the two kinetic energy values are experimentally indistinguishable. The fastest-moving macroscopic objects launched by human beings move sufficiently slowly compared to light that relativistic corrections to their energy are negligible.

P39.35 (a) $E_R = mc^2 = \left(1.67 \times 10^{-27} \text{ kg}\right)\left(2.998 \times 10^8 \text{ m/s}\right)^2 = 1.50 \times 10^{-10} \text{ J} = \boxed{938 \text{ MeV}}$

(b) $E = \gamma mc^2 = \dfrac{1.50 \times 10^{-10} \text{ J}}{\left[1 - (0.950c/c)^2\right]^{1/2}} = 4.81 \times 10^{-10} \text{ J} = \boxed{3.00 \times 10^3 \text{ MeV}}$

(c) $K = E - mc^2 = 4.81 \times 10^{-10} \text{ J} - 1.50 \times 10^{-10} \text{ J} = 3.31 \times 10^{-10} \text{ J} = \boxed{2.07 \times 10^3 \text{ MeV}}$

P39.36 We must conserve both energy and relativistic momentum of the system of fragments. With subscript 1 referring to the $0.868c$ particle and subscript 2 to the $0.987c$ particle,

$$\gamma_1 = \frac{1}{\sqrt{1 - (0.868)^2}} = 2.01 \text{ and } \gamma_2 = \frac{1}{\sqrt{1 - (0.987)^2}} = 6.22$$

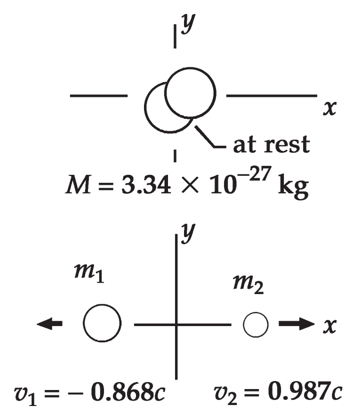


FIG. P39.36

Conservation of energy gives $E_1 + E_2 = E_{\text{total}}$

which is $\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 = m_{\text{total}} c^2$

or $2.01 m_1 + 6.22 m_2 = 3.34 \times 10^{-27} \text{ kg}$

This reduces to: $m_1 + 3.09 m_2 = 1.66 \times 10^{-27} \text{ kg}$ (1)

Since the final momentum of the system must equal zero, $p_1 = p_2$

gives $\gamma_1 m_1 u_1 = \gamma_2 m_2 u_2$

or $(2.01)(0.868c) m_1 = (6.22)(0.987c) m_2$

which becomes $m_1 = 3.52 m_2$ (2)

Solving (1) and (2) simultaneously, $3.52 m_2 + 3.09 m_2 = 1.66 \times 10^{-27} \text{ kg}$

$m_1 = \boxed{8.84 \times 10^{-28} \text{ kg}}$ and $m_2 = \boxed{2.51 \times 10^{-28} \text{ kg}}$

P39.37 $E = \gamma mc^2$ $\qquad p = \gamma mu$

$E^2 = \left(\gamma mc^2\right)^2$ $\qquad p^2 = (\gamma mu)^2$

$$E^2 - p^2c^2 = \left(\gamma mc^2\right)^2 - (\gamma mu)^2 c^2 = \gamma^2\left(\left(mc^2\right)^2 - (mc)^2 u^2\right) = \left(mc^2\right)^2\left(1 - \frac{u^2}{c^2}\right)\left(1 - \frac{u^2}{c^2}\right)^{-1} = \left(mc^2\right)^2$$

as was to be demonstrated.

P39.38 (a) $q(\Delta V) = K = (\gamma - 1) m_e c^2$

Thus, $\gamma = \dfrac{1}{\sqrt{1 - (u/c)^2}} = 1 + \dfrac{q(\Delta V)}{m_e c^2} = 1 + \dfrac{25\,000 \text{ eV}}{511\,000 \text{ eV}} = 1.0489$

so $1 - (u/c)^2 = 0.9089$ and $\boxed{u = 0.302c}$

(b) $K = (\gamma - 1) m_e c^2 = q(\Delta V) = \left(1.60 \times 10^{-19} \text{ C}\right)\left(2.50 \times 10^4 \text{ J/C}\right) = \boxed{4.00 \times 10^{-15} \text{ J}}$

***P39.39** From $K=(\gamma-1)mc^2=\left(\frac{1}{\sqrt{1-u^2/c^2}}-1\right)mc^2$ we have $\frac{K}{mc^2}+1=\frac{1}{\sqrt{1-u^2/c^2}}=\frac{K+mc^2}{mc^2}$.

$$1-\frac{u^2}{c^2}=\frac{m^2c^4}{\left(K+mc^2\right)^2}$$

$$\frac{u^2}{c^2}=1-\frac{\left(mc^2\right)^2}{\left(K+mc^2\right)^2}$$

$$u=c\left(1-\left(\frac{mc^2}{K+mc^2}\right)^2\right)^{1/2}$$

(a) Electron: $u=c\left(1-\left(\frac{0.511}{2.511}\right)^2\right)^{1/2}=\boxed{0.979c}$

(b) Proton: $u=c\left(1-\left(\frac{938}{940}\right)^2\right)^{1/2}=\boxed{0.065\ 2c}$

(c) $0.979c-0.065\ 2c=\boxed{0.914c}=2.74\times10^8\ \text{m/s}$

In this case the electron is moving relativistically, but the classical expression $\frac{1}{2}mv^2$ is accurate to two digits for the proton.

(d) Electron: $u=c\left(1-\left(\frac{0.511}{2\ 000.511}\right)^2\right)^{1/2}=\boxed{0.999\ 999\ 97c}$

Proton: $u=c\left(1-\left(\frac{938}{2\ 938}\right)^2\right)^{1/2}=\boxed{0.948c}$

Excess speed $=0.999\ 999\ 97c-0.948c=\boxed{0.052\ 3c}=1.57\times10^7\ \text{m/s}$

As the kinetic energies of both particles become large, the difference in their speeds approaches zero. By contrast, classically the speed difference would become large without any finite limit.

P39.40 (a) $E=\gamma mc^2=20.0$ GeV with $mc^2=0.511$ MeV for electrons.

Thus, $\gamma=\frac{20.0\times10^9\ \text{eV}}{0.511\times10^6\ \text{eV}}=\boxed{3.91\times10^4}$.

(b) $\gamma=\frac{1}{\sqrt{1-(u/c)^2}}=3.91\times10^4$ from which $\boxed{u=0.999\ 999\ 999\ 7c}$

(c) $L=L_p\sqrt{1-\left(\frac{u}{c}\right)^2}=\frac{L_p}{\gamma}=\frac{3.00\times10^3\ \text{m}}{3.91\times10^4}=7.67\times10^{-2}\ \text{m}=\boxed{7.67\ \text{cm}}$

P39.41 Conserving total momentum of the decaying particle system, $p_{\text{before decay}} = p_{\text{after decay}} = 0$

$$p_v = p_\mu = \gamma m_\mu u = \gamma(207m_e)u$$

Conservation of mass-energy for the system gives $E_\mu + E_v = E_\pi$: $\gamma\, m_\mu c^2 + p_v c = m_\pi c^2$

$$\gamma(207m_e) + \frac{p_v}{c} = 273m_e$$

Substituting from the momentum equation above, $\gamma(207m_e) + \gamma(207m_e)\frac{u}{c} = 273m_e$

or $\gamma\left(1+\frac{u}{c}\right) = \frac{273}{207} = 1.32$: $\frac{1+u/c}{1-u/c} = 1.74$ $\frac{u}{c} = 0.270$

Then, $K_\mu = (\gamma - 1)m_\mu c^2 = (\gamma-1)207(m_e c^2)$: $K_\mu = \left(\frac{1}{\sqrt{1-(0.270)^2}} - 1\right)207(0.511\text{ MeV})$

$$K_\mu = \boxed{4.08\text{ MeV}}$$

Also, $E_v = E_\pi - E_\mu$: $E_v = m_\pi c^2 - \gamma\, m_\mu c^2 = (273 - 207\gamma)m_e c^2$

$$E_v = \left(273 - \frac{207}{\sqrt{1-(0.270)^2}}\right)(0.511\text{ MeV})$$

$$E_v = \boxed{29.6\text{ MeV}}$$

***P39.42** $K = (\gamma-1)mc^2 = \left(\left(1-u^2/c^2\right)^{-1/2} - 1\right)mc^2$. We use the series expansion from Appendix B.5:

$K = mc^2\left[1 + \left(-\tfrac{1}{2}\right)(-u^2/c^2) + \left(-\tfrac{1}{2}\right)\left(-\tfrac{3}{2}\right)\tfrac{1}{2!}(-u^2/c^2)^2 + \cdots - 1\right]$

$K = \tfrac{1}{2}mu^2 + \tfrac{3}{8}m\frac{u^4}{c^2} + \cdots$ The actual kinetic energy, given by this relativistic equation, is $\boxed{\text{greater}}$ than the classical (1/2)mu^2. The difference, for m = 1 000 kg and u = 25 m/s, is

$\tfrac{3}{8}(1\,000\text{ kg})\frac{(25\text{ m/s})^4}{(3\times10^8\text{ m/s})^2} = 1.6\times10^{-9}\text{ J}$ $\boxed{\sim 10^{-9}\text{ J}}$

Section 39.9 **Mass and Energy**

***P39.43** $E = 2.86\times10^5$ J leaves the system so the final mass is $\boxed{\text{smaller}}$. The mass-energy relation says that $E = mc^2$. Therefore, $m = \frac{E}{c^2} = \frac{2.86\times10^5\text{ J}}{(3.00\times10^8\text{ m/s})^2} = \boxed{3.18\times10^{-12}\text{ kg}}$

A mass loss of this magnitude, as a fraction of a total of 9.00 g, $\boxed{\text{could not be detected}}$.

P39.44 $\Delta m = \frac{E}{c^2} = \frac{\mathcal{P}\Delta t}{c^2} = \frac{0.800(1.00\times10^9\text{ J/s})(3.00\text{ yr})(3.16\times10^7\text{ s/yr})}{(3.00\times10^8\text{ m/s})^2} = \boxed{0.842\text{ kg}}$

P39.45 $\mathcal{P} = \frac{dE}{dt} = \frac{d(mc^2)}{dt} = c^2\frac{dm}{dt} = 3.85\times10^{26}\ \text{W}$

Thus, $\frac{dm}{dt} = \frac{3.85\times10^{26}\ \text{J/s}}{(3.00\times10^8\ \text{m/s})^2} = \boxed{4.28\times10^9\ \text{kg/s}}$

P39.46 $2m_ec^2 = 1.02\ \text{MeV}$ $E_\gamma \geq \boxed{1.02\ \text{MeV}}$

Section 39.10 **The General Theory of Relativity**

P39.47 (a) For the satellite $\Sigma F = ma$: $\frac{GM_Em}{r^2} = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2$

$$GM_ET^2 = 4\pi^2r^3$$

$$r = \left(\frac{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2\left(5.98\times10^{24}\ \text{kg}\right)\left(43\ 080\ \text{s}\right)^2}{\text{kg}^2\,4\pi^2}\right)^{1/3}$$

$$r = \boxed{2.66\times10^7\ \text{m}}$$

(b) $v = \frac{2\pi r}{T} = \frac{2\pi\left(2.66\times10^7\ \text{m}\right)}{43\ 080\ \text{s}} = \boxed{3.87\times10^3\ \text{m/s}}$

(c) The small fractional decrease in frequency received is equal in magnitude to the fractional increase in period of the moving oscillator due to time dilation:

$$\text{fractional change in } f = -(\gamma-1) = -\left(\frac{1}{\sqrt{1-\left(3.87\times10^3/3\times10^8\right)^2}}-1\right)$$

$$= 1-\left(1-\frac{1}{2}\left[-\left(\frac{3.87\times10^3}{3\times10^8}\right)^2\right]\right) = \boxed{-8.34\times10^{-11}}$$

(d) The orbit altitude is large compared to the radius of the Earth, so we must use $U_g = -\frac{GM_Em}{r}$.

$$\Delta U_g = -\frac{6.67\times10^{-11}\ \text{Nm}^2\left(5.98\times10^{24}\ \text{kg}\right)m}{\text{kg}^2\ 2.66\times10^7\ \text{m}} + \frac{6.67\times10^{-11}\ \text{Nm}\left(5.98\times10^{24}\ \text{kg}\right)m}{\text{kg}\ 6.37\times10^6\ \text{m}}$$

$$= 4.76\times10^7\ \text{J/kg}\,m$$

$$\frac{\Delta f}{f} = \frac{\Delta U_g}{mc^2} = \frac{4.76\times10^7\ \text{m}^2/\text{s}^2}{\left(3\times10^8\ \text{m/s}\right)^2} = \boxed{+5.29\times10^{-10}}$$

(e) $-8.34\times10^{-11} + 5.29\times10^{-10} = \boxed{+4.46\times10^{-10}}$

Additional Problems

P39.48 (a) $d_{\text{earth}} = vt_{\text{earth}} = v\gamma\, t_{\text{astro}}$ so $\left(2.00\times 10^6\ \text{yr}\right)c = v\dfrac{1}{\sqrt{1-v^2/c^2}}30.0\ \text{yr}$

$$\sqrt{1-\frac{v^2}{c^2}} = \left(\frac{v}{c}\right)\left(1.50\times 10^{-5}\right) \qquad 1-\frac{v^2}{c^2} = \frac{v^2\left(2.25\times 10^{-10}\right)}{c^2}$$

$$1 = \frac{v^2}{c^2}\left(1+2.25\times 10^{-10}\right) \quad \text{so} \quad \frac{v}{c} = \left(1+2.25\times 10^{-10}\right)^{-1/2} = 1-\frac{1}{2}\left(2.25\times 10^{-10}\right)$$

$$\boxed{\frac{v}{c} = 1-1.12\times 10^{-10}}$$

(b) $$K = \left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2 = \left(\frac{2.00\times 10^6\ \text{yr}}{30\ \text{yr}}-1\right)(1\,000)(1\,000\ \text{kg})\left(3\times 10^8\ \text{m/s}\right)^2$$

$$= \boxed{6.00\times 10^{27}\ \text{J}}$$

(c) $$6.00\times 10^{27}\ \text{J} = 6.00\times 10^{27}\ \text{J}\left(\frac{13¢}{\text{kWh}}\right)\left(\frac{\text{k}}{10^3}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3\,600\ \text{s}}\right) = \boxed{\$2.17\times 10^{20}}$$

P39.49 (a) $10^{13}\ \text{MeV} = (\gamma-1)m_pc^2$ so $\gamma = 10^{10}$

$v_p \approx c$ $\qquad t' = \dfrac{t}{\gamma} = \dfrac{10^5\ \text{yr}}{10^{10}} = 10^{-5}\ \text{yr} \sim \boxed{10^2\ \text{s}}$

(b) $d' = ct'\ \boxed{\sim 10^8\ \text{km}}$

P39.50 (a) When $K_e = K_p$, $\quad m_ec^2(\gamma_e-1) = m_pc^2(\gamma_p-1)$

In this case, $\quad m_ec^2 = 0.511\ \text{MeV},\ m_pc^2 = 938\ \text{MeV}$

and $\quad \gamma_e = \left[1-(0.750)^2\right]^{-1/2} = 1.511\,9$

Substituting, $$\gamma_p = 1+\frac{m_ec^2(\gamma_e-1)}{m_pc^2} = 1+\frac{(0.511\ \text{MeV})(1.511\,9-1)}{938\ \text{MeV}}$$

$$= 1.000\,279$$

but $$\gamma_p = \frac{1}{\left[1-(u_p/c)^2\right]^{1/2}}$$

Therefore, $\quad u_p = c\sqrt{1-\gamma_p^{-2}} = \boxed{0.023\,6c}$

(b) When $p_e = p_p$ $\quad \gamma_pm_pu_p = \gamma_em_eu_e \quad$ or $\quad \gamma_pu_p = \dfrac{\gamma_em_eu_e}{m_p}$

Thus, $$\gamma_pu_p = \frac{(1.511\,9)\left(0.511\ \text{MeV}/c^2\right)(0.750c)}{938\ \text{MeV}/c^2} = 6.177\,2\times 10^{-4}c$$

and $$\frac{u_p}{c} = 6.177\,2\times 10^{-4}\sqrt{1-\left(\frac{u_p}{c}\right)^2}$$

which yields $\quad u_p = \boxed{6.18\times 10^{-4}c} = 185\ \text{km/s}$

***P39.51** (a) We let H represent K/mc^2. Then $H+1=\dfrac{1}{\sqrt{1-u^2/c^2}}$ so $1-u^2/c^2=\dfrac{1}{H^2+2H+1}$

$$\frac{u^2}{c^2}=1-\frac{1}{H^2+2H+1}=\frac{H^2+2H}{H^2+2H+1} \quad \text{and} \quad \boxed{u=c\left(\frac{H^2+2H}{H^2+2H+1}\right)^{1/2}}$$

(b) u goes to 0 as K goes to 0.

(c) u approaches c as K increases without limit.

(d) $a=\dfrac{du}{dt}$

$$=c\frac{1}{2}\left(\frac{H^2+2H}{H^2+2H+1}\right)^{-1/2}\left(\frac{\left[H^2+2H+1\right][2H+2]-\left[H^2+2H\right][2H+2]}{\left[H^2+2H+1\right]^2}\right)\frac{d(K/mc^2)}{dt}$$

$$=a=c\left(\frac{H^2+2H+1}{H^2+2H}\right)^{1/2}\left(\frac{H+1}{[H+1]^4}\right)\frac{\mathcal{P}}{mc^2}=\boxed{\frac{\mathcal{P}}{mcH^{1/2}(H+2)^{1/2}(H+1)^2}}$$

(e) When H is small we have approximately $a=\dfrac{\mathcal{P}}{mcH^{1/2}2^{1/2}}=\dfrac{\mathcal{P}}{(2mK)^{1/2}}$, in agreement with the nonrelativistic case.

(f) When H is large the acceleration approaches $\mathcal{P}/mcH^3=\mathcal{P}m^2c^5/K^3$.

(g) As energy is steadily imparted to particle, the particle's acceleration decreases. It decreases steeply, proportionally to $1/K^3$ at high energy. In this way the particle's speed cannot reach or surpass a certain upper limit, which is the speed of light in vacuum.

P39.52 (a) Since Mary is in the same reference frame, S', as Ted, she measures the ball to have the same speed Ted observes, namely $|u_x'|=\boxed{0.800c}$.

(b) $$\Delta t'=\frac{L_p}{|u_x'|}=\frac{1.80\times10^{12}\text{ m}}{0.800\left(3.00\times10^8\text{ m/s}\right)}=\boxed{7.50\times10^3\text{ s}}$$

(c) $$L=L_p\sqrt{1-\frac{v^2}{c^2}}=\left(1.80\times10^{12}\text{ m}\right)\sqrt{1-\frac{(0.600c)^2}{c^2}}=\boxed{1.44\times10^{12}\text{ m}}$$

Since $v=0.600c$ and $u_x'=-0.800c$, the velocity Jim measures for the ball is

$$u_x=\frac{u_x'+v}{1+u_x'v/c^2}=\frac{(-0.800c)+(0.600c)}{1+(-0.800)(0.600)}=\boxed{-0.385c}$$

(d) Jim measures the ball and Mary to be initially separated by 1.44×10^{12} m. Mary's motion at $0.600c$ and the ball's motion at $0.385c$ nibble into this distance from both ends. The gap closes at the rate $0.600c+0.385c=0.985c$, so the ball and catcher meet after a time

$$\Delta t=\frac{1.44\times10^{12}\text{ m}}{0.985\left(3.00\times10^8\text{ m/s}\right)}=\boxed{4.88\times10^3\text{ s}}$$

P39.53 $$\frac{\Delta mc^2}{mc^2}=\frac{4(938.78\text{ MeV})-3\,728.4\text{ MeV}}{4(938.78\text{ MeV})}\times100\%=\boxed{0.712\%}$$

P39.54 The energy of the first fragment is given by $E_1^2 = p_1^2c^2 + \left(m_1c^2\right)^2 = (1.75\text{ MeV})^2 + (1.00\text{ MeV})^2$

$E_1 = 2.02\text{ MeV}$

For the second, $E_2^2 = (2.00\text{ MeV})^2 + (1.50\text{ MeV})^2 \quad E_2 = 2.50\text{ MeV}$

(a) Energy is conserved, so the unstable object had $E = 4.52$ MeV. Each component of momentum is conserved, so the original object moved

with $p^2 = p_x^2 + p_y^2 = \left(\frac{1.75\text{ MeV}}{c}\right)^2 + \left(\frac{2.00\text{ MeV}}{c}\right)^2$. Then for

it $(4.52\text{ MeV})^2 = (1.75\text{ MeV})^2 + (2.00\text{ MeV})^2 + \left(mc^2\right)^2 \qquad \boxed{m = \frac{3.65\text{ MeV}}{c^2}}$

(b) Now $E = \gamma mc^2$ gives $4.52\text{ MeV} = \frac{1}{\sqrt{1 - v^2/c^2}} 3.65\text{ MeV} \quad 1 - \frac{v^2}{c^2} = 0.654$, $\boxed{v = 0.589c}$

P39.55 Look at the situation from the instructors' viewpoint since they are at rest relative to the clock, and hence measure the proper time. The Earth moves with velocity $v = -0.280c$ relative to the instructors while the students move with a velocity $u' = -0.600c$ relative to Earth. Using the velocity addition equation, the velocity of the students relative to the instructors (and hence the clock) is:

$$u = \frac{v + u'}{1 + vu'/c^2} = \frac{(-0.280c) - (0.600c)}{1 + (-0.280c)(-0.600c)/c^2} = -0.753c \text{ (students relative to clock)}$$

(a) With a proper time interval of $\Delta t_p = 50.0$ min, the time interval measured by the students is:

$$\Delta t = \gamma\, \Delta t_p \qquad \text{with} \qquad \gamma = \frac{1}{\sqrt{1 - (0.753c)^2/c^2}} = 1.52$$

Thus, the students measure the exam to last $T = 1.52(50.0\text{ min}) = \boxed{76.0\text{ minutes}}$.

(b) The duration of the exam as measured by observers on Earth is:

$$\Delta t = \gamma\, \Delta t_p \quad \text{with} \quad \gamma = \frac{1}{\sqrt{1 - (0.280c)^2/c^2}} \text{ so } T = 1.04(50.0\text{ min}) = \boxed{52.1\text{ minutes}}$$

***P39.56** (a) The speed of light in water is $c/1.33$, so the electron's speed is $1.1c/1.33$. Then

$$\gamma = \frac{1}{\sqrt{1 - (1.1/1.33)^2}} = 1.779 \quad \text{and the energy is } \gamma mc^2 = 1.779(0.511\text{ MeV}) \boxed{= 0.909\text{ MeV}}$$

(b) $K = 0.909\text{ MeV} - 0.511\text{ MeV} = \boxed{0.398\text{ MeV}}$

(c) $pc = \sqrt{E^2 - (mc^2)^2} = \sqrt{\gamma^2 - 1}\; mc^2 = \sqrt{1.779^2 - 1}\; 0.511\text{ MeV} = 0.752\text{ MeV}$

$p = \boxed{0.752\text{ MeV}/c = 4.01 \times 10^{-22}\text{ kg}\cdot\text{m/s}}$

(d) $\sin\theta = v/u$ where v is the wave speed, which is the speed of light in water, and u is the source speed. Then $\sin\theta = 1/1.1 = 0.909$ so $\theta = \boxed{65.4°}$

P39.57 (a) Take the spaceship as the primed frame, moving toward the right at $v = +0.600c$.

Then $u'_x = +0.800c$, and $$u_x = \frac{u'_x + v}{1 + (u'_x v)/c^2} = \frac{0.800c + 0.600c}{1 + (0.800)(0.600)} = \boxed{0.946c}$$

(b) $L = \frac{L_p}{\gamma}$: $$L = (0.200 \text{ ly})\sqrt{1 - (0.600)^2} = \boxed{0.160 \text{ ly}}$$

(c) The aliens observe the 0.160-ly distance closing because the probe nibbles into it from one end at $0.800c$ and the Earth reduces it at the other end at $0.600c$.

Thus, $$\text{time} = \frac{0.160 \text{ ly}}{0.800c + 0.600c} = \boxed{0.114 \text{ yr}}$$

(d) $K = \left(\frac{1}{\sqrt{1 - u^2/c^2}} - 1\right) mc^2$: $$K = \left(\frac{1}{\sqrt{1 - (0.946)^2}} - 1\right)(4.00 \times 10^5 \text{ kg})(3.00 \times 10^8 \text{ m/s})^2$$

$$K = \boxed{7.50 \times 10^{22} \text{ J}}$$

***P39.58** (a) From equation 39.18, the speed of light in the laboratory frame is

$$u = \frac{v + c/n}{1 + vc/nc^2} = \frac{c\ (1 + nv/c)}{n\ (1 + v/nc)}$$

(b) When v is much less than c we have

$$u = \frac{c}{n}\left(1 + \frac{nv}{c}\right)\left(1 + \frac{v}{nc}\right)^{-1} \approx \frac{c}{n}\left(1 + \frac{nv}{c}\right)\left(1 - \frac{v}{nc}\right) \approx \frac{c}{n}\left(1 + \frac{nv}{c} - \frac{v}{nc}\right) = \frac{c}{n} + v - \frac{v}{n^2}$$

The Galilean velocity transformation equation 4.20 would indeed give $c/n + v$ for the speed of light in the moving water. The third term $-v/n^2$ does represent a relativistic effect that was observed decades before the Michelson-Morley experiment. It is a piece of twentieth-century physics that dropped into the nineteenth century. We could say that light is intrinsically relativistic.

(c) To take the limit as v approaches c we must go back to

$$u = \frac{c\ (1 + nv/c)}{n\ (1 + v/nc)} \quad \text{to find } u \text{ approaches } \frac{c\ (1 + nc/c)}{n\ (1 + c/nc)} = \frac{c(1 + n)}{n + 1} = \boxed{c}$$

P39.59 The observer measures the proper length of the tunnel, 50.0 m, but measures the train contracted to length

$$L = L_p\sqrt{1 - \frac{v^2}{c^2}} = 100 \text{ m}\sqrt{1 - (0.950)^2} = 31.2 \text{ m}$$

shorter than the tunnel by $50.0 - 31.2 = \boxed{18.8 \text{ m}}$ so $\boxed{\text{it is completely within the tunnel}}$.

P39.60 If the energy required to remove a mass m from the surface is equal to its rest energy mc^2,

then $\dfrac{GM_s m}{R_g} = mc^2$

and $R_g = \dfrac{GM_s}{c^2} = \dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{(3.00\times10^8\ \text{m/s})^2}$

$R_g = 1.47\times10^3\ \text{m} = \boxed{1.47\ \text{km}}$

P39.61 (a) At any speed, the momentum of the particle is given by

$$p = \gamma mu = \frac{mu}{\sqrt{1-(u/c)^2}}$$

Since $F = qE = \dfrac{dp}{dt}$:

$$qE = \frac{d}{dt}\left[mu\left(1-\frac{u^2}{c^2}\right)^{-1/2}\right]$$

$$qE = m\left(1-\frac{u^2}{c^2}\right)^{-1/2}\frac{du}{dt} + \frac{1}{2}mu\left(1-\frac{u^2}{c^2}\right)^{-3/2}\left(\frac{2u}{c^2}\right)\frac{du}{dt}$$

So

$$\frac{qE}{m} = \frac{du}{dt}\left[\frac{1-u^2/c^2+u^2/c^2}{(1-u^2/c^2)^{3/2}}\right]$$

and

$$\boxed{a = \frac{du}{dt} = \frac{qE}{m}\left(1-\frac{u^2}{c^2}\right)^{3/2}}$$

(b) For u small compared to c, the relativistic expression reduces to the classical $a = \dfrac{qE}{m}$. As u approaches c, the acceleration approaches zero, so that the object can never reach the speed of light.

(c) $\displaystyle\int_0^u \frac{du}{(1-u^2/c^2)^{3/2}} = \int_{t=0}^t \frac{qE}{m}\,dt$ $\qquad \boxed{u = \dfrac{qEct}{\sqrt{m^2c^2+q^2E^2t^2}}}$

$\displaystyle x = \int_0^t u\,dt = qEc\int_0^t \frac{t\,dt}{\sqrt{m^2c^2+q^2E^2t^2}}$ $\qquad \boxed{x = \dfrac{c}{qE}\left(\sqrt{m^2c^2+q^2E^2t^2} - mc\right)}$

P39.62 (a) An observer at rest relative to the mirror sees the light travel a distance $D = 2d - x$, where $x = vt_S$ is the distance the ship moves toward the mirror in time t_S. Since this observer agrees that the speed of light is c, the time for it to travel distance D is

$$t_S = \frac{D}{c} = \frac{2d - vt_S}{c} \qquad t_S = \boxed{\frac{2d}{c+v}}$$

(b) The observer in the rocket measures a length-contracted initial distance to the mirror of

$$L = d\sqrt{1 - \frac{v^2}{c^2}}$$

and the mirror moving toward the ship at speed v. Thus, he measures the distance the light travels as $D = 2(L - y)$ where $y = \frac{vt}{2}$ is the distance the mirror moves toward the ship before the light reflects from it. This observer also measures the speed of light to be c, so the time for it to travel distance D is:

$$t = \frac{D}{c} = \frac{2}{c}\left[d\sqrt{1 - \frac{v^2}{c^2}} - \frac{vt}{2}\right] \text{ so } (c+v)t = \frac{2d}{c}\sqrt{(c+v)(c-v)} \text{ or } t = \boxed{\frac{2d}{c}\sqrt{\frac{c-v}{c+v}}}$$

***P39.63** (a) Conservation of momentum γmu:

$$\frac{mu}{\sqrt{1 - u^2/c^2}} + \frac{m(-u)}{3\sqrt{1 - u^2/c^2}} = \frac{Mv_f}{\sqrt{1 - v_f^2/c^2}} = \frac{2mu}{3\sqrt{1 - u^2/c^2}}$$

Conservation of energy γmc^2:

$$\frac{mc^2}{\sqrt{1 - u^2/c^2}} + \frac{mc^2}{3\sqrt{1 - u^2/c^2}} = \frac{Mc^2}{\sqrt{1 - v_f^2/c^2}} = \frac{4mc^2}{3\sqrt{1 - u^2/c^2}}$$

To start solving we can divide: $v_f = \frac{2u}{4} = \frac{u}{2}$. Then

$$\frac{M}{\sqrt{1 - u^2/4c^2}} = \frac{4m}{3\sqrt{1 - u^2/c^2}} = \frac{M}{(1/2)\sqrt{4 - u^2/c^2}}$$

$$\boxed{M = \frac{2m\sqrt{4 - u^2/c^2}}{3\sqrt{1 - u^2/c^2}}}$$

(b) When $u << c$, this reduces to $M = \frac{4m}{3}$, in agreement with the classical result, which is the arithmetic sum of the masses of the two colliding particles.

P39.64 Take the two colliding protons as the system

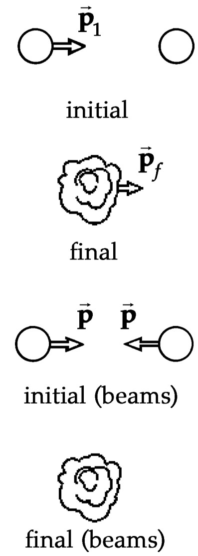


FIG. P39.64

$E_1 = K + mc^2$ $\quad$ $E_2 = mc^2$

$E_1^2 = p_1^2c^2 + m^2c^4$ $\quad$ $p_2 = 0$

In the final state, $E_f = K_f + Mc^2$: $\quad$ $E_f^2 = p_f^2c^2 + M^2c^4$

By energy conservation, $E_1 + E_2 = E_f$, so

$$E_1^2 + 2E_1E_2 + E_2^2 = E_f^2$$

$$p_1^2c^2 + m^2c^4 + 2\left(K + mc^2\right)mc^2 + m^2c^4$$

$$= p_f^2c^2 + M^2c^4$$

By conservation of momentum, $\quad$ $p_1 = p_f$

Then

$$M^2c^4 = 2Kmc^2 + 4m^2c^4$$

$$= \frac{4Km^2c^4}{2mc^2} + 4m^2c^4$$

$$Mc^2 = 2mc^2\sqrt{1 + \frac{K}{2mc^2}}$$

By contrast, for colliding beams we have

In the original state, $\quad$ $E_1 = K + mc^2$

$E_2 = K + mc^2$.

In the final state, $\quad$ $E_f = Mc^2$

$E_1 + E_2 = E_f$: $\quad$ $K + mc^2 + K + mc^2 = Mc^2$

$$Mc^2 = 2mc^2\left(1 + \frac{K}{2mc^2}\right)$$

P39.65 We choose to write down the answer to part (b) first.

(b) Consider a hermit who lives on an asteroid halfway between the Sun and Tau Ceti, stationary with respect to both. Just as our spaceship is passing him, he also sees the blast waves from both explosions. Judging both stars to be stationary, this observer concludes that the two stars blew up simultaneously.

(a) We in the spaceship moving past the hermit do not calculate the explosions to be simultaneous. We measure the distance we have traveled from the Sun as

$$L = L_p\sqrt{1 - \left(\frac{v}{c}\right)^2} = (6.00\text{ ly})\sqrt{1 - (0.800)^2} = 3.60\text{ ly}$$

We see the Sun flying away from us at $0.800c$ while the light from the Sun approaches at $1.00c$. Thus, the gap between the Sun and its blast wave has opened at $1.80c$, and the time we calculate to have elapsed since the Sun exploded is

$$\frac{3.60\text{ ly}}{1.80c} = 2.00\text{ yr}$$

continued on next page

We see Tau Ceti as moving toward us at 0.800*c*, while its light approaches at 1.00*c*, only 0.200*c* faster. We measure the gap between that star and its blast wave as 3.60 ly and growing at 0.200*c*. We calculate that it must have been opening for

$$\frac{3.60 \text{ ly}}{0.200c} = 18.0 \text{ yr}$$

and conclude that $\boxed{\text{Tau Ceti exploded 16.0 years before the Sun}}$.

P39.66 Take $m = 1.00$ kg.

The classical kinetic energy is $K_c = \frac{1}{2}mu^2 = \frac{1}{2}mc^2\left(\frac{u}{c}\right)^2 = \left(4.50\times10^{16}\ \text{J}\right)\left(\frac{u}{c}\right)^2$

and the actual kinetic energy is $K_r = \left(\frac{1}{\sqrt{1-(u/c)^2}} - 1\right)mc^2 = \left(9.00\times10^{16}\ \text{J}\right)\left(\frac{1}{\sqrt{1-(u/c)^2}} - 1\right)$

$\frac{u}{c}$	K_c (J)	K_r (J)
0.000	0.000	0.000
0.100	0.045×10^{16}	0.0453×10^{16}
0.200	0.180×10^{16}	0.186×10^{16}
0.300	0.405×10^{16}	0.435×10^{16}
0.400	0.720×10^{16}	0.820×10^{16}
0.500	1.13×10^{16}	1.39×10^{16}
0.600	1.62×10^{16}	2.25×10^{16}
0.700	2.21×10^{16}	3.60×10^{16}
0.800	2.88×10^{16}	6.00×10^{16}
0.900	3.65×10^{16}	11.6×10^{16}
0.990	4.41×10^{16}	54.8×10^{16}

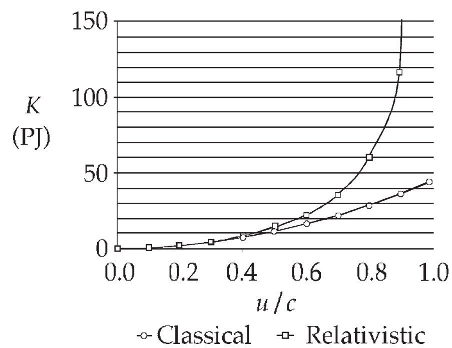


FIG. P39.66

$K_c = 0.990K_r$, when $\frac{1}{2}\left(\frac{u}{c}\right)^2 = 0.990\left[\frac{1}{\sqrt{1-(u/c)^2}} - 1\right]$, yielding $u = \boxed{0.115c}$.

Similarly, $K_c = 0.950K_r$ when $u = \boxed{0.257c}$

and $K_c = 0.500K_r$ when $u = \boxed{0.786c}$

P39.67 Since the total momentum is zero before decay, it is necessary that after the decay

$$p_{\text{nucleus}} = p_{\text{photon}} = \frac{E_\gamma}{c} = \frac{14.0\text{ keV}}{c}$$

Also, for the recoiling nucleus, $E^2 = p^2c^2 + \left(mc^2\right)^2$ with $mc^2 = 8.60 \times 10^{-9}\text{ J} = 53.8\text{ GeV}$.

Thus, $\left(mc^2 + K\right)^2 = (14.0\text{ keV})^2 + \left(mc^2\right)^2$ or $\left(1 + \frac{K}{mc^2}\right)^2 = \left(\frac{14.0\text{ keV}}{mc^2}\right)^2 + 1$

So $1 + \frac{K}{mc^2} = \sqrt{1 + \left(\frac{14.0\text{ keV}}{mc^2}\right)^2} \approx 1 + \frac{1}{2}\left(\frac{14.0\text{ keV}}{mc^2}\right)^2$ (Binomial Theorem)

and $K \approx \frac{(14.0\text{ keV})^2}{2mc^2} = \frac{\left(14.0 \times 10^3\text{ eV}\right)^2}{2\left(53.8 \times 10^9\text{ eV}\right)} = \boxed{1.82 \times 10^{-3}\text{ eV}}$

ANSWERS TO EVEN PROBLEMS

P39.2 See the solution.

P39.4 $0.866c$

P39.6 (a) 64.9/min (b) 10.6/min

P39.8 (a) 2.18 μs b) The muon sees the planet surface moving 649 m up toward it.

P39.10 (a) $\frac{cL_p}{\sqrt{c^2\Delta t^2 + L_p^2}}$ (b) 4.00 m/s (c) See the solution.

P39.12 $v = 0.140\,c$

P39.14 5.45 yr, Goslo is older

P39.16 (c) 2.00 kHz (d) ± 0.075 0 m/s ≈ 0.2 mi/h

P39.18 $0.220\,c = 6.59 \times 10^7$ m/s

P39.20 (a) 17.4 m (b) 3.30°

P39.22 (a) 2.50×10^8 m/s (b) 4.97 m (c) -1.33×10^{-8} s

P39.24 $0.357c$

P39.26 (a) $0.141c$ (b) $0.436c$

P39.28 (a) \$800 (b) \$$2.12 \times 10^9$

P39.30 (a) 3.07 MeV (b) $0.986c$

P39.32 See the solution.

P39.34 (a) 438 GJ (b) 438 GJ (c) The two kinetic energy values are experimentally indistinguishable, essentially identical. The fastest-moving macroscopic objects launched by human beings move sufficiently slowly compared to light that relativistic corrections to their energy are negligible.

P39.36 8.84×10^{-28} kg and 2.51×10^{-28} kg

P39.38 (a) $0.302c$ (b) 4.00 fJ

P39.40 (a) 3.91×10^{4} (b) $u = 0.999\,999\,999\,7\,c$ (c) 7.67 cm

P39.42 larger by ~10^{-9} J

P39.44 0.842 kg

P39.46 1.02 MeV

P39.48 (a) $\frac{v}{c} = 1 - 1.12 \times 10^{-10}$ (b) 6.00×10^{27} J (c) \$$2.17 \times 10^{20}$

P39.50 (a) $0.023\,6c$ (b) $6.18 \times 10^{-4}\,c$

P39.52 (a) $0.800\,c$ (b) 7.50 ks (c) 1.44 Tm, $-0.385\,c$ (d) 4.88 ks

P39.54 (a) $\frac{3.65\ \text{MeV}}{c^2}$ (b) $v = 0.589c$

P39.56 (a) 0.909 MeV (b) 0.398 MeV (c) $0.752\ \text{MeV}/c = 4.01 \times 10^{-22}\ \text{kg}\cdot\text{m/s}$ (d) 65.4°

P39.58 (c) $u \to c$

P39.60 1.47 km

P39.62 (a) $\frac{2d}{c+v}$ (b) $\frac{2d}{c}\sqrt{\frac{c-v}{c+v}}$

P39.64 See the solution.

P39.66 See the solution, $0.115c$, $0.257c$, $0.786c$.

40

Introduction to Quantum Physics

Note: In chapters 39, 40, and 41 we use u to represent the speed of a particle with mass, reserving v for the speeds associated with reference frames, wave functions, and photons.

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q40.1 The first flaw is that the Rayleigh–Jeans law predicts that the intensity of short wavelength radiation emitted by a blackbody approaches infinity as the wavelength decreases. This is known as the *ultraviolet catastrophe*. The second flaw is the prediction of much more power output from a blackbody than is shown experimentally. The intensity of radiation from the blackbody is given by the area under the red $I(\lambda, T)$ *vs.* λ curve in Figure 40.5 in the text, not by the area under the blue curve.

Planck's Law dealt with both of these issues and brought the theory into agreement with the experimental data by adding an exponential term to the denominator that depends on $\frac{1}{\lambda}$. This both keeps the predicted intensity from approaching infinity as the wavelength decreases and keeps the area under the curve finite.

Q40.2 Our eyes are not able to detect all frequencies of electromagnetic waves. For example, all objects that are above 0 K in temperature emit electromagnetic radiation in the infrared region. This describes *everything* in a dark room. We are only able to see objects that emit or reflect electromagnetic radiation in the visible portion of the spectrum.

***Q40.3** (i) The power input to the filament has increased by $8 \times 2 = 16$ times. The filament radiates this greater power according to Stefan's law, so its absolute temperature is higher by the fourth root of 16. It is two times higher. Answer (d).

(ii) Wien's displacement law then says that the wavelength emitted most strongly is half as large: answer (f).

Q40.4 No. The second metal may have a larger work function than the first, in which case the incident photons may not have enough energy to eject photoelectrons.

Q40.5 Comparing Equation 40.9 with the slope-intercept form of the equation for a straight line, $y = mx + b$, we see that the slope in Figure 40.11 in the text is Planck's constant h and that the y intercept is $-\phi$, the negative of the work function. If a different metal were used, the slope would remain the same but the work function would be different. Thus, data for different metals appear as parallel lines on the graph.

Q40.6 Wave theory predicts that the photoelectric effect should occur at any frequency, provided the light intensity is high enough. However, as seen in the photoelectric experiments, the light must have a sufficiently high frequency for the effect to occur.

Q40.7 The stopping voltage measures the kinetic energy of the most energetic photoelectrons. Each of them has gotten its energy from a single photon. According to Planck's $E = hf$, the photon energy depends on the frequency of the light. The intensity controls only the number of photons reaching a unit area in a unit time.

Q40.8 Ultraviolet light has shorter wavelength and higher photon energy than visible light.

***Q40.9** Answer (c). UV light has the highest frequency of the three, and hence each photon delivers more energy to a skin cell. This explains why you can become sunburned on a cloudy day: clouds block visible light and infrared, but not much ultraviolet. You usually do not become sunburned through window glass, even though you can see the visible light from the Sun coming through the window, because the glass absorbs much of the ultraviolet and reemits it as infrared.

Q40.10 The Compton effect describes the *scattering* of photons from electrons, while the photoelectric effect predicts the ejection of electrons due to the *absorption* of photons by a material.

***Q40.11** Answer (a). The x-ray photon transfers some of its energy to the electron. Thus, its frequency must decrease.

Q40.12 A few photons would only give a few dots of exposure, apparently randomly scattered.

***Q40.13** (i) a and c. Some people would say that electrons and protons possess mass and photons do not.

(ii) a and c (iii) a, b, and c (iv) a, b, and c (v) b (vi) a, b, and c

Q40.14 Light has both classical-wave and classical-particle characteristics. In single- and double-slit experiments light behaves like a wave. In the photoelectric effect light behaves like a particle. Light may be characterized as an electromagnetic wave with a particular wavelength or frequency, yet at the same time light may be characterized as a stream of photons, each carrying a discrete energy, hf. Since light displays *both* wave and particle characteristics, perhaps it would be fair to call light a "wavicle." It is customary to call a photon a *quantum particle*, different from a classical particle.

Q40.15 An electron has both classical-wave and classical-particle characteristics. In single- and double-slit diffraction and interference experiments, electrons behave like classical waves. An electron has mass and charge. It carries kinetic energy and momentum in parcels of definite size, as classical particles do. At the same time it has a particular wavelength and frequency. Since an electron displays characteristics of both classical waves and classical particles, it is neither a classical wave nor a classical particle. It is customary to call it a *quantum particle*, but another invented term, such as "wavicle," could serve equally well.

Q40.16 The discovery of electron diffraction by Davisson and Germer was a fundamental advance in our understanding of the motion of material particles. Newton's laws fail to properly describe the motion of an object with small mass. It moves as a wave, not as a classical particle. Proceeding from this recognition, the development of quantum mechanics made possible describing the motion of electrons in atoms; understanding molecular structure and the behavior of matter at the atomic scale, including electronics, photonics, and engineered materials; accounting for the motion of nucleons in nuclei; and studying elementary particles.

***Q40.17** Answer (a). If we set $\frac{p^2}{2m} = q\Delta V$, which is the same for both particles, then we see that the electron has the smaller momentum and therefore the longer wavelength $\left(\lambda = \frac{h}{p}\right)$.

Q40.18 Any object of macroscopic size—including a grain of dust—has an undetectably small wavelength and does not exhibit quantum behavior.

***Q40.19** The wavelength is described by $\lambda = h/p$ in all cases, so e, f, and g are all the same. For the photons the momentum is given by $p = E/c$, so a is also the same, and d has wavelength ten times larger. For the particles with mass, $pc = (E^2 - m^2c^4)^{1/2} = ([K + mc^2]^2 - m^2c^4)^{1/2} = (K^2 + 2Kmc^2)^{1/2}$. Thus a particle with larger mass has more momentum for the same kinetic energy, and a shorter wavelength. The ranking is then d > a = e = f = g > b > c.

Q40.20 The *intensity* of electron waves in some small region of space determines the *probability* that an electron will be found in that region.

Q40.21 The wavelength of violet light is on the order of $\frac{1}{2}$ μm, while the de Broglie wavelength of an electron can be 4 orders of magnitude smaller. Would your collar size be measured more precisely with an unruled meter stick or with one engraved with divisions down to $\frac{1}{10}$ mm?

***Q40.22** Answer (c). The proton has 1836 time more momentum, thus more momentum uncertainty, and thus possibly less position uncertainty.

Q40.23 The spacing between repeating structures on the surface of the feathers or scales is on the order of 1/2 the wavelength of light. An optical microscope would not have the resolution to see such fine detail, while an electron microscope can. The electrons can have much shorter wavelength.

Q40.24 (a) The slot is blacker than any black material or pigment. Any radiation going in through the hole will be absorbed by the walls or the contents of the box, perhaps after several reflections. Essentially none of that energy will come out through the hole again. Figure 40.1 in the text shows this effect if you imagine the beam getting weaker at each reflection.

(b) The open slots between the glowing tubes are brightest. When you look into a slot, you receive direct radiation emitted by the wall on the far side of a cavity enclosed by the fixture; and you also receive radiation that was emitted by other sections of the cavity wall and has bounced around a few or many times before escaping through the slot. In Figure 40.1 in the text, reverse all of the arrowheads and imagine the beam getting stronger at each reflection. Then the figure shows the extra efficiency of a cavity radiator. Here is the conclusion of Kirchhoff's thermodynamic argument: … energy radiated. A poor reflector—a good absorber—avoids rising in temperature by being an efficient emitter. Its emissivity is equal to its absorptivity: $e = a$. The slot in the box in part (a) of the question is a blackbody with reflectivity zero and absorptivity 1, so it must also be the most efficient possible radiator, to avoid rising in temperature above its surroundings in thermal equilibrium. Its emissivity in Stefan's law is $100\% = 1$, higher than perhaps 0.9 for black paper, 0.1 for light-colored paint, or 0.04 for shiny metal. Only in this way can the material objects underneath these different surfaces maintain equal temperatures after they come to thermal equilibrium and continue to exchange energy by electromagnetic radiation. By considering one blackbody facing another, Kirchhoff proved logically that the material forming the walls of the cavity made no difference to the radiation. By thinking about inserting color filters between two cavity radiators, he proved that the spectral distribution of blackbody radiation must be a universal function of wavelength, the same for all materials and depending only on the temperature. Blackbody radiation is a fundamental connection between the matter and the energy that physicists had previously studied separately.

SOLUTIONS TO PROBLEMS

Section 40.1 Blackbody Radiation and Planck's Hypothesis

P40.1 $T = \dfrac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{560 \times 10^{-9}\ \text{m}} = \boxed{5.18 \times 10^{3}\ \text{K}}$

***P40.2** (a) $\lambda_{\text{max}} = \dfrac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{2900\ \text{K}} = \boxed{999\ \text{nm}}$

(b) The wavelength emitted most strongly is infrared (greater than 700 nm), and much more energy is radiated at wavelengths longer than λ_{max} than at shorter wavelengths.

***P40.3** The peak radiation occurs at approximately 560 nm wavelength. From Wien's displacement law,

$$T = \frac{0.289\ 8 \times 10^{-2}\ \text{m} \cdot \text{K}}{\lambda_{\text{max}}} = \frac{0.289\ 8 \times 10^{-2}\ \text{m} \cdot \text{K}}{560 \times 10^{-9}\ \text{m}} \approx \boxed{5\ 200\ \text{K}}$$

Clearly, a firefly is not at this temperature, so $\boxed{\text{this is not blackbody radiation}}$.

P40.4 (a) $\lambda_{\text{max}} = \dfrac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{T} \sim \dfrac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{10^{4}\ \text{K}} \boxed{\sim 10^{-7}\ \text{m}}$ $\boxed{\text{ultraviolet}}$

(b) $\lambda_{\text{max}} \sim \dfrac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{10^{7}\ \text{K}} \boxed{\sim 10^{-10}\ \text{m}}$ $\boxed{\gamma\text{-ray}}$

P40.5 Planck's radiation law gives intensity-per-wavelength. Taking E to be the photon energy and n to be the number of photons emitted each second, we multiply by area and wavelength range to have energy-per-time leaving the hole:

$$\mathcal{P} = \frac{2\pi hc^2 (\lambda_2 - \lambda_1)\pi (d/2)^2}{\left[(\lambda_1 + \lambda_2)/2\right]^5 \left(e^{2hc/\left[(\lambda_1+\lambda_2)k_B T\right]} - 1\right)} = En = nhf \qquad \text{where} \qquad E = hf = \frac{2hc}{\lambda_1 + \lambda_2}$$

$$n = \frac{\mathcal{P}}{E} = \frac{8\pi^2 cd^2 (\lambda_2 - \lambda_1)}{(\lambda_1 + \lambda_2)^4 \left(e^{2hc/\left[(\lambda_1+\lambda_2)k_B T\right]} - 1\right)}$$

$$= \frac{8\pi^2 (3.00 \times 10^8\ \text{m/s})(5.00 \times 10^{-5}\ \text{m})^2 (1.00 \times 10^{-9}\ \text{m})}{(1\,001 \times 10^{-9}\ \text{m})^4 \left(e^{\left[2(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})\right]/\left[(1\,001\times10^{-9}\ \text{m})(1.38\times10^{-23}\ \text{J/K})(7.50\times10^{3}\ \text{K})\right]} - 1\right)}$$

$$n = \frac{5.90 \times 10^{16}/\text{s}}{(e^{3.84} - 1)} = \boxed{1.30 \times 10^{15}/\text{s}}$$

P40.6 (a) $\mathcal{P} = eA\sigma T^4 = 1(20.0\times10^{-4}\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(5\,000\ \text{K})^4 = \boxed{7.09\times10^4\ \text{W}}$

(b) $\lambda_{\max}T = \lambda_{\max}(5\,000\ \text{K}) = 2.898\times10^{-3}\ \text{m}\cdot\text{K} \Rightarrow \lambda_{\max} = \boxed{580\ \text{nm}}$

(c) We compute: $\dfrac{hc}{k_BT} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{(1.38\times10^{-23}\ \text{J/K})(5\,000\ \text{K})} = 2.88\times10^{-6}\ \text{m}$

The power per wavelength interval is $\mathcal{P}(\lambda) = AI(\lambda) = \dfrac{2\pi hc^2A}{\lambda^5\left[\exp(hc/\lambda k_BT)-1\right]}$, and

$$2\pi hc^2A = 2\pi(6.626\times10^{-34})(3.00\times10^8)^2(20.0\times10^{-4}) = 7.50\times10^{-19}\ \text{J}\cdot\text{m}^4/\text{s}$$

$$\mathcal{P}(580\ \text{nm}) = \frac{7.50\times10^{-19}\ \text{J}\cdot\text{m}^4/\text{s}}{(580\times10^{-9}\ \text{m})^5\left[\exp(2.88\ \mu\text{m}/0.580\ \mu\text{m})-1\right]} = \frac{1.15\times10^{13}\ \text{J/m}\cdot\text{s}}{e^{4.973}-1}$$

$$= \boxed{7.99\times10^{10}\ \text{W/m}}$$

(d)–(i) The other values are computed similarly:

	λ	$\dfrac{hc}{\lambda k_BT}$	$e^{hc/\lambda k_BT}-1$	$\dfrac{2\pi hc^2A}{\lambda^5}$	$\mathcal{P}(\lambda)$, W/m
(d)	1.00 nm	2882.6	7.96×10^{1251}	7.50×10^{26}	9.42×10^{-1226}
(e)	5.00 nm	576.5	2.40×10^{250}	2.40×10^{23}	1.00×10^{-227}
(f)	400 nm	7.21	1347	7.32×10^{13}	5.44×10^{10}
(c)	580 nm	4.97	143.5	1.15×10^{13}	7.99×10^{10}
(g)	700 nm	4.12	60.4	4.46×10^{12}	7.38×10^{10}
(h)	1.00 mm	0.00288	0.00289	7.50×10^{-4}	0.260
(i)	10.0 cm	2.88×10^{-5}	2.88×10^{-5}	7.50×10^{-14}	2.60×10^{-9}

(j) We approximate the area under the $\mathcal{P}(\lambda)$ versus λ curve, between 400 nm and 700 nm, as two trapezoids:

$$\mathcal{P} = \frac{\left[(5.44+7.99)\times10^{10}\ \text{W/m}\right]\left[(580-400)\times10^{-9}\ \text{m}\right]}{2} + \frac{\left[(7.99+7.38)\times10^{10}\ \text{W/m}\right]\left[(700-580)\times10^{-9}\ \text{m}\right]}{2}$$

$\mathcal{P} = 2.13\times10^4\ \text{W}$ so the power radiated as visible light is $\boxed{\text{approximately 20 kW}}$.

P40.7 (a) $\mathcal{P} = eA\sigma T^4$, so

$$T = \left(\frac{\mathcal{P}}{eA\sigma}\right)^{1/4} = \left[\frac{3.85\times10^{26}\ \text{W}}{1\left[4\pi(6.96\times10^8\ \text{m})^2\right](5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)}\right]^{1/4} = \boxed{5.78\times10^3\ \text{K}}$$

(b) $\lambda_{\max} = \dfrac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \dfrac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{5.78\times10^3\ \text{K}} = 5.01\times10^{-7}\ \text{m} = \boxed{501\ \text{nm}}$

P40.8 Energy of a single 500-nm photon:

$$E_\gamma = hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(500\times10^{-9}\ \text{m})} = 3.98\times10^{-19}\ \text{J}$$

The energy entering the eye each second

$$E = \mathcal{P}\Delta t = IA\Delta t = (4.00\times10^{-11}\ \text{W/m}^2)\left[\frac{\pi}{4}(8.50\times10^{-3}\ \text{m})^2\right](1.00\ \text{s}) = 2.27\times10^{-15}\ \text{J}$$

The number of photons required to yield this energy

$$n = \frac{E}{E_\gamma} = \frac{2.27\times10^{-15}\ \text{J}}{3.98\times10^{-19}\ \text{J/photon}} = \boxed{5.71\times10^{3}\ \text{photons}}$$

P40.9 (a) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(620\times10^{12}\ \text{s}^{-1})\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{2.57\ \text{eV}}$

(b) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.10\times10^{9}\ \text{s}^{-1})\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.28\times10^{-5}\ \text{eV}}$

(c) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(46.0\times10^{6}\ \text{s}^{-1})\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.91\times10^{-7}\ \text{eV}}$

(d) $\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{620\times10^{12}\ \text{Hz}} = 4.84\times10^{-7}\ \text{m} = \boxed{484\ \text{nm, visible light (blue)}}$

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{3.10\times10^{9}\ \text{Hz}} = 9.68\times10^{-2}\ \text{m} = \boxed{9.68\ \text{cm, radio wave}}$$

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{46.0\times10^{6}\ \text{Hz}} = \boxed{6.52\ \text{m, radio wave}}$$

P40.10 We take $\theta = 0.030\,0$ radians. Then the pendulum's total energy is

$$E = mgh = mg(L - L\cos\theta)$$

$$E = (1.00\ \text{kg})(9.80\ \text{m/s}^2)(1.00 - 0.999\,5) = 4.41\times10^{-3}\ \text{J}$$

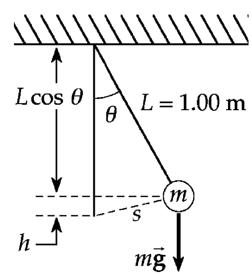


FIG. P40.10

The frequency of oscillation is $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = 0.498\ \text{Hz}$

The energy is quantized, $E = nhf$

Therefore,

$$n = \frac{E}{hf} = \frac{4.41\times10^{-3}\ \text{J}}{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(0.498\ \text{s}^{-1})}$$

$$= \boxed{1.34\times10^{31}}$$

P40.11 Each photon has an energy $E = hf = (6.626\times10^{-34})(99.7\times10^{6}) = 6.61\times10^{-26}\ \text{J}$

This implies that there are $\frac{150\times10^{3}\ \text{J/s}}{6.61\times10^{-26}\ \text{J/photon}} = \boxed{2.27\times10^{30}\ \text{photons/s}}$

***P40.12** (a) $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.02\text{ m})^3 = 3.35\times10^{-5}\text{ m}^3$

$$m = \rho V = 7.86\times10^3\ \text{kg/m}^3\ 3.35\times10^{-5}\ \text{m}^3 = \boxed{0.263\text{ kg}}$$

(b) $A = 4\pi r^2 = 4\pi(0.02\text{ m})^2 = 5.03\times10^{-3}\text{ m}^2$

$$\mathcal{P} = \sigma AeT^4 = \left(5.67\times10^{-8}\ \text{W/m}^2\,\text{K}^4\right)\ 5.03\times10^{-3}\ \text{m}^2\ 0.86(293\text{ K})^4 = \boxed{1.81\text{ W}}$$

(c) It emits but does not absorb radiation, so its temperature must drop according to

$$Q = mc\Delta T = mc\left(T_f - T_i\right) \qquad \frac{dQ}{dt} = mc\frac{dT_f}{dt}$$

$$\frac{dT_f}{dt} = \frac{dQ/dt}{mc} = \frac{-\mathcal{P}}{mc} = \frac{-1.81\ \text{J/s}}{0.263\text{ kg}\ 448\ \text{J/kg}\cdot\text{C}^\circ} = \boxed{-0.015\ 3\,^\circ\text{C/s}} = -0.919\,^\circ\text{C/min}$$

(d) $\lambda_{\max}T = 2.898\times10^{-3}\ \text{m}\cdot\text{K}$

$$\lambda_{\max} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{293\text{ K}} = \boxed{9.89\times10^{-6}\text{ m}}\ \text{infrared}$$

(e) $E = hf = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}\ 3\times10^8\ \text{m/s}}{9.89\times10^{-6}\text{ m}} = \boxed{2.01\times10^{-20}\text{ J}}$

(f) The energy output each second is carried by photons according to

$$\mathcal{P} = \left(\frac{N}{\Delta t}\right)E$$

$$\frac{N}{\Delta t} = \frac{\mathcal{P}}{E} = \frac{1.81\ \text{J/s}}{2.01\times10^{-20}\ \text{J/photon}} = \boxed{8.98\times10^{19}\ \text{photon/s}}$$

Matter is coupled to radiation quite strongly, in terms of photon numbers.

P40.13 Planck's radiation law is $I(\lambda, T) = \dfrac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda k_B T} - 1\right)}$

Using the series expansion $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots$

Planck's law reduces to $I(\lambda, T) = \dfrac{2\pi hc^2}{\lambda^5\left[\left(1 + hc/\lambda k_B T + \cdots\right) - 1\right]} \approx \dfrac{2\pi hc^2}{\lambda^5\left(hc/\lambda k_B T\right)} = \dfrac{2\pi ck_B T}{\lambda^4}$

which is the Rayleigh-Jeans law, for very long wavelengths.

Section 40.2 **The Photoelectric Effect**

P40.14 (a) $\lambda_c = \dfrac{hc}{\phi} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(4.20\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = \boxed{296\text{ nm}}$

$f_c = \dfrac{c}{\lambda_c} = \dfrac{3.00\times10^{8}\text{ m/s}}{296\times10^{-9}\text{ m}} = \boxed{1.01\times10^{15}\text{ Hz}}$

(b) $\dfrac{hc}{\lambda} = \phi + e\Delta V_S$: $\dfrac{(6.626\times10^{-34})(3.00\times10^{8})}{180\times10^{-9}} = (4.20\text{ eV})(1.60\times10^{-19}\text{ J/eV}) + (1.60\times10^{-19})\Delta V_S$

Therefore, $\boxed{\Delta V_S = 2.71\text{ V}}$

P40.15 (a) $e\Delta V_S = \dfrac{hc}{\lambda} - \phi \to \phi = \dfrac{1\,240\text{ nm}\cdot\text{eV}}{546.1\text{ nm}} - 0.376\text{ eV} = \boxed{1.90\text{ eV}}$

(b) $e\Delta V_S = \dfrac{hc}{\lambda} - \phi = \dfrac{1\,240\text{ nm}\cdot\text{eV}}{587.5\text{ nm}} - 1.90\text{ eV} \to \boxed{\Delta V_S = 0.216\text{ V}}$

P40.16 $K_{max} = \dfrac{1}{2}mu_{max}^2 = \dfrac{1}{2}(9.11\times10^{-31})(4.60\times10^{5})^2 = 9.64\times10^{-20}\text{ J} = 0.602\text{ eV}$

(a) $\phi = E - K_{max} = \dfrac{1\,240\text{ eV}\cdot\text{nm}}{625\text{ nm}} - 0.602\text{ nm} = \boxed{1.38\text{ eV}}$

(b) $f_c = \dfrac{\phi}{h} = \dfrac{1.38\text{ eV}}{6.626\times10^{-34}\text{ J}\cdot\text{s}}\left(\dfrac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right) = \boxed{3.34\times10^{14}\text{ Hz}}$

***P40.17** (a) We could find the energy of a photon with wavelength 400 nm and check whether it exceeds the work function. But we can instead use $\lambda_c = \dfrac{hc}{\phi}$ to find the threshold wavelength for each sample:

Li: $\lambda_c = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(2.30\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 540\text{ nm}$

Be: $\lambda_c = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(3.90\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 319\text{ nm}$

Hg: $\lambda_c = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(4.50\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 276\text{ nm}$

We must have $\lambda < \lambda_c$ for photocurrent to exist.

Thus, only lithium will exhibit the photoelectric effect.

(b) For lithium, $\dfrac{hc}{\lambda} = \phi + K_{max}$

$$\frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{400\times10^{-9}\text{ m}} = (2.30\text{ eV})(1.60\times10^{-19}) + K_{max}$$

$K_{max} = 1.29\times10^{-19}\text{ J} = \boxed{0.806\text{ eV}}$

P40.18 The energy needed is $E = 1.00\text{ eV} = 1.60\times10^{-19}\text{ J}$

The energy absorbed in time interval Δt is $E = \mathcal{P}\Delta t = IA\Delta t$

so $$\Delta t = \frac{E}{IA} = \frac{1.60\times10^{-19}\text{ J}}{(500\text{ J/s}\cdot\text{m}^2)\left[\pi(2.82\times10^{-15}\text{ m})^2\right]} = 1.28\times10^7\text{ s} = \boxed{148\text{ days}}$$

The success of quantum mechanics contrasts with the gross failure of the classical theory of the photoelectric effect.

P40.19 Ultraviolet photons will be absorbed to knock electrons out of the sphere with maximum kinetic energy $K_{\max} = hf - \phi$,

or $$K_{\max} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{200\times10^{-9}\text{ m}}\left(\frac{1.00\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) - 4.70\text{ eV} = 1.51\text{ eV}$$

The sphere is left with positive charge and so with positive potential relative to $V = 0$ at $r = \infty$. As its potential approaches 1.51 V, no further electrons will be able to escape, but will fall back onto the sphere. Its charge is then given by

$$V = \frac{k_e Q}{r} \quad\text{or}\quad Q = \frac{rV}{k_e} = \frac{(5.00\times10^{-2}\text{ m})(1.51\text{ N}\cdot\text{m/C})}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = \boxed{8.41\times10^{-12}\text{ C}}$$

P40.20 (a) By having the photon source move toward the metal, the incident photons are Doppler shifted to higher frequencies, and hence, higher energy.

(b) If $v = 0.280c$, $$f' = f\sqrt{\frac{1+v/c}{1-v/c}} = (7.00\times10^{14})\sqrt{\frac{1.28}{0.720}} = 9.33\times10^{14}\text{ Hz}$$

Therefore, $$\phi = (6.626\times10^{-34}\text{ J}\cdot\text{s})(9.33\times10^{14}\text{ Hz}) = 6.18\times10^{-19}\text{ J} = \boxed{3.87\text{ eV}}$$

(c) At $v = 0.900c$, $f = 3.05\times10^{15}\text{ Hz}$

and $$K_{\max} = hf - \phi = (6.626\times10^{-34}\text{ J}\cdot\text{s})(3.05\times10^{15}\text{ Hz})\left(\frac{1.00\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) - 3.87\text{ eV}$$

$$= \boxed{8.78\text{ eV}}$$

Section 40.3 **The Compton Effect**

P40.21 $$E = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{700\times10^{-9}\text{ m}} = 2.84\times10^{-19}\text{ J} = \boxed{1.78\text{ eV}}$$

$$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{700\times10^{-9}\text{ m}} = \boxed{9.47\times10^{-28}\text{ kg}\cdot\text{m/s}}$$

***P40.22** (a) and (b) From $\Delta\lambda = \dfrac{h}{m_e c}(1-\cos\theta)$ we calculate the wavelength of the scattered photon. For example, at $\theta = 30°$ we have

$$\lambda' + \Delta\lambda = 120\times10^{-12} + \frac{6.626\times10^{-34}}{(9.11\times10^{-31})(3.00\times10^{8})}(1-\cos 30.0°) = \boxed{120.3\times10^{-12}\ \text{m}}$$

The electron carries off the energy the photon loses:

$$K_e = \frac{hc}{\lambda_0} - \frac{hc}{\lambda'} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}\ 3\times10^{8}\ \text{m}}{(1.6\times10^{-19}\ \text{J/eV})\ \text{s}\ 10^{-12}\ \text{m}}\left(\frac{1}{120} - \frac{1}{120.3}\right) = 27.9\ \text{eV}$$

The other entries are computed similarly.

θ, degrees	0	30	60	90	120	150	180
λ', pm	120.0	120.3	121.2	122.4	123.6	124.5	124.8
K_e, eV	0	27.9	104	205	305	376	402

(c) 180°. We could answer like this: The photon imparts the greatest momentum to the originally stationary electron in a head-on collision. Here the photon recoils straight back and the electron has maximum kinetic energy.

P40.23 With $K_e = E'$, $K_e = E_0 - E'$ gives $E' = E_0 - E'$

$E' = \dfrac{E_0}{2}$ and $\lambda' = \dfrac{hc}{E'}$ $\qquad \lambda' = \dfrac{hc}{E_0/2} = 2\dfrac{hc}{E_0} = 2\lambda_0$

$\lambda' = \lambda_0 + \lambda_C(1-\cos\theta)$ $\qquad 2\lambda_0 = \lambda_0 + \lambda_C(1-\cos\theta)$

$$1-\cos\theta = \frac{\lambda_0}{\lambda_C} = \frac{0.001\,60}{0.002\,43} \qquad \theta = \boxed{70.0°}$$

P40.24 (a) $\Delta\lambda = \dfrac{h}{m_e c}(1-\cos\theta)$: $\Delta\lambda = \dfrac{6.626\times10^{-34}}{(9.11\times10^{-31})(3.00\times10^{8})}(1-\cos 37.0°) = \boxed{4.88\times10^{-13}\ \text{m}}$

(b) $E_0 = \dfrac{hc}{\lambda_0}$: $\qquad (300\times10^{3}\ \text{eV})(1.60\times10^{-19}\ \text{J/eV}) = \dfrac{(6.626\times10^{-34})(3.00\times10^{8}\ \text{m/s})}{\lambda_0}$

$$\lambda_0 = 4.14\times10^{-12}\ \text{m}$$

and $\qquad \lambda' = \lambda_0 + \Delta\lambda = 4.63\times10^{-12}\ \text{m}$

$$E' = \frac{hc}{\lambda'} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{4.63\times10^{-12}\ \text{m}} = 4.30\times10^{-14}\ \text{J} = \boxed{268\ \text{keV}}$$

(c) $K_e = E_0 - E' = 300\ \text{keV} - 268.5\ \text{keV} = \boxed{31.5\ \text{keV}}$

P40.25 (a) Conservation of momentum in the x direction gives: $p_\gamma = p'_\gamma \cos\theta + p_e \cos\phi$

or since $\theta = \phi$, $\dfrac{h}{\lambda_0} = \left(p_e + \dfrac{h}{\lambda'}\right)\cos\theta$ [1]

Conservation of momentum in the y direction gives: $0 = p'_\gamma \sin\theta - p_e \sin\theta$

which (neglecting the trivial solution $\theta = 0$) gives: $p_e = p'_\gamma = \dfrac{h}{\lambda'}$ [2]

Substituting [2] into [1] gives: $\dfrac{h}{\lambda_0} = \dfrac{2h}{\lambda'}\cos\theta$, or $\lambda' = 2\lambda_0 \cos\theta$ [3]

Then the Compton equation is $\lambda' - \lambda_0 = \dfrac{h}{m_e c}(1-\cos\theta)$

giving $2\lambda_0 \cos\theta - \lambda_0 = \dfrac{h}{m_e c}(1-\cos\theta)$

or $2\cos\theta - 1 = \dfrac{hc}{\lambda_0}\dfrac{1}{m_e c^2}(1-\cos\theta)$

Since $E_\gamma = \dfrac{hc}{\lambda_0}$, this may be written as: $2\cos\theta - 1 = \left(\dfrac{E_\gamma}{m_e c^2}\right)(1-\cos\theta)$

which reduces to: $\left(2 + \dfrac{E_\gamma}{m_e c^2}\right)\cos\theta = 1 + \dfrac{E_\gamma}{m_e c^2}$

or $\cos\theta = \dfrac{m_e c^2 + E_\gamma}{2m_e c^2 + E_\gamma} = \dfrac{0.511\text{ MeV} + 0.880\text{ MeV}}{1.02\text{ MeV} + 0.880\text{ MeV}} = 0.732$ so that $\boxed{\theta = \phi = 43.0°}$

(b) Using Equation (3): $E'_\gamma = \dfrac{hc}{\lambda'} = \dfrac{hc}{\lambda_0(2\cos\theta)} = \dfrac{E_\gamma}{2\cos\theta} = \dfrac{0.880\text{ MeV}}{2\cos 43.0°}$

$= 0.602\text{ MeV} = \boxed{602\text{ keV}}$

Then, $p'_\gamma = \dfrac{E'_\gamma}{c} = \dfrac{0.602\text{ MeV}}{c} = \boxed{3.21\times 10^{-22}\ \text{kg}\cdot\text{m/s}}$

(c) From Equation (2), $p_e = p'_\gamma = \dfrac{0.602\text{ MeV}}{c} = \boxed{3.21\times 10^{-22}\ \text{kg}\cdot\text{m/s}}$.

From energy conservation:

$K_e = E_\gamma - E'_\gamma = 0.880\text{ MeV} - 0.602\text{ MeV} = 0.278\text{ MeV} = \boxed{278\text{ keV}}$

P40.26 The energy of the incident photon is $E_0 = p_\gamma c = \dfrac{hc}{\lambda_0}$.

(a) Conserving momentum in the x direction gives

$$p_\lambda = p_e \cos\phi + p'_\gamma \cos\theta, \text{ or since } \phi = \theta, \ \frac{E_0}{c} = \left(p_e + p'_\gamma\right)\cos\theta \qquad [1]$$

Conserving momentum in the y direction (with $\phi = \theta$) yields

$$0 = p'_\gamma \sin\theta - p_e \sin\theta, \text{ or } p_e = p'_\gamma = \frac{h}{\lambda'} \qquad [2]$$

Substituting Equation [2] into Equation [1] gives

$$\frac{E_0}{c} = \left(\frac{h}{\lambda'} + \frac{h}{\lambda'}\right)\cos\theta, \text{ or } \lambda' = \frac{2hc}{E_0}\cos\theta \qquad [3]$$

By the Compton equation, $\lambda' - \lambda_0 = \dfrac{h}{m_e c}(1-\cos\theta)$, $\quad \dfrac{2hc}{E_0}\cos\theta - \dfrac{2hc}{E_0} = \dfrac{h}{m_e c}(1-\cos\theta)$

which reduces to $\quad \left(2m_e c^2 + E_0\right)\cos\theta = m_e c^2 + E_0$

Thus, $\quad \boxed{\phi = \theta = \cos^{-1}\left(\dfrac{m_e c^2 + E_0}{2m_e c^2 + E_0}\right)}$

(b) From Equation [3], $\quad \lambda' = \dfrac{2hc}{E_0}\cos\theta = \dfrac{2hc}{E_0}\left(\dfrac{m_e c^2 + E_0}{2m_e c^2 + E_0}\right)$

Therefore, $\quad E'_\gamma = \dfrac{hc}{\lambda'} = \dfrac{hc}{\left(2hc/E_0\right)\left(m_e c^2 + E_0\right)/\left(2m_e c^2 + E_0\right)}$

$$= \boxed{\frac{E_0}{2}\left(\frac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$$

and $\quad p'_\gamma = \dfrac{E'_\gamma}{c} = \boxed{\dfrac{E_0}{2c}\left(\dfrac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$

(c) From conservation of energy, $\quad K_e = E_0 - E'_\gamma = E_0 - \dfrac{E_0}{2}\left(\dfrac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)$

or $\quad K_e = \dfrac{E_0}{2}\left(\dfrac{2m_e c^2 + 2E_0 - 2m_e c^2 - E_0}{m_e c^2 + E_0}\right) = \boxed{\dfrac{E_0^2}{2\left(m_e c^2 + E_0\right)}}$

Finally, from Equation (2), $p_e = p'_\gamma = \boxed{\dfrac{E_0}{2c}\left(\dfrac{2m_e c^2 + E_0}{m_e c^2 + E_0}\right)}$.

***P40.27** (a) $K = \frac{1}{2}m_e u^2$: $K = \frac{1}{2}\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.40\times10^{6}\ \text{m/s}\right)^2 = 8.93\times10^{-19}\ \text{J} = 5.58\ \text{eV}$

$$E_0 = \frac{hc}{\lambda_0} = \frac{1\,240\ \text{eV}\cdot\text{nm}}{0.800\ \text{nm}} = 1\,550\ \text{eV}$$

$E' = E_0 - K$ and $\lambda' = \frac{hc}{E'} = \frac{1\,240\ \text{eV}\cdot\text{nm}}{1\,550\ \text{eV} - 5.58\ \text{eV}} = 0.803\ \text{nm}$

$$\Delta\lambda = \lambda' - \lambda_0 = 0.002\,89\ \text{nm} = \boxed{2.89\ \text{pm}}$$

(b) $\Delta\lambda = \lambda_C\left(1-\cos\theta\right)$: $\cos\theta = 1 - \frac{\Delta\lambda}{\lambda_C} = 1 - \frac{0.002\,89\ \text{nm}}{0.002\,43\ \text{nm}} = -0.189$

so $\boxed{\theta = 101°}$

P40.28 The electron's kinetic energy is

$$K = \frac{1}{2}mu^2 = \frac{1}{2}9.11\times10^{-31}\ \text{kg}\left(2.18\times10^{6}\ \text{m/s}\right)^2 = 2.16\times10^{-18}\ \text{J}$$

This is the energy lost by the photon, $hf_0 - hf'$

$\frac{hc}{\lambda_0} - \frac{hc}{\lambda'} = 2.16\times10^{-18}\ \text{J}$. We also have

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1-\cos\theta) = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}\cdot\text{s}}{9.11\times10^{-31}\ \text{kg}\left(3\times10^{8}\ \text{m}\right)}(1-\cos 17.4°\ )$$

$$\lambda' = \lambda_0 + 1.11\times10^{-13}\ \text{m}$$

(a) Combining the equations by substitution,

$$\frac{1}{\lambda_0} - \frac{1}{\lambda_0 + 0.111\ \text{pm}} = \frac{2.16\times10^{-18}\ \text{J}\cdot\text{s}}{6.63\times10^{-34}\ \text{J}\cdot\text{s}\left(3\times10^{8}\ \text{m}\right)} = 1.09\times10^{7}/\text{m}$$

$$\frac{\lambda_0 + 0.111\ \text{pm} - \lambda_0}{\lambda_0^2 + \lambda_0\left(0.111\ \text{pm}\right)} = 1.09\times10^{7}/\text{m}$$

$$0.111\ \text{pm} = \left(1.09\times10^{7}/\text{m}\right)\lambda_0^2 + 1.21\times10^{-6}\lambda_0$$

$$1.09\times10^{7}\lambda_0^2 + 1.21\times10^{-6}\ \text{m}\lambda_0 - 1.11\times10^{-13}\ \text{m}^2 = 0$$

$$\lambda_0 = \frac{-1.21\times10^{-6}\ \text{m} \pm \sqrt{\left(1.21\times10^{-6}\ \text{m}\right)^2 - 4\left(1.09\times10^{7}\right)\left(-1.11\times10^{-13}\ \text{m}^2\right)}}{2\left(1.09\times10^{7}\right)}$$

only the positive answer is physical: $\lambda_0 = \boxed{1.01\times10^{-10}\ \text{m}}$.

(b) Then $\lambda' = 1.01\times10^{-10}\ \text{m} + 1.11\times10^{-13}\ \text{m} = 1.01\times10^{-10}\ \text{m}$.
Conservation of momentum in the transverse direction:

$$0 = \frac{h}{\lambda'}\sin\theta - \gamma m_e u \sin\phi$$

$$\frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{1.01\times10^{-10}\ \text{m}}\sin 17.4° = \frac{9.11\times10^{-31}\ \text{kg}\left(2.18\times10^{6}\ \text{m/s}\right)\sin\phi}{\sqrt{1-\left(2.18\times10^{6}/3\times10^{8}\right)^2}}$$

$1.96\times10^{-24} = 1.99\times10^{-24}\sin\phi$ $\phi = \boxed{81.1°}$

***P40.29** It is, because Compton's equation and the conservation of vector momentum give three independent equations in the unknowns λ', λ_0, and u. They are

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1-\cos 90°) \qquad \text{so } \lambda' = \lambda_0 + \frac{h}{m_e c}$$

$$\frac{h}{\lambda_0} = \gamma m_e u \cos 20°$$

$$\frac{h}{\lambda'} = \gamma m_e u \sin 20°$$

Dividing the latter two equations gives $\frac{\lambda_0}{\lambda'} = \tan 20°$ Then substituting, $\lambda' = \lambda' \tan 20° + \frac{h}{m_e c}$

So $\lambda' = 2.43 \times 10^{-12}$ m/(1 – tan 20°) = $\boxed{3.82 \text{ pm}}$

P40.30 $\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$

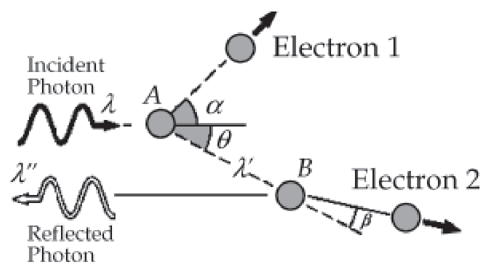


FIG. P40.30

$$\lambda'' - \lambda' = \frac{h}{m_e c}[1-\cos(\pi-\theta)]$$

$$\lambda'' - \lambda = \frac{h}{m_e c} - \frac{h}{m_e c}\cos(\pi-\theta) + \frac{h}{m_e c} - \frac{h}{m_e c}\cos\theta$$

Now $\cos(\pi-\theta) = -\cos\theta$, so

$$\lambda'' - \lambda = 2\frac{h}{m_e c} = \boxed{0.004\,86 \text{ nm}}$$

P40.31 Maximum energy loss appears as maximum increase in wavelength, which occurs for scattering angle 180°. Then $\Delta\lambda = (1-\cos 180°)\left(\frac{h}{mc}\right) = \frac{2h}{mc}$ where m is the mass of the target particle. The fractional energy loss is

$$\frac{E_0 - E'}{E_0} = \frac{hc/\lambda_0 - hc/\lambda'}{hc/\lambda_0} = \frac{\lambda' - \lambda_0}{\lambda'} = \frac{\Delta\lambda}{\lambda_0 + \Delta\lambda} = \frac{2h/mc}{\lambda_0 + 2h/mc}$$

Further, $\lambda_0 = \frac{hc}{E_0}$, so $\frac{E_0 - E'}{E_0} = \frac{2h/mc}{hc/E_0 + 2h/mc} = \frac{2E_0}{mc^2 + 2E_0}$.

(a) For scattering from a free electron, $mc^2 = 0.511$ MeV, so

$$\frac{E_0 - E'}{E_0} = \frac{2(0.511 \text{ MeV})}{0.511 \text{ MeV} + 2(0.511 \text{ MeV})} = \boxed{0.667}$$

(b) For scattering from a free proton, $mc^2 = 938$ MeV, and

$$\frac{E_0 - E'}{E_0} = \frac{2(0.511 \text{ MeV})}{938 \text{ MeV} + 2(0.511 \text{ MeV})} = \boxed{0.001\,09}$$

Section 40.4 **Photons and Electromagnetic Waves**

***P40.32** With photon energy $10.0\text{ eV} = hf$ a photon would have

$$f = \frac{10.0\left(1.6\times10^{-19}\text{ J}\right)}{6.63\times10^{-34}\text{ J}\cdot\text{s}} = 2.41\times10^{15}\text{ Hz and } \lambda = c/f = (3\times10^{8}\text{ m/s})/(2.41\times10^{15}/\text{s}) = 124\text{ nm}$$

> To have photon energy 10 eV or greater, according to this definition, ionizing radiation is the ultraviolet light, x-rays, and γ rays with wavelength shorter than 124 nm; that is, with frequency higher than 2.41×10^{15} Hz.

P40.33 The photon energy is $E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}\left(3\times10^{8}\text{ m/s}\right)}{633\times10^{-9}\text{ m}} = 3.14\times10^{-19}\text{ J}$. The power carried by the beam is $\left(2\times10^{18}\text{ photons/s}\right)\left(3.14\times10^{-19}\text{ J/photon}\right) = 0.628\text{ W}$. Its intensity is the average Poynting vector $I = S_{\text{av}} = \frac{\mathcal{P}}{\pi r^2} = \frac{0.628\text{ W}(4)}{\pi\left(1.75\times10^{-3}\text{ m}\right)^2} = 2.61\times10^{5}\text{ W/m}^2$.

(a) $S_{\text{av}} = \frac{1}{\mu_0}E_{\text{rms}}B_{\text{rms}}\sin 90° = \frac{1}{\mu_0}\frac{E_{\text{max}}}{\sqrt{2}}\frac{B_{\text{max}}}{\sqrt{2}}$. Also $E_{\text{max}} = B_{\text{max}}c$. So $S_{\text{av}} = \frac{E_{\text{max}}^2}{2\mu_0 c}$.

$$E_{\text{max}} = \left(2\mu_0 c S_{\text{av}}\right)^{1/2} = \left(2\left(4\pi\times10^{-7}\text{ Tm/A}\right)\left(3\times10^{8}\text{ m/s}\right)\left(2.61\times10^{5}\text{ W/m}^2\right)\right)^{1/2}$$

$$= \boxed{1.40\times10^{4}\text{ N/C}}$$

$$B_{\text{max}} = \frac{1.40\times10^{4}\text{ N/C}}{3\times10^{8}\text{ m/s}} = \boxed{4.68\times10^{-5}\text{ T}}$$

(b) Each photon carries momentum $\frac{E}{c}$. The beam transports momentum at the rate $\frac{\mathcal{P}}{c}$.

It imparts momentum to a perfectly reflecting surface at the rate

$$\frac{2\mathcal{P}}{c} = \text{force} = \frac{2(0.628\text{ W})}{3\times10^{8}\text{ m/s}} = \boxed{4.19\times10^{-9}\text{ N}}$$

(c) The block of ice absorbs energy $mL = \mathcal{P}\Delta t$ melting

$$m = \frac{\mathcal{P}\Delta t}{L} = \frac{0.628\text{ W}\left(1.5\times3\,600\text{ s}\right)}{3.33\times10^{5}\text{ J/kg}} = \boxed{1.02\times10^{-2}\text{ kg}}$$

Section 40.5 **The Wave Properties of Particles**

P40.34 $\lambda = \frac{h}{p} = \frac{h}{mu} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(1.67\times10^{-27}\ \text{kg})(1.00\times10^{6}\ \text{m/s})} = \boxed{3.97\times10^{-13}\ \text{m}}$

P40.35 (a) $\frac{p^2}{2m} = (50.0)(1.60\times10^{-19}\ \text{J})$

$p = 3.81\times10^{-24}\ \text{kg}\cdot\text{m/s}$

$\lambda = \frac{h}{p} = \boxed{0.174\ \text{nm}}$

(b) $\frac{p^2}{2m} = (50.0\times10^{3})(1.60\times10^{-19}\ \text{J})$

$p = 1.20\times10^{-22}\ \text{kg}\cdot\text{m/s}$

$\lambda = \frac{h}{p} = 5.49\times10^{-12}\ \text{m}$

The relativistic answer is slightly more precise:

$$\lambda = \frac{h}{p} = \frac{hc}{\left[(mc^2+K)^2 - m^2c^4\right]^{1/2}} = \boxed{5.37\times10^{-12}\ \text{m}}$$

P40.36 (a) Electron: $\lambda = \frac{h}{p}$ and $K = \frac{1}{2}m_e u^2 = \frac{m_e^2u^2}{2m_e} = \frac{p^2}{2m_e}$ so $p = \sqrt{2m_eK}$

and $\lambda = \frac{h}{\sqrt{2m_eK}} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(9.11\times10^{-31}\ \text{kg})(3.00)(1.60\times10^{-19}\ \text{J})}}$

$\lambda = 7.09\times10^{-10}\ \text{m} = \boxed{0.709\ \text{nm}}$

(b) Photon: $\lambda = \frac{c}{f}$ and $E = hf$ so $f = \frac{E}{h}$

and $\lambda = \frac{hc}{E} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{3(1.60\times10^{-19}\ \text{J})} = 4.14\times10^{-7}\ \text{m} = \boxed{414\ \text{nm}}$

***P40.37** (a) $\lambda \sim 10^{-14}$ m or less. $p = \frac{h}{\lambda} \sim \frac{6.6\times10^{-34}\ \text{J}\cdot\text{s}}{10^{-14}\ \text{m}} = 10^{-19}\ \text{kg}\cdot\text{m/s}$ or more

The energy of the electron is $E = \sqrt{p^2c^2 + m_e^2c^4} \sim \sqrt{(10^{-19})^2(3\times10^{8})^2 + (9\times10^{-31})^2(3\times10^{8})^4}$

or $E \sim 10^{-11}\ \text{J} \sim 10^{8}\ \text{eV}$ or more

so that $K = E - m_ec^2 \sim 10^{8}\ \text{eV} - (0.5\times10^{6}\ \text{eV})\ \boxed{\sim 10^{8}\ \text{eV}}$ or more

(b) If the nucleus contains ten protons, the electric potential energy of the electron-nucleus system would be

$$U_e = \frac{k_eq_1q_2}{r} \sim \frac{(9\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(10\times10^{-19}\ \text{C})(-e)}{5\times10^{-15}\ \text{m}}\ \boxed{\sim -10^{6}\ \text{eV}}$$

With its $K + U_e >> 0$ $\boxed{\text{the electron would immediately escape the nucleus}}$.

P40.38 From the condition for Bragg reflection,

$$m\lambda = 2d\sin\theta = 2d\cos\left(\frac{\phi}{2}\right)$$

But $\quad d = a\sin\left(\frac{\phi}{2}\right)$

where a is the lattice spacing.

Thus, with $m=1$, $\qquad \lambda = 2a\sin\left(\frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right) = a\sin\phi$

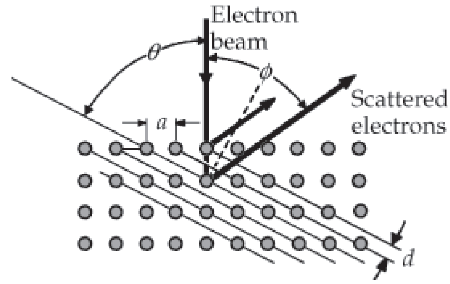


FIG. P40.38

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_eK}} \qquad \lambda = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2\left(9.11\times10^{-31}\ \text{kg}\right)\left(54.0\times1.60\times10^{-19}\ \text{J}\right)}} = 1.67\times10^{-10}\ \text{m}$$

Therefore, the lattice spacing is $a = \dfrac{\lambda}{\sin\phi} = \dfrac{1.67\times10^{-10}\ \text{m}}{\sin 50.0°} = 2.18\times10^{-10} = \boxed{0.218\ \text{nm}}$.

P40.39 (a) $E^2 = p^2c^2 + m^2c^4$

with $E = hf$, $\qquad p = \dfrac{h}{\lambda} \qquad$ and $\qquad mc = \dfrac{h}{\lambda_C}$

so $\qquad h^2f^2 = \dfrac{h^2c^2}{\lambda^2} + \dfrac{h^2c^2}{\lambda_C^2} \qquad$ and $\qquad \left(\dfrac{f}{c}\right)^2 = \dfrac{1}{\lambda^2} + \dfrac{1}{\lambda_C^2} \qquad$ (Eq. 1)

(b) For a photon $\qquad \dfrac{f}{c} = \dfrac{1}{\lambda}$

The third term $\dfrac{1}{\lambda_C}$ in Equation 1 for electrons and other massive particles shows that $\boxed{\text{they will always have a different frequency from photons of the same wavelength}}$.

***P40.40** (a) For the massive particle, $K = (\gamma-1)mc^2$ and $\lambda_m = \dfrac{h}{p} = \dfrac{h}{\gamma mu}$. For the photon (which we represent as γ), $E = K$ and $\lambda_\gamma = \dfrac{c}{f} = \dfrac{ch}{E} = \dfrac{ch}{K} = \dfrac{ch}{(\gamma-1)mc^2}$. Then the ratio is

$$\frac{\lambda_\gamma}{\lambda_m} = \frac{ch\gamma\,mu}{(\gamma-1)mc^2h} = \boxed{\frac{\gamma}{\gamma-1}\frac{u}{c} \qquad \text{where } \gamma = \frac{1}{\sqrt{1-u^2/c^2}}}$$

. The ratio can be written

$$\frac{u}{c\left(1-\sqrt{1-u^2/c^2}\right)}$$

(b) $\dfrac{\lambda_\gamma}{\lambda_m} = \dfrac{1(0.9)}{\sqrt{1-0.9^2}\left[\left(1/\sqrt{1-0.9^2}\right)-1\right]} = \boxed{1.60}$

(c) The ratio for a particular particle speed does not depend on the particle mass: There would be no change.

(d) $\dfrac{\lambda_\gamma}{\lambda_m} = \dfrac{1(0.001)}{\sqrt{1-(0.001)^2}\left[\left(1/\sqrt{1-(0.001)^2}\right)-1\right]} = \boxed{2.00\times10^3}$

(e) As $\dfrac{u}{c}\to 1$, $\gamma\to\infty$ and $\gamma-1$ becomes nearly equal to γ. Then $\dfrac{\lambda_\gamma}{\lambda_m}\to\dfrac{\gamma}{\gamma}1 = \boxed{1}$.

(f) As $\dfrac{u}{c}\to 0$, $\left(1-\dfrac{u^2}{c^2}\right)^{-1/2} - 1 \approx 1-\left(-\dfrac{1}{2}\right)\dfrac{u^2}{c^2} - 1 = \dfrac{1}{2}\dfrac{u^2}{c^2}$ and $\dfrac{\lambda_\gamma}{\lambda_m}\to 1\dfrac{u/c}{(1/2)\left(u^2/c^2\right)} = \dfrac{2c}{u}\to\boxed{\infty}$.

P40.41 $\lambda = \dfrac{h}{p}$ $p = \dfrac{h}{\lambda} = \dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{1.00\times10^{-11}\ \text{m}} = 6.63\times10^{-23}\ \text{kg}\cdot\text{m/s}$

(a) electrons: $K_e = \dfrac{p^2}{2m_e} = \dfrac{\left(6.63\times10^{-23}\right)^2}{2\left(9.11\times10^{-31}\right)}\ \text{J} = 15.1\ \text{keV}$

The relativistic answer is more precisely correct:

$$K_e = \sqrt{p^2c^2 + m_e^2c^4} - m_ec^2 = \boxed{14.9\ \text{keV}}$$

(b) photons: $E_\gamma = pc = \left(6.63\times10^{-23}\right)\left(3.00\times10^{8}\right) = \boxed{124\ \text{keV}}$

***P40.42** (a) The wavelength of the student is $\lambda = \dfrac{h}{p} = \dfrac{h}{mu}$. If w is the width of the diffracting aperture,

then we need $w \le 10.0\lambda = 10.0\left(\dfrac{h}{mu}\right)$

so that $u \le 10.0\dfrac{h}{mw} = 10.0\left(\dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(80.0\ \text{kg})(0.750\ \text{m})}\right) = \boxed{1.10\times10^{-34}\ \text{m/s}}$

(b) Using $\Delta t = \dfrac{d}{u}$ we get: $\Delta t \ge \dfrac{0.150\ \text{m}}{1.10\times10^{-34}\ \text{m/s}} = \boxed{1.36\times10^{33}\ \text{s}}$, which is more than 10^{15} times the age of the Universe.

(c) $\boxed{\text{He should not worry.}}$ The student cannot be diffracted to a measurable extent. Even if he tries to stand still, molecular bombardment will give him a sufficiently high speed to make his wavelength immeasurably small.

***P40.43** (a) $E = \gamma mc^2$ $\gamma = E/mc^2 = 20\,000\ \text{MeV}/0.511\ \text{MeV} = \boxed{3.91\times10^4}$

(b) $pc = [E^2 - (mc^2)^2]^{1/2} = [(20\,000\ \text{MeV})^2 - (0.511\ \text{MeV})^2]^{1/2} = 20.0\ \text{GeV}$

$p = \boxed{20.0\ \text{GeV}/c = 1.07\times10^{-17}\ \text{kg}\cdot\text{m/s}}$

(c) $\lambda = h/p = 6.626\times10^{-34}\ \text{J}\cdot\text{s}/1.07\times10^{-17}\ \text{kg}\cdot\text{m/s} = \boxed{6.21\times10^{-17}\ \text{m}}$, small compared to the size of the nucleus. The scattering of the electrons can give information about the particles forming the nucleus.

Section 40.6 **The Quantum Particle**

P40.44 $E = K = \dfrac{1}{2}mu^2 = hf$ and $\lambda = \dfrac{h}{mu}$

$$v_{\text{phase}} = f\lambda = \frac{mu^2}{2h}\frac{h}{mu} = \boxed{\frac{u}{2} = v_{\text{phase}}}$$

This is different from the speed u at which the particle transports mass, energy, and momentum.

P40.45 As a bonus, we begin by proving that the phase speed $v_p = \frac{\omega}{k}$ is not the speed of the particle.

$$v_p = \frac{\omega}{k} = \frac{\sqrt{p^2c^2 + m^2c^4}\hbar}{\hbar\gamma mu} = \frac{\sqrt{\gamma^2 m^2 u^2 c^2 + m^2 c^4}}{\sqrt{\gamma^2 m^2 u^2}} = c\sqrt{1 + \frac{c^2}{\gamma^2 u^2}} = c\sqrt{1 + \frac{c^2}{u^2}\left(1 - \frac{u^2}{c^2}\right)}$$

$$= c\sqrt{1 + \frac{c^2}{u^2} - 1} = \frac{c^2}{u}$$

In fact, the phase speed is larger than the speed of light. A point of constant phase in the wave function carries no mass, no energy, and no information.

Now for the group speed:

$$v_g = \frac{d\omega}{dk} = \frac{d\hbar\omega}{d\hbar k} = \frac{dE}{dp} = \frac{d}{dp}\sqrt{m^2c^4 + p^2c^2}$$

$$v_g = \frac{1}{2}\left(m^2c^4 + p^2c^2\right)^{-1/2}\left(0 + 2pc^2\right) = \sqrt{\frac{p^2c^4}{p^2c^2 + m^2c^4}}$$

$$v_g = c\sqrt{\frac{\gamma^2 m^2 u^2}{\gamma^2 m^2 u^2 + m^2 c^2}} = c\sqrt{\frac{u^2/\left(1 - u^2/c^2\right)}{u^2/\left(1 - u^2/c^2\right) + c^2}} = c\sqrt{\frac{u^2/\left(1 - u^2/c^2\right)}{\left(u^2 + c^2 - u^2\right)/\left(1 - u^2/c^2\right)}} = u$$

It is this speed at which mass, energy, and momentum are transported.

Section 40.7 **The Double-Slit Experiment Revisited**

P40.46 Consider the first bright band away from the center:

$$d\sin\theta = m\lambda \qquad \left(6.00\times10^{-8}\text{ m}\right)\sin\left(\tan^{-1}\left[\frac{0.400}{200}\right]\right) = (1)\,\lambda = 1.20\times10^{-10}\text{ m}$$

$$\lambda = \frac{h}{m_e u} \quad \text{so} \qquad m_e u = \frac{h}{\lambda}$$

and

$$K = \frac{1}{2}m_e u^2 = \frac{m_e^2 u^2}{2m_e} = \frac{h^2}{2m_e\lambda^2} = e\Delta V$$

$$\Delta V = \frac{h^2}{2em_e\lambda^2} \qquad \Delta V = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(1.60\times10^{-19}\text{ C}\right)\left(9.11\times10^{-31}\text{ kg}\right)\left(1.20\times10^{-10}\text{ m}\right)^2} = \boxed{105\text{ V}}$$

P40.47 (a) $\lambda = \frac{h}{mu} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{(1.67 \times 10^{-27}\ \text{kg})(0.400\ \text{m/s})} = \boxed{9.92 \times 10^{-7}\ \text{m}}$

(b) For destructive interference in a multiple-slit experiment, $d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$, with $m = 0$ for the first minimum. Then

$$\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right) = 0.028\,4°$$

$$\frac{y}{L} = \tan\theta \qquad y = L\tan\theta = (10.0\ \text{m})(\tan 0.028\,4°) = \boxed{4.96\ \text{mm}}$$

(c) We cannot say the neutron passed through one slit. If its detection forms part of an interference pattern, we can only say it passed through the array of slits. If we test to see which slit a particular neutron passes through, it will not form part of the interference pattern.

P40.48 We find the speed of each electron from energy conservation in the firing process:

$$0 = K_f + U_f = \frac{1}{2}mu^2 - \text{eV}$$

$$u = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19}\ \text{C}(45\ \text{V})}{9.11 \times 10^{-31}\ \text{kg}}} = 3.98 \times 10^{6}\ \text{m/s}$$

The time of flight is $\Delta t = \frac{\Delta x}{u} = \frac{0.28\ \text{m}}{3.98 \times 10^{6}\ \text{m/s}} = 7.04 \times 10^{-8}\ \text{s}$. The current when electrons are 28 cm apart is $I = \frac{q}{t} = \frac{e}{\Delta t} = \frac{1.6 \times 10^{-19}\ \text{C}}{7.04 \times 10^{-8}\ \text{s}} = \boxed{2.27 \times 10^{-12}\ \text{A}}$.

Section 40.8 **The Uncertainty Principle**

P40.49 For the electron, $\Delta p = m_e \Delta u = (9.11 \times 10^{-31}\ \text{kg})(500\ \text{m/s})(1.00 \times 10^{-4}) = 4.56 \times 10^{-32}\ \text{kg} \cdot \text{m/s}$

$$\Delta x = \frac{h}{4\pi\,\Delta p} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{4\pi(4.56 \times 10^{-32}\ \text{kg} \cdot \text{m/s})} = \boxed{1.16\ \text{mm}}$$

For the bullet, $\Delta p = m\Delta u = (0.020\,0\ \text{kg})(500\ \text{m/s})(1.00 \times 10^{-4}) = 1.00 \times 10^{-3}\ \text{kg} \cdot \text{m/s}$

$$\Delta x = \frac{h}{4\pi\,\Delta p} = \boxed{5.28 \times 10^{-32}\ \text{m}}$$

P40.50 (a) $\Delta p \Delta x = m\Delta u \Delta x \geq \frac{\hbar}{2}$ so $\Delta u \geq \frac{h}{4\pi\, m\Delta x} = \frac{2\pi\ \text{J} \cdot \text{s}}{4\pi(2.00\ \text{kg})(1.00\ \text{m})} = \boxed{0.250\ \text{m/s}}$

(b) The duck might move by $(0.25\ \text{m/s})(5\ \text{s}) = 1.25\ \text{m}$. With original position uncertainty of 1.00 m, we can think of Δx growing to $1.00\ \text{m} + 1.25\ \text{m} = \boxed{2.25\ \text{m}}$.

P40.51 $\frac{\Delta y}{x}=\frac{\Delta p_y}{p_x}$ and $d\Delta p_y \geq \frac{h}{4\pi}$

Eliminate Δp_y and solve for x.

$$x = 4\pi\, p_x(\Delta y)\frac{d}{h}: \qquad x = 4\pi\left(1.00\times10^{-3}\ \text{kg}\right)\left(100\ \text{m/s}\right)\left(1.00\times10^{-2}\ \text{m}\right)\frac{\left(2.00\times10^{-3}\ \text{m}\right)}{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)}$$

The answer, $x = \boxed{3.79\times10^{28}\ \text{m}}$, is 190 times greater than the diameter of the observable Universe.

P40.52 With $\Delta x = 2\times10^{-15}$ m, the uncertainty principle requires $\Delta p_x \geq \frac{\hbar}{2\Delta x} = 2.6\times10^{-20}\ \text{kg}\cdot\text{m/s}$.

The average momentum of the particle bound in a stationary nucleus is zero. The uncertainty in momentum measures the root-mean-square momentum, so we take $p_{rms} \approx 3\times10^{-20}\ \text{kg}\cdot\text{m/s}$. For an electron, the non-relativistic approximation $p = m_e u$ would predict $u \approx 3\times10^{10}$ m/s, while u cannot be greater than c.

Thus, a better solution would be $$E = \left[\left(m_e c^2\right)^2 + (pc)^2\right]^{1/2} \approx 56\ \text{MeV} = \gamma m_e c^2$$

$$\gamma \approx 110 = \frac{1}{\sqrt{1-v^2/c^2}} \qquad \text{so} \qquad u \approx 0.999\,96c$$

For a proton, $u = \frac{p}{m}$ gives $u = 1.8\times10^7$ m/s, less than one-tenth the speed of light.

P40.53 (a) At the top of the ladder, the woman holds a pellet inside a small region Δx_i. Thus, the uncertainty principle requires her to release it with typical horizontal momentum $\Delta p_x = m\Delta u_x = \frac{\hbar}{2\Delta x_i}$. It falls to the floor in a travel time given by $H = 0 + \frac{1}{2}gt^2$ as $t = \sqrt{\frac{2H}{g}}$, so the total width of the impact points is

$$\Delta x_f = \Delta x_i + \left(\Delta u_x\right)t = \Delta x_i + \left(\frac{\hbar}{2m\Delta x_i}\right)\sqrt{\frac{2H}{g}} = \Delta x_i + \frac{A}{\Delta x_i}$$

where $$A = \frac{\hbar}{2m}\sqrt{\frac{2H}{g}}$$

To minimize Δx_f, we require $$\frac{d\left(\Delta x_f\right)}{d\left(\Delta x_i\right)} = 0 \qquad \text{or} \qquad 1 - \frac{A}{\Delta x_i^2} = 0$$

so $$\Delta x_i = \sqrt{A}$$

The minimum width of the impact points is

$$\left(\Delta x_f\right)_{\min} = \left(\Delta x_i + \frac{A}{\Delta x_i}\right)\Bigg|_{\Delta x_i=\sqrt{A}} = 2\sqrt{A} = \boxed{\sqrt{\frac{2\hbar}{m}}\left(\frac{2H}{g}\right)^{1/4}}$$

(b) $$\left(\Delta x_f\right)_{\min} = \left[\frac{2\left(1.054\,6\times10^{-34}\ \text{J}\cdot\text{s}\right)}{5.00\times10^{-4}\ \text{kg}}\right]^{1/2}\left[\frac{2(2.00\ \text{m})}{9.80\ \text{m/s}^2}\right]^{1/4} = \boxed{5.19\times10^{-16}\ \text{m}}$$

Additional Problems

***P40.54** The condition on electric power delivered to the filament is

$$\mathcal{P} = I\,\Delta V = \frac{(\Delta V)^2}{R} = \frac{(\Delta V)^2 A}{\rho \ell} = \frac{(\Delta V)^2 \pi r^2}{\rho \ell} \text{ so } r = \left(\frac{\mathcal{P}\rho \ell}{\pi(\Delta V)^2}\right)^{1/2}. \text{ Here } \mathcal{P} = 75 \text{ W},$$

$\rho = 7.13\times10^{-7}\,\Omega\cdot\text{m}$, and $\Delta V = 120$ V. As the filament radiates in steady state, it must emit all of this power through its lateral surface area $\mathcal{P} = \sigma eAT^4 = \sigma e 2\pi r \ell T^4$. We combine the conditions by substitution:

$$\mathcal{P} = \sigma e 2\pi \left(\frac{\mathcal{P}\rho\ell}{\pi(\Delta V)^2}\right)^{1/2} \ell T^4$$

$$(\Delta V)\mathcal{P}^{1/2} = \sigma e 2\pi^{1/2}\rho^{1/2}\ell^{3/2}T^4$$

$$\ell = \left(\frac{(\Delta V)\mathcal{P}^{1/2}}{\sigma e 2\pi^{1/2}\rho^{1/2}T^4}\right)^{2/3}$$

$$= \left(\frac{120\text{ V}(75\text{ W})^{1/2}\ \text{m}^2\text{K}^4}{5.67\times10^{-8}\ \text{W } 0.450(2)\pi^{1/2}\left(7.13\times10^{-7}\,\Omega\cdot\text{m}\right)^{1/2}(2\,900\text{ K})^4}\right)^{2/3}$$

$$= \left(0.192\text{ m}^{3/2}\right)^{2/3} = \boxed{0.333\text{ m} = \ell}$$

$$\text{and} \quad r = \left(\frac{\mathcal{P}\rho\ell}{\pi(\Delta V)^2}\right)^{1/2} = \left(\frac{75\text{ W } 7.13\times10^{-7}\,\Omega\cdot\text{m } 0.333\text{ m}}{\pi(120\text{ V})^2}\right)^{1/2} = \boxed{r = 1.98\times10^{-5}\text{ m}}$$

P40.55 We want an Einstein plot of K_{max} versus f

λ, nm	f, 10^{14} Hz	K_{max}, eV
588	5.10	0.67
505	5.94	0.98
445	6.74	1.35
399	7.52	1.63

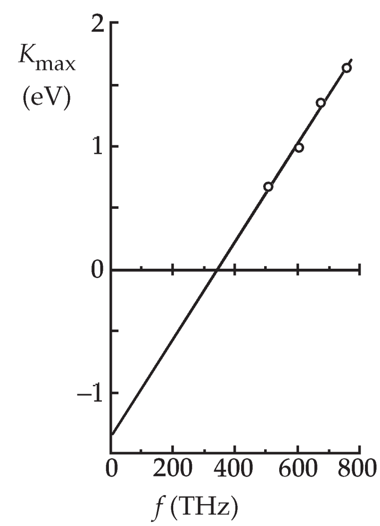


FIG. P40.55

(a) slope $= \dfrac{0.402\text{ eV}}{10^{14}\text{ Hz}} \pm 8\%$

(b) $e\Delta V_S = hf - \phi$

$$h = (0.402)\left(\frac{1.60\times10^{-19}\ \text{J}\cdot\text{s}}{10^{14}}\right) = \boxed{6.4\times10^{-34}\ \text{J}\cdot\text{s} \pm 8\%}$$

(c) $K_{max} = 0$

at $f \approx 344\times10^{12}$ Hz

$$\phi = hf = 2.32\times10^{-19}\text{ J} = \boxed{1.4\text{ eV}}$$

P40.56 $\Delta V_S = \left(\frac{h}{e}\right) f - \frac{\phi}{e}$

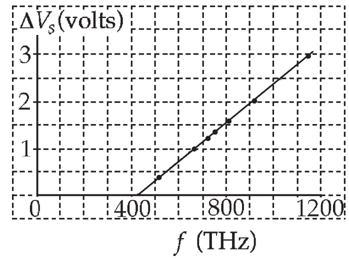


FIG. P40.56

From two points on the graph $0 = \left(\frac{h}{e}\right)\left(4.1\times10^{14}\text{ Hz}\right) - \frac{\phi}{e}$

and $3.3\text{ V} = \left(\frac{h}{e}\right)\left(12\times10^{14}\text{ Hz}\right) - \frac{\phi}{e}$

Combining these two expressions we find:

(a) $\phi = \boxed{1.7\text{ eV}}$

(b) $\frac{h}{e} = \boxed{4.2\times10^{-15}\text{ V}\cdot\text{s}}$

(c) At the cutoff wavelength $\frac{hc}{\lambda_c} = \phi = \left(\frac{h}{e}\right)\frac{ec}{\lambda_c}$

$$\lambda_c = \left(4.2\times10^{-15}\text{ V}\cdot\text{s}\right)\left(1.6\times10^{-19}\text{ C}\right)\frac{\left(3\times10^{8}\text{ m/s}\right)}{(1.7\text{ eV})\left(1.6\times10^{-19}\text{ J/eV}\right)} = \boxed{730\text{ nm}}$$

P40.57 From the path the electrons follow in the magnetic field, the maximum kinetic energy is seen to be:

$$K_{\max} = \frac{e^2B^2R^2}{2m_e}$$

From the photoelectric equation, $K_{\max} = hf - \phi = \frac{hc}{\lambda} - \phi$

Thus, the work function is $\phi = \frac{hc}{\lambda} - K_{\max} = \boxed{\frac{hc}{\lambda} - \frac{e^2B^2R^2}{2m_e}}$

***P40.58** (a) We find the energy of one photon:

$$3.44\text{ eV}\left(\frac{1.6\times10^{-19}\text{ J}}{1\text{ eV}}\right) + \tfrac{1}{2}9.11\times10^{-31}\text{ kg}\left(420\times10^{3}\text{ m/s}\right)^2 = 5.50\times10^{-19}\text{ J} + 0.804\times10^{-19}\text{ J}$$

$$= 6.31\times10^{-19}\text{ J}$$

The number intensity of photon bombardment is

$$\frac{550\text{ J/s}\cdot\text{m}^2}{6.31\times10^{-19}\text{ J}}\left(\frac{1\text{ m}^2}{10^4\text{ cm}^2}\right) = \boxed{8.72\times10^{16}\text{ 1/s}\cdot\text{cm}^2}$$

(b) The density of the current the imagined electrons comprise is

$$8.72\times10^{16}\frac{1}{\text{s}\cdot\text{cm}^2}1.6\times10^{-19}\text{ C} = \boxed{14.0\text{ mA/cm}^2}$$

(c) Many photons are likely reflected or give their energy to the metal as internal energy, so the actual current is probably a small fraction of 14.0 mA.

P40.59 Isolate the terms involving ϕ in Equations 40.13 and 40.14. Square and add to eliminate ϕ.

$$h^2\left[\frac{1}{\lambda_0^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda_0\lambda'}\right]=\gamma^2 m_e^2 u^2$$

Solve for $\frac{u^2}{c^2}=\frac{b}{(b+c^2)}$ where we define $b=\frac{h^2}{m_e^2}\left[\frac{1}{\lambda_0^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda_0\lambda'}\right]$

Substitute into Eq. 40.12: $1+\left(\frac{h}{m_e c}\right)\left[\frac{1}{\lambda_0}-\frac{1}{\lambda'}\right]=\gamma=\left(1-\frac{b}{b+c^2}\right)^{-1/2}=\sqrt{\frac{c^2+b}{c^2}}$

Square each side:
$$c^2+\frac{2hc}{m_e}\left[\frac{1}{\lambda_0}-\frac{1}{\lambda'}\right]+\frac{h^2}{m_e^2}\left[\frac{1}{\lambda_0}-\frac{1}{\lambda'}\right]^2$$
$$=c^2+\left(\frac{h^2}{m_e^2}\right)\left[\frac{1}{\lambda_0^2}+\frac{1}{\lambda'^2}-\frac{2\cos\theta}{\lambda_0\lambda'}\right]$$

From this we get Eq. 40.11: $\lambda'-\lambda_0=\left(\frac{h}{m_e c}\right)[1-\cos\theta]$

P40.60 We show that if all of the energy of a photon is transmitted to an electron, momentum will not be conserved.

Energy: $$\frac{hc}{\lambda_0}=\frac{hc}{\lambda'}+K_e=m_e c^2(\gamma-1) \quad \text{if} \quad \frac{hc}{\lambda'}=0 \qquad (1)$$

Momentum: $$\frac{h}{\lambda_0}=\frac{h}{\lambda'}+\gamma m_e u=\gamma m_e u \quad \text{if} \quad \lambda'=\infty \qquad (2)$$

From (1), $$\gamma=\frac{h}{\lambda_0 m_e c}+1 \qquad (3)$$

$$u=c\sqrt{1-\left(\frac{\lambda_0 m_e c}{h+\lambda_0 m_e c}\right)^2} \qquad (4)$$

Substitute (3) and (4) into (2) and show the inconsistency:

$$\frac{h}{\lambda_0}=\left(1+\frac{h}{\lambda_0 m_e c}\right)m_e c\sqrt{1-\left(\frac{\lambda_0 m_e c}{h+\lambda_0 m_e c}\right)^2}=\frac{\lambda_0 m_e c+h}{\lambda_0}\sqrt{\frac{h(h+2\lambda_0 m_e c)}{(h+\lambda_0 m_e c)^2}}=\frac{h}{\lambda_0}\sqrt{\frac{h+2\lambda_0 m_e c}{h}}$$

This is impossible, so all of the energy of a photon cannot be transmitted to an electron.

P40.61 (a) Starting with Planck's law, $I(\lambda, T) = \dfrac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_B T} - 1\right]}$

the total power radiated per unit area $\displaystyle\int_0^\infty I(\lambda, T)\,d\lambda = \int_0^\infty \frac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_B T} - 1\right]}\,d\lambda$

Change variables by letting $x = \dfrac{hc}{\lambda k_B T}$

and $dx = -\dfrac{hc\,d\lambda}{k_B T\lambda^2}$

Note that as λ varies from $0 \to \infty$, x varies from $\infty \to 0$.

Then $\displaystyle\int_0^\infty I(\lambda, T)\,d\lambda = -\frac{2\pi k_B^4 T^4}{h^3c^2}\int_\infty^0 \frac{x^3}{(e^x - 1)}\,dx = \frac{2\pi k_B^4 T^4}{h^3c^2}\left(\frac{\pi^4}{15}\right)$

Therefore, $\boxed{\displaystyle\int_0^\infty I(\lambda, T)\,d\lambda = \left(\frac{2\pi^5 k_B^4}{15h^3c^2}\right)T^4 = \sigma T^4}$

(b) From part (a), $\sigma = \dfrac{2\pi^5 k_B^4}{15h^3c^2} = \dfrac{2\pi^5\left(1.38\times10^{-23}\ \text{J/K}\right)^4}{15\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^3\left(3.00\times10^8\ \text{m/s}\right)^2}$

$\sigma = \boxed{5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4}$

P40.62 Planck's law states $I(\lambda, T) = \dfrac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_B T} - 1\right]} = 2\pi hc^2\lambda^{-5}\left[e^{hc/\lambda k_B T} - 1\right]^{-1}$.

To find the wavelength at which this distribution has a maximum, compute

$$\frac{dI}{d\lambda} = 2\pi hc^2\left\{-5\lambda^{-6}\left[e^{hc/\lambda k_B T} - 1\right]^{-1} - \lambda^{-5}\left[e^{hc/\lambda k_B T} - 1\right]^{-2} e^{hc/\lambda k_B T}\left(-\frac{hc}{\lambda^2 k_B T}\right)\right\} = 0$$

$$\frac{dI}{d\lambda} = \frac{2\pi hc^2}{\lambda^6\left[e^{hc/\lambda k_B T} - 1\right]}\left\{-5 + \frac{hc}{\lambda k_B T}\frac{e^{hc/\lambda k_B T}}{\left[e^{hc/\lambda k_B T} - 1\right]}\right\} = 0$$

Letting $x = \dfrac{hc}{\lambda k_B T}$, the condition for a maximum becomes $\dfrac{xe^x}{e^x - 1} = 5$.

We zero in on the solution to this transcendental equation by iterations as shown in the table below.

continued on next page

x	$xe^x/(e^x-1)$
4.000 00	4.074 629 4
4.500 00	4.550 552 1
5.000 00	5.033 918 3
4.900 00	4.936 762 0
4.950 00	4.985 313 0
4.975 00	5.009 609 0
4.963 00	4.997 945 2
4.969 00	5.003 776 7
4.966 00	5.000 860 9

x	$xe^x/(e^x-1)$
4.964 50	4.999 403 0
4.965 50	5.000 374 9
4.965 00	4.999 889 0
4.965 25	5.000 132 0
4.965 13	5.000 015 3
4.965 07	4.999 957 0
4.965 10	4.999 986 2
4.965 115	5.000 000 8

The solution is found to be

$$x = \frac{hc}{\lambda_{\max} k_B T} = 4.965\,115 \text{ and } \qquad \lambda_{\max} T = \frac{hc}{4.965\,115 k_B}$$

Thus, $\lambda_{\max} T = \dfrac{(6.626\,075\times10^{-34}\text{ J}\cdot\text{s})(2.997\,925\times10^{8}\text{ m/s})}{4.965\,115(1.380\,658\times10^{-23}\text{ J/K})} = \boxed{2.897\,755\times10^{-3}\text{ m}\cdot\text{K}}$.

This result agrees with Wien's experimental value of $\lambda_{\max} T = 2.898\times10^{-3}\text{ m}\cdot\text{K}$ for this constant.

P40.63 $p = mu = \sqrt{2mE} = \sqrt{2(1.67\times10^{-27}\text{ kg})(0.040\,0\text{ eV})(1.60\times10^{-19}\text{ J/eV})}$

$$\lambda = \frac{h}{mu} = 1.43\times10^{-10}\text{ m} = \boxed{0.143\text{ nm}}$$

This is of the same order of magnitude as the spacing between atoms in a crystal, so diffraction should appear. A diffraction pattern with maxima and minima at the same angles can be produced with x-rays, with neutrons, and with electrons of much higher kinetic energy, by using incident quantum particles with the same wavelength.

P40.64 (a) $mgy_i = \frac{1}{2}mu_f^2$

$$u_f = \sqrt{2gy_i} = \sqrt{2(9.80\text{ m/s}^2)(50.0\text{ m})} = 31.3\text{ m/s}$$

$$\lambda = \frac{h}{mu} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(75.0\text{ kg})(31.3\text{ m/s})} = \boxed{2.82\times10^{-37}\text{ m}} \quad \text{(not observable)}$$

(b) $\Delta E\Delta t \geq \frac{\hbar}{2}$

so $\Delta E \geq \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi(5.00\times10^{-3}\text{ s})} = \boxed{1.06\times10^{-32}\text{ J}}$

(c) $\dfrac{\Delta E}{E} = \dfrac{1.06\times10^{-32}\text{ J}}{(75.0\text{ kg})(9.80\text{ m/s}^2)(50.0\text{ m})} = \boxed{2.87\times10^{-35}\%}$

P40.65 $\lambda_C = \dfrac{h}{m_e c}$ and $\lambda = \dfrac{h}{p}$: $\qquad \dfrac{\lambda_C}{\lambda} = \dfrac{h/m_e c}{h/p} = \dfrac{p}{m_e c}$

$$E^2 = c^2 p^2 + (m_e c^2)^2: \qquad p = \sqrt{\frac{E^2}{c^2} - (m_e c)^2}$$

$$\frac{\lambda_C}{\lambda} = \frac{1}{m_e c}\sqrt{\frac{E^2}{c^2} - (m_e c)^2} = \sqrt{\frac{1}{(m_e c)^2}\left[\frac{E^2}{c^2} - (m_e c)^2\right]} = \sqrt{\left(\frac{E}{m_e c^2}\right)^2 - 1}$$

P40.66 $\Delta\lambda = \dfrac{h}{m_e c}(1-\cos\theta) = \lambda' - \lambda_0$

$$E' = \frac{hc}{\lambda'} = \frac{hc}{\lambda_0 + \Delta\lambda} = hc\left[\lambda_0 + \frac{h}{m_e c}(1-\cos\theta)\right]^{-1}$$

$$E' = \frac{hc}{\lambda_0}\left[1 + \frac{hc}{m_e c^2 \lambda_0}(1-\cos\theta)\right]^{-1}$$

$$E' = \frac{hc}{\lambda_0}\left[1 + \frac{hc}{m_e c^2 \lambda_0}(1-\cos\theta)\right]^{-1} = E_0\left[1 + \frac{E_0}{m_e c^2}(1-\cos\theta)\right]^{-1}$$

P40.67 From the uncertainty principle $\quad \Delta E \Delta t \geq \dfrac{\hbar}{2}$

or $\quad \Delta(mc^2)\Delta t = \dfrac{\hbar}{2}$

Therefore, $\quad \dfrac{\Delta m}{m} = \dfrac{h}{4\pi c^2 (\Delta t) m} = \dfrac{h}{4\pi(\Delta t) E_R}$

$$\frac{\Delta m}{m} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi(8.70\times10^{-17}\ \text{s})(135\ \text{MeV})}\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right)$$

$$= \boxed{2.81\times10^{-8}}$$

P40.68 Let u' represent the final speed of the electron and let $\gamma' = \left(1 - \dfrac{u'^2}{c^2}\right)^{-1/2}$. We must eliminate β and u' from the three conservation equations:

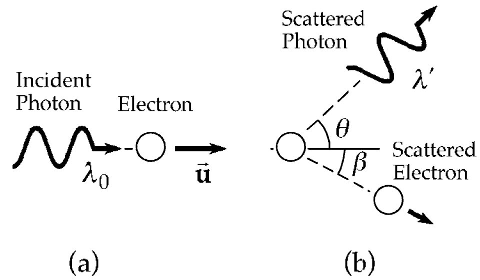


FIG. P40.68

$$\frac{hc}{\lambda_0} + \gamma m_e c^2 = \frac{hc}{\lambda'} + \gamma' m_e c^2 \qquad [1]$$

$$\frac{h}{\lambda_0} + \gamma m_e u - \frac{h}{\lambda'}\cos\theta = \gamma' m_e u' \cos\beta \qquad [2]$$

$$\frac{h}{\lambda'}\sin\theta = \gamma' m_e u' \sin\beta \qquad [3]$$

Square Equations [2] and [3] and add:

$$\frac{h^2}{\lambda_0^2} + \gamma^2 m_e^2 u^2 + \frac{h^2}{\lambda'^2} + \frac{2h\gamma\, m_e u}{\lambda_0} - \frac{2h^2\cos\theta}{\lambda_0\lambda'} - \frac{2h\gamma\, m_e u\cos\theta}{\lambda'} = \gamma'^2 m_e^2 u'^2$$

$$\frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda'^2} + \gamma^2 m_e^2 u^2 + \frac{2h\gamma\, m_e u}{\lambda_0} - \frac{2h\gamma\, m_e u\cos\theta}{\lambda'} - \frac{2h^2\cos\theta}{\lambda_0\lambda'} = \frac{m_e^2 u'^2}{1 - u'^2/c^2}$$

Call the left-hand side b. Then $b - \dfrac{bu'^2}{c^2} = m_e^2 u'^2$ and $u'^2 = \dfrac{b}{m_e^2 + b/c^2} = \dfrac{c^2 b}{m_e^2 c^2 + b}$.

Now square Equation [1] and substitute to eliminate γ':

$$\frac{h^2}{\lambda_0^2} + \gamma^2 m_e^2 c^2 + \frac{h^2}{\lambda'^2} + \frac{2h\gamma\, m_e c}{\lambda_0} - \frac{2h^2}{\lambda_0\lambda'} - \frac{2h\gamma\, m_e c}{\lambda'} = \frac{m_e^2 c^2}{1 - u'^2/c^2} = m_e^2 c^2 + b$$

So we have $\dfrac{h^2}{\lambda_0^2} + \dfrac{h^2}{\lambda'^2} + \gamma^2 m_e^2 c^2 + \dfrac{2h\gamma\, m_e c}{\lambda_0} - \dfrac{2h\gamma\, m_e c}{\lambda'} - \dfrac{2h^2}{\lambda_0\lambda'}$

$$= m_e c^2 + \frac{h^2}{\lambda_0^2} + \frac{h^2}{\lambda'^2} + \gamma^2 m_e^2 u^2 + \frac{2h\gamma\, m_e u}{\lambda_0} - \frac{2h\gamma\, m_e u\cos\theta}{\lambda'} - \frac{2h^2\cos\theta}{\lambda_0\lambda'}$$

Multiply through by $\dfrac{\lambda_0\lambda'}{m_e^2 c^2}$

$$\lambda_0\lambda'\gamma^2 + \frac{2h\lambda'\gamma}{m_e c} - \frac{2h\lambda_0\gamma}{m_e c} - \frac{2h^2}{m_e^2c^2} = \lambda_0\lambda' + \frac{\lambda_0\lambda'\gamma^2 u^2}{m_e c^2} + \frac{2h\lambda' u\gamma}{m_e c^2} - \frac{2h\gamma\lambda_0 u\cos\theta}{m_e c^2} - \frac{2h^2\cos\theta}{m_e^2 c^2}$$

$$\lambda_0\lambda'\left(\gamma^2 - 1 - \frac{\gamma^2 u^2}{c^2}\right) + \frac{2h\gamma\lambda'}{m_e c}\left(1 - \frac{u}{c}\right) = \frac{2h\gamma\lambda_0}{m_e c}\left(1 - \frac{u\cos\theta}{c}\right) + \frac{2h^2}{m_e^2 c^2}(1 - \cos\theta)$$

The first term is zero. Then

$$\lambda' = \lambda_0\left(\frac{1 - (u\cos\theta)/c}{1 - u/c}\right) + \frac{h\gamma^{-1}}{m_e c}\left(\frac{1}{1 - u/c}\right)(1 - \cos\theta)$$

Since

$$\gamma^{-1} = \sqrt{1 - \left(\frac{u}{c}\right)^2} = \sqrt{\left(1 - \frac{u}{c}\right)\left(1 + \frac{u}{c}\right)}$$

this result may be written as

$$\boxed{\lambda' = \lambda_0\left(\frac{1 - (u\cos\theta)/c}{1 - u/c}\right) + \frac{h}{m_e c}\sqrt{\frac{1 + u/c}{1 - u/c}}\,(1 - \cos\theta)}$$

It shows a specific combination of what looks like a Doppler shift and a Compton shift. This problem is about the same as the first problem in Albert Messiah's graduate text on quantum mechanics.

ANSWERS TO EVEN PROBLEMS

P40.2 (a) 999 nm (b) The wavelength emitted most strongly is infrared, and much more energy is radiated at wavelengths longer than $\lambda_{\max}$ than at shorter wavelengths.

P40.4 (a) $\sim 10^{-7}$ m ultraviolet (b) $\sim 10^{-10}$ m gamma ray

P40.6 (a) 70.9 kW (b) 580 nm (c) 7.99×10^{10} W/m (d) 9.42×10^{-1226} W/m
(e) 1.00×10^{-227} W/m (f) 5.44×10^{10} W/m (g) 7.38×10^{10} W/m (h) 0.260 W/m
(i) 2.60×10^{-9} W/m (j) 20 kW

P40.8 5.71×10^{3} photons

P40.10 1.34×10^{31}

P40.12 (a) 0.263 kg (b) 1.81 W (c) –0.015 3°C/s = –0.919°C/min (d) 9.89 μm (e) 2.01×10^{-20} J
(f) 8.98×10^{19} photon/s

P40.14 (a) 296 nm, 1.01 PHz (b) 2.71 V

P40.16 (a) 1.38 eV (b) 334 THz

P40.18 148 d, the classical theory is a gross failure

P40.20 (a) The incident photons are Doppler shifted to higher frequencies, and hence, higher energy.
(b) 3.87 eV (c) 8.78 eV

P40.22 (a) and (b)

θ, degrees	0	30	60	90	120	150	180
λ', pm	120.0	120.3	121.2	122.4	123.6	124.5	124.8
K_e, eV	0	27.9	104	205	305	376	402

(c) 180°. We could answer like this: The photon imparts the greatest momentum to the originally stationary electron in a head-on collision. Here the photon recoils straight back and the electron has maximum kinetic energy.

P40.24 (a) 488 fm (b) 268 keV (c) 31.5 keV

P40.26 (a) $\cos^{-1}\left(\dfrac{m_ec^2+E_0}{2m_ec^2+E_0}\right)$ (b) $E'_\gamma = \dfrac{E_0}{2}\left(\dfrac{2m_ec^2+E_0}{m_ec^2+E_0}\right)$, $p'_\gamma = \dfrac{E_0}{2c}\left(\dfrac{2m_ec^2+E_0}{m_ec^2+E_0}\right)$

(c) $K_e = \dfrac{E_0^2}{2\left(m_ec^2+E_0\right)}$, $p_e = \dfrac{E_0}{2c}\left(\dfrac{2m_ec^2+E_0}{m_ec^2+E_0}\right)$

P40.28 (a) 0.101 nm (b) 81.1°

P40.30 0.004 86 nm

P40.32 To have photon energy 10 eV or greater, according to this definition, ionizing radiation is the ultraviolet light, x-rays, and γ rays with wavelength shorter than 124 nm; that is, with frequency higher than 2.41×10^{15} Hz.

P40.34 397 fm

P40.36 (a) 0.709 nm (b) 414 nm

P40.38 0.218 nm

P40.40 (a) $\frac{\gamma}{\gamma-1}\frac{u}{c}$ where $\gamma = \frac{1}{\sqrt{1-u^2/c^2}}$ (b) 1.60 (c) no change (d) 2.00×10^3 (e) 1 (f) ∞

P40.42 (a) 1.10×10^{-34} m/s (b) 1.36×10^{33} s is more than 10^{15} times the age of the Universe. (c) The student cannot be diffracted to a measurable extent. Even if he tries to stand still, molecular bombardment will give him a sufficiently high speed to make his wavelength immeasurably small.

P40.44 $v_{\text{phase}} = u/2$

P40.46 105 V

P40.48 2.27 pA

P40.50 (a) 0.250 m/s (b) 2.25 m

P40.52 The electron energy must be ~100 mc^2 or larger. The proton energy can be as small as 1.001 mc^2, which is within the range well described classically.

P40.54 length 0.333 m, radius 19.8 μm

P40.56 (a) 1.7 eV (b) 4.2×10^{-15} V · s (c) 730 nm

P40.58 (a) 8.72×10^{16}/s (b) 14.0 mA (c) Many photons are likely reflected or give their energy to the metal as internal energy, so the actual current is probably a small fraction of 14.0 mA.

P40.60 See the solution.

P40.62 See the solution.

P40.64 (a) 2.82×10^{-37} m (b) 1.06×10^{-32} J (c) 2.87×10^{-35}%

P40.66 See the solution.

P40.68 See the solution.

41

Quantum Mechanics

Note: In chapters 39, 40, and 41 we use u to represent the speed of a particle with mass, reserving v for the speeds associated with reference frames, wave functions, and photons.

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q41.1 A particle's wave function represents its state, containing all the information there is about its location and motion. The squared absolute value of its wave function tells where we would classically think of the particle as spending most its time. $|\Psi|^2$ is the probability distribution function for the position of the particle.

***Q41.2** For the squared wave function to be the probability per length of finding the particle, we require

$$|\psi|^2 = \frac{0.48}{7\text{ nm} - 4\text{ nm}} = \frac{0.16}{\text{nm}} \quad \text{and} \quad \psi = 0.4/\sqrt{\text{nm}}$$

(i) Answer (e). (ii) Answer (e).

***Q41.3** **(i)** For a photon a and b are true, c false, d, e, f, and g true, h false, i and j true.

(ii) For an electron a is true, b false, c, d, e, f true, g false, h, i and j true.

Note that statements a, d, e, f, i, and j are true for both.

***Q41.4** We consider the quantity $h^2n^2/8mL^2$.

In (a) it is $h^2 1/8m_1(3\text{ nm})^2 = h^2/72\, m_1\text{ nm}^2$.
In (b) it is $h^2 4/8m_1(3\text{ nm})^2 = h^2/18\, m_1\text{ nm}^2$.
In (c) it is $h^2 1/16m_1(3\text{ nm})^2 = h^2/144\, m_1\text{ nm}^2$.
In (d) it is $h^2 1/8m_1(6\text{ nm})^2 = h^2/288\, m_1\text{ nm}^2$.
In (e) it is $0^2 1/8m_1(3\text{ nm})^2 = 0$.
The ranking is then b > a > c > d > e.

Q41.5 The motion of the quantum particle does not consist of moving through successive points. The particle has no definite position. It can sometimes be found on one side of a node and sometimes on the other side, but never at the node itself. There is no contradiction here, for the quantum particle is moving as a wave. It is not a classical particle. In particular, the particle does not speed up to infinite speed to cross the node.

Q41.6 Consider a particle bound to a restricted region of space. If its minimum energy were zero, then the particle could have zero momentum and zero uncertainty in its momentum. At the same time, the uncertainty in its position would not be infinite, but equal to the width of the region. In such a case, the uncertainty product $\Delta x \Delta p_x$ would be zero, violating the uncertainty principle. This contradiction proves that the minimum energy of the particle is not zero.

***Q41.7** Compare Figures 41.4 and 41.7 in the text. In the square well with infinitely high walls, the particle's simplest wave function has strict nodes separated by the length *L* of the well. The particle's wavelength is 2*L*, its momentum $\frac{h}{2L}$, and its energy $\frac{p^2}{2m} = \frac{h^2}{8mL^2}$. Now in the well with walls of only finite height, the wave function has nonzero amplitude at the walls. In this finite-depth well …

(i) The particle's wavelength is longer, answer (a).

(ii) The particle's momentum in its ground state is smaller, answer (b).

(iii) The particle has less energy, answer (b).

Q41.8 As Newton's laws are the rules which a particle of large mass follows in its motion, so the Schrödinger equation describes the motion of a quantum particle, a particle of small or large mass. In particular, the states of atomic electrons are confined-wave states with wave functions that are solutions to the Schrödinger equation.

***Q41.9** Answer (b). The reflected amplitude decreases as *U* decreases. The amplitude of the reflected wave is proportional to the reflection coefficient, *R*, which is $1-T$, where *T* is the transmission coefficient as given in equation 41.22. As *U* decreases, *C* decreases as predicted by equation 41.23, *T* increases, and *R* decreases.

***Q41.10** Answer (a). Because of the exponential tailing of the wave function within the barrier, the tunneling current is more sensitive to the width of the barrier than to its height.

Q41.11 Consider the Heisenberg uncertainty principle. It implies that electrons initially moving at the same speed and accelerated by an electric field through the same distance *need not* all have the same measured speed after being accelerated. Perhaps the philosopher could have said "it is necessary for the very existence of science that the same conditions always produce the same results within the uncertainty of the measurements."

Q41.12 In quantum mechanics, particles are treated as wave functions, not classical particles. In classical mechanics, the kinetic energy is never negative. That implies that $E \geq U$. Treating the particle as a wave, the Schrödinger equation predicts that there is a nonzero probability that a particle can tunnel through a barrier—a region in which $E < U$.

***Q41.13** Answer (c). Other points see a wider potential-energy barrier and carry much less tunneling current.

SOLUTIONS TO PROBLEMS

Section 41.1 An Interpretation of Quantum Mechanics

P41.1 (a) $\psi(x) = Ae^{i(5.00\times10^{10}x)} = A\cos(5\times10^{10}x) + Ai\sin(5\times10^{10}x) = A\cos(kx) + Ai\sin(kx)$ goes through a full cycle when x changes by λ and when kx changes by 2π. Then $k\lambda = 2\pi$ where $k = 5.00\times10^{10}\ \text{m}^{-1} = \dfrac{2\pi}{\lambda}$. Then $\lambda = \dfrac{2\pi\ \text{m}}{(5.00\times10^{10})} = \boxed{1.26\times10^{-10}\ \text{m}}$.

(b) $$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{1.26\times10^{-10}\ \text{m}} = \boxed{5.27\times10^{-24}\ \text{kg}\cdot\text{m/s}}$$

(c) $m_e = 9.11\times10^{-31}$ kg

$$K = \frac{m_e^2u^2}{2m_e} = \frac{p^2}{2m} = \frac{(5.27\times10^{-24}\ \text{kg}\cdot\text{m/s})^2}{(2\times9.11\times10^{-31}\ \text{kg})} = 1.52\times10^{-17}\ \text{J} = \frac{1.52\times10^{-17}\ \text{J}}{1.60\times10^{-19}\ \text{J/eV}} = \boxed{95.5\ \text{eV}}$$

P41.2 Probability $$P = \int_{-a}^{a}|\psi(x)|^2 = \int_{-a}^{a}\frac{a}{\pi(x^2+a^2)}dx = \left(\frac{a}{\pi}\right)\left(\frac{1}{a}\right)\tan^{-1}\left(\frac{x}{a}\right)\Big|_{-a}^{a}$$

$$P = \frac{1}{\pi}\left[\tan^{-1}1 - \tan^{-1}(-1)\right] = \frac{1}{\pi}\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \boxed{\frac{1}{2}}$$

Section 41.2 The Quantum Particle under Boundary Conditions

P41.3 $E_1 = 2.00\ \text{eV} = 3.20\times10^{-19}$ J

For the ground state, $$E_1 = \frac{h^2}{8m_eL^2}$$

(a) $$L = \frac{h}{\sqrt{8m_eE_1}} = 4.34\times10^{-10}\ \text{m} = \boxed{0.434\ \text{nm}}$$

(b) $$\Delta E = E_2 - E_1 = 4\left(\frac{h^2}{8m_eL^2}\right) - \left(\frac{h^2}{8m_eL^2}\right) = \boxed{6.00\ \text{eV}}$$

P41.4 For an electron wave to "fit" into an infinitely deep potential well, an integral number of half-wavelengths must equal the width of the well.

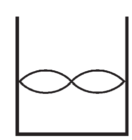


FIG. P41.4

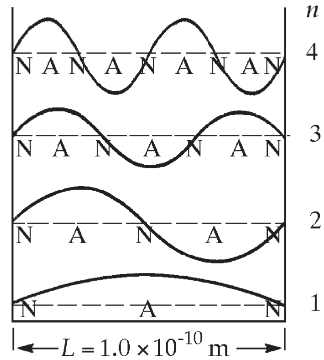
$$\frac{n\lambda}{2} = 1.00\times10^{-9}\ \text{m} \quad \text{so} \quad \lambda = \frac{2.00\times10^{-9}}{n} = \frac{h}{p}$$

(a) Since $$K = \frac{p^2}{2m_e} = \frac{(h^2/\lambda^2)}{2m_e} = \frac{h^2}{2m_e}\frac{n^2}{(2\times10^{-9})^2} = (0.377n^2)\ \text{eV}$$

For $K \approx 6$ eV $\boxed{n = 4}$

(b) With $n = 4$, $\boxed{K = 6.03\ \text{eV}}$

P41.5 (a) We can draw a diagram that parallels our treatment of standing mechanical waves. In each state, we measure the distance d from one node to another (N to N), and base our solution upon that:



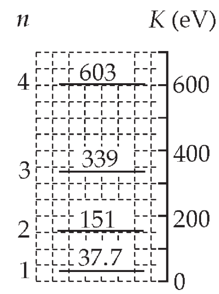


FIG. P41.5

Since $$d_{\text{N to N}} = \frac{\lambda}{2} \text{ and } \lambda = \frac{h}{p}$$

$$p = \frac{h}{\lambda} = \frac{h}{2d}$$

Next, $$K = \frac{p^2}{2m_e} = \frac{h^2}{8m_e d^2} = \frac{1}{d^2}\left[\frac{\left(6.626\times 10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(9.11\times 10^{-31}\ \text{kg}\right)}\right]$$

Evaluating, $$K = \frac{6.02\times 10^{-38}\ \text{J}\cdot\text{m}^2}{d^2} \qquad K = \frac{3.77\times 10^{-19}\ \text{eV}\cdot\text{m}^2}{d^2}$$

In state 1, $d = 1.00\times 10^{-10}$ m $\quad K_1 = 37.7$ eV

In state 2, $d = 5.00\times 10^{-11}$ m $\quad K_2 = 151$ eV

In state 3, $d = 3.33\times 10^{-11}$ m $\quad K_3 = 339$ eV

In state 4, $d = 2.50\times 10^{-11}$ m $\quad K_4 = 603$ eV

(b) When the electron falls from state 2 to state 1, it puts out energy

$$E = 151\ \text{eV} - 37.7\ \text{eV} = 113\ \text{eV} = hf = \frac{hc}{\lambda}$$

into emitting a photon of wavelength

$$\lambda = \frac{hc}{E} = \frac{\left(6.626\times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times 10^{8}\ \text{m/s}\right)}{(113\ \text{eV})\left(1.60\times 10^{-19}\ \text{J/eV}\right)} = 11.0\ \text{nm}$$

The wavelengths of the other spectral lines we find similarly:

Transition	$4\to 3$	$4\to 2$	$4\to 1$	$3\to 2$	$3\to 1$	$2\to 1$
E(eV)	264	452	565	188	302	113
λ(nm)	4.71	2.75	2.20	6.60	4.12	11.0

***P41.6** For the bead's energy we have both $(1/2)mu^2$ and $h^2n^2/8mL^2$. Then

$$n = \sqrt{\frac{1}{2}mu^2\frac{8mL^2}{h^2}} = \frac{2muL}{h} \text{ note that this expression can be thought of as } \frac{2L}{\lambda} = \frac{L}{d_{NN}}$$

Evaluating, $$n = \frac{2(0.005\ \text{kg})(10^{-10}\ \text{m})\ 0.2\ \text{m}}{3.156\times 10^{7}\ \text{s}\ (6.626\times 10^{-34}\ \text{J}\cdot\text{s})} = \boxed{9.56\times 10^{12}}$$

P41.7 $$\Delta E = \frac{hc}{\lambda} = \left(\frac{h^2}{8m_e L^2}\right)\left[2^2 - 1^2\right] = \frac{3h^2}{8m_e L^2}$$

$$L = \sqrt{\frac{3h\lambda}{8m_e c}} = 7.93\times 10^{-10}\ \text{m} = \boxed{0.793\ \text{nm}}$$

P41.8 $\Delta E = \frac{hc}{\lambda} = \left(\frac{h^2}{8m_e L^2}\right)\left[2^2 - 1^2\right] = \frac{3h^2}{8m_e L^2}$

so $L = \boxed{\sqrt{\frac{3h\lambda}{8m_e c}}}$

P41.9 The confined proton can be described in the same way as a standing wave on a string. At level 1, the node-to-node distance of the standing wave is 1.00×10^{-14} m, so the wavelength is twice this distance:

$$\frac{h}{p} = 2.00 \times 10^{-14} \text{ m}$$

The proton's kinetic energy is

$$K = \frac{1}{2}mu^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{\left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right)^2}{2\left(1.67 \times 10^{-27} \text{ kg}\right)\left(2.00 \times 10^{-14} \text{ m}\right)^2}$$

$$= \frac{3.29 \times 10^{-13} \text{ J}}{1.60 \times 10^{-19} \text{ J/eV}} = 2.05 \text{ MeV}$$

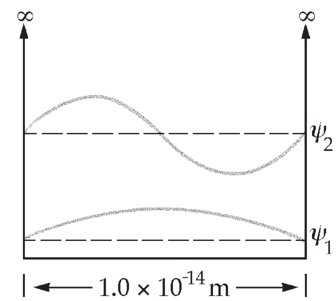


FIG. P41.9

In the first excited state, level 2, the node-to-node distance is half as long as in state 1. The momentum is two times larger and the energy is four times larger: $K = 8.22$ MeV.

The proton has mass, has charge, moves slowly compared to light in a standing wave state, and stays inside the nucleus. When it falls from level 2 to level 1, its energy change is

$$2.05 \text{ MeV} - 8.22 \text{ MeV} = -6.16 \text{ MeV}$$

Therefore, we know that a photon (a traveling wave with no mass and no charge) is emitted at the speed of light, and that it has an energy of $\boxed{+6.16 \text{ MeV}}$.

Its frequency is $f = \frac{E}{h} = \frac{\left(6.16 \times 10^6 \text{ eV}\right)\left(1.60 \times 10^{-19} \text{ J/eV}\right)}{6.626 \times 10^{-34} \text{ J} \cdot \text{s}} = 1.49 \times 10^{21} \text{ Hz}$

And its wavelength is $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{1.49 \times 10^{21} \text{ s}^{-1}} = \boxed{2.02 \times 10^{-13} \text{ m}}$

This is a $\boxed{\text{gamma ray}}$, according to the electromagnetic spectrum chart in Chapter 34.

P41.10 The ground state energy of a particle (mass *m*) in a 1-dimensional box of width *L* is $E_1 = \frac{h^2}{8mL^2}$.

(a) For a proton $\left(m = 1.67 \times 10^{-27} \text{ kg}\right)$ in a 0.200-nm wide box:

$$E_1 = \frac{\left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right)^2}{8\left(1.67 \times 10^{-27} \text{ kg}\right)\left(2.00 \times 10^{-10} \text{ m}\right)^2} = 8.22 \times 10^{-22} \text{ J} = \boxed{5.13 \times 10^{-3} \text{ eV}}$$

(b) For an electron $\left(m = 9.11 \times 10^{-31} \text{ kg}\right)$ in the same size box:

$$E_1 = \frac{\left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right)^2}{8\left(9.11 \times 10^{-31} \text{ kg}\right)\left(2.00 \times 10^{-10} \text{ m}\right)^2} = 1.51 \times 10^{-18} \text{ J} = \boxed{9.41 \text{ eV}}$$

(c) The electron has a much higher energy because it is much less massive.

***P41.11** $E_n = \left(\dfrac{h^2}{8mL^2}\right)n^2$

$$E_1 = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(1.67\times10^{-27}\ \text{kg}\right)\left(2.00\times10^{-14}\ \text{m}\right)^2} = 8.22\times10^{-14}\ \text{J}$$

$$E_1 = \boxed{0.513\ \text{MeV}} \qquad E_2 = 4E_1 = \boxed{2.05\ \text{MeV}} \qquad E_3 = 9E_1 = \boxed{4.62\ \text{MeV}}$$

Yes; the energy differences are ~1 MeV, which is a typical energy for a γ-ray photon as radiated by an atomic nucleus in an excited state.

P41.12 (a) The energies of the confined electron are $E_n = \dfrac{h^2}{8m_eL^2}n^2$. Its energy gain in the quantum jump from state 1 to state 4 is $\dfrac{h^2}{8m_eL^2}\left(4^2-1^2\right)$ and this is the photon energy: $\dfrac{h^2 15}{8m_eL^2} = hf = \dfrac{hc}{\lambda}$. Then $8m_ecL^2 = 15h\lambda$ and $\boxed{L = \left(\dfrac{15h\lambda}{8m_ec}\right)^{1/2}}$.

(b) Let λ' represent the wavelength of the photon emitted: $\dfrac{hc}{\lambda'} = \dfrac{h^2}{8m_eL^2}4^2 - \dfrac{h^2}{8m_eL^2}2^2 = \dfrac{12h^2}{8m_eL^2}$.

Then $\dfrac{hc}{\lambda}\dfrac{\lambda'}{hc} = \dfrac{h^2 15\left(8m_eL^2\right)}{8m_eL^2 12h^2} = \dfrac{5}{4}$ and $\boxed{\lambda' = 1.25\lambda}$.

***P41.13** (a) From $\Delta x\Delta p \ge \hbar/2$ with $\Delta x = L$, the uncertainty in momentum must be at least $\boxed{\Delta p \approx \hbar/2L}$.

(b) Its energy is all kinetic, $E = p^2/2m = (\Delta p)^2/2m \approx \boxed{\hbar^2/8mL^2} = h^2/(4\pi)^2 8mL^2$.
Compared to the actual $h^2/8mL^2$, this estimate is too low by $4\pi^2 \approx 40$ times. The actual wave function does not have the particular (Gaussian) shape of a minimum-uncertainty wave function. The result correctly displays the pattern of dependence of the energy on the mass and on the length of the well.

P41.14 (a) $\langle x\rangle = \displaystyle\int_0^L x\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx = \frac{2}{L}\int_0^L x\left(\frac{1}{2}-\frac{1}{2}\cos\frac{4\pi x}{L}\right)dx$

$$\langle x\rangle = \frac{1}{L}\frac{x^2}{2}\bigg|_0^L - \frac{1}{L}\frac{L^2}{16\pi^2}\left[\frac{4\pi x}{L}\sin\frac{4\pi x}{L} + \cos\frac{4\pi x}{L}\right]_0^L = \boxed{\frac{L}{2}}$$

(b) Probability $= \displaystyle\int_{0.490L}^{0.510L}\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx = \left[\frac{1}{L}x - \frac{1}{L}\frac{L}{4\pi}\sin\frac{4\pi x}{L}\right]_{0.490L}^{0.510L}$

Probability $= 0.020 - \dfrac{1}{4\pi}\left(\sin 2.04\pi - \sin 1.96\pi\right) = \boxed{5.26\times10^{-5}}$

(c) Probability $\left[\dfrac{x}{L} - \dfrac{1}{4\pi}\sin\dfrac{4\pi x}{L}\right]_{0.240L}^{0.260L} = \boxed{3.99\times10^{-2}}$

(d) In the $n = 2$ graph in the text's Figure 41.4(b), it is more probable to find the particle either near $x = \dfrac{L}{4}$ or $x = \dfrac{3L}{4}$ than at the center, where the probability density is zero. Nevertheless, the symmetry of the distribution means that the average position is $\dfrac{L}{2}$.

P41.15 Normalization requires

$$\int_{\text{all space}} |\psi|^2\,dx = 1 \qquad \text{or} \qquad \int_0^L A^2\sin^2\left(\frac{n\pi x}{L}\right)dx = 1$$

$$\int_0^L A^2\sin^2\left(\frac{n\pi x}{L}\right)dx = A^2\left(\frac{L}{2}\right) = 1 \qquad \text{or} \qquad \boxed{A = \sqrt{\frac{2}{L}}}$$

***P41.16** (a) The probability is $\int_0^{L/3} |\psi_1|^2\,dx = \frac{2}{L}\int_0^{L/3}\sin^2\left(\frac{\pi x}{L}\right)dx = \frac{1}{L}\int_0^{L/3}\left[1-\cos\left(\frac{2\pi x}{L}\right)\right]dx$

$$= \frac{1}{L}\left[x - \frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\right]_0^{L/3} = \frac{1}{3} - \frac{1}{2\pi}\sin\left(\frac{2\pi}{3}\right) = \frac{1}{3} - \frac{0.866}{2\pi} = \boxed{0.196}$$

(b) Classically, the particle moves back and forth steadily, spending equal time intervals in each third of the line. Then the classical probability is $\boxed{\text{0.333, significantly larger}}$.

(c) The probability is $\int_0^{L/3} |\psi_{99}|^2\,dx = \frac{2}{L}\int_0^{L/3}\sin^2\left(\frac{99\pi x}{L}\right)dx = \frac{1}{L}\int_0^{L/3}\left[1-\cos\left(\frac{198\pi x}{L}\right)\right]dx$

$$= \frac{1}{L}\left[x - \frac{L}{198\pi}\sin\left(\frac{198\pi x}{L}\right)\right]_0^{L/3} = \frac{1}{3} - \frac{1}{198\pi}\sin(66\pi) = \frac{1}{3} - 0 = \boxed{0.333}$$

$\boxed{\text{in agreement with the classical model}}$.

***P41.17** In $0 \le x \le L$, the argument $\frac{2\pi x}{L}$ of the sine function ranges from 0 to 2π. The probability density $\left(\frac{2}{L}\right)\sin^2\left(\frac{2\pi x}{L}\right)$ reaches maxima at $\sin\theta = 1$ and $\sin\theta = -1$. These points are at

$$\frac{2\pi x}{L} = \frac{\pi}{2} \quad \text{and} \quad \frac{2\pi x}{L} = \frac{3\pi}{2}.$$

Therefore the most probable positions of the particle are $\boxed{\text{at } x = \frac{L}{4} \text{ and } x = \frac{3L}{4}}$.

***P41.18** (a) Probability $= \int_0^{\ell} |\psi_1|^2\,dx = \frac{2}{L}\int_0^{\ell}\sin^2\left(\frac{\pi x}{L}\right)dx = \frac{1}{L}\int_0^{\ell}\left[1-\cos\left(\frac{2\pi x}{L}\right)\right]dx$

$$= \frac{1}{L}\left[x - \frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\right]_0^{\ell} = \boxed{\frac{\ell}{L} - \frac{1}{2\pi}\sin\left(\frac{2\pi\ell}{L}\right)}$$

(b)

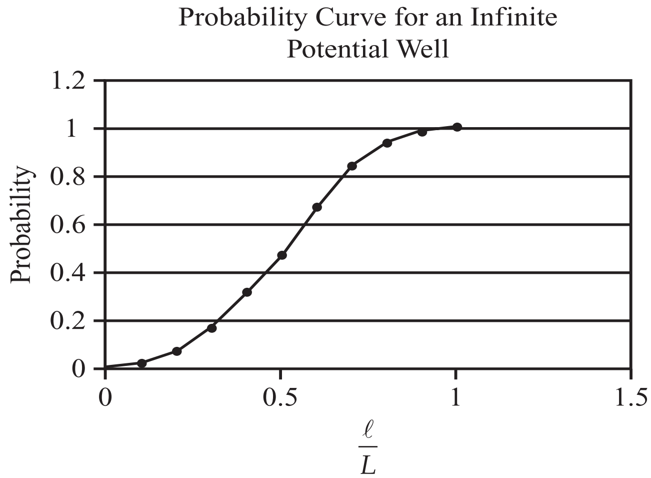


FIG. P41.18(b)

(c) The wave function is zero for $x < 0$ and for $x > L$. The probability at $\ell = 0$ must be zero because the particle is never found at $x < 0$ or exactly at $x = 0$. The probability at $\ell = L$ must be 1 for normalization: the particle is always found somewhere at $x < L$.

(d) The probability of finding the particle between $x = 0$ and $x = \ell$ is $\frac{2}{3}$, and between $x = \ell$ and $x = L$ is $\frac{1}{3}$.

Thus, $\int_0^{\ell} |\psi_1|^2\,dx = \frac{2}{3}$

$\therefore \frac{\ell}{L} - \frac{1}{2\pi}\sin\left(\frac{2\pi\ell}{L}\right) = \frac{2}{3}$, or $u - \frac{1}{2\pi}\sin 2\pi u = \frac{2}{3}$

This equation for $\frac{\ell}{L}$ can be solved by homing in on the solution with a calculator, the result being $\frac{\ell}{L} = 0.585$, or $\ell = \boxed{0.585L}$ to three digits.

P41.19 (a) The probability is $P = \int_0^{L/3} |\psi|^2\, dx = \int_0^{L/3} \frac{2}{L}\sin^2\left(\frac{\pi x}{L}\right)dx = \frac{2}{L}\int_0^{L/3}\left(\frac{1}{2} - \frac{1}{2}\cos\frac{2\pi x}{L}\right)dx$

$$P = \left(\frac{x}{L} - \frac{1}{2\pi}\sin\frac{2\pi x}{L}\right)\Bigg|_0^{L/3} = \left(\frac{1}{3} - \frac{1}{2\pi}\sin\frac{2\pi}{3}\right) = \left(\frac{1}{3} - \frac{\sqrt{3}}{4\pi}\right) = \boxed{0.196}$$

(b) The probability density is symmetric about $x = \frac{L}{2}$. Thus, the probability of finding the particle between $x = \frac{2L}{3}$ and $x = L$ is the same 0.196. Therefore, the probability of finding it in the range $\frac{L}{3} \le x \le \frac{2L}{3}$ is $P = 1.00 - 2(0.196) = \boxed{0.609}$.

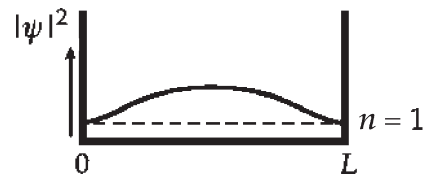


FIG. P41.19(b)

(c) Classically, the electron moves back and forth with constant speed between the walls, and the probability of finding the electron is the same for all points between the walls. Thus, the classical probability of finding the electron in any range equal to one-third of the available space is $P_{\text{classical}} = \frac{1}{3}$. $\boxed{\text{The result of part (a) is significantly smaller}}$, because of the curvature of the graph of the probability density.

Section 41.3 **The Schrödinger Equation**

P41.20 $\psi(x) = A\cos kx + B\sin kx$ $\qquad \frac{\partial\psi}{\partial x} = -kA\sin kx + kB\cos kx$

$\frac{\partial^2\psi}{\partial x^2} = -k^2 A\cos kx - k^2 B\sin kx$ $\qquad -\frac{2m}{\hbar}(E-U)\psi = -\frac{2mE}{\hbar^2}(A\cos kx + B\sin kx)$

Therefore the Schrödinger equation is satisfied if

$$\frac{\partial^2\psi}{\partial x^2} = \left(-\frac{2m}{\hbar^2}\right)(E-U)\psi \quad \text{or} \quad -k^2(A\cos kx + B\sin kx) = \left(-\frac{2mE}{\hbar^2}\right)(A\cos kx + B\sin kx)$$

This is true as an identity (functional equality) for all x if $\boxed{E = \frac{\hbar^2 k^2}{2m}}$.

P41.21 We have $\psi = Ae^{i(kx-\omega t)}$ so $\frac{\partial\psi}{\partial x} = ik\psi$ and $\frac{\partial^2\psi}{\partial x^2} = -k^2\psi$.

We test by substituting into Schrödinger's equation: $\frac{\partial^2\psi}{\partial x^2} = -k^2\psi = -\frac{2m}{\hbar^2}(E-U)\psi$.

Since $k^2 = \frac{(2\pi)^2}{\lambda^2} = \frac{(2\pi p)^2}{h^2} = \frac{p^2}{\hbar^2}$ and $E - U = \frac{p^2}{2m}$

$\boxed{\text{Thus this equation balances.}}$

P41.22 (a) Setting the total energy E equal to zero and rearranging the Schrödinger equation to isolate the potential energy function gives

$$U(x)=\left(\frac{\hbar^2}{2m}\right)\frac{1}{\psi}\frac{d^2\psi}{dx^2}$$

If $\qquad \psi(x)=Axe^{-x^2/L^2}$

Then $\qquad \dfrac{d^2\psi}{dx^2}=\left(4Ax^3-6AxL^2\right)\dfrac{e^{-x^2/L^2}}{L^4}$

or $\qquad \dfrac{d^2\psi}{dx^2}=\dfrac{\left(4x^2-6L^2\right)}{L^4}\psi(x)$

and $\qquad \boxed{U(x)=\dfrac{\hbar^2}{2mL^2}\left(\dfrac{4x^2}{L^2}-6\right)}$

(b) $\boxed{\text{See the figure to the right.}}$

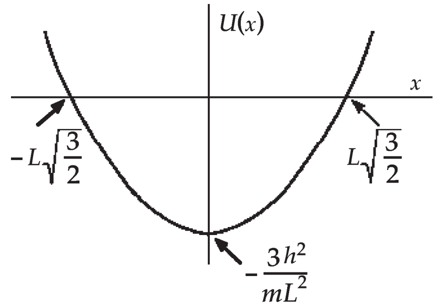


FIG. P41.22(b)

P41.23 Problem 41 in Chapter 16 helps students to understand how to draw conclusions from an identity.

(a) $\psi(x)=A\left(1-\dfrac{x^2}{L^2}\right) \qquad \dfrac{d\psi}{dx}=-\dfrac{2Ax}{L^2} \qquad \dfrac{d^2\psi}{dx^2}=-\dfrac{2A}{L^2}$

Schrödinger's equation $\qquad \dfrac{d^2\psi}{dx^2}=-\dfrac{2m}{\hbar^2}(E-U)\psi$

becomes $\qquad -\dfrac{2A}{L^2}=-\dfrac{2m}{\hbar^2}EA\left(1-\dfrac{x^2}{L^2}\right)+\dfrac{2m}{\hbar^2}\dfrac{\left(-\hbar^2x^2\right)A\left(1-x^2/L^2\right)}{mL^2\left(L^2-x^2\right)}$

$$-\frac{1}{L^2}=-\frac{mE}{\hbar^2}+\frac{mEx^2}{\hbar^2L^2}-\frac{x^2}{L^4}$$

This will be true for all x if both $\qquad \dfrac{1}{L^2}=\dfrac{mE}{\hbar^2}$

and $\qquad \dfrac{mE}{\hbar^2L^2}-\dfrac{1}{L^4}=0$

both these conditions are satisfied for a particle of energy $\qquad \boxed{E=\dfrac{\hbar^2}{L^2m}}$

(b) For normalization, $\qquad 1=\displaystyle\int_{-L}^{L}A^2\left(1-\frac{x^2}{L^2}\right)^2dx=A^2\int_{-L}^{L}\left(1-\frac{2x^2}{L^2}+\frac{x^4}{L^4}\right)dx$

$$1=A^2\left[x-\frac{2x^3}{3L^2}+\frac{x^5}{5L^4}\right]_{-L}^{L}=A^2\left[L-\frac{2}{3}L+\frac{L}{5}+L-\frac{2}{3}L+\frac{L}{5}\right]=A^2\left(\frac{16L}{15}\right) \qquad \boxed{A=\sqrt{\frac{15}{16L}}}$$

(c) $P=\displaystyle\int_{-L/3}^{L/3}\psi^2dx=\frac{15}{16L}\int_{-L/3}^{L/3}\left(1-\frac{2x^2}{L^2}+\frac{x^4}{L^4}\right)dx=\frac{15}{16L}\left[x-\frac{2x^3}{3L^2}+\frac{x^5}{5L^5}\right]_{-L/3}^{L/3}$

$$=\frac{30}{16L}\left[\frac{L}{3}-\frac{2L}{81}+\frac{L}{1\,215}\right]$$

$$P=\frac{47}{81}=\boxed{0.580}$$

P41.24 (a) $\psi_1(x)=\sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)$; $\quad P_1(x)=|\psi_1(x)|^2=\frac{2}{L}\cos^2\left(\frac{\pi x}{L}\right)$

$\psi_2(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$; $\quad P_2(x)=|\psi_2(x)|^2=\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)$

$\psi_3(x)=\sqrt{\frac{2}{L}}\cos\left(\frac{3\pi x}{L}\right)$; $\quad P_3(x)=|\psi_3(x)|^2=\frac{2}{L}\cos^2\left(\frac{3\pi x}{L}\right)$

(b)

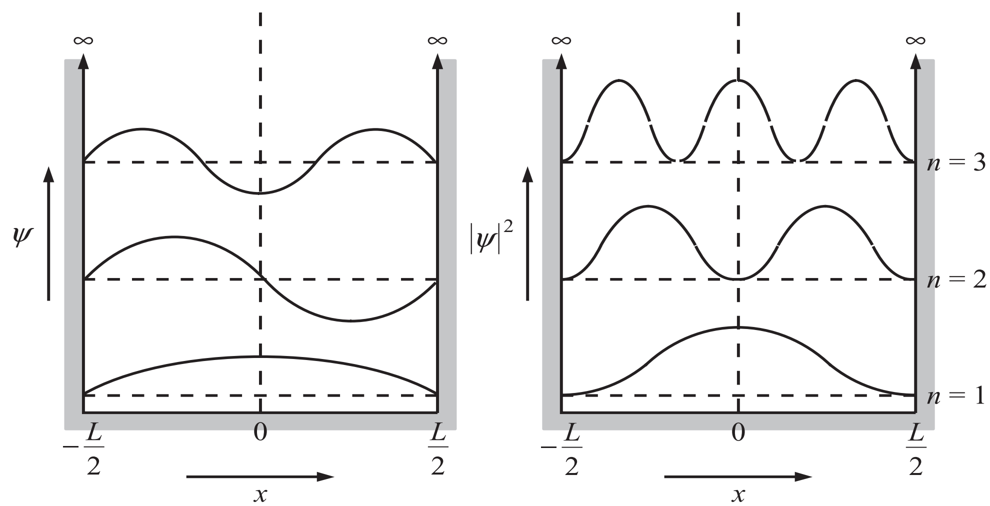


FIG. P41.24(b)

P41.25 (a) With $\psi(x)=A\sin(kx)$

$$\frac{d}{dx}A\sin kx=Ak\cos kx \qquad \text{and} \qquad \frac{d^2}{dx^2}\psi=-Ak^2\sin kx$$

Then $$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}=+\frac{\hbar^2k^2}{2m}A\sin kx=\frac{h^2\left(4\pi^2\right)}{4\pi^2\left(\lambda^2\right)(2m)}\psi=\frac{p^2}{2m}\psi=\frac{m^2u^2}{2m}\psi=\frac{1}{2}mu^2\psi=K\psi$$

(b) With $\psi(x)=A\sin\left(\frac{2\pi x}{\lambda}\right)=A\sin kx$, the proof given in part (a) applies again.

Section 41.4 **A Particle in a Well of Finite Height**

P41.26

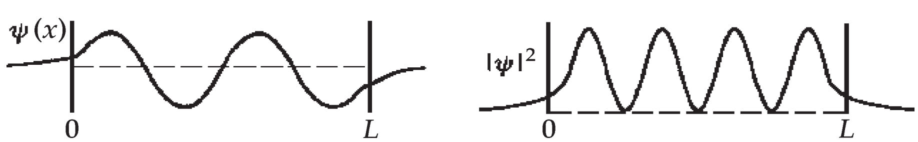


FIG. P41.26

P41.27 (a) $\boxed{\text{See figure to the right.}}$

(b) The wavelength of the transmitted wave traveling to the left is the same as the original wavelength, which equals $\boxed{2L}$.

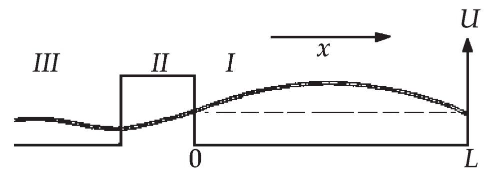


FIG. P41.27(a)

Section 41.5 Tunneling Through a Potential Energy Barrier

P41.28 $$C=\frac{\sqrt{2\left(9.11\times10^{-31}\right)(5.00-4.50)\left(1.60\times10^{-19}\right)}\ \text{kg}\cdot\text{m/s}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}}$$
$$=3.62\times10^{9}\ \text{m}^{-1}$$

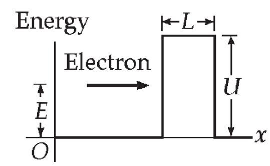


FIG. P41.28

$$T=e^{-2CL}=\exp\left[-2\left(3.62\times10^{9}\ \text{m}^{-1}\right)\left(950\times10^{-12}\ \text{m}\right)\right]=\exp(-6.88)$$
$$T=\boxed{1.03\times10^{-3}}$$

P41.29 From problem 28, $C=3.62\times10^{9}\ \text{m}^{-1}$

We require $10^{-6}=\exp\left[-2\left(3.62\times10^{9}\ \text{m}^{-1}\right)L\right]$.

Taking logarithms, $-13.816=-2\left(3.62\times10^{9}\ \text{m}^{-1}\right)L$

New $L=1.91\ \text{nm}$

$\Delta L=1.91\ \text{nm}-0.950\ \text{nm}=\boxed{0.959\ \text{nm}}$

***P41.30** $T=e^{-2CL}$ where $C=\dfrac{\sqrt{2m(U-E)}}{\hbar}$

(a) $$2CL=\frac{2\sqrt{2\left(9.11\times10^{-31}\right)\left(0.01\times1.6\times10^{-19}\right)}}{1.055\times10^{-34}}\left(10^{-10}\right)=0.102\qquad T=e^{-0.102}=\boxed{0.903}$$

(b) $$2CL=\frac{2\sqrt{2\left(9.11\times10^{-31}\right)\left(1.6\times10^{-19}\right)}}{1.055\times10^{-34}}\left(10^{-10}\right)=1.02\qquad T=e^{-1.02}=\boxed{0.359}$$

(c) $$2CL=\frac{2\sqrt{2\left(6.65\times10^{-27}\right)\left(10^{6}\times1.6\times10^{-19}\right)}}{1.055\times10^{-34}}\left(10^{-15}\right)=0.875\qquad T=e^{-0.875}=\boxed{0.417}$$

(d) $$2CL=\frac{2\sqrt{2(8)(1)}}{1.055\times10^{-34}}(0.02)=1.52\times10^{33}\qquad T=e^{-1.52\times10^{33}}=e^{(\ln10)(-1.52\times10^{33}/\ln10)}=\boxed{10^{-6.59\times10^{32}}}$$

P41.31 $T = e^{-2CL}$ where $C = \dfrac{\sqrt{2m(U-E)}}{\hbar}$

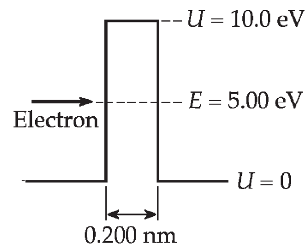


FIG. P41.31

$$2CL = \frac{2\sqrt{2\left(9.11\times10^{-31}\right)\left(8.00\times10^{-19}\right)}}{1.055\times10^{-34}}\left(2.00\times10^{-10}\right) = 4.58$$

(a) $T = e^{-4.58} = \boxed{0.010\,3}$, a 1% chance of transmission.

(b) $R = 1 - T = \boxed{0.990}$, a 99% chance of reflection.

P41.32 The original tunneling probability is $T = e^{-2CL}$ where

$$C = \frac{\left(2m(U-E)\right)^{1/2}}{\hbar} = \frac{2\pi\left(2\times9.11\times10^{-31}\text{ kg}(20-12)1.6\times10^{-19}\text{ J}\right)^{1/2}}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = 1.448\,1\times10^{10}\text{ m}^{-1}$$

The photon energy is $hf = \dfrac{hc}{\lambda} = \dfrac{1\,240\text{ eV}\cdot\text{nm}}{546\text{ nm}} = 2.27$ eV, to make the electron's new kinetic energy $12 + 2.27 = 14.27$ eV and its decay coefficient inside the barrier

$$C' = \frac{2\pi\left(2\times9.11\times10^{-31}\text{ kg}(20-14.27)1.6\times10^{-19}\text{ J}\right)^{1/2}}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = 1.225\,5\times10^{10}\text{ m}^{-1}$$

Now the factor of increase in transmission probability is

$$\frac{e^{-2C'L}}{e^{-2CL}} = e^{2L(C-C')} = e^{2\times10^{-9}\text{ m}\times0.223\times10^{10}\text{ m}^{-1}} = e^{4.45} = \boxed{85.9}$$

Section 41.6 **Applications of Tunneling**

P41.33 With the wave function proportional to e^{-CL}, the transmission coefficient and the tunneling current are proportional to $|\psi|^2$, to e^{-2CL}.

Then, $$\frac{I(0.500\text{ nm})}{I(0.515\text{ nm})} = \frac{e^{-2(10.0/\text{nm})(0.500\text{ nm})}}{e^{-2(10.0/\text{nm})(0.515\text{ nm})}} = e^{20.0(0.015)} = \boxed{1.35}$$

P41.34 With transmission coefficient e^{-2CL}, the fractional change in transmission is

$$\frac{e^{-2(10.0/\text{nm})L} - e^{-2(10.0/\text{nm})(L+0.002\,00\text{ nm})}}{e^{-2(10.0/\text{nm})L}} = 1 - e^{-20.0(0.002\,00)} = 0.0392 = \boxed{3.92\%}$$

Section 41.7 **The Simple Harmonic Oscillator**

P41.35 $\psi = Be^{-(m\omega/2\hbar)x^2}$ so $\frac{d\psi}{dx} = -\left(\frac{m\omega}{\hbar}\right)x\psi$ and $\frac{d^2\psi}{dx^2} = \left(\frac{m\omega}{\hbar}\right)^2 x^2\psi + \left(-\frac{m\omega}{\hbar}\right)\psi$

Substituting into the Schrödinger equation gives

$$\left(\frac{m\omega}{\hbar}\right)^2 x^2\psi + \left(-\frac{m\omega}{\hbar}\right)\psi = -\left(\frac{2mE}{\hbar^2}\right)\psi + \left(\frac{m\omega}{\hbar}\right)^2 x^2\psi$$

which is satisfied provided that $E = \frac{\hbar\omega}{2}$.

P41.36 Problem 41 in Chapter 16 helps students to understand how to draw conclusions from an identity.

$\psi = Axe^{-bx^2}$ so $\frac{d\psi}{dx} = Ae^{-bx^2} - 2bx^2Ae^{-bx^2}$

and $\frac{d^2\psi}{dx^2} = -2bxAe^{-bx^2} - 4bxAe^{-bx^2} + 4b^2x^3e^{-bx^2} = -6b\psi + 4b^2x^2\psi$

Substituting into the Schrödinger equation, $-6b\psi + 4b^2x^2\psi = -\left(\frac{2mE}{\hbar}\right)\psi + \left(\frac{m\omega}{\hbar}\right)^2 x^2\psi$

For this to be true as an identity, it must be true for all values of x.

So we must have both $-6b = -\frac{2mE}{\hbar^2}$ and $4b^2 = \left(\frac{m\omega}{\hbar}\right)^2$

(a) Therefore $\boxed{b = \frac{m\omega}{2\hbar}}$

(b) and $E = \frac{3b\hbar^2}{m} = \boxed{\frac{3}{2}\hbar\omega}$

(c) The wave function is that of the $\boxed{\text{first excited state}}$.

P41.37 The longest wavelength corresponds to minimum photon energy, which must be equal to the spacing between energy levels of the oscillator:

$$\frac{hc}{\lambda} = \hbar\omega = \hbar\sqrt{\frac{k}{m}} \text{ so } \lambda = 2\pi c\sqrt{\frac{m}{k}} = 2\pi\left(3.00\times10^8 \text{ m/s}\right)\left(\frac{9.11\times10^{-31} \text{ kg}}{8.99 \text{ N/m}}\right)^{1/2} = \boxed{600 \text{ nm}}$$

P41.38 (a) With $\psi = Be^{-(m\omega/2\hbar)x^2}$, the normalization condition $\int_{\text{all } x} |\psi|^2\, dx = 1$

becomes $1 = \int_{-\infty}^{\infty} B^2 e^{-2(m\omega/2\hbar)x^2} dx = 2B^2 \int_0^{\infty} e^{-(m\omega/\hbar)x^2} dx = 2B^2 \frac{1}{2}\sqrt{\frac{\pi}{m\omega/\hbar}}$

where Table B.6 in Appendix B was used to evaluate the integral.

Thus, $1 = B^2 \sqrt{\frac{\pi\hbar}{m\omega}}$ and $\boxed{B = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}}$.

(b) For small δ, the probability of finding the particle in the range $-\frac{\delta}{2} < x < \frac{\delta}{2}$ is

$$\int_{-\delta/2}^{\delta/2} |\psi|^2\, dx = \delta|\psi(0)|^2 = \delta B^2 e^{-0} = \boxed{\delta\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}}$$

P41.39 (a) For the center of mass to be fixed, $m_1u_1 + m_2u_2 = 0$. Then

$$u = |u_1| + |u_2| = |u_1| + \frac{m_1}{m_2}|u_1| = \frac{m_2 + m_1}{m_2}|u_1| \quad \text{and} \quad |u_1| = \frac{m_2 u}{m_1 + m_2}$$

Similarly, $u = \frac{m_2}{m_1}|u_2| + |u_2|$

and $|u_2| = \frac{m_1 u}{m_1 + m_2}$. Then

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 + \frac{1}{2}kx^2 = \frac{1}{2}\frac{m_1 m_2^2 u^2}{(m_1+m_2)^2} + \frac{1}{2}\frac{m_2 m_1^2 u^2}{(m_1+m_2)^2} + \frac{1}{2}kx^2$$

$$= \frac{1}{2}\frac{m_1 m_2 (m_1+m_2)}{(m_1+m_2)^2}u^2 + \frac{1}{2}kx^2 = \frac{1}{2}\mu u^2 + \frac{1}{2}kx^2$$

(b) $\frac{d}{dx}\left(\frac{1}{2}\mu u^2 + \frac{1}{2}kx^2\right) = 0$ because energy is constant

$$0 = \frac{1}{2}\mu\, 2u\frac{du}{dx} + \frac{1}{2}k2x = \mu\frac{dx}{dt}\frac{du}{dx} + kx = \mu\frac{du}{dt} + kx$$

Then $\mu a = -kx$, $a = -\frac{kx}{\mu}$. This is the condition for simple harmonic motion, that the acceleration of the equivalent particle be a negative constant times the excursion from equilibrium. By identification with $a = -\omega^2 x$, $\omega = \sqrt{\frac{k}{\mu}} = 2\pi f$ and $\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}}}$.

P41.40 (a) With $\langle x\rangle = 0$ and $\langle p_x\rangle = 0$, the average value of x^2 is $(\Delta x)^2$ and the average value of p_x^2 is $(\Delta p_x)^2$. Then $\Delta x \geq \frac{\hbar}{2\Delta p_x}$ requires

$$E \geq \frac{p_x^2}{2m} + \frac{k}{2}\frac{\hbar^2}{4p_x^2} = \boxed{\frac{p_x^2}{2m} + \frac{k\hbar^2}{8p_x^2}}$$

(b) To minimize this as a function of p_x^2, we require $\frac{dE}{dp_x^2} = 0 = \frac{1}{2m} + \frac{k\hbar^2}{8}(-1)\frac{1}{p_x^4}$

Then $\frac{k\hbar^2}{8p_x^4} = \frac{1}{2m}$ so $p_x^2 = \left(\frac{2mk\hbar^2}{8}\right)^{1/2} = \frac{\hbar\sqrt{mk}}{2}$

and $E \geq \frac{\hbar\sqrt{mk}}{2(2m)} + \frac{k\hbar^2 2}{8\hbar\sqrt{mk}} = \frac{\hbar}{4}\sqrt{\frac{k}{m}} + \frac{\hbar}{4}\sqrt{\frac{k}{m}}$

$$E_{\min} = \frac{\hbar}{2}\sqrt{\frac{k}{m}} = \boxed{\frac{\hbar\omega}{2}}$$

Additional Problems

P41.41 Suppose the marble has mass 20 g. Suppose the wall of the box is 12 cm high and 2 mm thick. While it is inside the wall,

$$U = mgy = (0.02\text{ kg})(9.8\text{ m/s}^2)(0.12\text{ m}) = 0.023\,5\text{ J}$$

and $E = K = \frac{1}{2}mu^2 = \frac{1}{2}(0.02\text{ kg})(0.8\text{ m/s})^2 = 0.006\,4\text{ J}$

Then $C = \frac{\sqrt{2m(U-E)}}{\hbar} = \frac{\sqrt{2(0.02\text{ kg})(0.017\,1\text{ J})}}{1.055\times10^{-34}\text{ J}\cdot\text{s}} = 2.5\times10^{32}\text{ m}^{-1}$

and the transmission coefficient is

$$e^{-2CL} = e^{-2(2.5\times10^{32})(2\times10^{-3})} = e^{-10\times10^{29}} = e^{-2.30(4.3\times10^{29})} = 10^{-4.3\times10^{29}} = \boxed{\sim 10^{-10^{30}}}$$

P41.42 (a) $\lambda = 2L = \boxed{2.00\times10^{-10}\text{ m}}$

(b) $p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{2.00\times10^{-10}\text{ m}} = \boxed{3.31\times10^{-24}\text{ kg}\cdot\text{m/s}}$

(c) $E = \frac{p^2}{2m} = \boxed{0.172\text{ eV}}$

P41.43 (a) See the figure. (b) See the figure.

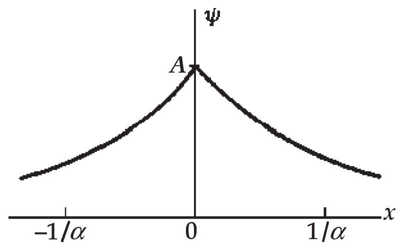


FIG. P41.43(a)

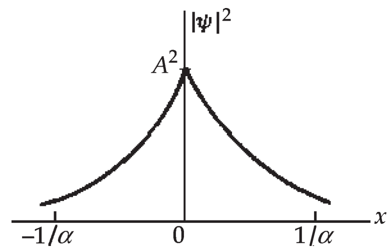


FIG. P41.43(b)

(c) ψ is continuous and $\psi \to 0$ as $x \to \pm\infty$. The function can be normalized. It describes a particle bound near $x = 0$.

(d) Since ψ is symmetric,

$$\int_{-\infty}^{\infty} |\psi|^2 \, dx = 2\int_{0}^{\infty} |\psi|^2 \, dx = 1$$

or
$$2A^2 \int_0^{\infty} e^{-2\alpha x} dx = \left(\frac{2A^2}{-2\alpha}\right)\left(e^{-\infty} - e^0\right) = 1$$

This gives $\boxed{A = \sqrt{\alpha}}$.

(e) $$P_{(-1/2\alpha)\to(1/2\alpha)} = 2\left(\sqrt{a}\right)^2 \int_{x=0}^{1/2\alpha} e^{-2\alpha x} dx = \left(\frac{2\alpha}{-2\alpha}\right)\left(e^{-2\alpha/2\alpha} - 1\right) = \left(1 - e^{-1}\right) = \boxed{0.632}$$

***P41.44** If we had $n = 0$ for a quantum particle in a box, its momentum would be zero. The uncertainty in its momentum would be zero. The uncertainty in its position would not be infinite, but just equal to the width of the box. Then the uncertainty product would be zero, to violate the uncertainty principle. The contradiction shows that the quantum number cannot be zero. In its ground state the particle has some nonzero zero-point energy.

***P41.45** (a) With ground state energy 0.3 eV, the energy in the $n = 2$ state is $2^2 \times 0.3$ eV = 1.2 eV. The energy in state 3 is 9×0.3 eV = 2.7 eV. The energy in state 4 is 16×0.3 eV = 4.8 eV. For the transition from the $n = 3$ level to the $n = 1$ level, the electron loses energy (2.7 – 0.3) eV = 2.4 eV. The photon carries off this energy and has wavelength hc/E = 1240 eV·nm/2.4 eV = $\boxed{517\text{ nm}}$.

(b) For the transition from level 2 to level 1, the photon energy is 0.9 eV and the photon wavelength is $\lambda = hc/E = 1240\text{ eV}\cdot\text{nm}/0.9\text{ eV} = \boxed{1.38\ \mu\text{m}}$. This photon, with wavelength greater than 700 nm, is $\boxed{\text{infrared}}$.

For level 4 to 1, $E = 4.5$ eV and $\lambda = \boxed{276\text{ nm ultraviolet}}$.

For 3 to 2, $E = 1.5$ eV and $\boxed{\lambda = 827\text{ nm infrared}}$.

For 4 to 2, $E = 3.6$ eV and $\boxed{\lambda = 344\text{ nm near ultraviolet}}$.

For 4 to 3, $E = 2.1$ eV and $\boxed{\lambda = 590\text{ nm yellow-orange visible}}$.

P41.46 (a) Use Schrödinger's equation

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{2m}{\hbar^2}(E-U)\psi$$

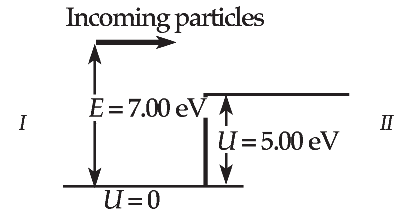


FIG. P41.46(a)

with solutions

$$\psi_1 = Ae^{ik_1x} + Be^{-ik_1x} \qquad \text{[region } I\text{]}$$

$$\psi_2 = Ce^{ik_2x} \qquad \text{[region } II\text{]}$$

Where $k_1 = \frac{\sqrt{2mE}}{\hbar}$

and $k_2 = \frac{\sqrt{2m(E-U)}}{\hbar}$

Then, matching functions and derivatives at $x = 0$

$(\psi_1)_0 = (\psi_2)_0$ gives $A + B = C$

and $\left(\frac{d\psi_1}{dx}\right)_0 = \left(\frac{d\psi_2}{dx}\right)_0$ gives $k_1(A-B) = k_2C$

Then $B = \frac{1-k_2/k_1}{1+k_2/k_1}A$

and $C = \frac{2}{1+k_2/k_1}A$

Incident wave Ae^{ikx} reflects Be^{-ikx}, with probability $R = \frac{B^2}{A^2} = \frac{(1-k_2/k_1)^2}{(1+k_2/k_1)^2}$

$$= \boxed{\frac{(k_1-k_2)^2}{(k_1+k_2)^2}}$$

(b) With $E = 7.00 \text{ eV}$

and $U = 5.00 \text{ eV}$

$$\frac{k_2}{k_1} = \sqrt{\frac{E-U}{E}} = \sqrt{\frac{2.00}{7.00}} = 0.535$$

The reflection probability is $R = \frac{(1-0.535)^2}{(1+0.535)^2} = \boxed{0.092\,0}$

The probability of transmission is $T = 1 - R = \boxed{0.908}$

P41.47 $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2} = \frac{(1 - k_2/k_1)^2}{(1 + k_2/k_1)^2}$

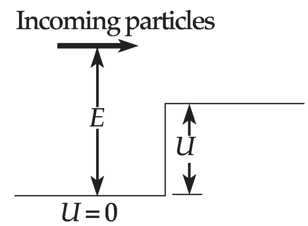


FIG. P41.47

$\frac{\hbar^2 k^2}{2m} = E - U$ for constant U

$\frac{\hbar^2 k_1^2}{2m} = E$ since $U = 0$ (1)

$\frac{\hbar^2 k_2^2}{2m} = E - U$ (2)

Dividing (2) by (1), $\frac{k_2^2}{k_1^2} = 1 - \frac{U}{E} = 1 - \frac{1}{2} = \frac{1}{2}$ so $\frac{k_2}{k_1} = \frac{1}{\sqrt{2}}$

and therefore, $R = \frac{(1 - 1/\sqrt{2})^2}{(1 + 1/\sqrt{2})^2} = \frac{(\sqrt{2} - 1)^2}{(\sqrt{2} + 1)^2} = \boxed{0.029\ 4}$

P41.48 (a) The wave functions and probability densities are the same as those shown in the two lower curves in Figure 41.4 of the textbook.

(b) $P_1 = \int_{0.150\text{ nm}}^{0.350\text{ nm}} |\psi_1|^2\, dx = \left(\frac{2}{1.00\text{ nm}}\right) \int_{0.150}^{0.350} \sin^2\left(\frac{\pi x}{1.00\text{ nm}}\right) dx$

$= (2.00/\text{nm}) \left[\frac{x}{2} - \frac{1.00\text{ nm}}{4\pi} \sin\left(\frac{2\pi x}{1.00\text{ nm}}\right)\right]_{0.150\text{ nm}}^{0.350\text{ nm}}$

In the above result we used $\int \sin^2 ax dx = \left(\frac{x}{2}\right) - \left(\frac{1}{4a}\right)\sin(2ax)$.

Therefore, $P_1 = (1.00/\text{nm})\left[x - \frac{1.00\text{ nm}}{2\pi} \sin\left(\frac{2\pi x}{1.00\text{ nm}}\right)\right]_{0.150\text{ nm}}^{0.350\text{ nm}}$

$P_1 = (1.00/\text{nm})\left\{0.350\text{ nm} - 0.150\text{ nm} - \frac{1.00\text{ nm}}{2\pi}[\sin(0.700\pi) - \sin(0.300\pi)]\right\} = \boxed{0.200}$

(c) $P_2 = \frac{2}{1.00} \int_{0.150}^{0.350} \sin^2\left(\frac{2\pi x}{1.00}\right) dx = 2.00\left[\frac{x}{2} - \frac{1.00}{8\pi}\sin\left(\frac{4\pi x}{1.00}\right)\right]_{0.150}^{0.350}$

$P_2 = 1.00\left[x - \frac{1.00}{4\pi}\sin\left(\frac{4\pi x}{1.00}\right)\right]_{0.150}^{0.350}$

$= 1.00\left\{(0.350 - 0.150) - \frac{1.00}{4\pi}[\sin(1.40\pi) - \sin(0.600\pi)]\right\}$

$= \boxed{0.351}$

(d) Using $E_n = \frac{n^2 h^2}{8mL^2}$, we find that $E_1 = \boxed{0.377\text{ eV}}$ and $E_2 = \boxed{1.51\text{ eV}}$.

P41.49 (a) $f = \frac{E}{h} = \frac{(1.80\text{ eV})}{(6.626\times10^{-34}\text{ J}\cdot\text{s})}\left(\frac{1.60\times10^{-19}\text{ J}}{1.00\text{ eV}}\right) = \boxed{4.34\times10^{14}\text{ Hz}}$

(b) $\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\text{ m/s}}{4.34\times10^{14}\text{ Hz}} = 6.91\times10^{-7}\text{ m} = \boxed{691\text{ nm}}$

(c) $\Delta E\Delta t \geq \frac{\hbar}{2}$ so $\Delta E \geq \frac{\hbar}{2\Delta t} = \frac{h}{4\pi(\Delta t)} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi(2.00\times10^{-6}\text{ s})} = 2.64\times10^{-29}\text{ J} = \boxed{1.65\times10^{-10}\text{ eV}}$

P41.50 (a) Taking $L_x = L_y = L$, we see that the expression for E becomes

$$E = \frac{h^2}{8m_eL^2}\left(n_x^2 + n_y^2\right)$$

For a normalizable wave function describing a particle, neither n_x nor n_y can be zero. The ground state, corresponding to $n_x = n_y = 1$, has an energy of

$$E_{1,1} = \frac{h^2}{8m_eL^2}\left(1^2 + 1^2\right) = \boxed{\frac{h^2}{4m_eL^2}}$$

The first excited state, corresponding to either $n_x = 2$, $n_y = 1$ or $n_x = 1$, $n_y = 2$, has an energy

$$E_{2,1} = E_{1,2} = \frac{h^2}{8m_eL^2}\left(2^2 + 1^2\right) = \boxed{\frac{5h^2}{8m_eL^2}}$$

The second excited state, corresponding to $n_x = 2$, $n_y = 2$, has an energy of

$$E_{2,2} = \frac{h^2}{8m_eL^2}\left(2^2 + 2^2\right) = \boxed{\frac{h^2}{m_eL^2}}$$

Finally, the third excited state, corresponding to either $n_x = 1$, $n_y = 3$ or $n_x = 3$, $n_x = 1$, has an energy

$$E_{1,3} = E_{3,1} = \frac{h^2}{8m_eL^2}\left(1^2 + 3^2\right) = \boxed{\frac{5h^2}{4m_eL^2}}$$

(b) The energy difference between the second excited state and the ground state is given by

$$\Delta E = E_{2,2} - E_{1,1} = \frac{h^2}{m_eL^2} - \frac{h^2}{4m_eL^2}$$

$$= \boxed{\frac{3h^2}{4m_eL^2}}$$

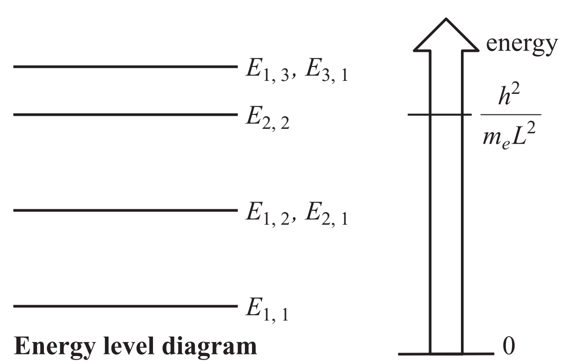


FIG. P41.50(b)

P41.51 $\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 |\psi|^2 \, dx$

For a one-dimensional box of width L, $\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$.

Thus, $\langle x^2 \rangle = \frac{2}{L} \int_0^L x^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = \boxed{\frac{L^2}{3} - \frac{L^2}{2n^2\pi^2}}$ (from integral tables).

P41.52 (a) $\int_{-\infty}^{\infty} |\psi|^2 \, dx = 1$ becomes

$$A^2 \int_{-L/4}^{L/4} \cos^2\left(\frac{2\pi x}{L}\right) dx = A^2 \left(\frac{L}{2\pi}\right) \left[\frac{\pi x}{L} + \frac{1}{4} \sin\left(\frac{4\pi x}{L}\right)\right]_{-L/4}^{L/4} = A^2 \left(\frac{L}{2\pi}\right) \left[\frac{\pi}{2}\right] = 1$$

or $A^2 = \frac{4}{L}$ and $\boxed{A = \frac{2}{\sqrt{L}}}$.

(b) The probability of finding the particle between 0 and $\frac{L}{8}$ is

$$\int_0^{L/8} |\psi|^2 \, dx = A^2 \int_0^{L/8} \cos^2\left(\frac{2\pi x}{L}\right) dx = \frac{1}{4} + \frac{1}{2\pi} = \boxed{0.409}$$

P41.53 For a particle with wave function

$$\psi(x) = \sqrt{\frac{2}{a}} e^{-x/a} \qquad \text{for } x > 0$$

and 0 for $x < 0$

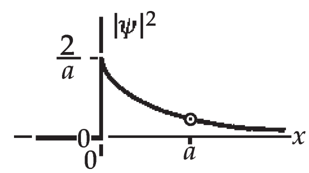


FIG. P41.53

(a) $|\psi(x)|^2 = 0,\ x < 0$ and $|\psi^2(x)| = \frac{2}{a} e^{-2x/a},\ x > 0$ as shown

(b) $\text{Prob}(x < 0) = \int_{-\infty}^{0} |\psi(x)|^2 \, dx = \int_{-\infty}^{0} (0) \, dx = \boxed{0}$

(c) Normalization $\int_{-\infty}^{\infty} |\psi(x)|^2 \, dx = \int_{-\infty}^{0} |\psi|^2 \, dx + \int_0^{\infty} |\psi|^2 \, dx = 1$

$$\int_{-\infty}^{0} 0 dx + \int_0^{\infty} \left(\frac{2}{a}\right) e^{-2x/a} dx = 0 - e^{-2x/a}\Big|_0^{\infty} = -\left(e^{-\infty} - 1\right) = 1$$

$$\text{Prob}(0 < x < a) = \int_0^a |\psi|^2 \, dx = \int_0^a \left(\frac{2}{a}\right) e^{-2x/a} dx$$

$$= -e^{-2x/a}\Big|_0^a = 1 - e^{-2} = \boxed{0.865}$$

P41.54 (a) The requirement that $\frac{n\lambda}{2} = L$ so $p = \frac{h}{\lambda} = \frac{nh}{2L}$ is still valid.

$$E = \sqrt{(pc)^2 + (mc^2)^2} \Rightarrow E_n = \sqrt{\left(\frac{nhc}{2L}\right)^2 + (mc^2)^2}$$

$$K_n = E_n - mc^2 = \boxed{\sqrt{\left(\frac{nhc}{2L}\right)^2 + (mc^2)^2} - mc^2}$$

(b) Taking $L = 1.00 \times 10^{-12}$ m, $m = 9.11 \times 10^{-31}$ kg, and $n = 1$, we find $K_1 = \boxed{4.69 \times 10^{-14} \text{ J}}$.

Nonrelativistic, $E_1 = \frac{h^2}{8mL^2} = \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})^2}{8(9.11 \times 10^{-31} \text{ kg})(1.00 \times 10^{-12} \text{ m})^2} = 6.02 \times 10^{-14}$ J.

Comparing this to K_1, we see that this value is too large by $\boxed{28.6\%}$.

P41.55 (a) $U = \frac{e^2}{4\pi \in_0 d}\left[-1 + \frac{1}{2} - \frac{1}{3} + \left(-1 + \frac{1}{2}\right) + (-1)\right] = \frac{(-7/3)e^2}{4\pi \in_0 d} = \boxed{-\frac{7k_e e^2}{3d}}$

(b) From Equation 41.14, $K = 2E_1 = \frac{2h^2}{8m_e(9d^2)} = \boxed{\frac{h^2}{36m_e d^2}}$.

(c) $E = U + K$ and $\frac{dE}{dd} = 0$ for a minimum: $\frac{7k_e e^2}{3d^2} - \frac{h^2}{18m_e d^3} = 0$

$$d = \frac{3h^2}{(7)(18k_e e^2 m_e)} = \frac{h^2}{42m_e k_e e^2} = \frac{(6.626 \times 10^{-34})^2}{(42)(9.11 \times 10^{-31})(8.99 \times 10^9)(1.60 \times 10^{-19} \text{ C})^2}$$

$$= \boxed{0.049\,9 \text{ nm}}$$

(d) Since the lithium spacing is a, where $Na^3 = V$, and the density is $\frac{Nm}{V}$, where m is the mass of one atom, we get:

$$a = \left(\frac{Vm}{Nm}\right)^{1/3} = \left(\frac{m}{\text{density}}\right)^{1/3} = \left(\frac{1.66 \times 10^{-27} \text{ kg} \times 7}{530 \text{ kg}}\right)^{1/3} \text{ m} = 2.80 \times 10^{-10} \text{ m} = \boxed{0.280 \text{ nm}}$$

The lithium interatomic spacing of 280 pm is 5.62 times larger than the answer to (c). Thus it is of the same order of magnitude as the interatomic spacing $2d$ here.

P41.56 (a) $\psi = Bxe^{-(m\omega/2\hbar)x^2}$

$$\frac{d\psi}{dx} = Be^{-(m\omega/2\hbar)x^2} + Bx\left(-\frac{m\omega}{2\hbar}\right)2xe^{-(m\omega/2\hbar)x^2} = Be^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)x^2 e^{-(m\omega/2\hbar)x^2}$$

$$\frac{d^2\psi}{dx^2} = Bx\left(-\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)2xe^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)x^2\left(-\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2}$$

$$\frac{d^2\psi}{dx^2} = -3B\left(\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} + B\left(\frac{m\omega}{\hbar}\right)^2 x^3 e^{-(m\omega/2\hbar)x^2}$$

Substituting into the Schrödinger equation, we have

$$-3B\left(\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} + B\left(\frac{m\omega}{\hbar}\right)^2 x^3 e^{-(m\omega/2\hbar)x^2} = -\frac{2mE}{\hbar^2}Bxe^{-(m\omega/2\hbar)x^2} + \left(\frac{m\omega}{\hbar}\right)^2 x^2 Bxe^{-(m\omega/2\hbar)x^2}$$

This is true if $-3\omega = -\frac{2E}{\hbar}$; it is true if $\boxed{E = \frac{3\hbar\omega}{2}}$.

(b) We never find the particle at $\boxed{x = 0}$ because $\psi = 0$ there.

(c) ψ is maximized if $\frac{d\psi}{dx} = 0 = 1 - x^2\left(\frac{m\omega}{\hbar}\right)$, which is true at $\boxed{x = \pm\sqrt{\frac{\hbar}{m\omega}}}$.

(d) We require $\int_{-\infty}^{\infty}|\psi|^2\,dx = 1$:

$$1 = \int_{-\infty}^{\infty} B^2x^2e^{-(m\omega/\hbar)x^2}dx = 2B^2\int x^2e^{-(m\omega/\hbar)x^2}dx = 2B^2\frac{1}{4}\sqrt{\frac{\pi}{(m\omega/\hbar)^3}} = \frac{B^2}{2}\frac{\pi^{1/2}\hbar^{3/2}}{(m\omega)^{3/2}}$$

Then $B = \frac{2^{1/2}}{\pi^{1/4}}\left(\frac{m\omega}{\hbar}\right)^{3/4} = \boxed{\left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/4}}$.

(e) At $x = 2\sqrt{\frac{\hbar}{m\omega}}$, the potential energy is $\frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2\left(\frac{4\hbar}{m\omega}\right) = 2\hbar\omega$. This is larger than the total energy $\frac{3\hbar\omega}{2}$, so there is $\boxed{\text{zero}}$ classical probability of finding the particle here.

(f) Probability $= |\psi|^2\,dx = \left(Bxe^{-(m\omega/2\hbar)x^2}\right)^2\delta = \delta B^2x^2e^{-(m\omega/\hbar)x^2}$

Probability $= \delta\frac{2}{\pi^{1/2}}\left(\frac{m\omega}{\hbar}\right)^{3/2}\left(\frac{4\hbar}{m\omega}\right)e^{-(m\omega/\hbar)4(\hbar/m\omega)} = \boxed{8\delta\left(\frac{m\omega}{\hbar\pi}\right)^{1/2}e^{-4}}$

P41.57 (a) $\int_0^L |\psi|^2\,dx = 1$: $\quad A^2\int_0^L\left[\sin^2\left(\frac{\pi x}{L}\right)+16\sin^2\left(\frac{2\pi x}{L}\right)+8\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)\right]dx = 1$

$$A^2\left[\left(\frac{L}{2}\right)+16\left(\frac{L}{2}\right)+8\int_0^L \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)dx\right]=1$$

$$A^2\left[\frac{17L}{2}+16\int_0^L \sin^2\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right)dx\right]=A^2\left[\frac{17L}{2}+\frac{16L}{3\pi}\sin^3\left(\frac{\pi x}{L}\right)\Big|_{x=0}^{x=L}\right]=1$$

$A^2=\dfrac{2}{17L}$, so the normalization constant is $\boxed{A=\sqrt{\dfrac{2}{17L}}}$.

(b) $\int_{-a}^{a}|\psi|^2\,dx=1$: $\quad \int_{-a}^{a}\left[|A|^2\cos^2\left(\frac{\pi x}{2a}\right)+|B|^2\sin^2\left(\frac{\pi x}{a}\right)+2|A||B|\cos\left(\frac{\pi x}{2a}\right)\sin\left(\frac{\pi x}{a}\right)\right]dx=1$

The first two terms are $|A|^2 a$ and $|B|^2 a$. The third term is:

$$2|A||B|\int_{-a}^{a}\cos\left(\frac{\pi x}{2a}\right)\left[2\sin\left(\frac{\pi x}{2a}\right)\cos\left(\frac{\pi x}{2a}\right)\right]dx=4|A||B|\int_{-a}^{a}\cos^2\left(\frac{\pi x}{2a}\right)\sin\left(\frac{\pi x}{2a}\right)dx$$

$$=\frac{8a|A||B|}{3\pi}\cos^3\left(\frac{\pi x}{2a}\right)\Big|_{-a}^{a}=0$$

so that $a\left(|A|^2+|B|^2\right)=1$, giving $\boxed{|A|^2+|B|^2=\dfrac{1}{a}}$.

P41.58 (a) $\langle x\rangle_0=\int_{-\infty}^{\infty}x\left(\frac{a}{\pi}\right)^{1/2}e^{-ax^2}dx=\boxed{0}$, since the integrand is an odd function of x.

(b) $\langle x\rangle_1=\int_{-\infty}^{\infty}x\left(\frac{4a^3}{\pi}\right)^{1/2}x^2e^{-ax^2}dx=\boxed{0}$, since the integrand is an odd function of x.

(c) $\langle x\rangle_{01}=\int_{-\infty}^{\infty}x\frac{1}{2}\left(\psi_0+\psi_1\right)^2dx=\frac{1}{2}\langle x\rangle_0+\frac{1}{2}\langle x\rangle_1+\int_{-\infty}^{\infty}x\psi_0(x)\psi_1(x)\,dx$

The first two terms are zero, from (a) and (b). Thus:

$$\langle x\rangle_{01}=\int_{-\infty}^{\infty}x\left(\frac{a}{\pi}\right)^{1/4}e^{-ax^2/2}\left(\frac{4a^3}{\pi}\right)^{1/4}xe^{-ax^2/2}dx=2\left(\frac{2a^2}{\pi}\right)^{1/2}\int_0^{\infty}x^2e^{-ax^2}dx$$

$$=2\left(\frac{2a^2}{\pi}\right)^{1/2}\frac{1}{4}\left(\frac{\pi}{a^3}\right)^{1/2}\text{, from Table B.6}$$

$$=\boxed{\frac{1}{\sqrt{2a}}}$$

P41.59 With one slit open $P_1 = |\psi_1|^2$ or $P_2 = |\psi_2|^2$

With both slits open, $P = |\psi_1 + \psi_2|^2$

At a maximum, the wave functions are in phase $P_{\max} = \left(|\psi_1| + |\psi_2|\right)^2$

At a minimum, the wave functions are out of phase $P_{\min} = \left(|\psi_1| - |\psi_2|\right)^2$

Now $\dfrac{P_1}{P_2} = \dfrac{|\psi_1|^2}{|\psi_2|^2} = 25.0$, so $\dfrac{|\psi_1|}{|\psi_2|} = 5.00$

and $\dfrac{P_{\max}}{P_{\min}} = \dfrac{\left(|\psi_1| + |\psi_2|\right)^2}{\left(|\psi_1| - |\psi_2|\right)^2} = \dfrac{\left(5.00|\psi_2| + |\psi_2|\right)^2}{\left(5.00|\psi_2| - |\psi_2|\right)^2} = \dfrac{(6.00)^2}{(4.00)^2} = \dfrac{36.0}{16.0} = \boxed{2.25}$

ANSWERS TO EVEN PROBLEMS

P41.2 $\dfrac{1}{2}$

P41.4 (a) 4 (b) 6.03 eV

P41.6 9.56×10^{12}

P41.8 $\left(\dfrac{3h\lambda}{8m_e c}\right)^{1/2}$

P41.10 (a) 5.13 meV (b) 9.41 eV (c) The much smaller mass of the electron requires it to have much more energy to have the same momentum.

P41.12 (a) $\left(\dfrac{15h\lambda}{8m_e c}\right)^{1/2}$ (b) 1.25λ

P41.14 (a) $\dfrac{L}{2}$ (b) 5.26×10^{-5} (c) 3.99×10^{-2} (d) See the solution.

P41.16 (a) 0.196 (b) The classical probability is 0.333, significantly larger. (c) 0.333 for both classical and quantum models.

P41.18 (a) $\dfrac{\ell}{L} - \dfrac{1}{2\pi}\sin\left(\dfrac{2\pi\ell}{L}\right)$ (b) See the solution. (c) The wave function is zero for $x < 0$ and for $x > L$. The probability at $\ell = 0$ must be zero because the particle is never found at $x < 0$ or exactly at $x = 0$. The probability at $\ell = L$ must be 1 for normalization. This statement means that the particle is always found somewhere at $x < L$. (d) $\ell = 0.585L$

P41.20 See the solution; $\dfrac{\hbar^2 k^2}{2m}$

P41.22 (a) $\dfrac{\hbar^2}{2mL^2}\left(\dfrac{4x^2}{L^2} - 6\right)$ (b) See the solution.

P41.24 (a) $\psi_1(x)=\sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)$ $P_1(x)=\frac{2}{L}\cos^2\left(\frac{\pi x}{L}\right)$ $\psi_2(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$

$P_2(x)=\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)$ $\psi_3(x)=\sqrt{\frac{2}{L}}\cos\left(\frac{3\pi x}{L}\right)$ $P_3(x)=\frac{2}{L}\cos^2\left(\frac{3\pi x}{L}\right)$

(b) See the solution.

P41.26 See the solution.

P41.28 1.03×10^{-3}

P41.30 (a) 0.903 (b) 0.359 (c) 0.417 (d) $10^{-6.59\times10^{32}}$

P41.32 85.9

P41.34 3.92%

P41.36 (a) See the solution. $b=\frac{m\omega}{2\hbar}$ (b) $E=\frac{3}{2}\hbar\omega$ (c) first excited state

P41.38 (a) $B=\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}$ (b) $\delta\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}$

P41.40 See the solution.

P41.42 (a) 2.00×10^{-10} m (b) 3.31×10^{-24} kg·m/s (c) 0.172 eV

P41.44 See the solution.

P41.46 (a) See the solution. (b) 0.092 0, 0.908

P41.48 (a) See the solution. (b) 0.200 (c) 0.351 (d) 0.377 eV, 1.51 eV

P41.50 (a) $\frac{h^2}{4m_eL^2}$, $\frac{5h^2}{8m_eL^2}$, $\frac{h^2}{m_eL^2}$, $\frac{5h^2}{4m_eL^2}$ (b) See the solution, $\frac{3h^2}{4m_eL^2}$

P41.52 (a) $\frac{2}{\sqrt{L}}$ (b) 0.409

P41.54 (a) $\sqrt{\left(\frac{nhc}{2L}\right)^2+m^2c^4}-mc^2$ (b) 46.9 fJ; 28.6%

P41.56 (a) $\frac{3\hbar\omega}{2}$ (b) $x=0$ (c) $\pm\sqrt{\frac{\hbar}{m\omega}}$ (d) $\left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/4}$ (e) 0 (f) $8\delta\left(\frac{m\omega}{\hbar\pi}\right)^{1/2}e^{-4}$

P41.58 (a) 0 (b) 0 (c) $(2a)^{-1/2}$

42

Atomic Physics

Note: In chapters 39, 40, and 41 we used u to represent the speed of a particle with mass. In this chapter 42 and the remaining chapters we go back to using v for the symbol for speed.

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q42.1 If an electron moved like a hockey puck, it could have any arbitrary frequency of revolution around an atomic nucleus. If it behaved like a charge in a radio antenna, it would radiate light with frequency equal to its own frequency of oscillation. Thus, the electron in hydrogen atoms would emit a continuous spectrum, electromagnetic waves of all frequencies smeared together.

***Q42.2** (a) Yes, provided that the energy of the photon is *precisely* enough to put the electron into one of the allowed energy states. Strangely—more precisely non-classically—enough, if the energy of the photon is not sufficient to put the electron into a particular excited energy level, the photon will not interact with the atom at all!

(b) Yes, a photon of any energy greater than 13.6 eV will ionize the atom. Any "extra" energy will go into kinetic energy of the newly liberated electron.

***Q42.3** Answer (a). The 10.5-eV bombarding energy does not match the 10.2-eV excitation energy required to lift the atom from state 1 to state 2. But the atom can be excited into state 2 and the bombarding particle can carry off the excess energy.

***Q42.4** (i) b (ii) g From Equations 42.7, 42.8 and 42.9, we have $-|E| = -\frac{k_e e^2}{2r} = +\frac{k_e e^2}{2r} - \frac{k_e e^2}{r} = K + U_e$.

Then $K = |E|$ and $U_e = -2|E|$.

***Q42.5** In $\Delta E = (13.6 \text{ eV})\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$ for $\Delta E > 0$ we have absorption and for $\Delta E < 0$ we have emission.

(a) for $n_i = 2$ and $n_f = 5$, $\Delta E = 2.86$ eV (absorption)

(b) for $n_i = 5$ and $n_f = 3$, $\Delta E = -0.967$ eV (emission)

(c) for $n_i = 7$ and $n_f = 4$, $\Delta E = -0.572$ eV (emission)

(d) for $n_i = 4$ and $n_f = 7$, $\Delta E = 0.572$ eV (absorption)

(i) In order of energy change, the ranking is a > d > c > b

(ii) $E = \frac{hc}{\lambda}$ so the ranking in order of decreasing wavelength of the associated photon is c = d > b > a.

***Q42.6** (a) Yes. (b) No. The greatest frequency is that of the Lyman series limit. (c) Yes. We can imagine arbitrarily low photon energies for transitions between adjacent states with *n* large.

Q42.7 Bohr modeled the electron as moving in a perfect circle, with zero uncertainty in its radial coordinate. Then its radial velocity is always zero with zero uncertainty. Bohr's theory violates the uncertainty principle by making the uncertainty product $\Delta r \Delta p_r$ be zero, less than the minimum allowable $\frac{\hbar}{2}$.

***Q42.8** (i) d (ii) c and d (iii) b and c. Having *n* start from 1 and ℓ start from zero, with ℓ always less than *n*, is a way of reminding ourselves that the minimum kinetic energy of a bound quantum particle is greater than zero and the minimum angular momentum is precisely zero.

Q42.9 Fundamentally, three quantum numbers describe an orbital wave function because we live in three-dimensional space. They arise mathematically from boundary conditions on the wave function, expressed as a product of a function of *r*, a function of θ, and a function of ϕ.

Q42.10 Bohr's theory pictures the electron as moving in a flat circle like a classical particle described by $\Sigma F = ma$. Schrödinger's theory pictures the electron as a cloud of probability amplitude in the three-dimensional space around the hydrogen nucleus, with its motion described by a wave equation. In the Bohr model, the ground-state angular momentum is $1\hbar$; in the Schrödinger model the ground-state angular momentum is zero. Both models predict that the electron's energy is limited to discrete energy levels, given by $\frac{-13.606 \text{ eV}}{n^2}$ with $n = 1, 2, 3.$

Q42.11 Practically speaking, no. Ions have a net charge and the magnetic force $q(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$ would deflect the beam, making it difficult to separate the atoms with different orientations of magnetic moments.

Q42.12 The deflecting force on an atom with a magnetic moment is proportional to the *gradient* of the magnetic field. Thus, atoms with oppositely directed magnetic moments would be deflected in *opposite* directions in an inhomogeneous magnetic field.

Q42.13 If the exclusion principle were not valid, the elements and their chemical behavior would be grossly different because every electron would end up in the lowest energy level of the atom. All matter would be nearly alike in its chemistry and composition, since the shell structures of all elements would be identical. Most materials would have a much higher density. The spectra of atoms and molecules would be very simple, and there would be very little color in the world.

Q42.14 In a neutral helium atom, one electron can be modeled as moving in an electric field created by the nucleus and the other electron. According to Gauss's law, if the electron is above the ground state it moves in the electric field of a net charge of $+2e-1e=+1e$. We say the nuclear charge is *screened* by the inner electron. The electron in a He^+ ion moves in the field of the unscreened nuclear charge of 2 protons. Then the potential energy function for the electron is about double that of one electron in the neutral atom.

Q42.15 The three elements have similar electronic configurations. Each has filled inner shells plus one electron in an *s* orbital. Their single outer electrons largely determine their chemical interactions with other atoms.

Q42.16 Each of the electrons must have at least one quantum number different from the quantum numbers of each of the other electrons. They can differ (in m_s) by being spin-up or spin-down. They can also differ (in ℓ) in angular momentum and in the general shape of the wave function. Those electrons with $\ell=1$ can differ (in m_ℓ) in orientation of angular momentum—look at Figure Q42.16.

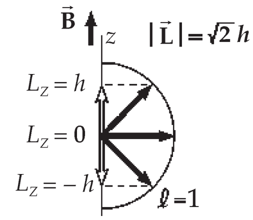


FIG. Q42.16

***Q42.17** The M means that the electron falls down into the M shell. The final principal quantum number is 3. M_α would refer to 4→3 and M_β refers to 5→3. Answers: (i) e (ii) c

Q42.18 No. Laser light is collimated. The energy generally travels in the same direction. The intensity of a laser beam stays remarkably constant, independent of the distance it has traveled.

Q42.19 Stimulated emission coerces atoms to emit photons along a specific axis, rather than in the random directions of spontaneously emitted photons. The photons that are emitted through stimulation can be made to accumulate over time. The fraction allowed to escape constitutes the intense, collimated, and coherent laser beam. If this process relied solely on spontaneous emission, the emitted photons would not exit the laser tube or crystal in the same direction. Neither would they be coherent with one another.

Q42.20 (a) The terms "I define" and "this part of the universe" seem vague, in contrast to the precision of the rest of the statement. But the statement is true in the sense of being experimentally verifiable. The way to test the orientation of the magnetic moment of an electron is to apply a magnetic field to it. When that is done for any electron, it has precisely a 50% chance of being either spin-up or spin-down. Its spin magnetic moment vector must make one of two allowed angles with the applied magnetic field. They are given by $\cos\theta=\frac{S_z}{S}=\frac{1/2}{\sqrt{3/2}}$ and $\cos\theta=\frac{-1/2}{\sqrt{3/2}}$. You can calculate as many digits of the two angles allowed by "space quantization" as you wish.

(b) This statement may be true. There is no reason to suppose that an ant can comprehend the cosmos, and no reason to suppose that a human can comprehend all of it. Our experience with macroscopic objects does not prepare us to understand quantum particles. On the other hand, what seems strange to us now may be the common knowledge of tomorrow. Looking back at the past 150 years of physics, great strides in understanding the Universe—from the quantum to the galactic scale—have been made. Think of trying to explain the photoelectric effect using Newtonian mechanics. What seems strange sometimes just has an underlying structure that has not yet been described fully. On the other hand still, it has been demonstrated that a "hidden-variable" theory, that would model quantum uncertainty as caused by some determinate but fluctuating quantity, cannot agree with experiment.

***Q42.21** People commonly say that science is difficult to learn. Many other branches of knowledge are constructed by humans. Learning them can require getting your mind to follow the path of someone else's mind. A well-developed science does not mirror human patterns of thought, but the way nature works. Thus we agree with the view stated in the problem. A scientific discovery can be like a garbled communication in the sense that it does not explain itself. It can seem fragmentary and in need of context. It can seem unfriendly in the sense that it has no regard for human desires or ease of human comprehension. Evolution has adapted the human mind for catching rabbits and outwitting bison and leopards, but not necessarily for understanding atoms. Education in science gives the student opportunities for understanding a wide variety of ideas; for evaluating new information; and for recognizing that extraordinary evidence must be adduced for extraordinary claims.

SOLUTIONS TO PROBLEMS

Section 42.1 **Atomic Spectra of Gases**

P42.1 (a) Lyman series $\frac{1}{\lambda} = R\left(1 - \frac{1}{n_i^2}\right)$ $\qquad n_i = 2, 3, 4, \ldots$

$$\frac{1}{\lambda} = \frac{1}{94.96 \times 10^{-9}} = \left(1.097 \times 10^7\right)\left(1 - \frac{1}{n_i^2}\right) \qquad \boxed{n_i = 5}$$

(b) Paschen series: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right)$ $\qquad n_i = 4, 5, 6, \ldots$

The shortest wavelength for this series corresponds to $n_i = \infty$ for ionization

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{9} - \frac{1}{n_i^2}\right)$$

For $n_i = \infty$, this gives $\lambda = 820$ nm

This is larger than 94.96 nm, so this wave length $\boxed{\text{cannot be associated with the Paschen series}}$.

Balmer series: $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$ $\qquad n_i = 3, 4, 5, \ldots$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{n_i^2}\right)$$ with $n_i = \infty$ for ionization, $\lambda_{\min} = 365$ nm

Once again the shorter given wavelength $\boxed{\text{cannot be associated with the Balmer series}}$.

***P42.2** (a) The fifth excited state must lie above the second excited state by the photon energy

$$E_{52} = hf = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3\times10^{8}\ \text{m/s}\right)}{520\times10^{-9}\ \text{m}} = 3.82\times10^{-19}\ \text{J}$$

The sixth excited state exceeds the second in energy by

$$E_{62} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3\times10^{8}\ \text{m/s}\right)}{410\times10^{-9}\ \text{m}} = 4.85\times10^{-19}\ \text{J}$$

Then the sixth excited state is above the fifth by $(4.85-3.82)10^{-19}\ \text{J} = 1.03\times10^{-19}\ \text{J}$. In the 6 to 5 transition the atom emits a photon with the infrared wavelength

$$\lambda = \frac{hc}{E_{62}} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3\times10^{8}\ \text{m/s}\right)}{1.03\times10^{-19}\ \text{J}} = \boxed{1.94\times10^{-6}\ \text{m}}$$

(b) The same steps solve the symbolic problem.

$$E_{CA} = E_{CB} + E_{BA}$$

$$\frac{hc}{\lambda_{CA}} = \frac{hc}{\lambda_{CB}} + \frac{hc}{\lambda_{BA}}$$

$$\frac{1}{\lambda_{CB}} = \frac{1}{\lambda_{CA}} - \frac{1}{\lambda_{BA}}$$

$$\boxed{\lambda_{CB} = \left(\frac{1}{\lambda_{CA}} - \frac{1}{\lambda_{BA}}\right)^{-1}}$$

Section 42.2 Early Models of the Atom

P42.3 (a) For a classical atom, the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{1}{4\pi\in_0}\frac{e^2}{r^2 m_e}$$

$$E = -\frac{e^2}{4\pi\in_0 r} + \frac{m_e v^2}{2} = -\frac{e^2}{8\pi\in_0 r}$$

so

$$\frac{dE}{dt} = \frac{e^2}{8\pi\in_0 r^2}\frac{dr}{dt} = \frac{-1}{6\pi\in_0}\frac{e^2 a^2}{c^3}$$

$$= \frac{-e^2}{6\pi\in_0 c^3}\left(\frac{e^2}{4\pi\in_0 r^2 m_e}\right)^2$$

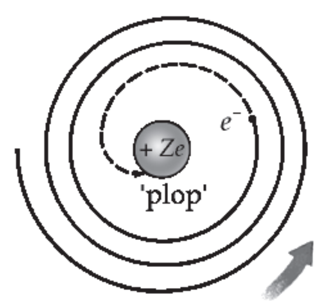


FIG. P42.3

Therefore, $\dfrac{dr}{dt} = -\dfrac{e^4}{12\pi^2 \in_0^2 r^2 m_e^2 c^3}$.

(b) $$-\int_{2.00\times10^{-10}\ \text{m}}^{0} 12\pi^2 \in_0^2 r^2 m_e^2 c^3\,dr = e^4 \int_0^T dt \qquad \frac{12\pi^2 \in_0^2 m_e^2 c^3}{e^4}\left.\frac{r^3}{3}\right|_0^{2.00\times10^{-10}} = T = \boxed{8.46\times10^{-10}\ \text{s}}$$

Since atoms last a lot longer than 0.8 ns, the classical laws (fortunately!) do not hold for systems of atomic size.

P42.4 (a) The point of closest approach is found when

$$E = K + U = 0 + \frac{k_e q_\alpha q_{\text{Au}}}{r}$$

or $$r_{\min} = \frac{k_e(2e)(79e)}{E}$$

$$r_{\min} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(158)\left(1.60\times10^{-19}\ \text{C}\right)^2}{(4.00\ \text{MeV})\left(1.60\times10^{-13}\ \text{J/MeV}\right)} = \boxed{5.68\times10^{-14}\ \text{m}}$$

(b) The maximum force exerted on the alpha particle is

$$F_{\max} = \frac{k_e q_\alpha q_{\text{Au}}}{r_{\min}^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(158)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(5.68\times10^{-14}\ \text{m}\right)^2} = \boxed{11.3\ \text{N}}$$ away from the nucleus.

Section 42.3 **Bohr's Model of the Hydrogen Atom**

***P42.5** (a) Longest wavelength implies lowest frequency and smallest energy:

the atom falls from $n = 3$ to $n = 2$

losing energy $$-\frac{13.6\ \text{eV}}{3^2} + \frac{13.6\ \text{eV}}{2^2} = \boxed{1.89\ \text{eV}}$$

The photon frequency is $f = \dfrac{\Delta E}{h}$

and its wavelength is

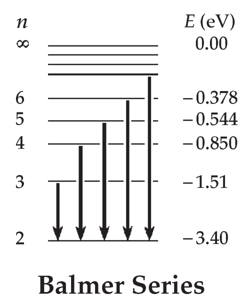


FIG. P42.5

$$\lambda = \frac{c}{f} = \frac{hc}{\Delta E} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(2.998\times10^8\ \text{m/s}\right)}{(1.89\ \text{eV})}\left(\frac{\text{eV}}{1.602\times10^{-19}\ \text{J}}\right)$$

$$\lambda = \boxed{656\ \text{nm}}$$

This is the red Balmer-alpha line, which gives its characteristic color to the chromosphere of the Sun and to photographs of the Orion nebula.

(b) The biggest energy loss is for an atom to fall from an ionized configuration,

$n = \infty$ to the $n = 2$ state

It loses energy $$-\frac{13.6\ \text{eV}}{\infty} + \frac{13.6\ \text{eV}}{2^2} = \boxed{3.40\ \text{eV}}$$

to emit light of wavelength $$\lambda = \frac{hc}{\Delta E} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^8\ \text{m/s}\right)}{(3.40\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)} = \boxed{365\ \text{nm}}$$

This is the Balmer series limit, in the near ultraviolet.

P42.6 (a) $v_1 = \sqrt{\dfrac{k_e e^2}{m_e r_1}}$

where $r_1 = (1)^2 a_0 = 0.005\,29 \text{ nm} = 5.29 \times 10^{-11} \text{ m}$

$$v_1 = \sqrt{\frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(9.11 \times 10^{-31} \text{ kg})(5.29 \times 10^{-11} \text{ m})}} = \boxed{2.19 \times 10^6 \text{ m/s}}$$

(b) $K_1 = \frac{1}{2} m_e v_1^2 = \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(2.19 \times 10^6 \text{ m/s})^2 = 2.18 \times 10^{-18} \text{ J} = \boxed{13.6 \text{ eV}}$

(c) $U_1 = -\dfrac{k_e e^2}{r_1} = -\dfrac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{5.29 \times 10^{-11} \text{ m}} = -4.35 \times 10^{-18} \text{ J} = \boxed{-27.2 \text{ eV}}$

P42.7 (a) $r_2^2 = (0.052\,9 \text{ nm})(2)^2 = \boxed{0.212 \text{ nm}}$

(b) $m_e v_2 = \sqrt{\dfrac{m_e k_e e^2}{r_2}} = \sqrt{\dfrac{(9.11 \times 10^{-31} \text{ kg})(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{0.212 \times 10^{-9} \text{ m}}}$

$m_e v_2 = \boxed{9.95 \times 10^{-25} \text{ kg}\cdot\text{m/s}}$

(c) $L_2 = m_e v_2 r_2 = (9.95 \times 10^{-25} \text{ kg}\cdot\text{m/s})(0.212 \times 10^{-9} \text{ m}) = \boxed{2.11 \times 10^{-34} \text{ kg}\cdot\text{m}^2/\text{s}}$

(d) $K_2 = \dfrac{1}{2} m_e v_2^2 = \dfrac{(m_e v_2)^2}{2m_e} = \dfrac{(9.95 \times 10^{-25} \text{ kg}\cdot\text{m/s})^2}{2(9.11 \times 10^{-31} \text{ kg})} = 5.43 \times 10^{-19} \text{ J} = \boxed{3.40 \text{ eV}}$

(e) $U_2 = -\dfrac{k_e e^2}{r_2} = -\dfrac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{0.212 \times 10^{-9} \text{ m}} = -1.09 \times 10^{-18} \text{ J} = \boxed{-6.80 \text{ eV}}$

(f) $E_2 = K_2 + U_2 = 3.40 \text{ eV} - 6.80 \text{ eV} = \boxed{-3.40 \text{ eV}}$

***P42.8** (a) The batch of excited atoms must make these six transitions to get back to state one: $2 \to 1$, and also $3 \to 2$ and $3 \to 1$, and also $4 \to 3$ and $4 \to 2$ and $4 \to 1$. Thus, the incoming light must have just enough energy to produce the $1 \to 4$ transition. It must be the third line of the Lyman series in the absorption spectrum of hydrogen. The absorbing atom changes from energy

$$E_i = -\frac{13.6 \text{ eV}}{1^2} = -13.6 \text{ eV} \text{ to } E_f = -\frac{13.6 \text{ eV}}{4^2} = -0.850 \text{ eV}$$

so the incoming photons have wavelength $\lambda = c/f = hc/E_{\text{photon}}$

$$\lambda = \frac{hc}{E_f - E_i} = \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})}{-0.850 \text{ eV} - (-13.6 \text{ eV})}\left(\frac{1.00 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) .$$

$$= 9.74 \times 10^{-8} \text{ m} = \boxed{97.4 \text{ nm}}$$

(b) The longest of the six wavelengths corresponds to the lowest photon energy, emitted in the $4 \to 3$ transition. Here $E_i = -\dfrac{13.6 \text{ eV}}{4^2} = -0.850 \text{ eV}$ and $E_f = -\dfrac{13.6 \text{ eV}}{3^2} = -1.51 \text{ eV}$,

so $\lambda = \dfrac{hc}{E_f - E_i} = \dfrac{1240 \text{ eV}\cdot\text{nm}}{-0.850 \text{ eV} - (-1.51 \text{ eV})} = \boxed{1.88\ \mu\text{m}}$. This infrared wavelength is part of the Paschen series, since the lower state has $n = 3$.

(c) The shortest wavelength emitted is the same as the wavelength absorbed: 97.4 nm, ultraviolet, Lyman series.

P42.9 (a) The energy levels of a hydrogen-like ion whose charge number is Z are given by

$$E_n = (-13.6 \text{ eV})\frac{Z^2}{n^2}$$

Thus for Helium $(Z=2)$, the energy levels are

$$\boxed{E_n = -\frac{54.4 \text{ eV}}{n^2} \quad n = 1,\ 2,\ 3,\ \ldots}$$

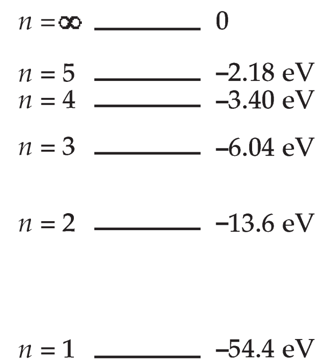


FIG. P42.9

(b) For He^+, $Z = 2$, so we see that the ionization energy (the energy required to take the electron from the $n=1$ to the $n=\infty$ state) is

$$E = E_\infty - E_1 = 0 - \frac{(-13.6 \text{ eV})(2)^2}{(1)^2} = \boxed{54.4 \text{ eV}}$$

***P42.10** (a) The photon has energy 2.28 eV.

And $\frac{13.6 \text{ eV}}{2^2} = 3.40 \text{ eV}$ is required to ionize a hydrogen atom from state $n = 2$. So while the photon cannot ionize a hydrogen atom pre-excited to $n = 2$, it can ionize a hydrogen atom in the $n = \boxed{3}$ state, with energy $-\frac{13.6 \text{ eV}}{3^2} = -1.51 \text{ eV}$.

(b) The electron thus freed can have kinetic energy $K_e = 2.28 \text{ eV} - 1.51 \text{ eV} = 0.769 \text{ eV} = \frac{1}{2}m_e v^2$.

Therefore $v = \sqrt{\frac{2(0.769)\left(1.60\times10^{-19}\right) \text{ J}}{9.11\times10^{-31} \text{ kg}}} = \boxed{520 \text{ km/s}}$.

P42.11 Let r represent the distance between the electron and the positron. The two move in a circle of radius $\frac{r}{2}$ around their center of mass with opposite velocities. The total angular momentum of the electron-positron system is quantized according to

$$L_n = \frac{mvr}{2} + \frac{mvr}{2} = n\hbar$$

where $n = 1,\ 2,\ 3,\ \ldots$

For each particle, $\Sigma F = ma$ expands to $\qquad \frac{k_e e^2}{r^2} = \frac{mv^2}{r/2}$

We can eliminate $v = \frac{n\hbar}{mr}$ to find $\qquad \frac{k_e e^2}{r} = \frac{2mn^2\hbar^2}{m^2 r^2}$

So the separation distances are $\qquad r = \frac{2n^2\hbar^2}{mk_e e^2} = 2a_0 n^2 = \boxed{\left(1.06\times10^{-10} \text{ m}\right)n^2}$

The orbital radii are $\frac{r}{2} = a_0 n^2$, the same as for the electron in hydrogen.

The energy can be calculated from $\qquad E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 - \frac{k_e e^2}{r}$

Since $mv^2 = \frac{k_e e^2}{2r}$, $\qquad E = \frac{k_e e^2}{2r} - \frac{k_e e^2}{r} = -\frac{k_e e^2}{2r} = \frac{-k_e e^2}{4a_0 n^2} = \boxed{-\frac{6.80 \text{ eV}}{n^2}}$

***P42.12** (a) From the Bohr theory we have for the speed of the electron $v = \frac{n\hbar}{m_e r}$.

The period of its orbital motion is $T = \frac{2\pi r}{v} = \frac{2\pi r m_e r}{n\hbar}$. Substituting the orbital radius

$r = \frac{n^2\hbar^2}{m_e k_e e^2}$ gives $T = \frac{2\pi m_e n^4 \hbar^4}{n\hbar m_e^2 k_e^2 e^4} = \frac{2\pi\hbar^3}{m_e k_e^2 e^4} n^3$. Thus we have the periods determined in terms of the ground-state period

$$t_0 = \frac{2\pi\hbar^3}{m_e k_e^2 e^4} = \frac{2\pi(1.055\times10^{-34}\ \text{J}\cdot\text{s})^3}{9.11\times10^{-31}\ \text{kg}(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)^2(1.6\times10^{-19}\ \text{C})^4} = \boxed{153\ \text{as}}$$

(b) In the $n = 2$ state the period is $(153\times10^{-18}\ \text{s})2^3 = 1.22\times10^{-15}$ s so the number of orbits completed in the excited state is $10\times10^{-6}\ \text{s}/1.22\times10^{-15}\ \text{s} = \boxed{8.18\times10^9\ \text{revolutions}}$.

(c) Its lifetime in electron years is comparable to the lifetime of the Sun in Earth years, so we can think of it as a long time.

Section 42.4 **The Quantum Model of the Hydrogen Atom**

P42.13 The reduced mass of positronium is **less** than hydrogen, so the photon energy will be **less** for positronium than for hydrogen. This means that the wavelength of the emitted photon will be **longer** than 656.3 nm. On the other hand, helium has about the same reduced mass but more charge than hydrogen, so its transition energy will be **larger**, corresponding to a wavelength **shorter** than 656.3 nm.

All the factors in the given equation are constant for this problem except for the reduced mass and the nuclear charge. Therefore, the wavelength corresponding to the energy difference for the transition can be found simply from the ratio of mass and charge variables.

For hydrogen, $\mu = \frac{m_p m_e}{m_p + m_e} \approx m_e$ The photon energy is $\Delta E = E_3 - E_2$

Its wavelength is $\lambda = 656.3$ nm where $\lambda = \frac{c}{f} = \frac{hc}{\Delta E}$

(a) For positronium, $\mu = \frac{m_e m_e}{m_e + m_e} = \frac{m_e}{2}$

so the energy of each level is one half as large as in hydrogen, which we could call "protonium." The photon energy is inversely proportional to its wavelength, so for positronium,

$$\lambda_{32} = 2(656.3\ \text{nm}) = \boxed{1.31\ \mu\text{m}}\ \text{(in the infrared region)}$$

(b) For He^+, $\mu \approx m_e$, $q_1 = e$, and $q_2 = 2e$

so the transition energy is $2^2 = 4$ times larger than hydrogen

Then, $\lambda_{32} = \left(\frac{656}{4}\right)\text{nm} = \boxed{164\ \text{nm}}$ (in the ultraviolet region)

***P42.14** (a) For a particular transition from n_i to n_f, $\Delta E_{\text{H}} = -\dfrac{\mu_{\text{H}} k_e^{\,2} e^4}{2\hbar^2}\left(\dfrac{1}{n_f^{\,2}} - \dfrac{1}{n_i^{\,2}}\right) = \dfrac{hc}{\lambda_{\text{H}}}$ and

$$\Delta E_{\text{D}} = -\frac{\mu_{\text{D}} k_e^{\,2} e^4}{2\hbar^2}\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right) = \frac{hc}{\lambda_{\text{D}}} \text{ where } \mu_{\text{H}} = \frac{m_e m_p}{m_e + m_p} \text{ and } \mu_{\text{D}} = \frac{m_e m_{\text{D}}}{m_e + m_{\text{D}}}.$$

By division, $\dfrac{\Delta E_{\text{H}}}{\Delta E_{\text{D}}} = \dfrac{\mu_{\text{H}}}{\mu_{\text{D}}} = \dfrac{\lambda_{\text{D}}}{\lambda_{\text{H}}}$ or $\lambda_{\text{D}} = \left(\dfrac{\mu_{\text{H}}}{\mu_{\text{D}}}\right)\lambda_{\text{H}}$. Then, $\boxed{\lambda_{\text{H}} - \lambda_{\text{D}} = \left(1 - \dfrac{\mu_{\text{H}}}{\mu_{\text{D}}}\right)\lambda_{\text{H}}}$.

(b) $$\frac{\mu_{\text{H}}}{\mu_{\text{D}}} = \left(\frac{m_e m_p}{m_e + m_p}\right)\left(\frac{m_e + m_{\text{D}}}{m_e m_{\text{D}}}\right) = \frac{(1.007\ 276\text{ u})(0.000\ 549\text{ u} + 2.013\ 553\text{ u})}{(0.000\ 549\text{ u} + 1.007\ 276\text{ u})(2.013\ 553\text{ u})} = 0.999\ 728$$

$$\lambda_{\text{H}} - \lambda_{\text{D}} = (1 - 0.999\ 728)(656.3\text{ nm}) = \boxed{0.179\text{ nm}}$$

***P42.15** (a) $\Delta x \Delta p \ge \dfrac{\hbar}{2}$ so if $\Delta x = r$, $\Delta p \ge \boxed{\dfrac{\hbar}{2r}}$

(b) Arbitrarily choosing $\Delta p \approx \dfrac{\hbar}{r}$, we find $K = \dfrac{p^2}{2m_e} \approx \dfrac{(\Delta p)^2}{2m_e} = \boxed{\dfrac{\hbar^2}{2m_e r^2}}$

$$U = \frac{-k_e e^2}{r}\text{, so } E = K + U \approx \boxed{\frac{\hbar^2}{2m_e r^2} - \frac{k_e e^2}{r}}$$

(c) To minimize E,

$$\frac{dE}{dr} = -\frac{\hbar^2}{m_e r^3} + \frac{k_e e^2}{r^2} = 0 \to r = \boxed{\frac{\hbar^2}{m_e k_e e^2} = a_0} \text{ (the Bohr radius)}$$

Then, $E = \dfrac{\hbar^2}{2m_e}\left(\dfrac{m_e k_e e^2}{\hbar^2}\right)^2 - k_e e^2\left(\dfrac{m_e k_e e^2}{\hbar^2}\right) = -\dfrac{m_e k_e^2 e^4}{2\hbar^2} = \boxed{-13.6\text{ eV}}$. With our particular choice for the momentum uncertainty as double its minimum possible value, we find precisely the Bohr results for the orbital radius and for the ground-state energy.

Section 42.5 The Wave Functions of Hydrogen

P42.16 $\psi_{1s}(r) = \dfrac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$ is the ground state hydrogen wave function.

$P_{1s}(r) = \dfrac{4r^2}{a_0^3} e^{-2r/a_0}$ is the ground state radial probability distribution function.

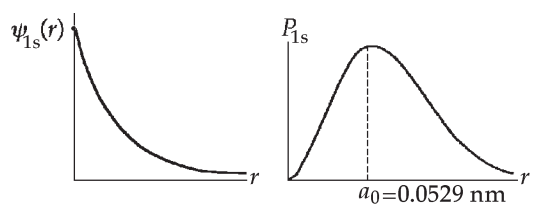


FIG. P42.16

P42.17 (a) $\int |\psi|^2 \, dV = 4\pi \int_0^\infty |\psi|^2 \, r^2 dr = 4\pi \left(\frac{1}{\pi a_0^3} \right) \int_0^\infty r^2 e^{-2r/a_0} dr$

Using integral tables, $\int |\psi|^2 \, dV = -\frac{2}{a_0^2} \left[e^{-2r/a_0} \left(r^2 + a_0 r + \frac{a_0^2}{2} \right) \right]_0^\infty = \left(-\frac{2}{a_0^2} \right) \left(-\frac{a_0^2}{2} \right) = \boxed{1}$

so the wave function as given is normalized.

(b) $P_{a_0/2 \to 3a_0/2} = 4\pi \int_{a_0/2}^{3a_0/2} |\psi|^2 \, r^2 dr = 4\pi \left(\frac{1}{\pi a_0^3} \right) \int_{a_0/2}^{3a_0/2} r^2 e^{-2r/a_0} dr$

Again, using integral tables,

$$P_{a_0/2 \to 3a_0/2} = -\frac{2}{a_0^2} \left[e^{-2r/a_0} \left(r^2 + a_0 r + \frac{a_0^2}{2} \right) \right]_{a_0/2}^{3a_0/2} = -\frac{2}{a_0^2} \left[e^{-3} \left(\frac{17a_0^2}{4} \right) - e^{-1} \left(\frac{5a_0^2}{4} \right) \right] = \boxed{0.497}$$

P42.18 $\psi = \frac{1}{\sqrt{3}} \frac{1}{(2a_0)^{3/2}} \frac{r}{a_0} e^{-r/2a_0}$

so $P_r = 4\pi r^2 \left| \psi^2 \right| = 4\pi r^2 \frac{r^2}{24 a_0^5} e^{-r/a_0}$

Set $\frac{dP}{dr} = \frac{4\pi}{24 a_0^5} \left[4r^3 e^{-r/a_0} + r^4 \left(-\frac{1}{a_0} \right) e^{-r/a_0} \right] = 0$

Solving for *r*, this is a maximum at $\boxed{r = 4a_0}$.

P42.19 $\psi = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$ $\qquad \frac{2}{r}\frac{d\psi}{dr} = \frac{-2}{r\sqrt{\pi a_0^5}} e^{-r/a_0} = -\frac{2}{r a_0} \psi$

$$\frac{d^2\psi}{dr^2} = \frac{1}{\sqrt{\pi a_0^7}} e^{-r/a_0} = \frac{1}{a_0^2} \psi$$

Substitution into the Schrödinger equation to test the validity of the solution yields

$$-\frac{\hbar^2}{2m_e} \left(\frac{1}{a_0^2} - \frac{2}{r a_0} \right) \psi - \frac{e^2}{4\pi \in_0 r} \psi = E\psi$$

But $a_0 = \frac{\hbar^2 (4\pi \in_0)}{m_e e^2}$

so $-\frac{e^2}{8\pi \in_0 a_0} = E$ or $\boxed{E = -\frac{k_e e^2}{2a_0}}$

This is true, so the Schrödinger equation is satisfied.

P42.20 The hydrogen ground-state radial probability density is

$$P(r) = 4\pi r^2 \left| \psi_{1s} \right|^2 = \frac{4r^2}{a_0^3} \exp\left(-\frac{2r}{a_0} \right)$$

The number of observations at $2a_0$ is, by proportion

$$N = 1\,000 \frac{P(2a_0)}{P(a_0/2)} = 1\,000 \frac{(2a_0)^2}{(a_0/2)^2} \frac{e^{-4a_0/a_0}}{e^{-a_0/a_0}} = 1\,000(16) e^{-3} = \boxed{797 \text{ times}}$$

Section 42.6 Physical Interpretation of the Quantum Numbers

Note: Problems 31 and 36 in Chapter 29 and Problem 62 in Chapter 30 can be assigned with this section.

P42.21 (a) In the 3d subshell, $n = 3$ and $\ell = 2$,

we have n	3	3	3	3	3	3	3	3	3	3
ℓ	2	2	2	2	2	2	2	2	2	2
m_ℓ	+2	+2	+1	+1	0	0	–1	–1	–2	–2
m_s	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2

(A total of 10 states)

(b) In the 3p subshell, $n = 3$ and $\ell = 1$,

we have n	3	3	3	3	3	3
ℓ	1	1	1	1	1	1
m_ℓ	+1	+1	0	0	–1	–1
m_s	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2

(A total of 6 states)

P42.22 (a) For the d state, $\ell = 2$, $L = \boxed{\sqrt{6}\hbar} = 2.58 \times 10^{-34}\ \text{J} \cdot \text{s}$

(b) For the f state, $\ell = 3$, $L = \sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{12}\hbar} = 3.65 \times 10^{-34}\ \text{J} \cdot \text{s}$

***P42.23** (a) The problem: Find the orbital quantum number of a hydrogen atom in a state in which it has orbital angular momentum $4.714 \times 10^{-34}\ \text{J} \cdot \text{s}$.

(b) The solution: $L = \sqrt{\ell(\ell+1)}\hbar$ $\qquad 4.714 \times 10^{-34} = \sqrt{\ell(\ell+1)}\left(\dfrac{6.626 \times 10^{-34}}{2\pi}\right)$

$$\ell(\ell+1) = \frac{\left(4.714 \times 10^{-34}\right)^2 (2\pi)^2}{\left(6.626 \times 10^{-34}\right)^2} = 1.998 \times 10^1 \approx 20 = 4(4+1)$$

so the orbital quantum number is $\boxed{\ell = 4}$.

P42.24 The 5th excited state has $n = 6$, energy $\dfrac{-13.6\ \text{eV}}{36} = -0.378\ \text{eV}$.

The atom loses this much energy: $\dfrac{hc}{\lambda} = \dfrac{\left(6.626 \times 10^{-34}\ \text{J} \cdot \text{s}\right)\left(3.00 \times 10^8\ \text{m/s}\right)}{\left(1\,090 \times 10^{-9}\ \text{m}\right)\left(1.60 \times 10^{-19}\ \text{J/eV}\right)} = 1.14\ \text{eV}$

to end up with energy $-0.378\ \text{eV} - 1.14\ \text{eV} = -1.52\ \text{eV}$

which is the energy in state 3: $-\dfrac{13.6\ \text{eV}}{3^3} = -1.51\ \text{eV}$

While $n = 3$, ℓ can be as large as 2, giving angular momentum $\sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{6}\hbar}$.

P42.25 (a) $n = 1$: For $n=1,\ \ell=0,\ m_\ell=0,\ m_s=\pm\frac{1}{2}$

n	ℓ	m_ℓ	m_s
1	0	0	–1/2
1	0	0	+1/2

Yields 2 sets; $2n^2 = 2(1)^2 = \boxed{2}$

(b) $n = 2$: For $n = 2$,

we have

n	ℓ	m_ℓ	m_s
2	0	0	±1/2
2	1	–1	±1/2
2	1	0	±1/2
2	1	1	±1/2

Yields 8 sets; $2n^2 = 2(2)^2 = \boxed{8}$

Note that the number is twice the number of m_ℓ values. Also, for each ℓ there are $(2\ell+1)$ different m_ℓ values. Finally, ℓ can take on values ranging from 0 to $n-1$.

So the general expression is $\quad \text{number} = \sum_0^{n-1} 2(2\ell+1)$

The series is an arithmetic progression: $\quad 2+6+10+14\ldots$

the sum of which is $\quad \text{number} = \frac{n}{2}[2a+(n-1)d]$

where $a = 2,\ d = 4$: $\quad \text{number} = \frac{n}{2}[4+(n-1)4] = 2n^2$

(c) $n = 3$: $\quad 2(1)+2(3)+2(5) = 2+6+10 = 18 \quad 2n^2 = 2(3)^2 = \boxed{18}$

(d) $n = 4$: $\quad 2(1)+2(3)+2(5)+2(7) = 32 \quad 2n^2 = 2(4)^2 = \boxed{32}$

(e) $n = 5$: $\quad 32+2(9) = 32+18 = 50 \quad 2n^2 = 2(5)^2 = \boxed{50}$

P42.26 For a 3d state, $\quad n = 3$ and $\ell = 2$

Therefore, $\quad L = \sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{6}\hbar} = 2.58\times10^{-34}\ \text{J}\cdot\text{s}$

m_ℓ can have the values $\quad$ –2, –1, 0, 1, and 2

so $\quad \boxed{L_z \text{ can have the values } -2\hbar,\ -\hbar,\ 0,\ \hbar \text{ and } 2\hbar}$.

Using the relation $\quad \cos\theta = \frac{L_z}{L}$

we find the possible values of θ $\quad \boxed{145°,\ 114°,\ 90.0°,\ 65.9°,\text{ and } 35.3°}$

***P42.27** (a) Density of a proton: $\rho = \dfrac{m}{V} = \dfrac{1.67\times10^{-27}\ \text{kg}}{(4/3)\pi\left(1.00\times10^{-15}\ \text{m}\right)^3} = \boxed{3.99\times10^{17}\ \text{kg/m}^3}$

(b) Size of model electron: $r = \left(\dfrac{3m}{4\pi\rho}\right)^{1/3} = \left(\dfrac{3\left(9.11\times10^{-31}\ \text{kg}\right)}{4\pi\left(3.99\times10^{17}\ \text{kg/m}^3\right)}\right)^{1/3} = \boxed{8.17\times10^{-17}\ \text{m}}$

(c) Moment of inertia: $I = \dfrac{2}{5}mr^2 = \dfrac{2}{5}\left(9.11\times10^{-31}\ \text{kg}\right)\left(8.17\times10^{-17}\ \text{m}\right)^2$

$$= 2.43\times10^{-63}\ \text{kg}\cdot\text{m}^2$$

$$L_z = I\omega = \frac{\hbar}{2} = \frac{Iv}{r}$$

Therefore, $v = \dfrac{\hbar r}{2I} = \dfrac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(8.17\times10^{-17}\ \text{m}\right)}{2\pi\left(2\times2.43\times10^{-63}\ \text{kg}\cdot\text{m}^2\right)} = \boxed{1.77\times10^{12}\ \text{m/s}}$

(d) This is $\boxed{5.91\times10^3\ \text{times larger}}$ than the speed of light. So the spinning-solid-ball model of an electron with spin angular momentum is absurd.

P42.28 In the N shell, $n = 4$. For $n = 4$, ℓ can take on values of 0, 1, 2, and 3. For each value of ℓ, m_ℓ can be $-\ell$ to ℓ in integral steps. Thus, the maximum value for m_ℓ is 3. Since $L_z = m_\ell\hbar$, the maximum value for L_z is $L_z = \boxed{3\hbar}$.

P42.29 The 3d subshell has $\ell = 2$, and $n = 3$. Also, we have $s = 1$.

Therefore, we can have $\boxed{n = 3,\ \ell = 2;\ m_\ell = -2,\ -1,\ 0,\ 1,\ 2;\ s = 1;\ \text{and}\ m_s = -1,\ 0,\ 1}$

leading to the following table:

n	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
ℓ	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
m_ℓ	–2	–2	–2	–1	–1	–1	0	0	0	1	1	1	2	2	2
s	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
m_s	–1	0	1	–1	0	1	–1	0	1	–1	0	1	–1	0	1

Section 42.7 The Exclusion Principle and the Periodic Table

P42.30 (a) $\boxed{1s^2 2s^2 2p^4}$

(b) For the $1s$ electrons, $n = 1,\ \ell = 0,\ m_\ell = 0,$ $m_s = +\frac{1}{2}$ and $-\frac{1}{2}$

For the two $2s$ electrons, $n = 2,\ \ell = 0,\ m_\ell = 0,$ $m_s = +\frac{1}{2}$ and $-\frac{1}{2}$

For the four $2p$ electrons, $n = 2;\ \ell = 1;\ m_\ell = -1$, 0, or 1; and $m_s = +\frac{1}{2}$ or $-\frac{1}{2}$

So one possible set of quantum numbers is

n	1	1	2	2	2	2	2	2
ℓ	0	0	0	0	1	1	1	1
m_ℓ	0	0	0	0	–1	0	1	–1
m_s	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

P42.31 The $\boxed{4s \text{ subshell fills first}}$, for potassium and calcium, before the $3d$ subshell starts to fill for scandium through zinc. Thus, we would first suppose that $[\text{Ar}]3d^4 4s^2$ would have lower energy than $[\text{Ar}]3d^5 4s^1$. But the latter has more unpaired spins, six instead of four, and Hund's rule suggests that this could give the latter configuration lower energy. In fact it must, for $[\text{Ar}]3d^5 4s^1$ is the ground state for chromium.

P42.32 Electronic configuration: Sodium to Argon

$[1s^2 2s^2 2p^6]$	$+3s^1$	$\rightarrow$	Na^{11}
	$+3s^2$	$\rightarrow$	Mg^{12}
	$+3s^2 3p^1$	$\rightarrow$	Al^{13}
	$+3s^2 3p^2$	$\rightarrow$	Si^{14}
	$+3s^2 3p^3$	$\rightarrow$	P^{15}
	$+3s^2 3p^4$	$\rightarrow$	S^{16}
	$+3s^2 3p^5$	$\rightarrow$	Cl^{17}
	$+3s^2 3p^6$	$\rightarrow$	Ar^{18}
$[1s^2 2s^2 2p^6 3s^2 3p^6]4s^1$		$\rightarrow$	K^{19}

P42.33 In the table of electronic configurations in the text, or on a periodic table, we look for the element whose last electron is in a $3p$ state and which has three electrons outside a closed shell. Its electron configuration then ends in $3s^2 3p^1$. The element is $\boxed{\text{aluminum}}$.

P42.34 (a) For electron one and also for electron two, $n = 3$ and $\ell = 1$. The possible states are listed here in columns giving the other quantum numbers:

electron	m_ℓ	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
one	m_s	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
electron	m_ℓ	1	0	0	–1	–1	1	0	0	–1	–1	1	1	0	–1	–1
two	m_s	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

electron	m_ℓ	0	0	0	0	0	–1	–1	–1	–1	–1	–1	–1	–1	–1	–1
one	m_s	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
electron	m_ℓ	1	1	0	–1	–1	1	1	0	0	–1	1	1	0	0	–1
two	m_s	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$

There are thirty allowed states, since electron one can have any of three possible values for m_ℓ for both spin up and spin down, amounting to six states, and the second electron can have any of the other five states.

(b) Were it not for the exclusion principle, there would be $\boxed{36}$ possible states, six for each electron independently.

P42.35 (a)

$n+\ell$	1	2	3	4	5	6	7
subshell	1*s*	2*s*	2*p*, 3*s*	3*p*, 4*s*	3*d*, 4*p*, 5*s*	4*d*, 5*p*, 6*s*	4*f*, 5*d*, 6*p*, 7*s*

(b) $Z = 15$: Filled subshells: 1*s*, 2*s*, 2*p*, 3*s* (12 electrons)
Valence subshell: 3 electrons in 3*p* subshell
Prediction: Valence $= -3$ or $+5$
Element is phosphorus, Valence $= -3$ or $+5$ (Prediction correct)

$Z = 47$: Filled subshells: 1*s*, 2*s*, 2*p*, 3*s*, 3*p*, 4*s*, 3*d*, 4*p*, 5*s* (38 electrons)
Outer subshell: 9 electrons in 4*d* subshell
Prediction: Valence $= -1$
Element is silver, (Prediction fails) Valence is $+1$

$Z = 86$: Filled subshells: 1*s*, 2*s*, 2*p*, 3*s*, 3*p*, 4*s*, 3*d*, 4*p*, 5*s*, 4*d*, 5*p*, 6*s*, 4*f*, 5*d*, 6*p* (86 electrons)
Prediction Outer subshell is full: inert gas
Element is radon, inert (Prediction correct)

P42.36 Listing subshells in the order of filling, we have for element 110,

$$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^{10} 5p^6 6s^2 4f^{14} 5d^{10} 6p^6 7s^2 5f^{14} 6d^8$$

In order of increasing principal quantum number, this is

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 5f^{14} 6s^2 6p^6 6d^8 7s^2}$$

P42.37 In the ground state of sodium, the outermost electron is in an s state. This state is spherically symmetric, so it generates no magnetic field by orbital motion, and has the same energy no matter whether the electron is spin-up or spin-down. The energies of the states $3p\uparrow$ and $3p\downarrow$ above $3s$ are $hf_1 = \frac{hc}{\lambda}$ and $hf_2 = \frac{hc}{\lambda_2}$.

The energy difference is

$$2\mu_B B = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

so

$$B = \frac{hc}{2\mu_B}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$= \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3\times10^8\ \text{m/s})}{2(9.27\times10^{-24}\ \text{J/T})}\left(\frac{1}{588.995\times10^{-9}\ \text{m}} - \frac{1}{589.592\times10^{-9}\ \text{m}}\right)$$

$$B = \boxed{18.4\ \text{T}}$$

Section 42.8 More on Atomic Spectra: Visible and X-ray

P42.38 (a) $\boxed{n = 3,\ \ell = 0,\ m_\ell = 0}$

$\boxed{n = 3,\ \ell = 1,\ m_\ell = -1,\ 0,\ 1}$

For $\boxed{n = 3,\ \ell = 2,\ m_\ell = -2,\ -1,\ 0,\ 1,\ 2}$

(b) ψ_{300} corresponds to $E_{300} = -\frac{Z^2 E_0}{n^2} = -\frac{2^2(13.6)}{3^2} = \boxed{-6.05\ \text{eV}}$.

ψ_{31-1}, ψ_{310}, ψ_{311} have the same energy since n is the same.

ψ_{32-2}, ψ_{32-1}, ψ_{320}, ψ_{321}, ψ_{322} have the same energy since n is the same.

All states are degenerate.

***P42.39** For the 3p state, $E_n = \dfrac{-13.6 \text{ eV } Z_{eff}^2}{n^2}$ becomes $-3.0 \text{ eV} = \dfrac{-13.6 \text{ eV } Z_{eff}^2}{3^2}$ so $Z_{eff} = \boxed{1.4}$

For the 3d state $-1.5 \text{ eV} = \dfrac{-13.6 \text{ eV } Z_{eff}^2}{3^2}$ so $Z_{eff} = \boxed{1.0}$

When the outermost electron in sodium is promoted from the 3s state into a 3p state, its wave function still overlaps somewhat with the ten electrons below it. It therefore sees the +11e nuclear charge not fully screened, and on the average moves in an electric field like that created by a particle with charge +11e – 9.6e = 1.4e. When this valence electron is lifted farther to a 3p state, it is essentially entirely outside the cloud of ten electrons below it, and moves in the field of a net charge +11e – 10e = 1e.

***P42.40** (a) Picture all of the energy of an electron after its acceleration going into producing a single photon. Then we have $E = \dfrac{hc}{\lambda} = e\Delta V$ and

$$\Delta V = \frac{hc}{\lambda} = \frac{6.626\times10^{-34} \text{ J}\cdot\text{s } 2.998\times10^{8} \text{ m/s}}{\lambda \quad 1.602\times10^{-19} \text{ J/eV}} = \boxed{\frac{1240 \text{ V}\cdot\text{nm}}{\lambda}}$$

(b) The potential difference is inversely proportional to the wavelength.

(c) Yes. It predicts a minimum wavelength of 33.5 pm when the accelerating voltage is 37 keV, in agreement with the minimum wavelength in the figure.

(d) Yes, but it might be unlikely for a very high-energy electron to stop in a single interaction to produce a high-energy gamma ray; and it might be difficult to observe the very low-intensity radio waves produced as bremsstrahlung by low-energy electrons. The potential difference goes to infinity as the wavelength goes to zero. The potential difference goes to zero as the wavelength goes to infinity.

P42.41 Following Example 42.5 $E_\gamma = \dfrac{3}{4}(42-1)^2(13.6 \text{ eV}) = 1.71\times10^4 \text{ eV} = 2.74\times10^{-15} \text{ J}$

$f = 4.14\times10^{18} \text{ Hz}$

and $\lambda = \boxed{0.072\,5 \text{ nm}}$

P42.42 $E = \dfrac{hc}{\lambda} = \dfrac{1\,240 \text{ eV}\cdot\text{nm}}{\lambda} = \dfrac{1.240 \text{ keV}\cdot\text{nm}}{\lambda}$

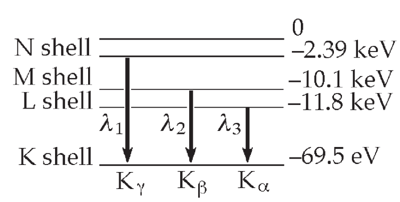


FIG. P42.42

For $\lambda_1 = 0.018\,5 \text{ nm}$, $E = 67.11 \text{ keV}$

$\lambda_2 = 0.020\,9 \text{ nm}$, $E = 59.4 \text{ keV}$

$\lambda_3 = 0.021\,5 \text{ nm}$, $E = 57.7 \text{ keV}$

The ionization energy for the K shell is 69.5 keV, so the ionization energies for the other shells are:

$\boxed{\text{L shell} = 11.8 \text{ keV}}$ $\boxed{\text{M shell} = 10.1 \text{ keV}}$ $\boxed{\text{N shell} = 2.39 \text{ keV}}$

P42.43 The K_β x-rays are emitted when there is a vacancy in the $(n=1)$ K shell and an electron from the $(n=3)$ M shell falls down to fill it. Then this electron is shielded by nine electrons originally and by one in its final state.

$$\frac{hc}{\lambda} = -\frac{13.6(Z-9)^2}{3^2}\ \text{eV} + \frac{13.6(Z-1)^2}{1^2}\ \text{eV}$$

$$\frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(0.152\times10^{-9}\ \text{m})(1.60\times10^{-19}\ \text{J/eV})} = (13.6\ \text{eV})\left(-\frac{Z^2}{9}+\frac{18Z}{9}-\frac{81}{9}+Z^2-2Z+1\right)$$

$$8.17\times10^{3}\ \text{eV} = (13.6\ \text{eV})\left(\frac{8Z^2}{9}-8\right)$$

so $\quad 601 = \dfrac{8Z^2}{9} - 8$

and $\quad Z = 26 \qquad \boxed{\text{Iron}}$

Section 42.9 **Spontaneous and Stimulated Transitions**

Section 42.10 **Lasers**

P42.44 The photon energy is $\quad E_4 - E_3 = (20.66 - 18.70)\ \text{eV} = 1.96\ \text{eV} = \dfrac{hc}{\lambda}$

$$\lambda = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{1.96(1.60\times10^{-19}\ \text{J})} = \boxed{633\ \text{nm}}$$

P42.45 $f = \dfrac{E}{h} = \dfrac{0.117\ \text{eV}}{6.630\times10^{-34}\ \text{J}\cdot\text{s}}\left(\dfrac{1.60\times10^{-19}\ \text{C}}{e}\right)\left(\dfrac{1\ \text{J}}{1\ \text{V}\cdot\text{C}}\right) = \boxed{2.82\times10^{13}\ \text{s}^{-1}}$

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{2.82\times10^{13}\ \text{s}^{-1}} = \boxed{10.6\ \mu\text{m}},\ \boxed{\text{infrared}}$$

P42.46 (a) $I = \dfrac{(3.00\times10^{-3}\ \text{J})}{(1.00\times10^{-9}\ \text{s})\left[\pi(15.0\times10^{-6}\ \text{m})^2\right]} = \boxed{4.24\times10^{15}\ \text{W/m}^2}$

(b) $(3.00\times10^{-3}\ \text{J})\dfrac{(0.600\times10^{-9}\ \text{m})^2}{(30.0\times10^{-6}\ \text{m})^2} = \boxed{1.20\times10^{-12}\ \text{J}} = 7.50\ \text{MeV}$

P42.47 $E = \mathcal{P}\,\Delta t = (1.00\times10^{6}\ \text{W})(1.00\times10^{-8}\ \text{s}) = 0.010\,0\ \text{J}$

$$E_\gamma = hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34})(3.00\times10^{8})}{694.3\times10^{-9}}\ \text{J} = 2.86\times10^{-19}\ \text{J}$$

$$N = \frac{E}{E_\gamma} = \frac{0.010\,0}{2.86\times10^{-19}} = \boxed{3.49\times10^{16}\ \text{photons}}$$

P42.48 (a) $$\frac{N_3}{N_2}=\frac{N_g e^{-E_3/(k_B\cdot 300\text{ K})}}{N_g e^{-E_2/(k_B\cdot 300\text{ K})}}=e^{-(E_3-E_2)/(k_B\cdot 300\text{ K})}=e^{-hc/\lambda(k_B\cdot 300\text{ K})}$$

where λ is the wavelength of light radiated in the $3\to 2$ transition.

$$\frac{N_3}{N_2}=e^{-(6.63\times 10^{-34}\text{ J}\cdot\text{s})(3\times 10^{8}\text{ m/s})/(632.8\times 10^{-9}\text{ m})(1.38\times 10^{-23}\text{ J/K})(300\text{ K})}$$

$$\frac{N_3}{N_2}=e^{-75.9}=\boxed{1.07\times 10^{-33}}$$

(b) $$\frac{N_u}{N_\ell}=e^{-(E_u-E_\ell)/k_BT}$$

where the subscript u refers to an upper energy state and the subscript ℓ to a lower energy state.

Since $E_u-E_\ell=E_{\text{photon}}=\dfrac{hc}{\lambda}$ $\qquad \dfrac{N_u}{N_\ell}=e^{-hc/\lambda k_BT}$

Thus, we require $\qquad 1.02=e^{-hc/\lambda k_BT}$

or $$\ln(1.02)=-\frac{(6.63\times 10^{-34}\text{ J}\cdot\text{s})(3\times 10^{8}\text{ m/s})}{(632.8\times 10^{-9}\text{ m})(1.38\times 10^{-23}\text{ J/K})T}$$

$$T=-\frac{2.28\times 10^{4}}{\ln(1.02)}=\boxed{-1.15\times 10^{6}\text{ K}}$$

A negative-temperature state is not achieved by cooling the system below 0 K, but by heating it above $T=\infty$, for as $T\to\infty$ the populations of upper and lower states approach equality.

(c) Because $E_u-E_\ell>0$, and in any real equilibrium state $T>0$,

$e^{-(E_u-E_\ell)/k_BT}<1$ and $N_u<N_\ell$

Thus, a population inversion cannot happen in thermal equilibrium.

P42.49 (a) The light in the cavity is incident perpendicularly on the mirrors, although the diagram shows a large angle of incidence for clarity. We ignore the variation of the index of refraction with wavelength. To minimize reflection at a vacuum wavelength of 632.8 nm, the net phase difference between rays (1) and (2) should be 180°. There is automatically a 180° shift in one of the two rays upon reflection, so the extra distance traveled by ray (2) should be one whole wavelength:

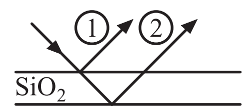


FIG. P42.49

$$2t=\frac{\lambda}{n}$$

$$t=\frac{\lambda}{2n}=\frac{632.8\text{ nm}}{2(1.458)}=\boxed{217\text{ nm}}$$

(b) The total phase difference should be 360°, including contributions of 180° by reflection and 180° by extra distance traveled

$$2t=\frac{\lambda}{2n}$$

$$t=\frac{\lambda}{4n}=\frac{543\text{ nm}}{4(1.458)}=\boxed{93.1\text{ nm}}$$

Additional Problems

P42.50 (a) Using the same procedure that was used in the Bohr model of the hydrogen atom, we apply Newton's second law to the Earth. We simply replace the Coulomb force by the gravitational force exerted by the Sun on the Earth and find

$$G\frac{M_S M_E}{r^2} = M_E \frac{v^2}{r} \qquad (1)$$

where v is the orbital speed of the Earth. Next, we apply the postulate that angular momentum of the Earth is quantized in multiples of $\hbar$:

$$M_E v r = n\hbar \qquad (n = 1,\ 2,\ 3,\ \ldots)$$

Solving for v gives

$$v = \frac{n\hbar}{M_E r} \qquad (2)$$

Substituting (2) into (1), we find

$$r = \frac{n^2\hbar^2}{GM_S M_E^2} \qquad (3)$$

(b) Solving (3) for n gives

$$n = \sqrt{GM_S r}\,\frac{M_E}{\hbar} \qquad (4)$$

Taking $M_S = 1.99 \times 10^{30}$ kg, and $M_E = 5.98 \times 10^{24}$ kg, $r = 1.496 \times 10^{11}$ m, $G = 6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$, and $\hbar = 1.055 \times 10^{-34}$ Js, we find

$$n = \boxed{2.53 \times 10^{74}}$$

(c) We can use (3) to determine the radii for the orbits corresponding to the quantum numbers n and $n+1$:

$$r_n = \frac{n^2\hbar^2}{GM_S M_E^2} \qquad \text{and} \qquad r_{n+1} = \frac{(n+1)^2\hbar^2}{GM_S M_E^2}$$

Hence, the separation between these two orbits is

$$\Delta r = \frac{\hbar^2}{GM_S M_E^2}\left[(n+1)^2 - n^2\right] = \frac{\hbar^2}{GM_S M_E^2}(2n+1)$$

Since n is very large, we can neglect the number 1 in the parentheses and express the separation as

$$\Delta r \approx \frac{\hbar^2}{GM_S M_E^2}(2n) = \boxed{1.18 \times 10^{-63}\ \text{m}}$$

This number is *much smaller* than the radius of an atomic nucleus $\left(\sim 10^{-15}\ \text{m}\right)$, so the distance between quantized orbits of the Earth is too small to observe.

P42.51 (a) $\Delta E = \dfrac{e\hbar B}{m_e} = \dfrac{1.60\times10^{-19}\ \text{C}\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)(5.26\ \text{T})}{2\pi\left(9.11\times10^{-31}\ \text{kg}\right)}\left(\dfrac{\text{N}\cdot\text{s}}{\text{T}\cdot\text{C}\cdot\text{m}}\right)\left(\dfrac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right) = 9.75\times10^{-23}\ \text{J}$

$= \boxed{609\ \mu\text{eV}}$

(b) $k_BT = \left(1.38\times10^{-23}\ \text{J/K}\right)\left(80\times10^{-3}\ \text{K}\right) = 1.10\times10^{-24}\ \text{J} = \boxed{6.9\ \mu\text{eV}}$

(c) $f = \dfrac{\Delta E}{h} = \dfrac{9.75\times10^{-23}\ \text{J}}{6.63\times10^{-34}\ \text{J}\cdot\text{s}} = \boxed{1.47\times10^{11}\ \text{Hz}}$

$\lambda = \dfrac{c}{f} = \dfrac{3\times10^{8}\ \text{m/s}}{1.47\times10^{11}\ \text{Hz}} = \boxed{2.04\times10^{-3}\ \text{m}}$

P42.52 (a) Probability $= \int_r^\infty P_{1s}(r')\,dr' = \dfrac{4}{a_0^3}\int_r^\infty r'^2 e^{-2r'/a_0}\,dr' = \left[-\left(\dfrac{2r'^2}{a_0^2} + \dfrac{2r'}{a_0} + 1\right)e^{-2r'/a_0}\right]_r^\infty$

using integration by parts, we find

$= \boxed{\left(\dfrac{2r^2}{a_0^2} + \dfrac{2r}{a_0} + 1\right)e^{-2r/a_0}}$

(b)

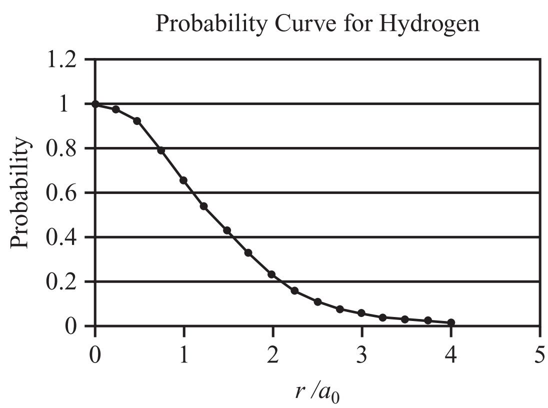


FIG. P42.52

(c) The probability of finding the electron inside or outside the sphere of radius r is $\dfrac{1}{2}$.

$\therefore \left(\dfrac{2r^2}{a_0^2} + \dfrac{2r}{a_0} + 1\right)e^{-2r/a_0} = \dfrac{1}{2}$ or $z^2 + 2z + 2 = e^z$ where $z = \dfrac{2r}{a_0}$

One can home in on a solution to this transcendental equation for r on a calculator, the result being $r = \boxed{1.34a_0}$ to three digits.

P42.53 $hf = \Delta E = \dfrac{4\pi^2 m_e k_e^2 e^4}{2h^2}\left(\dfrac{1}{(n-1)^2} - \dfrac{1}{n^2}\right)$ $\qquad f = \dfrac{2\pi^2 m_e k_e^2 e^4}{h^3}\left(\dfrac{2n-1}{(n-1)^2 n^2}\right)$

As n approaches infinity, we have f approaching $\dfrac{2\pi^2 m_e k_e^2 e^4}{h^3}\dfrac{2}{n^3}$

The classical frequency is $\qquad f = \dfrac{v}{2\pi r} = \dfrac{1}{2\pi}\sqrt{\dfrac{k_e e^2}{m_e}}\dfrac{1}{r^{3/2}}$

where $\qquad r = \dfrac{n^2 h^2}{4\pi m_e k_e e^2}$

Using this equation to eliminate r from the expression for f, we find $f = \dfrac{2\pi^2 m_e k_e^2 e^4}{h^3}\dfrac{2}{n^3}$ in agreement with the Bohr result for large n.

P42.54 (a) The energy difference between these two states is equal to the energy that is absorbed.

Thus, $\qquad E = E_2 - E_1 = \dfrac{(-13.6\text{ eV})}{4} - \dfrac{(-13.6\text{ eV})}{1} = 10.2\text{ eV} = \boxed{1.63\times10^{-18}\text{ J}}$

(b) $E = \dfrac{3}{2}k_B T$ or $\qquad T = \dfrac{2E}{3k_B} = \dfrac{2(1.63\times10^{-18}\text{ J})}{3(1.38\times10^{-23}\text{ J/K})} = \boxed{7.88\times10^4\text{ K}}$

P42.55 (a) The energy of the ground state is: $\qquad E_1 = -\dfrac{hc}{\lambda_{\text{series limit}}} = -\dfrac{1\,240\text{ eV}\cdot\text{nm}}{152.0\text{ nm}}$

$$= \boxed{-8.16\text{ eV}}$$

From the wavelength of the Lyman α line: $\qquad E_2 - E_1 = \dfrac{hc}{\lambda} = \dfrac{1\,240\text{ nm}\cdot\text{eV}}{202.6\text{ nm}} = 6.12\text{ eV}$

$$E_2 = E_1 + 6.12\text{ eV} = \boxed{-2.04\text{ eV}}$$

The wavelength of the Lyman β line gives: $\qquad E_3 - E_1 = \dfrac{1\,240\text{ nm}\cdot\text{eV}}{170.9\text{ nm}} = 7.26\text{ eV}$

so $\qquad E_3 = \boxed{-0.902\text{ eV}}$

Next, using the Lyman γ line gives: $\qquad E_4 - E_1 = \dfrac{1\,240\text{ nm}\cdot\text{eV}}{162.1\text{ nm}} = 7.65\text{ eV}$

and $\qquad E_4 = \boxed{-0.508\text{ eV}}$

From the Lyman δ line, $\qquad E_5 - E_1 = \dfrac{1\,240\text{ nm}\cdot\text{eV}}{158.3\text{ nm}} = 7.83\text{ eV}$

so $\qquad E_5 = \boxed{-0.325\text{ eV}}$

continued on next page

(b) For the Balmer series, $\dfrac{hc}{\lambda} = E_i - E_2$, or $\lambda = \dfrac{1\,240 \text{ nm}\cdot\text{eV}}{E_i - E_2}$

For the α line, $E_i = E_3$ and so $\lambda_a = \dfrac{1\,240 \text{ nm}\cdot\text{eV}}{(-0.902 \text{ eV}) - (-2.04 \text{ eV})}$
$= \boxed{1\,090 \text{ nm}}$

Similarly, the wavelengths of the β line, γ line, and the short wavelength limit are found to be: $\boxed{811 \text{ nm}}$, $\boxed{724 \text{ nm}}$, and $\boxed{609 \text{ nm}}$.

(c) Computing 60.0% of the wavelengths of the spectral lines shown on the energy-level diagram gives:

$0.600(202.6 \text{ nm}) = \boxed{122 \text{ nm}}$, $0.600(170.9 \text{ nm}) = \boxed{103 \text{ nm}}$,

$0.600(162.1 \text{ nm}) = \boxed{97.3 \text{ nm}}$, $0.600(158.3 \text{ nm}) = \boxed{95.0 \text{ nm}}$,

and $0.600(152.0 \text{ nm}) = \boxed{91.2 \text{ nm}}$

These are seen to be the wavelengths of the α, β, γ, and δ lines as well as the short wavelength limit for the Lyman series in Hydrogen.

(d) The observed wavelengths could be the result of Doppler shift when the source moves away from the Earth. The required speed of the source is found from

$$\frac{f'}{f} = \frac{\lambda}{\lambda'} = \sqrt{\frac{1-(v/c)}{1+(v/c)}} = 0.600 \quad \text{yielding} \quad \boxed{v = 0.471c}$$

***P42.56** (a) The energy emitted by the atom is $\Delta E = E_4 - E_2 = -13.6 \text{ eV}\left(\dfrac{1}{4^2} - \dfrac{1}{2^2}\right) = 2.55 \text{ eV}$. The wavelength of the photon produced is then

$$\lambda = \frac{hc}{E_\gamma} = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^{8} \text{ m/s})}{(2.55 \text{ eV})(1.60\times10^{-19} \text{ J/eV})} = 4.87\times10^{-7} \text{ m} = \boxed{487 \text{ nm}}$$

(b) Since momentum must be conserved, the photon and the atom go in opposite directions with equal magnitude momenta. Thus, $p = m_{\text{atom}} v = \dfrac{h}{\lambda}$ or

$$v = \frac{h}{m_{\text{atom}}\lambda} = \frac{6.626\times10^{-34} \text{ J}\cdot\text{s}}{(1.67\times10^{-27} \text{ kg})(4.87\times10^{-7} \text{ m})} = \boxed{0.814 \text{ m/s}}$$

P42.57 The wave function for the 2*s* state is given by Equation 42.26:

$$\psi_{2s}(r) = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left[2 - \frac{r}{a_0}\right]e^{-r/2a_0}.$$

(a) Taking $r = a_0 = 0.529\times10^{-10} \text{ m}$

we find $\psi_{2s}(a_0) = \dfrac{1}{4\sqrt{2\pi}}\left(\dfrac{1}{0.529\times10^{-10} \text{ m}}\right)^{3/2}[2-1]e^{-1/2} = \boxed{1.57\times10^{14} \text{ m}^{-3/2}}$

(b) $|\psi_{2s}(a_0)|^2 = (1.57\times10^{14} \text{ m}^{-3/2})^2 = \boxed{2.47\times10^{28} \text{ m}^{-3}}$

(c) Using Equation 42.24 and the results to (b) gives $P_{2s}(a_0) = 4\pi a_0^2 |\psi_{2s}(a_0)|^2 = \boxed{8.69\times10^{8} \text{ m}^{-1}}$.

***P42.58** The average squared separation distance is

$$\langle r^2 \rangle = \int_{\text{all space}} \psi_{1s}^* r^2 \psi_{1s} dV = \int_{r=0}^{\infty} \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} r^2 \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} 4\pi r^2 dr = \frac{4\pi}{\pi a_0^3} \int_0^{\infty} r^4 e^{-2r/a_0} dr$$

We use $\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$ from Table B.6.

$$\langle r^2 \rangle = \frac{4}{a_0^3} \frac{4!}{(2/a_0)^5} = \frac{a_0^2 96}{32} = 3a_0^2$$

$$\Delta r = \sqrt{\langle r^2 \rangle - \langle r \rangle^2} = \left(3a_0^2 - \left(\frac{3a_0}{2}\right)^2\right)^{1/2} = \left(3a_0^2 - \frac{9a_0^2}{4}\right)^{1/2} = \boxed{\left(\frac{3}{4}\right)^{1/2} a_0}$$

P42.59 (a) $(3.00 \times 10^8 \text{ m/s})(14.0 \times 10^{-12} \text{ s}) = \boxed{4.20 \text{ mm}}$

(b) $E = \frac{hc}{\lambda} = 2.86 \times 10^{-19} \text{ J}$

$$N = \frac{3.00 \text{ J}}{2.86 \times 10^{-19} \text{ J}} = \boxed{1.05 \times 10^{19} \text{ photons}}$$

(c) $V = (4.20 \text{ mm})\left[\pi (3.00 \text{ mm})^2\right] = 119 \text{ mm}^3$

$$n = \frac{1.05 \times 10^{19}}{119} = \boxed{8.82 \times 10^{16} \text{ mm}^{-3}}$$

P42.60 (a) The length of the pulse is $\Delta L = \boxed{c\Delta t}$.

(b) The energy of each photon is $E_\gamma = \frac{hc}{\lambda}$ so $N = \frac{E}{E_\gamma} = \boxed{\frac{E\lambda}{hc}}$

(c) $V = \Delta L \pi \frac{d^2}{4}$ $\quad n = \frac{N}{V} = \boxed{\left(\frac{4}{c\Delta t \pi d^2}\right)\left(\frac{E\lambda}{hc}\right)}$

P42.61 The fermions are described by the exclusion principle. Two of them, one spin-up and one spin-down, will be in the ground energy level, with

$$d_{\text{NN}} = L = \frac{1}{2}\lambda,\ \lambda = 2L = \frac{h}{p}, \text{ and } p = \frac{h}{2L} \qquad K = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{8mL^2}$$

The third must be in the next higher level, with

$$d_{\text{NN}} = \frac{L}{2} = \frac{\lambda}{2},\ \lambda = L, \text{ and } p = \frac{h}{L} \qquad K = \frac{p^2}{2m} = \frac{h^2}{2mL^2}$$

The total energy is then $\frac{h^2}{8mL^2} + \frac{h^2}{8mL^2} + \frac{h^2}{2mL^2} = \boxed{\frac{3h^2}{4mL^2}}$

P42.62 $\Delta z = \frac{at^2}{2} = \frac{1}{2}\left(\frac{F_z}{m_0}\right)t^2 = \frac{\mu_z\left(dB_z/dz\right)}{2m_0}\left(\frac{\Delta x}{v}\right)^2$ and $\mu_z = \frac{e\hbar}{2m_e}$

$$\frac{dB_z}{dz} = \frac{2m_0(\Delta z)v^2(2m_e)}{\Delta x^2 e\hbar} = \frac{2(108)\left(1.66\times10^{-27}\text{ kg}\right)\left(10^{-3}\text{ m}\right)\left(10^4\text{ m}^2/\text{s}^2\right)\left(2\times9.11\times10^{-31}\text{ kg}\right)}{\left(1.00\text{ m}^2\right)\left(1.60\times10^{-19}\text{ C}\right)\left(1.05\times10^{-34}\text{ J}\cdot\text{s}\right)}$$

$$\frac{dB_z}{dz} = \boxed{0.389\text{ T/m}}$$

P42.63 We use $\psi_{2s}(r) = \frac{1}{4}\left(2\pi a_0^3\right)^{-1/2}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$

By Equation 42.24, $P(r) = 4\pi r^2\psi^2 = \frac{1}{8}\left(\frac{r^2}{a_0^3}\right)\left(2-\frac{r}{a_0}\right)^2 e^{-r/a_0}$

(a) $$\frac{dP(r)}{dr} = \frac{1}{8}\left[\frac{2r}{a_0^3}\left(2-\frac{r}{a_0}\right)^2 - \frac{2r^2}{a_0^3}\left(\frac{1}{a_0}\right)\left(2-\frac{r}{a_0}\right) - \frac{r^2}{a_0^3}\left(2-\frac{r}{a_0}\right)^2\left(\frac{1}{a_0}\right)\right]e^{-r/a_0} = 0$$

or $$\frac{1}{8}\left(\frac{r}{a_0^3}\right)\left(2-\frac{r}{a_0}\right)\left[2\left(2-\frac{r}{a_0}\right) - \frac{2r}{a_0} - \frac{r}{a_0}\left(2-\frac{r}{a_0}\right)\right]e^{-r/a_0} = 0$$

The roots of $\frac{dP}{dr} = 0$ at $r = 0$, $r = 2a_0$, and $r = \infty$ are minima with $P(r) = 0$.

Therefore we require $[.....] = 4 - \left(\frac{6r}{a_0}\right) + \left(\frac{r}{a_0}\right)^2 = 0$

with solutions $r = \left(3\pm\sqrt{5}\right)a_0$

We substitute the last two roots into $P(r)$ to determine the most probable value:

When $r = \left(3-\sqrt{5}\right)a_0 = 0.763\,9a_0$, $P(r) = \frac{0.051\,9}{a_0}$

When $r = \left(3+\sqrt{5}\right)a_0 = 5.236a_0$, $P(r) = \frac{0.191}{a_0}$

Therefore, the most probable value of r is $\left(3+\sqrt{5}\right)a_0 = \boxed{5.236a_0}$

(b) $$\int_0^\infty P(r)\,dr = \int_0^\infty \frac{1}{8}\left(\frac{r^2}{a_0^3}\right)\left(2-\frac{r}{a_0}\right)^2 e^{-r/a_0}\,dr$$

Let $u = \frac{r}{a_0}$, $dr = a_0 du$,

$$\int_0^\infty P(r)\,dr = \int_0^\infty \frac{1}{8}u^2\left(4-4u+u^2\right)e^{-u}dr = \int_0^\infty \frac{1}{8}\left(u^4-4u^3+4u^2\right)e^{-u}du$$

$$= -\frac{1}{8}\left(u^4+4u^2+8u+8\right)e^{-u}\Big|_0^\infty = 1$$

$\boxed{\text{This is as required for normalization.}}$

P42.64 (a) Suppose the atoms move in the $+x$ direction. The absorption of a photon by an atom is a completely inelastic collision, described by

$$mv_i\hat{\mathbf{i}}+\frac{h}{\lambda}\left(-\hat{\mathbf{i}}\right)=mv_f\hat{\mathbf{i}} \qquad \text{so} \qquad v_f-v_i=-\frac{h}{m\lambda}$$

This happens promptly every time an atom has fallen back into the ground state, so it happens every $10^{-8}\text{ s}=\Delta t$. Then,

$$a=\frac{v_f-v_i}{\Delta t}=-\frac{h}{m\lambda\Delta t}\sim-\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\left(10^{-25}\text{ kg}\right)\left(500\times10^{-9}\text{ m}\right)\left(10^{-8}\text{ s}\right)}\sim\boxed{-10^{6}\text{ m/s}^2}$$

(b) With constant average acceleration,

$$v_f^2=v_i^2+2a\Delta x \qquad 0\sim\left(10^{3}\text{ m/s}\right)^2+2\left(-10^{6}\text{ m/s}^2\right)\Delta x$$

so

$$\Delta x\sim\frac{\left(10^{3}\text{ m/s}\right)^2}{10^{6}\text{ m/s}^2}\boxed{\sim1\text{ m}}$$

P42.65 With one vacancy in the K shell, excess energy

$$\Delta E\approx-(Z-1)^2(13.6\text{ eV})\left(\frac{1}{2^2}-\frac{1}{1^2}\right)=5.40\text{ keV}$$

We suppose the outermost 4*s* electron is shielded by 22 electrons inside its orbit:

$$E_{\text{ionization}}\approx\frac{2^2(13.6\text{ eV})}{4^2}=3.40\text{ eV}$$

As evidence that this is of the right order of magnitude, note that the experimental ionization energy is 6.76 eV.

$$K=\Delta E-E_{\text{ionization}}\approx\boxed{5.39\text{ keV}}$$

P42.66 $$E=\frac{hc}{\lambda}=\frac{1\,240\text{ eV}\cdot\text{nm}}{\lambda}=\Delta E$$

$\lambda_1=310\text{ nm}$, so $\Delta E_1=4.00\text{ eV}$

$\lambda_2=400\text{ nm}$, $\Delta E_2=3.10\text{ eV}$

$\lambda_3=1\,378\text{ nm}$, $\Delta E_3=0.900\text{ eV}$

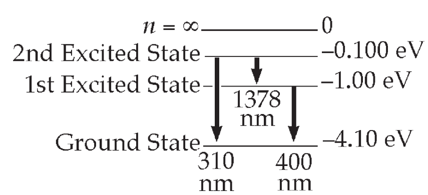


FIG. P42.66

and the ionization energy $=4.10$ eV.

The energy level diagram having the fewest levels and consistent with these energies is shown at the right.

P42.67 $$P=\int_{2.50a_0}^{\infty}\frac{4r^2}{a_0^3}e^{-2r/a_0}dr=\frac{1}{2}\int_{5.00}^{\infty}z^2e^{-z}dz \quad \text{where } z\equiv\frac{2r}{a_0}$$

$$P=-\frac{1}{2}\left(z^2+2z+2\right)e^{-z}\Big|_{5.00}^{\infty}=-\frac{1}{2}[0]+\frac{1}{2}(25.0+10.0+2.00)e^{-5}=\left(\frac{37}{2}\right)(0.006\,74)=\boxed{0.125}$$

P42.68 (a) One molecule's share of volume

Al: $V = \dfrac{\text{mass per molecule}}{\text{density}} = \left(\dfrac{27.0\ \text{g/mol}}{6.02\times 10^{23}\ \text{molecules/mol}}\right)\left(\dfrac{1.00\times 10^{-6}\ \text{m}^3}{2.70\ \text{g}}\right)$

$$= 1.66\times 10^{-29}\ \text{m}^3$$

$$\sqrt[3]{V} = \boxed{2.55\times 10^{-10}\ \text{m} \sim 10^{-1}\ \text{nm}}$$

U: $V = \left(\dfrac{238\ \text{g}}{6.02\times 10^{23}\ \text{molecules}}\right)\left(\dfrac{1.00\times 10^{-6}\ \text{m}^3}{18.9\ \text{g}}\right) = 2.09\times 10^{-29}\ \text{m}^3$

$$\sqrt[3]{V} = \boxed{2.76\times 10^{-10}\ \text{m} \sim 10^{-1}\ \text{nm}}$$

(b) The outermost electron in any atom sees the nuclear charge screened by all the electrons below it. If we can visualize a single outermost electron, it moves in the electric field of net charge $+Ze-(Z-1)e=+e$, the charge of a single proton, as felt by the electron in hydrogen. So the Bohr radius sets the scale for the outside diameter of every atom. An innermost electron, on the other hand, sees the nuclear charge unscreened, and the scale size of its (K-shell) orbit is $\dfrac{a_0}{Z}$.

P42.69 $\Delta E = 2\mu_B B = hf$

so $2\left(9.27\times 10^{-24}\ \text{J/T}\right)(0.350\ \text{T}) = \left(6.626\times 10^{-34}\ \text{J}\cdot\text{s}\right) f$

and $f = \boxed{9.79\times 10^{9}\ \text{Hz}}$

ANSWERS TO EVEN PROBLEMS

P42.2 (a) 1.94 μm (b) $\lambda_{CB} = \dfrac{1}{1/\lambda_{CA} - 1/\lambda_{BA}}$

P42.4 (a) 56.8 fm (b) 11.3 N away from the nucleus

P42.6 (a) 2.19 Mm/s (b) 13.6 eV (c) –27.2 eV

P42.8 (a) The atoms must be excited to energy level $n = 4$, to emit six different photon energies in the downward transitions $4\to 3$, $4\to 2$, $4\to 1$, $3\to 2$, $3\to 1$, and $2\to 1$. The photon energy absorbed in the $1\to 4$ transition is 12.8 eV, making the wavelength 97.4 nm. (b) 1.88 μm, infrared, Paschen (c) 97.4 nm, ultraviolet, Lyman

P42.10 (a) 3 (b) 520 km/s

P42.12 (a) 153 as (b) 8.18×10^9 revolutions (c) Its lifetime in electron years is comparable to the lifetime of the Sun in Earth years, so you can think of it as a long time.

P42.14 (a) See the solution. (b) 0.179 nm

P42.16 See the solution.

P42.18 $4a_0$

P42.20 797 times

P42.22 (a) $\sqrt{6}\hbar = 2.58 \times 10^{-34}\ \text{J}\cdot\text{s}$ (b) $\sqrt{12}\hbar = 3.65 \times 10^{-34}\ \text{J}\cdot\text{s}$

P42.24 $\sqrt{6}\hbar$

P42.26 $\sqrt{6}\hbar$; $-2\hbar$, $-\hbar$, 0, $\hbar$, $2\hbar$; 145°, 114°, 90.0°, 65.9°, 35.3°

P42.28 $3\hbar$

P42.30 (a) $1s^2 2s^2 2p^4$

(b)

n	1	1	2	2	2	2	2	2
ℓ	0	0	0	0	1	1	1	1
m_ℓ	0	0	0	0	1	1	0	−1
m_s	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

P42.32 See the solution.

P42.34 (a) See the solution. (b) 36 states instead of 30

P42.36 $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10}\ 4f^{14} 5s^2 5p^6 5d^{10} 5f^{14} 6s^2 6p^6 6d^8 7s^2$

P42.38 (a) $\ell = 0$ with $m_\ell = 0$; $\ell = 1$ with $m_\ell = 1$, 0, or 1; and $\ell = 2$ with $m_\ell = -2, -1, 0, 1, 2$ (b) –6.05 eV

P42.40 (a) $\Delta V = 1240\ \text{V}\cdot\text{nm}/\lambda$. (b) The potential difference is inversely proportional to the wavelength. (c) Yes. It predicts a minimum wavelength of 33.5 pm when the accelerating voltage is 37 keV, in agreement with the minimum wavelength in the figure. (d) Yes, but it might be unlikely for a very high-energy electron to stop in a single interaction to produce a high-energy gamma ray; and it might be difficult to observe the very low-intensity radio waves produced as bremsstrahlung by low-energy electrons. The potential difference goes to infinity as the wavelength goes to zero. The potential difference goes to zero as the wavelength goes to infinity.

P42.42 L shell 11.8 keV, M shell 10.1 keV, N shell 2.39 keV. See the solution.

P42.44 See the solution.

P42.46 (a) $4.24\ \text{PW/m}^2$ (b) $1.20\ \text{pJ} = 7.50\ \text{MeV}$

P42.48 (a) 1.07×10^{-33} (b) $-1.15 \times 10^{6}\ \text{K}$ (c) Negative temperatures do not describe systems in thermal equilibrium.

P42.50 (a) See the solution. (b) 2.53×10^{74} (c) $1.18 \times 10^{-63}\ \text{m}$, unobservably small

P42.52 (a) Probability $= \left(\dfrac{2r^2}{a_0^2} + \dfrac{2r}{a_0} + 1\right)e^{-2r/a_0}$ (b) See the solution. (c) $1.34a_0$

P42.54 (a) $10.2\ \text{eV} = 1.63\ \text{aJ}$ (b) $7.88 \times 10^4\ \text{K}$

P42.56 (a) 487 nm (b) $0.814\ \text{m/s}$

P42.58 $\sqrt{3/4}\ a_0 = 0.866\ a_0$

P42.60 (a) $c\Delta t$ (b) $\frac{E\lambda}{hc}$ (c) $\frac{4E\lambda}{\Delta t\pi d^2 hc^2}$

P42.62 0.389 T/m

P42.64 (a) $\sim -10^6\ \text{m/s}^2$ (b) ~1 m

P42.66 Energy levels at 0, –0.100 eV, –1.00 eV, and –4.10 eV

P42.68 (a) diameter $\sim 10^{-1}$ nm for both (b) A K-shell electron moves in an orbit with size on the order of $\frac{a_0}{Z}$.

43

Molecules and Solids

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q43.1 Ionic bonds are ones between oppositely charged ions. A simple model of an ionic bond is the electrostatic attraction of a negatively charged latex balloon to a positively charged Mylar balloon.

Covalent bonds are ones in which atoms share electrons. Classically, two children playing a short-range game of catch with a ball models a covalent bond. On a quantum scale, the two atoms are sharing a wave function, so perhaps a better model would be two children using a single hula hoop.

Van der Waals bonds are weak electrostatic forces: the dipole-dipole force is analogous to the attraction between the opposite poles of two bar magnets, the dipole-induced dipole force is similar to a bar magnet attracting an iron nail or paper clip, and the dispersion force is analogous to an alternating-current electro-magnet attracting a paper clip.

A hydrogen atom in a molecule is not ionized, but its electron can spend more time elsewhere than it does in the hydrogen atom. The hydrogen atom can be a location of net positive charge, and can weakly attract a zone of negative charge in another molecule.

Q43.2 Rotational, vibrational, and electronic (as discussed in Chapter 42) are the three major forms of excitation. Rotational energy for a diatomic molecule is on the order of $\frac{\hbar^2}{2I}$, where I is the moment of inertia of the molecule. A typical value for a small molecule is on the order of $1\text{ meV} = 10^{-3}\text{ eV}$. Vibrational energy is on the order of hf, where f is the vibration frequency of the molecule. A typical value is on the order of 0.1 eV. Electronic energy depends on the state of an electron in the molecule and is on the order of a few eV. The rotational energy can be zero, but neither the vibrational nor the electronic energy can be zero.

***Q43.3** If you start with a solid sample and raise its temperature, it will typically melt first, then start emitting lots of far infrared light, then emit light with a spectrum peaking in the near infrared, and later have its molecules dissociate into atoms. Rotation of a diatomic molecule involves less energy than vibration. Absorption and emission of microwave photons, of frequency $\sim 10^{11}$ Hz, accompany excitation and de-excitation of rotational motion, while infrared photons, of frequency $\sim 10^{13}$ Hz, accompany changes in the vibration state of typical simple molecules. The ranking is then b > d > c > a.

Q43.4 From the rotational spectrum of a molecule, one can easily calculate the moment of inertia of the molecule using Equation 43.7 in the text. Note that with this method, only the spacing between adjacent energy levels needs to be measured. From the moment of inertia, the size of the molecule can be calculated, provided that the structure of the molecule is known.

***Q43.5** Answer (b). At higher temperature, molecules are typically in higher rotational energy levels before as well as after infrared absorption.

***Q43.6** (i) Answer (a). An example is NaCl, table salt.
(ii) Answer (b). Examples are elemental silicon and carborundum (silicon carbide).
(iii) Answer (c). Think of aluminum foil.

***Q43.7** (i) Answer (b). The density of states is proportional to the energy to the one-half power.
(ii) Answer (a). Most states well above the Fermi energy are unoccupied.

Q43.8 In a metal, there is no energy gap between the valence and conduction bands, or the conduction band is partly full even at absolute zero in temperature. Thus an applied electric field is able to inject a tiny bit of energy into an electron to promote it to a state in which it is moving through the metal as part of an electric current. In an insulator, there is a large energy gap between a full valence band and an empty conduction band. An applied electric field is unable to give electrons in the valence band enough energy to jump across the gap into the higher energy conduction band. In a semiconductor, the energy gap between valence and conduction bands is smaller than in an insulator. At absolute zero the valence band is full and the conduction band is empty, but at room temperature thermal energy has promoted some electrons across the gap. Then there are some mobile holes in the valence band as well as some mobile electrons in the conduction band.

***Q43.9** Answer (b). First consider electric conduction in a metal. The number of conduction electrons is essentially fixed. They conduct electricity by having drift motion in an applied electric field superposed on their random thermal motion. At higher temperature, the ion cores vibrate more and scatter more efficiently the conduction electrons flying among them. The mean time between collisions is reduced. The electrons have time to develop only a lower drift speed. The electric current is reduced, so we see the resistivity increasing with temperature.

Now consider an intrinsic semiconductor. At absolute zero its valence band is full and its conduction band is empty. It is an insulator, with very high resistivity. As the temperature increases, more electrons are promoted to the conduction band, leaving holes in the valence band. Then both electrons and holes move in response to an applied electric field. Thus we see the resistivity decreasing as temperature goes up.

Q43.10 The energy of the photon is given to the electron. The energy of a photon of visible light is sufficient to promote the electron from the lower-energy valence band to the higher-energy conduction band. This results in the additional electron in the conduction band and an additional hole—the energy state that the electron used to occupy—in the valence band.

Q43.11 Along with arsenic (As), any other element in group V, such as phosphorus (P), antimony (Sb), and bismuth (Bi), would make good donor atoms. Each has 5 valence electrons. Any element in group III would make good acceptor atoms, such as boron (B), aluminum, (Al), gallium (Ga), and indium (In). They all have only 3 valence electrons.

Q43.12 The two assumptions in the free-electron theory are that the conduction electrons are not bound to any particular atom, and that the nuclei of the atoms are fixed in a lattice structure. In this model, it is the "soup" of free electrons that are conducted through metals. The energy band model is more comprehensive than the free-electron theory. The energy band model includes an account of the more tightly bound electrons as well as the conduction electrons. It can be developed into a theory of the structure of the crystal and its mechanical and thermal properties.

Q43.13 A molecule containing two atoms of ^2H, deuterium, has twice the mass of a molecule containing two atoms of ordinary hydrogen ^1H. The atoms have the same electronic structure, so the molecules have the same interatomic spacing, and the same spring constant. Then the moment of inertia of the double-deuteron is twice as large and the rotational energies one-half as large as for ordinary hydrogen. Each vibrational energy level for D_2 is $\frac{1}{\sqrt{2}}$ times that of H_2.

Q43.14 Yes. A material can absorb a photon of energy greater than the energy gap, as an electron jumps into a higher energy state. If the photon does not have enough energy to raise the energy of the electron by the energy gap, then the photon will not be absorbed.

***Q43.15** (i) and (ii) Answer (a) for both. Either kind of doping contributes more mobile charge carriers, either holes or electrons.

***Q43.16** (a) false (b) false (c) true (d) true (e) true

SOLUTIONS TO PROBLEMS

Section 43.1 **Molecular Bonds**

P43.1 (a) $F = \dfrac{q^2}{4\pi \in_0 r^2} = \dfrac{\left(1.60\times10^{-19}\right)^2\left(8.99\times10^9\right)}{\left(5.00\times10^{-10}\right)^2}\text{ N} = \boxed{0.921\times10^{-9}\text{ N}}$ toward the other ion.

(b) $U = \dfrac{-q^2}{4\pi \in_0 r} = -\dfrac{\left(1.60\times10^{-19}\right)^2\left(8.99\times10^9\right)}{5.00\times10^{-10}}\text{ J} \approx \boxed{-2.88\text{ eV}}$

P43.2 We are told $\quad K + Cl + 0.7\text{ eV} \to K^+ + Cl^-$

and $\quad Cl + e^- \to Cl^- + 3.6\text{ eV}$

or $\quad Cl^- \to Cl + e^- - 3.6\text{ eV}$

By substitution, $\quad K + Cl + 0.7\text{ eV} \to K^+ + Cl + e^- - 3.6\text{ eV}$

$$K + 4.3\text{ eV} \to K^+ + e^-$$

or the ionization energy of potassium is $\boxed{4.3\text{ eV}}$.

P43.3 (a) Minimum energy of the molecule is found from

$$\frac{dU}{dr} = -12Ar^{-13} + 6Br^{-7} = 0 \text{ yielding } \boxed{r_0 = \left[\frac{2A}{B}\right]^{1/6}}$$

(b) $E = U\big|_{r=\infty} - U\big|_{r=r_0} = 0 - \left[\dfrac{A}{4A^2/B^2} - \dfrac{B}{2A/B}\right] = -\left[\dfrac{1}{4} - \dfrac{1}{2}\right]\dfrac{B^2}{A} = \boxed{\dfrac{B^2}{4A}}$

This is also the equal to the binding energy, the amount of energy given up by the two atoms as they come together to form a molecule.

(c) $r_0 = \left[\dfrac{2\left(0.124\times10^{-120}\text{ eV}\cdot\text{m}^{12}\right)}{1.488\times10^{-60}\text{ eV}\cdot\text{m}^6}\right]^{1/6} = 7.42\times10^{-11}\text{ m} = \boxed{74.2\text{ pm}}$

$$E = \frac{\left(1.488\times10^{-60}\text{ eV}\cdot\text{m}^6\right)^2}{4\left(0.124\times10^{-120}\text{ eV}\cdot\text{m}^{12}\right)} = \boxed{4.46\text{ eV}}$$

P43.4 (a) We add the reactions $K + 4.34\text{ eV} \to K^+ + e^-$

and $I + e^- \to I^- + 3.06\text{ eV}$

to obtain $K + I \to K^+ + I^- + (4.34 - 3.06)\text{ eV}$

The activation energy is $\boxed{1.28\text{ eV}}$.

(b) $$\frac{dU}{dr} = \frac{4\in}{\sigma}\left[-12\left(\frac{\sigma}{r}\right)^{13} + 6\left(\frac{\sigma}{r}\right)^{7}\right]$$

At $r = r_0$ we have $\frac{dU}{dr} = 0$. Here $\left(\frac{\sigma}{r_0}\right)^{13} = \frac{1}{2}\left(\frac{\sigma}{r_0}\right)^{7}$

$\frac{\sigma}{r_0} = 2^{-1/6}$ $\quad \sigma = 2^{-1/6}(0.305)\text{ nm} = \boxed{0.272\text{ nm} = \sigma}$

Then also

$$U(r_0) = 4\in\left[\left(\frac{2^{-1/6}r_0}{r_0}\right)^{12} - \left(\frac{2^{-1/6}r_0}{r_0}\right)^{6}\right] + E_a = 4\in\left[\frac{1}{4} - \frac{1}{2}\right] + E_a = -\in + E_a$$

$$\in = E_a - U(r_0) = 1.28\text{ eV} + 3.37\text{ eV} = \boxed{4.65\text{ eV} = \in}$$

(c) $$F(r) = -\frac{dU}{dr} = \frac{4\in}{\sigma}\left[12\left(\frac{\sigma}{r}\right)^{13} - 6\left(\frac{\sigma}{r}\right)^{7}\right]$$

To find the maximum force we calculate $\frac{dF}{dr} = \frac{4\in}{\sigma^2}\left[-156\left(\frac{\sigma}{r}\right)^{14} + 42\left(\frac{\sigma}{r}\right)^{8}\right] = 0$ when

$$\frac{\sigma}{r_{\text{rupture}}} = \left(\frac{42}{156}\right)^{1/6}$$

$$F_{\max} = \frac{4(4.65\text{ eV})}{0.272\text{ nm}}\left[12\left(\frac{42}{156}\right)^{13/6} - 6\left(\frac{42}{156}\right)^{7/6}\right] = -41.0\text{ eV/nm} = -41.0\frac{1.6\times10^{-19}\text{ Nm}}{10^{-9}\text{ m}}$$

$$= -6.55\text{ nN}$$

Therefore the applied force required to rupture the molecule is $\boxed{+6.55\text{ nN}}$ away from the center.

(d) $$U(r_0 + s) = 4\in\left[\left(\frac{\sigma}{r_0 + s}\right)^{12} - \left(\frac{\sigma}{r_0 + s}\right)^{6}\right] + E_a = 4\in\left[\left(\frac{2^{-1/6}r_0}{r_0 + s}\right)^{12} - \left(\frac{2^{-1/6}}{r_0 + s}\right)^{6}\right] + E_a$$

$$= 4\in\left[\frac{1}{4}\left(1 + \frac{s}{r_0}\right)^{-12} - \frac{1}{2}\left(1 + \frac{s}{r_0}\right)^{-6}\right] + E_a$$

$$= 4\in\left[\frac{1}{4}\left(1 - 12\frac{s}{r_0} + 78\frac{s^2}{r_0^2} - \cdots\right) - \frac{1}{2}\left(1 - 6\frac{s}{r_0} + 21\frac{s^2}{r_0^2} - \cdots\right)\right] + E_a$$

$$= \in - 12\in\frac{s}{r_0} + 78\in\frac{s^2}{r_0^2} - 2\in + 12\in\frac{s}{r_0} - 42\in\frac{s^2}{r_0^2} + E_a + \cdots$$

$$= -\in + E_a + 0\left(\frac{s}{r_0}\right) + 36\in\frac{s^2}{r_0^2} + \cdots$$

$$U(r_0 + s) \approx U(r_0) + \frac{1}{2}ks^2$$

where $k = \frac{72\in}{r_0^2} = \frac{72(4.65\text{ eV})}{(0.305\text{ nm})^2} = 3\,599\text{ eV/nm}^2 = \boxed{576\text{ N/m}}$

P43.5 At the boiling or condensation temperature, $k_BT \approx 10^{-3}\ \text{eV} = 10^{-3}\left(1.6\times10^{-19}\ \text{J}\right)$

$$T \approx \frac{1.6\times10^{-22}\ \text{J}}{1.38\times10^{-23}\ \text{J/K}}\ \boxed{\sim 10\ \text{K}}$$

Section 43.2 **Energy States and Spectra of Molecules**

***P43.6** (a) With r representing the distance of each atom from the center of mass, the moment of inertia is

$$mr^2 + mr^2 = 2\left(1.67\times10^{-27}\ \text{kg}\right)\left(\frac{0.75\times10^{-10}\ \text{m}}{2}\right)^2 = 4.70\times10^{-48}\ \text{kg}\cdot\text{m}^2$$

The rotational energy is $E_{\text{rot}} = \dfrac{\hbar^2}{2I}J(J+1)$ or it is zero for $J = 0$ and for $J = 1$ it is

$$\frac{h^2 1(2)}{4\pi^2 2I} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{4\pi^2(4.70\times10^{-48}\ \text{kg}\cdot\text{m}^2)}\left(\frac{1\ \text{eV}}{1.6\times10^{-19}\ \text{J}}\right) = \boxed{0.0148\ \text{eV}}$$

(b) $$\lambda = \frac{c}{f} = \frac{ch}{E} = \frac{(2.998\times10^8\ \text{m/s})(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{0.0148\ \text{eV}}\frac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}} = \frac{1240\ \text{eV}\cdot\text{nm}}{0.0148\ \text{eV}} = \boxed{83.8\ \mu\text{m}}$$

P43.7 $$\mu = \frac{m_1m_2}{m_1+m_2} = \frac{132.9(126.9)}{132.9+126.9}\left(1.66\times10^{-27}\ \text{kg}\right) = 1.08\times10^{-25}\ \text{kg}$$

$$I = \mu r^2 = \left(1.08\times10^{-25}\ \text{kg}\right)\left(0.127\times10^{-9}\ \text{m}\right)^2 = 1.74\times10^{-45}\ \text{kg}\cdot\text{m}^2$$

(a) $$E = \frac{1}{2}I\omega^2 = \frac{(I\omega)^2}{2I} = \frac{J(J+1)\hbar^2}{2I}$$

$J = 0$ gives $E = 0$

$J = 1$ gives $$E = \frac{\hbar^2}{I} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{4\pi^2\left(1.74\times10^{-45}\ \text{kg}\cdot\text{m}^2\right)} = 6.41\times10^{-24}\ \text{J} = \boxed{40.0\ \mu\text{eV}}$$

$hf = 6.41\times10^{-24}\ \text{J} - 0$ gives $f = \boxed{9.66\times10^9\ \text{Hz}}$

(b) $$f = \frac{E_1}{h} = \frac{\hbar^2}{hI} = \frac{h}{4\pi^2\mu r^2} \propto r^{-2}$$ $\boxed{\text{If } r \text{ is 10\% too small, } f \text{ is 20\% too large.}}$

***P43.8** $$\Delta E_{vib} = \frac{h}{2\pi}\sqrt{\frac{k}{\mu}} = hf \qquad \text{so} \qquad k = 4\pi^2 f^2\mu$$

$$\mu = \frac{k}{4\pi^2 f^2} = \frac{1530\ \text{N/m}}{4\pi^2(56.3\times10^{12}/\text{s})^2} = \boxed{1.22\times10^{-26}\,\text{kg}}$$

$$\mu = \frac{m_1m_2}{m_1+m_2} = \frac{14.007\ \text{u}\ 15.999\ \text{u}}{14.007\ \text{u} + 15.999\ \text{u}}\frac{1.66\times10^{-27}\ \text{kg}}{1\ \text{u}} = \boxed{1.24\times10^{-26}\ \text{kg}}$$

The reduced masses agree, because the small apparent difference can be attributed to uncertainty in the data.

P43.9 For the HCl molecule in the $J=1$ rotational energy level, we are given $r_0 = 0.127\,5$ nm.

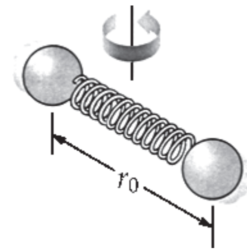


FIG. P43.9

$$E_{\text{rot}} = \frac{\hbar^2}{2I}J(J+1)$$

Taking $J=1$, we have $E_{\text{rot}} = \frac{\hbar^2}{I} = \frac{1}{2}I\omega^2$ or $\omega = \sqrt{\frac{2\hbar^2}{I^2}} = \sqrt{2}\frac{\hbar}{I}$

The moment of inertia of the molecule is given by Equation 43.3.

$$I = \mu r_0^2 = \left(\frac{m_1 m_2}{m_1+m_2}\right)r_0^2$$

$$I = \left[\frac{(1.01\text{ u})(35.5\text{ u})}{1.01\text{ u}+35.5\text{ u}}\right]r_0^2 = (0.982\text{ u})(1.66\times10^{-27}\text{ kg/u})(1.275\times10^{-10}\text{ m})^2 = 2.65\times10^{-47}\text{ kg}\cdot\text{m}^2$$

Therefore, $\omega = \sqrt{2}\frac{\hbar}{I} = \frac{\sqrt{2}\ 6.626\times10^{-34}\text{ J}\cdot\text{s}}{2\pi(2.65\times10^{-47}\text{ kg}\cdot\text{m}^2)} = \boxed{5.63\times10^{12}\text{ rad/s}}$

P43.10 $hf = \Delta E = \frac{\hbar^2}{2I}[2(2+1)] - \frac{\hbar^2}{2I}[1(1+1)] = \frac{\hbar^2}{2I}(4)$

$$I = \frac{4(h/2\pi)^2}{2hf} = \frac{h}{2\pi^2 f} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{2\pi^2(2.30\times10^{11}\text{ Hz})} = \boxed{1.46\times10^{-46}\text{ kg}\cdot\text{m}^2}$$

P43.11 $I = m_1 r_1^2 + m_2 r_2^2$ where $m_1 r_1 = m_2 r_2$ and $r_1 + r_2 = r$

Then $r_1 = \frac{m_2 r_2}{m_1}$ so $\frac{m_2 r_2}{m_1} + r_2 = r$ and $r_2 = \frac{m_1 r}{m_1+m_2}$

Also, $r_2 = \frac{m_1 r_1}{m_2}$. Thus, $r_1 + \frac{m_1 r_1}{m_2} = r$ and $r_1 = \frac{m_2 r}{m_1+m_2}$

$$I = m_1\frac{m_2^2 r^2}{(m_1+m_2)^2} + \frac{m_2 m_1^2 r^2}{(m_1+m_2)^2} = \frac{m_1 m_2 r^2(m_2+m_1)}{(m_1+m_2)^2} = \frac{m_1 m_2 r^2}{m_1+m_2} = \boxed{\mu r^2}$$

P43.12 (a) $\mu = \frac{22.99(35.45)}{(22.99+35.45)}(1.66\times10^{-27}\text{ kg}) = 2.32\times10^{-26}\text{ kg}$

$$I = \mu r^2 = (2.32\times10^{-26}\text{ kg})(0.280\times10^{-9}\text{ m})^2 = \boxed{1.81\times10^{-45}\text{ kg}\cdot\text{m}^2}$$

(b) $\frac{hc}{\lambda} = \frac{\hbar^2}{2I}2(2+1) - \frac{\hbar^2}{2I}1(1+1) = \frac{3\hbar^2}{I} - \frac{\hbar^2}{I} = \frac{2\hbar^2}{I} = \frac{2h^2}{4\pi^2 I}$

$$\lambda = \frac{c4\pi^2 I}{2h} = \frac{(3.00\times10^8\text{ m/s})4\pi^2(1.81\times10^{-45}\text{ kg}\cdot\text{m}^2)}{2(6.626\times10^{-34}\text{ J}\cdot\text{s})} = \boxed{1.62\text{ cm}}$$

P43.13 The energy of a rotational transition is $\Delta E = \left(\frac{\hbar^2}{I}\right)J$ where J is the rotational quantum number of the higher energy state (see Equation 43.7). We do not know J from the data. However,

$$\Delta E = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\lambda}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right).$$

For each observed wavelength,

λ (mm)	ΔE (eV)
0.120 4	0.010 32
0.096 4	0.012 88
0.080 4	0.015 44
0.069 0	0.018 00
0.060 4	0.020 56

The $\Delta E'$s consistently increase by 0.002 56 eV. $E_1 = \frac{\hbar^2}{I} = 0.002\ 56\text{ eV}$

and $I = \frac{\hbar^2}{E_1} = \frac{\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{(0.002\ 56\text{ eV})}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \boxed{2.72\times10^{-47}\text{ kg}\cdot\text{m}^2}$

For the HCl molecule, the internuclear radius is $r = \sqrt{\frac{I}{\mu}} = \sqrt{\frac{2.72\times10^{-47}}{1.62\times10^{-27}}}\text{ m} = 0.130\text{ nm}$

***P43.14** (a) Minimum amplitude of vibration of HI is characterized by

$$\frac{1}{2}kA^2 = \frac{1}{2}\hbar\omega = \frac{h}{4\pi}\sqrt{\frac{k}{\mu}} \qquad \text{so} \qquad A = \sqrt{\frac{h}{2\pi}}\left(\frac{1}{k\mu}\right)^{1/4}$$

$$A = \sqrt{\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{2\pi}}\left(\frac{1}{(320\text{ N/m})(127/128)(1.66\times10^{-27}\text{ kg})}\right)^{1/4} = \boxed{12.1\text{ pm}}$$

(b) For HF, $A = \sqrt{\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{2\pi}}\left(\frac{1}{(970\text{ N/m})(19/20)(1.66\times10^{-27}\text{ kg})}\right)^{1/4} = \boxed{9.23\text{ pm}}$

(c) Since HI has the smaller k, it is more weakly bound.

P43.15 $\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{35}{36}\times1.66\times10^{-27}\text{ kg} = 1.61\times10^{-27}\text{ kg}$

$$\Delta E_{\text{vib}} = \hbar\sqrt{\frac{k}{\mu}} = \left(1.055\times10^{-34}\right)\sqrt{\frac{480}{1.61\times10^{-27}}} = 5.74\times10^{-20}\text{ J} = \boxed{0.358\text{ eV}}$$

P43.16 (a) The reduced mass of the O_2 is

$$\mu = \frac{(16\text{ u})(16\text{ u})}{(16\text{ u})+(16\text{ u})} = 8\text{ u} = 8\left(1.66\times10^{-27}\text{ kg}\right) = 1.33\times10^{-26}\text{ kg}$$

The moment of inertia is then

$$I = \mu r^2 = \left(1.33\times10^{-26}\text{ kg}\right)\left(1.20\times10^{-10}\text{ m}\right)^2$$
$$= 1.91\times10^{-46}\text{ kg}\cdot\text{m}^2$$

The rotational energies are $E_{\text{rot}} = \frac{\hbar^2}{2I}J(J+1) = \frac{\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(1.91\times10^{-46}\text{ kg}\cdot\text{m}^2\right)}J(J+1)$

Thus $E_{\text{rot}} = \left(2.91\times10^{-23}\text{ J}\right)J(J+1)$

And for $J = 0, 1, 2,$ $E_{\text{rot}} = \boxed{0,\ 3.64\times10^{-4}\text{ eV},\ 1.09\times10^{-3}\text{ eV}}$

(b) $$E_{\text{vib}} = \left(v+\frac{1}{2}\right)\hbar\sqrt{\frac{k}{\mu}} = \left(v+\frac{1}{2}\right)\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)\sqrt{\frac{1177\text{ N/m}}{8\left(1.66\times10^{-27}\text{ kg}\right)}}$$

$$E_{\text{vib}} = \left(v+\frac{1}{2}\right)\left(3.14\times10^{-20}\text{ J}\right)\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \left(v+\frac{1}{2}\right)(0.196\text{ eV})$$

For $v = 0, 1, 2,$ $\boxed{E_{\text{vib}} = 0.098\,2\text{ eV},\ 0.295\text{ eV},\ 0.491\text{ eV}}$.

P43.17 In Benzene, the carbon atoms are each 0.110 nm from the axis and each hydrogen atom is $(0.110+0.100\text{ nm}) = 0.210\text{ nm}$ from the axis. Thus, $I = \Sigma mr^2$:

$$I = 6\left(1.99\times10^{-26}\text{ kg}\right)\left(0.110\times10^{-9}\text{ m}\right)^2 + 6\left(1.67\times10^{-27}\text{ kg}\right)\left(0.210\times10^{-9}\text{ m}\right)^2$$
$$= 1.89\times10^{-45}\text{ kg}\cdot\text{m}^2$$

The allowed rotational energies are then

$$E_{\text{rot}} = \frac{\hbar^2}{2I}J(J+1) = \frac{\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(1.89\times10^{-45}\text{ kg}\cdot\text{m}^2\right)}J(J+1) = \left(2.95\times10^{-24}\text{ J}\right)J(J+1)$$
$$= \left(18.4\times10^{-6}\text{ eV}\right)J(J+1)$$

$$E_{\text{rot}} = \boxed{(18.4\ \mu\text{eV})J(J+1)\text{ where }J = 0, 1, 2, 3, \ldots}$$

The first five of these allowed energies are: $E_{\text{rot}} = 0$, 36.9 μeV, 111 μeV, 221 μeV, and 369 μeV.

P43.18 We carry extra digits through the solution because part (c) involves the subtraction of two close numbers. The longest wavelength corresponds to the smallest energy difference between the rotational energy levels. It is between $J=0$ and $J=1$, namely $\frac{\hbar^2}{I}$

$\lambda = \frac{hc}{\Delta E_{\min}} = \frac{hc}{\hbar^2/I} = \frac{4\pi^2 Ic}{h}$. If μ is the reduced mass, then

$$I = \mu r^2 = \mu\left(0.127\ 46\times 10^{-9}\ \text{m}\right)^2 = \left(1.624\ 605\times 10^{-20}\ \text{m}^2\right)\mu$$

$$\text{Therefore } \lambda = \frac{4\pi^2\left(1.624\ 605\times 10^{-20}\ \text{m}^2\right)\mu\left(2.997\ 925\times 10^{8}\ \text{m/s}\right)}{6.626\ 075\times 10^{-34}\ \text{J}\cdot\text{s}} = \left(2.901\ 830\times 10^{23}\ \text{m/kg}\right)\mu \quad (1)$$

(a) $\mu_{35} = \frac{(1.007\ 825\text{u})(34.968\ 853\text{u})}{1.007\ 825\text{u} + 34.968\ 853\text{u}} = 0.979\ 593\text{u} = 1.626\ 653\times 10^{-27}\ \text{kg}$

From (1): $\lambda_{35} = \left(2.901\ 830\times 10^{23}\ \text{m/kg}\right)\left(1.626\ 653\times 10^{-27}\ \text{kg}\right) = \boxed{472\ \mu\text{m}}$

(b) $\mu_{37} = \frac{(1.007\ 825\text{u})(36.965\ 903\text{u})}{1.007\ 825\text{u} + 36.965\ 903\text{u}} = 0.981\ 077\text{u} = 1.629\ 118\times 10^{-27}\ \text{kg}$

From (1): $\lambda_{37} = \left(2.901\ 830\times 10^{23}\ \text{m/kg}\right)\left(1.629\ 118\times 10^{-27}\ \text{kg}\right) = \boxed{473\ \mu\text{m}}$

(c) $\lambda_{37} - \lambda_{35} = 472.742\ 4\ \mu\text{m} - 472.027\ 0\ \mu\text{m} = \boxed{0.715\ \mu\text{m}}$

P43.19 We find an average spacing between peaks by counting 22 gaps between 7.96×10^{13} Hz and 9.24×10^{13} Hz:

$$\Delta f = \frac{(9.24-7.96)10^{13}\ \text{Hz}}{22} = 0.058\ 2\times 10^{13}\ \text{Hz} = \frac{1}{h}\left(\frac{h^2}{4\pi^2 I}\right)$$

$$I = \frac{h}{4\pi^2\Delta f} = \frac{6.63\times 10^{-34}\ \text{J}\cdot\text{s}}{4\pi^2 5.82\times 10^{11}/\text{s}} = \boxed{2.9\times 10^{-47}\ \text{kg}\cdot\text{m}^2}$$

P43.20 We carry extra digits through the solution because the given wavelengths are close together.

(a) $E_{vJ}=\left(v+\frac{1}{2}\right)hf+\frac{\hbar^2}{2I}J(J+1)$

$$\therefore E_{00}=\frac{1}{2}hf, \qquad E_{11}=\frac{3}{2}hf+\frac{\hbar^2}{I}, \quad E_{02}=\frac{1}{2}hf+\frac{3\hbar^2}{I}$$

$$\therefore E_{11}-E_{00}=hf+\frac{\hbar^2}{I}=\frac{hc}{\lambda}=\frac{(6.626\,075\times10^{-34}\text{ J}\cdot\text{s})(2.997\,925\times10^{8}\text{ m/s})}{2.211\,2\times10^{-6}\text{ m}}$$

$$\therefore hf+\frac{\hbar^2}{I}=8.983\,573\times10^{-20}\text{ J} \tag{1}$$

$$E_{11}-E_{02}=hf-\frac{2\hbar^2}{I}=\frac{hc}{\lambda}=\frac{(6.626\,075\times10^{-34}\text{ J}\cdot\text{s})(2.997\,925\times10^{8}\text{ m/s})}{2.405\,4\times10^{-6}\text{ m}}$$

$$\therefore hf-\frac{2\hbar^2}{I}=8.258\,284\times10^{-20}\text{ J} \tag{2}$$

Subtract (2) from (1): $\frac{3\hbar^2}{I}=7.252\,89\times10^{-21}\text{ J}$

$$\therefore I=\frac{3(1.054\,573\times10^{-34}\text{ J}\cdot\text{s})^2}{7.252\,89\times10^{-21}\text{ J}}=\boxed{4.60\times10^{-48}\text{ kg}\cdot\text{m}^2}$$

(b) From (1):

$$f=\frac{8.983\,573\times10^{-20}\text{ J}}{6.626\,075\times10^{-34}\text{ J}\cdot\text{s}}-\frac{(1.054\,573\times10^{-34}\text{ J}\cdot\text{s})^2}{(4.600\,060\times10^{-48}\text{ kg}\cdot\text{m}^2)(6.626\,075\times10^{-34}\text{ J}\cdot\text{s})}$$

$$=\boxed{1.32\times10^{14}\text{ Hz}}$$

(c) $I=\mu r^2$, where μ is the reduced mass:

$$\mu=\frac{1}{2}m_H=\frac{1}{2}(1.007\,825\text{u})=8.367\,669\times10^{-28}\text{ kg}$$

So $r=\sqrt{\frac{I}{\mu}}=\sqrt{\frac{4.600\,060\times10^{-48}\text{ kg}\cdot\text{m}^2}{8.367\,669\times10^{-28}\text{ kg}}}=\boxed{0.074\,1\text{ nm}}$.

P43.21 The emission energies are the same as the absorption energies, but the final state must be below $(v=1,\ J=0)$. The transition must satisfy $\Delta J=\pm1$, so it must end with $J=1$. To be lower in energy, it must be $(v=0,\ J=1)$. The emitted photon energy is therefore

$$hf_{\text{photon}}=\left(E_{\text{vib}}\big|_{v=1}+E_{\text{rot}}\big|_{J=0}\right)-\left(E_{\text{vib}}\big|_{v=0}+E_{\text{rot}}\big|_{J=1}\right)=\left(E_{\text{vib}}\big|_{v=1}-E_{\text{vib}}\big|_{v=0}\right)-\left(E_{\text{rot}}\big|_{J=1}-E_{\text{rot}}\big|_{J=0}\right)$$

$$hf_{\text{photon}}=hf_{\text{vib}}-hf_{\text{rot}}$$

Thus, $f_{\text{photon}}=f_{\text{vib}}-f_{\text{rot}}=6.42\times10^{13}\text{ Hz}-1.15\times10^{11}\text{ Hz}=\boxed{6.41\times10^{13}\text{ Hz}}$.

P43.22 The moment of inertia about the molecular axis is $I_x=\frac{2}{5}mr^2+\frac{2}{5}mr^2=\frac{4}{5}m(2.00\times10^{-15}\text{ m})^2$.

The moment of inertia about a perpendicular axis is $I_y=m\left(\frac{R}{2}\right)^2+m\left(\frac{R}{2}\right)^2=\frac{m}{2}(2.00\times10^{-10}\text{ m})^2$.

The allowed rotational energies are $E_{\text{rot}}=\left(\frac{\hbar^2}{2I}\right)J(J+1)$, so the energy of the first excited state is

$E_1=\frac{\hbar^2}{I}$. The ratio is therefore

$$\frac{E_{1,x}}{E_{1,y}}=\frac{(\hbar^2/I_x)}{(\hbar^2/I_y)}=\frac{I_y}{I_x}=\frac{(1/2)m(2.00\times10^{-10}\text{ m})^2}{(4/5)m(2.00\times10^{-15}\text{ m})^2}=\frac{5}{8}(10^5)^2=\boxed{6.25\times10^9}$$

Section 43.3 **Bonding in Solids**

P43.23 $U = -\frac{\alpha k_e e^2}{r_0}\left(1 - \frac{1}{m}\right) = -(1.747\,6)\left(8.99\times10^9\right)\frac{\left(1.60\times10^{-19}\right)^2}{\left(0.281\times10^{-9}\right)}\left(1 - \frac{1}{8}\right) = -1.25\times10^{-18}\text{ J} = \boxed{-7.84\text{ eV}}$

P43.24 Consider a cubical salt crystal of edge length 0.1 mm.

The number of atoms is $\left(\frac{10^{-4}\text{ m}}{0.261\times10^{-9}\text{ m}}\right)^3 \boxed{\sim 10^{17}}$

This number of salt crystals would have volume $\left(10^{-4}\text{ m}\right)^3\left(\frac{10^{-4}\text{ m}}{0.261\times10^{-9}\text{ m}}\right)^3 \boxed{\sim 10^5\text{ m}^3}$

If it is cubic, it has edge length 40 m.

P43.25 $U = -\frac{k_e e^2}{r} - \frac{k_e e^2}{r} + \frac{k_e e^2}{2r} + \frac{k_e e^2}{2r} - \frac{k_e e^2}{3r} - \frac{k_e e^2}{3r} + \frac{k_e e^2}{4r} + \frac{k_e e^2}{4r} - \cdots$

$= -\frac{2k_e e^2}{r}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\right)$

But, $\ln(1+x) = 1 - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

so, $U = -\frac{2k_e e^2}{r}\ln 2$, or $\boxed{U = -k_e\alpha\frac{e^2}{r}\text{ where }\alpha = 2\ln 2}$

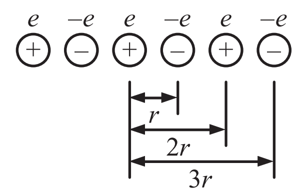


FIG. P43.25

P43.26 Visualize a K^+ ion at the center of each shaded cube, a Cl^- ion at the center of each white one.

The distance *ab* is $\sqrt{2}(0.314\text{ nm}) = \boxed{0.444\text{ nm}}$

Distance *ac* is $2(0.314\text{ nm}) = \boxed{0.628\text{ nm}}$

Distance *ad* is $\sqrt{2^2 + \left(\sqrt{2}\right)^2}(0.314\text{ nm}) = \boxed{0.769\text{ nm}}$

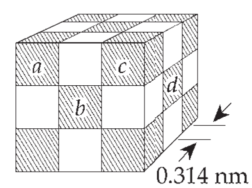


FIG. P43.26

Section 43.4 **Free-Electron Theory of Metals**

Section 43.5 **Band Theory of Solids**

P43.27 The density of conduction electrons n is given by $E_F = \frac{h^2}{2m}\left(\frac{3n_e}{8\pi}\right)^{2/3}$

or $n_e = \frac{8\pi}{3}\left(\frac{2mE_F}{h^2}\right)^{3/2} = \frac{8\pi}{3}\frac{\left[2\left(9.11\times10^{-31}\text{ kg}\right)(5.48)\left(1.60\times10^{-19}\text{ J}\right)\right]^{3/2}}{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^3} = 5.80\times10^{28}\text{ m}^{-3}$

The number-density of silver atoms is

$$n_{Ag} = \left(10.6\times10^3\text{ kg/m}^3\right)\left(\frac{1\text{ atom}}{108\text{ u}}\right)\left(\frac{1\text{ u}}{1.66\times10^{-27}\text{ kg}}\right) = 5.91\times10^{28}\text{ m}^{-3}$$

So an average atom contributes $\frac{5.80}{5.91} = \boxed{0.981\text{ electron to the conduction band}}$.

***P43.28** (a) The Fermi energy is proportional to the spatial concentration of free electrons to the two-thirds power.

(b) $E_F = \frac{h^2}{2m}\left(\frac{3n_e}{8\pi}\right)^{2/3} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}{2(9.11\times10^{-31}\text{ kg})(1.60\times10^{-19}\text{ J/eV})}\left(\frac{3}{8\pi}\right)^{2/3} n_e^{2/3}$ becomes

$E_F = (3.65\times10^{-19})n_e^{2/3}$ eV with n measured in electrons/m^3.

(c) Copper has the greater concentration of free electrons by a factor of $8.49\times10^{28}/1.40\times10^{28} = 6.06$

Copper has the greater Fermi energy by a factor of 7.05 eV/2.12 eV = 3.33. This behavior agrees with the proportionality because $6.06^{2/3} = 3.33$.

P43.29 (a) $\frac{1}{2}mv^2 = 7.05$ eV

$$v = \sqrt{\frac{2(7.05\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{ kg}}} = \boxed{1.57\times10^6\text{ m/s}}$$

(b) $\boxed{\text{Larger than }10^{-4}\text{ m/s by ten orders of magnitude.}}$ However, the energy of an electron at room temperature is typically $k_BT = \frac{1}{40}$ eV.

***P43.30** The occupation probability is

$$f(E) = \frac{1}{e^{(E-E_F)/k_BT}+1} = \frac{1}{e^{(0.99E_F-E_F)/k_BT}+1}$$

$$= \frac{1}{\exp[-0.01(7.05\text{ eV})(1.6\times10^{-19}\text{ J/eV})/(1.38\times10^{-23}\text{ J/K})300\text{ K}]+1} = \frac{1}{e^{-2.72}+1} = \boxed{0.938}$$

***P43.31** (a) $E_{av} = \frac{3}{5}E_F = 0.6(7.05\text{ eV}) = \boxed{4.23\text{ eV}}$

(b) The average energy of a molecule in an ideal gas is $\frac{3}{2}k_BT$ so we have

$$T = \frac{2}{3}\frac{4.23\text{ eV}}{1.38\times10^{-23}\text{ J/K}}\frac{1.6\times10^{-19}\text{ J}}{1\text{ eV}} = \boxed{3.27\times10^4\text{ K}}$$

P43.32 For sodium, $M = 23.0$ g/mol and $\rho = 0.971$ g/cm^3.

(a) $n_e = \frac{N_A\rho}{M} = \frac{(6.02\times10^{23}\text{ electrons/mol})(0.971\text{ g/cm}^3)}{23.0\text{ g/mol}}$

$n_e = 2.54\times10^{22}\text{ electrons/cm}^3 = \boxed{2.54\times10^{28}\text{ electrons/m}^3}$

(b) $E_F = \left(\frac{h^2}{2m}\right)\left(\frac{3n_e}{8\pi}\right)^{2/3} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}{2(9.11\times10^{-31}\text{ kg})}\left[\frac{3(2.54\times10^{28}\text{ m}^{-3})}{8\pi}\right]^{2/3}$

$= 5.05\times10^{-19}\text{ J} = \boxed{3.15\text{ eV}}$

P43.33 Taking $E_F = 5.48$ eV for sodium at 800 K,

$$f = \left[e^{(E-E_F)/k_BT}+1\right]^{-1} = 0.950$$

$$e^{(E-E_F)/k_BT} = \frac{1}{0.950} - 1 = 0.052\,6$$

$$\frac{E-E_F}{k_BT} = \ln(0.052\,6) = -2.94$$

$$E - E_F = -2.94\frac{(1.38\times10^{-23})(800)\text{ J}}{1.60\times10^{-19}\text{ J/eV}} = -0.203\text{ eV or } \boxed{E = 5.28\text{ eV}}$$

P43.34 The melting point of silver is 1 234 K. Its Fermi energy at 300 K is 5.48 eV. The approximate fraction of electrons excited is

$$\frac{k_BT}{E_F} = \frac{\left(1.38\times10^{-23}\text{ J/K}\right)\left(1\,234\text{ K}\right)}{\left(5.48\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)} \approx \boxed{2\%}$$

P43.35 $E_{av} = \dfrac{1}{n_e}\displaystyle\int_0^\infty EN(E)\,dE$

At $T = 0$, $\quad N(E) = 0$ for $E > E_F$

Since $f(E) = 1$ for $E < E_{EF}$ and $f(E) = 0$ for $E > E_F$,

we can take $\quad N(E) = CE^{1/2} = \dfrac{8\sqrt{2}\pi m_e^{3/2}}{h^3}E^{1/2}$

$$E_{av} = \frac{1}{n_e}\int_0^{E_F} CE^{3/2}dE = \frac{C}{n_e}\int_0^{E_F} E^{3/2}dE = \frac{2C}{5n_e}E_F^{5/2}$$

But from Equation 43.25, $\dfrac{C}{n_e} = \dfrac{3}{2}E_F^{-3/2}$, so that $\quad E_{av} = \left(\dfrac{2}{5}\right)\left(\dfrac{3}{2}E_F^{-3/2}\right)E_F^{5/2} = \boxed{\dfrac{3}{5}E_F}$

P43.36 $d = 1.00$ mm, so $\quad V = \left(1.00\times10^{-3}\text{ m}\right)^3 = 1.00\times10^{-9}\text{ m}^3$

The density of states is $\quad g(E) = CE^{1/2} = \dfrac{8\sqrt{2}\pi m^{3/2}}{h^3}E^{1/2}$

or $\quad g(E) = \dfrac{8\sqrt{2}\pi\left(9.11\times10^{-31}\text{ kg}\right)^{3/2}}{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^3}\sqrt{\left(4.00\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}$

$$g(E) = 8.50\times10^{46}\text{ m}^{-3}\cdot\text{J}^{-1} = 1.36\times10^{28}\text{ m}^{-3}\cdot\text{eV}^{-1}$$

So, the total number of electrons is

$$N = \left[g(E)\right](\Delta E)V = \left(1.36\times10^{28}\text{ m}^{-3}\cdot\text{eV}^{-1}\right)\left(0.025\,0\text{ eV}\right)\left(1.00\times10^{-9}\text{ m}^3\right)$$
$$= \boxed{3.40\times10^{17}\text{ electrons}}$$

***P43.37** (a) The density of states at energy E is $\quad g(E) = CE^{1/2}$

Hence, the required ratio is $\quad \dfrac{g(8.50\text{ eV})}{g(7.00\text{ eV})} = \dfrac{C(8.50)^{1/2}}{C(7.00)^{1/2}} = \boxed{1.10}$

(b) From Equation 43.22, the number of occupied states having energy E is

$$N(E) = \frac{CE^{1/2}}{e^{(E-E_F)/k_BT}+1}$$

Hence, the required ratio is $\quad \dfrac{N(8.50\text{ eV})}{N(7.00\text{ eV})} = \dfrac{(8.50)^{1/2}}{(7.00)^{1/2}}\left[\dfrac{e^{(7.00-7.00)/k_BT}+1}{e^{(8.50-7.00)/k_BT}+1}\right]$

At $T = 300$ K, $k_BT = 4.14\times10^{-21}$ J $= 0.025\,9$ eV,

$$\frac{N(8.50\text{ eV})}{N(7.00\text{ eV})} = \frac{(8.50)^{1/2}}{(7.00)^{1/2}}\left[\frac{2.00}{e^{(1.50)/0.025\,9}+1}\right]$$

And $\quad \dfrac{N(8.50\text{ eV})}{N(7.00\text{ eV})} = \boxed{1.47\times10^{-25}}$

This result is vastly smaller than that in part (a). We conclude that very few states well above the Fermi energy are occupied at room temperature.

P43.38 Consider first the wave function in *x*. At $x = 0$ and $x = L$, $\psi = 0$.

Therefore, $\sin k_x L = 0$ and $k_x L = \pi,\ 2\pi,\ 3\pi,\ \ldots$

Similarly, $\sin k_y L = 0$ and $k_y L = \pi,\ 2\pi,\ 3\pi,\ \ldots$

$\sin k_z L = 0$ and $k_z L = \pi,\ 2\pi,\ 3\pi,\ \ldots$

$$\psi = A\sin\left(\frac{n_x \pi x}{L}\right)\sin\left(\frac{n_y \pi y}{L}\right)\sin\left(\frac{n_z \pi z}{L}\right)$$

From $\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}=\frac{2m_e}{\hbar^2}(U-E)\psi,$ we have inside the box, where $U = 0$,

$$\left(-\frac{n_x^2\pi^2}{L^2}-\frac{n_y^2\pi^2}{L^2}-\frac{n_z^2\pi^2}{L^2}\right)\psi = \frac{2m_e}{\hbar^2}(-E)\psi \qquad \boxed{E=\frac{\hbar^2\pi^2}{2m_eL^2}\left(n_x^2+n_y^2+n_z^2\right) \quad n_x,\ n_y,\ n_z = 1,\ 2,\ 3,\ \ldots}$$

Outside the box we require $\psi = 0$.

The minimum energy state inside the box is $n_x = n_y = n_z = 1$, with $E = \frac{3\hbar^2\pi^2}{2m_eL^2}$

Section 43.6 **Electrical Conduction in Metals, Insulators, and Semiconductors**

P43.39 (a) $E_g = 1.14$ eV for Si

$$hf = 1.14\text{ eV} = (1.14\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right) = 1.82\times10^{-19}\text{ J} \quad \text{so} \quad f \geq \boxed{2.75\times10^{14}\text{ Hz}}$$

(b) $$c = \lambda f;\ \lambda = \frac{c}{f} = \frac{3.00\times10^{8}\text{ m/s}}{2.75\times10^{14}\text{ Hz}} = 1.09\times10^{-6}\text{ m} = \boxed{1.09\ \mu\text{m}}$$ (in the infrared region)

P43.40 Photons of energy greater than 2.42 eV will be absorbed. This means wavelength shorter than

$$\lambda = \frac{hc}{E} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{2.42\times1.60\times10^{-19}\text{ J}} = 514\text{ nm}$$

All the hydrogen Balmer lines except for the red line at 656 nm will be absorbed.

***P43.41** If $\lambda \leq 1.00\times10^{-6}$ m, then photons of sunlight have energy

$$E \geq \frac{hc}{\lambda_{\text{max}}} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{1.00\times10^{-6}\text{ m}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 1.24\text{ eV}$$

Thus, the energy gap for the collector material should be $\boxed{E_g \leq 1.24\text{ eV}}$. Since Si has an energy gap $E_g \approx 1.14$ eV, it can absorb nearly all of the photons in sunlight. Therefore, $\boxed{\text{Si is an appropriate material}}$ for a solar collector.

P43.42 $$E_g = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{650\times10^{-9}\text{ m}}\text{ J} \approx \boxed{1.91\text{ eV}}$$

P43.43 If the photon energy is 5.5 eV or higher, the diamond window will absorb. Here,

$$(hf)_{\max} = \frac{hc}{\lambda_{\min}} = 5.5 \text{ eV}: \qquad \lambda_{\min} = \frac{hc}{5.5 \text{ eV}} = \frac{(6.626\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^{8} \text{ m/s})}{(5.5 \text{ eV})(1.60\times10^{-19} \text{ J/eV})}$$

$$\lambda_{\min} = 2.26\times10^{-7} \text{ m} = \boxed{226 \text{ nm}}$$

P43.44 In the Bohr model we replace k_e by $\frac{k_e}{\kappa}$ and m_e by m^*. Then the radius of the first Bohr orbit,

$a_0 = \frac{\hbar^2}{m_e k_e e^2}$ in hydrogen, changes to

$$a' = \frac{\hbar^2 \kappa}{m^* k_e e^2} = \left(\frac{m_e}{m^*}\right)\kappa\frac{\hbar^2}{m_e k_e e^2} = \left(\frac{m_e}{m^*}\right)\kappa a_0 = \left(\frac{m_e}{0.220 m_e}\right)11.7(0.052\,9 \text{ nm}) = \boxed{2.81 \text{ nm}}$$

The energy levels are in hydrogen $E_n = -\frac{k_e e^2}{2a_0}\frac{1}{n^2}$ and here

$$E_n' = -\frac{k_e e^2}{\kappa 2a'}\frac{1}{n^2} = -\frac{k_e e^2}{\kappa 2(m_e/m^*)\kappa a_0} = -\left(\frac{m^*}{m_e}\right)\frac{E_n}{\kappa^2}$$

For $n = 1$, $E_1' = -0.220\frac{13.6 \text{ eV}}{11.7^2} = \boxed{-0.021\,9 \text{ eV}}$.

Section 43.7 **Semiconductor Devices**

P43.45 $I = I_0\left(e^{e(\Delta V)/k_B T} - 1\right)$ Thus, $e^{e(\Delta V)/k_B T} = 1 + \frac{I}{I_0}$

and $\Delta V = \frac{k_B T}{e}\ln\left(1 + \frac{I}{I_0}\right)$

At $T = 300$ K, $\Delta V = \frac{(1.38\times10^{-23} \text{ J/K})(300 \text{ K})}{1.60\times10^{-19} \text{ C}}\ln\left(1+\frac{I}{I_0}\right) = (25.9 \text{ mV})\ln\left(1+\frac{I}{I_0}\right)$

(a) If $I = 9.00 I_0$, $\Delta V = (25.9 \text{ mV})\ln(10.0) = \boxed{59.5 \text{ mV}}$

(b) If $I = -0.900 I_0$, $\Delta V = (25.9 \text{ mV})\ln(0.100) = \boxed{-59.5 \text{ mV}}$

The basic idea behind a semiconductor device is that a large current or charge can be controlled by a small control voltage.

P43.46 The voltage across the diode is about 0.6 V. The voltage drop across the resistor is $(0.025 \text{ A})(150\ \Omega) = 3.75$ V. Thus, $\mathcal{E} - 0.6 \text{ V} - 3.8 \text{ V} = 0$ and $\mathcal{E} = \boxed{4.4 \text{ V}}$.

P43.47 First, we evaluate I_0 in $I = I_0\left(e^{e(\Delta V)/k_B T} - 1\right)$, given that $I = 200$ mA when $\Delta V = 100$ mV and $T = 300$ K.

$$\frac{e(\Delta V)}{k_B T} = \frac{(1.60\times10^{-19} \text{ C})(0.100 \text{ V})}{(1.38\times10^{-23} \text{ J/K})(300 \text{ K})} = 3.86 \text{ so } I_0 = \frac{I}{e^{e(\Delta V)/k_B T} - 1} = \frac{200 \text{ mA}}{e^{3.86} - 1} = 4.28 \text{ mA}$$

If $\Delta V = -100$ mV, $\frac{e(\Delta V)}{k_B T} = -3.86$; and the current will be

$$I = I_0\left(e^{e(\Delta V)/k_B T} - 1\right) = (4.28 \text{ mA})\left(e^{-3.86} - 1\right) = \boxed{-4.19 \text{ mA}}$$

P43.48 (a) The currents to be plotted are

$$I_D = \left(10^{-6}\ \text{A}\right)\left(e^{\Delta V/0.025\ \text{V}} - 1\right),$$

$$I_W = \frac{2.42\ \text{V} - \Delta V}{745\ \Omega}$$

The two graphs intersect at $\Delta V = 0.200$ V. The currents are then

$$I_D = \left(10^{-6}\ \text{A}\right)\left(e^{0.200\ \text{V}/0.025\ \text{V}} - 1\right) = 2.98\ \text{mA}$$

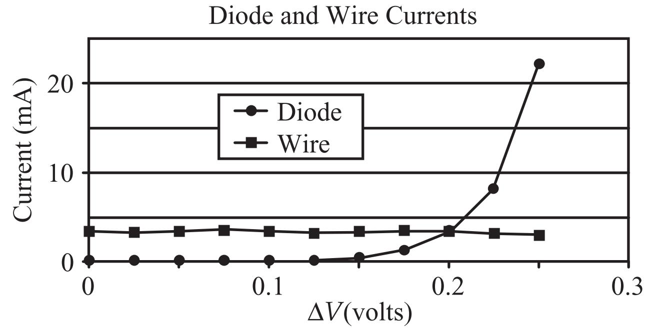


FIG. P43.48

$$I_W = \frac{2.42\ \text{V} - 0.200\ \text{V}}{745\ \Omega} = 2.98\ \text{mA}.$$ They agree to three digits.

$$\therefore I_D = I_W = \boxed{2.98\ \text{mA}}$$

(b) $$\frac{\Delta V}{I_D} = \frac{0.200\ \text{V}}{2.98\times10^{-3}\ \text{A}} = \boxed{67.1\ \Omega}$$

(c) $$\frac{d(\Delta V)}{dI_D} = \left[\frac{dI_D}{d(\Delta V)}\right]^{-1} = \left[\frac{10^{-6}\ \text{A}}{0.025\ \text{V}}e^{0.200\ \text{V}/0.025\ \text{V}}\right]^{-1} = \boxed{8.39\ \Omega}$$

Section 43.8 **Superconductivity**

P43.49 By Faraday's law (from Chapter 32) $$\frac{\Delta\Phi_B}{\Delta t} = L\frac{\Delta I}{\Delta t} = A\frac{\Delta B}{\Delta t}$$

Thus, $$\Delta I = \frac{A(\Delta B)}{L} = \frac{\pi(0.010\,0\ \text{m})^2(0.020\,0\ \text{T})}{3.10\times10^{-8}\ \text{H}} = \boxed{203\ \text{A}}$$

The direction of the induced current is such as to maintain the B-field through the ring.

***P43.50** (a) In the definition of resistance $\Delta V = IR$, if R is zero then $\Delta V = 0$ for any value of current.

(b) The graph shows a direct proportionality with resistance given by the reciprocal of the slope:

$$\text{Slope} = \frac{1}{R} = \frac{\Delta I}{\Delta V} = \frac{(155 - 57.8)\ \text{mA}}{(3.61 - 1.356)\ \text{mV}} = 43.1\ \Omega^{-1}$$

$$R = \boxed{0.023\,2\ \Omega}$$

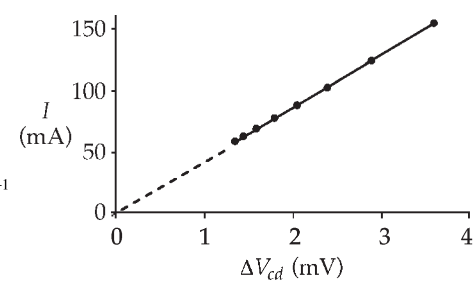


FIG. P43.50

(c) Expulsion of magnetic flux and therefore fewer current-carrying paths could explain the decrease in current.

P43.51 (a) $\boxed{\text{See the figure at right.}}$

(b) For a surface current around the outside of the cylinder as shown,

$$B = \frac{N\mu_0 I}{\ell} \quad \text{or} \quad NI = \frac{B\ell}{\mu_0} = \frac{(0.540\ \text{T})(2.50\times10^{-2}\ \text{m})}{(4\pi\times10^{-7})\ \text{T}\cdot\text{m/A}} = \boxed{10.7\ \text{kA}}$$

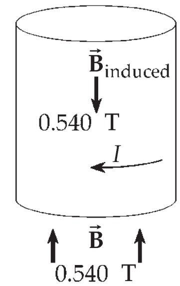


FIG. P43.51

Additional Problems

P43.52 For the N_2 molecule, $k = 2\,297\ \text{N/m}$, $m = 2.32\times10^{-26}\ \text{kg}$, $r = 1.20\times10^{-10}\ \text{m}$, $\mu = \frac{m}{2}$

$$\omega = \sqrt{\frac{k}{\mu}} = 4.45\times10^{14}\ \text{rad/s}, \quad I = \mu r^2 = (1.16\times10^{-26}\ \text{kg})(1.20\times10^{-10}\ \text{m})^2 = 1.67\times10^{-46}\ \text{kg}\cdot\text{m}^2$$

For a rotational state sufficient to allow a transition to the first exited vibrational state,

$$\frac{\hbar^2}{2I}J(J+1) = \hbar\omega \text{ so } J(J+1) = \frac{2I\omega}{\hbar} = \frac{2(1.67\times10^{-46})(4.45\times10^{14})}{1.055\times10^{-34}} = 1\,410.$$

Thus $\boxed{J = 37}$.

***P43.53** (a) Since the interatomic potential is the same for both molecules, the spring constant is the same.

Then $f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}}$ where $\mu_{12} = \frac{(12\ \text{u})(16\ \text{u})}{12\ \text{u} + 16\ \text{u}} = 6.86\ \text{u}$ and $\mu_{14} = \frac{(14\ \text{u})(16\ \text{u})}{14\ \text{u} + 16\ \text{u}} = 7.47\ \text{u}$.

Therefore,

$$f_{14} = \frac{1}{2\pi}\sqrt{\frac{k}{\mu_{14}}} = \frac{1}{2\pi}\sqrt{\frac{k}{\mu_{12}}\left(\frac{\mu_{12}}{\mu_{14}}\right)} = f_{12}\sqrt{\frac{\mu_{12}}{\mu_{14}}} = (6.42\times10^{13}\ \text{Hz})\sqrt{\frac{6.86\ \text{u}}{7.47\ \text{u}}} = \boxed{6.15\times10^{13}\ \text{Hz}}$$

(b) The equilibrium distance is the same for both molecules.

$$I_{14} = \mu_{14}r^2 = \left(\frac{\mu_{14}}{\mu_{12}}\right)\mu_{12}r^2 = \left(\frac{\mu_{14}}{\mu_{12}}\right)I_{12}$$

$$I_{14} = \left(\frac{7.47\ \text{u}}{6.86\ \text{u}}\right)(1.46\times10^{-46}\ \text{kg}\cdot\text{m}^2) = \boxed{1.59\times10^{-46}\ \text{kg}\cdot\text{m}^2}$$

(c) The molecule can move to the $(v = 1,\ J = 9)$ state or to the $(v = 1, J = 11)$ state. The energy it can absorb is either

$$\Delta E = \frac{hc}{\lambda} = \left[\left(1+\frac{1}{2}\right)hf_{14} + 9(9+1)\frac{\hbar^2}{2I_{14}}\right] - \left[\left(0-\frac{1}{2}\right)hf_{14} + 10(10+1)\frac{\hbar^2}{2I_{14}}\right]$$

or

$$\Delta E = \frac{hc}{\lambda} = \left[\left(1+\frac{1}{2}\right)hf_{14} + 11(11+1)\frac{\hbar^2}{2I_{14}}\right] - \left[\left(0+\frac{1}{2}\right)hf_{14} + 10(10+1)\frac{\hbar^2}{2I_{14}}\right]$$

The wavelengths it can absorb are then

$$\lambda = \frac{c}{f_{14} - 10\hbar/(2\pi I_{14})} \quad \text{or} \quad \lambda = \frac{c}{f_{14} + 11\hbar/(2\pi I_{14})}$$

These are:

$$\lambda = \frac{3.00\times10^8\ \text{m/s}}{6.15\times10^{13}\ \text{Hz} - [10(1.055\times10^{-34}\ \text{J}\cdot\text{s})]/[2\pi(1.59\times10^{-46}\ \text{kg}\cdot\text{m}^2)]} = \boxed{4.96\ \mu\text{m}}$$

and

$$\lambda = \frac{3.00\times10^8\ \text{m/s}}{6.15\times10^{13}\ \text{Hz} + [11(1.055\times10^{-34}\ \text{J}\cdot\text{s})]/[2\pi(1.59\times10^{-46}\ \text{kg}\cdot\text{m}^2)]} = \boxed{4.79\ \mu\text{m}}$$

P43.54 With 4 van der Waals bonds per atom pair or 2 electrons per atom, the total energy of the solid is

$$E = 2\left(1.74\times10^{-23}\ \text{J/atom}\right)\left(\frac{6.02\times10^{23}\ \text{atoms}}{4.00\ \text{g}}\right) = \boxed{5.23\ \text{J/g}}$$

P43.55 $\Delta E_{\max} = 4.5\ \text{eV} = \left(v+\frac{1}{2}\right)\hbar\omega$ so $\frac{(4.5\ \text{eV})\left(1.6\times10^{-19}\ \text{J/eV}\right)}{\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(8.28\times10^{14}\ \text{s}^{-1}\right)} \geq \left(v+\frac{1}{2}\right)$

$8.25 > 7.5$ $\boxed{v = 7}$

P43.56 Suppose it is a harmonic-oscillator potential well. Then, $\frac{1}{2}hf + 4.48\ \text{eV} = \frac{3}{2}hf + 3.96\ \text{eV}$ is the depth of the well below the dissociation point. We see $hf = 0.520\ \text{eV}$, so the depth of the well is $\frac{1}{2}hf + 4.48\ \text{eV} = \frac{1}{2}(0.520\ \text{eV}) + 4.48\ \text{eV} = \boxed{4.74\ \text{eV}}$.

P43.57 The total potential energy is given by Equation 43.17: $U_{\text{total}} = -\alpha\frac{k_e e^2}{r} + \frac{B}{r^m}$.

The total potential energy has its minimum value U_0 at the equilibrium spacing, $r = r_0$. At this point, $\left.\frac{dU}{dr}\right|_{r=r_0} = 0$,

or $$\left.\frac{dU}{dr}\right|_{r=r_0} = \frac{d}{dr}\left(-\alpha\frac{k_e e^2}{r} + \frac{B}{r^m}\right)\Bigg|_{r=r_0} = \alpha\frac{k_e e^2}{r_0^2} - \frac{mB}{r_0^{m+1}} = 0$$

Thus, $B = \alpha\frac{k_e e^2}{m} r_0^{m-1}$

Substituting this value of B into U_{total}, $U_0 = -\alpha\frac{k_e e^2}{r_0} + \alpha\frac{k_e e^2}{m} r_0^{m-1}\left(\frac{1}{r_0^m}\right) = \boxed{-\alpha\frac{k_e e^2}{r_0}\left(1-\frac{1}{m}\right)}$

***P43.58** (a) The total potential energy $-\alpha\frac{k_e e^2}{r} + \frac{B}{r^m}$ has its minimum value at the equilibrium spacing, $r = r_0$. At this point, $F = -\left.\frac{dU}{dr}\right|_{r=r_0} = 0$, or

$$F = -\frac{d}{dr}\left(-\alpha\frac{k_e e^2}{r} + \frac{B}{r^m}\right)\Bigg|_{r=r_0} = -\alpha\frac{k_e e^2}{r_0^2} + \frac{mB}{r_0^{m+1}} = 0$$

Thus, $B = \alpha\frac{k_e e^2}{m} r_0^{m-1}$.

Substituting this value of B into F, $F = -\alpha\frac{k_e e^2}{r^2} + \frac{m}{r^{m+1}}\alpha\frac{k_e e^2}{m} r_0^{m-1} = \boxed{-\alpha\frac{k_e e^2}{r^2}\left[1-\left(\frac{r_0}{r}\right)^{m-1}\right]}$.

(b) Let $r = r_0 + x$ so $r_0 = r - x$ Then assuming x is small we have

$$F = -\alpha\frac{k_e e^2}{r^2}\left[1-\left(\frac{r-x}{r}\right)^{m-1}\right] = -\alpha\frac{k_e e^2}{r^2}\left[1-\left(1-\frac{x}{r}\right)^{m-1}\right] \approx -\alpha\frac{k_e e^2}{r^2}\left[1-1+(m-1)\frac{x}{r}\right]$$

$$\approx -\alpha\frac{k_e e^2}{r_0^3}(m-1)x$$

This is of the form of Hooke's law with spring constant $K = k_e\alpha e^2(m-1)/r_0^3$.

(c) Section 38.5 on electron diffraction gives the interatomic spacing in NaCl as (0.562 737 nm)/2. Other problems in this chapter give the same information, or we could calculate it from the statement in the chapter text that the ionic cohesive energy for this crystal is –7.84 eV.
The stiffness constant is then

$$K = \alpha\frac{k_e e^2}{r_0^3}(m-1) = 1.7476\frac{8.99\times10^9\ \text{N}\cdot\text{m}^2(1.6\times10^{-19}\ \text{C})^2(8-1)}{\text{C}^2(2.81\times10^{-10}\ \text{m})^3} = 127\ \text{N/m}$$

The vibration frequency of a sodium ion within the crystal is

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{127\ \text{N/m}}{23.0\times1.66\times10^{-27}\ \text{kg}}} = \boxed{9.18\ \text{THz}}$$

P43.59 (a) For equilibrium, $\frac{dU}{dx}=0$: $\frac{d}{dx}\left(Ax^{-3}-Bx^{-1}\right)=-3Ax^{-4}+Bx^{-2}=0$

$x\to\infty$ describes one equilibrium position, but the stable equilibrium position is at $3Ax_0^{-2}=B$.

$$x_0=\sqrt{\frac{3A}{B}}=\sqrt{\frac{3\left(0.150\text{ eV}\cdot\text{nm}^3\right)}{3.68\text{ eV}\cdot\text{nm}}}=\boxed{0.350\text{ nm}}$$

(b) The depth of the well is given by $U_0=U\big|_{x=x_0}=\frac{A}{x_0^3}-\frac{B}{x_0}=\frac{AB^{3/2}}{3^{3/2}A^{3/2}}-\frac{BB^{1/2}}{3^{1/2}A^{1/2}}$

$$U_0=U\big|_{x=x_0}=-\frac{2B^{3/2}}{3^{3/2}A^{1/2}}=-\frac{2\left(3.68\text{ eV}\cdot\text{nm}\right)^{3/2}}{3^{3/2}\left(0.150\text{ eV}\cdot\text{nm}^3\right)^{1/2}}=\boxed{-7.02\text{ eV}}$$

(c) $F_x=-\frac{dU}{dx}=3Ax^{-4}-Bx^{-2}$

To find the maximum force, we determine finite x_m such that $\frac{dF}{dx}\bigg|_{x=x_m}=0$.

Thus, $\left[-12Ax^{-5}+2Bx^{-3}\right]_{x=x_m}=0$ so that $x_m=\left(\frac{6A}{B}\right)^{1/2}$

Then $F_{\max}=3A\left(\frac{B}{6A}\right)^2-B\left(\frac{B}{6A}\right)=-\frac{B^2}{12A}=-\frac{(3.68\text{ eV}\cdot\text{nm})^2}{12\left(0.150\text{ eV}\cdot\text{nm}^3\right)}$

or $F_{\max}=-7.52\text{ eV/nm}\left(\frac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right)=-1.20\times10^{-9}\text{ N}=\boxed{-1.20\text{ nN}}$

P43.60 (a) For equilibrium, $\frac{dU}{dx}=0$: $\frac{d}{dx}\left(Ax^{-3}-Bx^{-1}\right)=-3Ax^{-4}+Bx^{-2}=0$

$x\to\infty$ describes one equilibrium position, but the stable equilibrium position is at

$3Ax_0^{-2}=B$ or $\boxed{x_0=\sqrt{\frac{3A}{B}}}$

(b) The depth of the well is given by $U_0=U\big|_{x=x_0}=\frac{A}{x_0^3}-\frac{B}{x_0}=\frac{AB^{3/2}}{3^{3/2}A^{3/2}}-\frac{BB^{1/2}}{3^{1/2}A^{1/2}}=\boxed{-2\sqrt{\frac{B^3}{27A}}}$.

(c) $F_x=-\frac{dU}{dx}=3Ax^{-4}-Bx^{-2}$

To find the maximum force, we determine finite x_m such that

$\frac{dF_x}{dx}\bigg|_{x=x_m}=\left[-12Ax^{-5}+2Bx^{-3}\right]_{x=x_0}=0$ then $F_{\max}=3A\left(\frac{B}{6A}\right)^2-B\left(\frac{B}{6A}\right)=\boxed{-\frac{B^2}{12A}}$

P43.61 (a) At equilibrium separation, $r = r_e$, $\left.\frac{dU}{dr}\right|_{r=r_e} = -2aB\left[e^{-a(r_e-r_0)}-1\right]e^{-a(r_e-r_0)} = 0$

We have neutral equilibrium as $r_e \to \infty$ and stable equilibrium at $e^{-a(r_e-r_0)} = 1$,

or $r_e = \boxed{r_0}$

(b) At $r = r_0$, $U = 0$. As $r \to \infty$, $U \to B$. The depth of the well is $\boxed{B}$.

(c) We expand the potential in a Taylor series about the equilibrium point:

$$U(r) \approx U(r_0) + \left.\frac{dU}{dr}\right|_{r=r_0}(r-r_0) + \frac{1}{2}\left.\frac{d^2U}{dr^2}\right|_{r=r_0}(r-r_0)^2$$

$$U(r) \approx 0 + 0 + \frac{1}{2}(-2Ba)\left[-ae^{-2(r-r_0)} - ae^{-(r-r_0)}\left(e^{-2(r-r_0)}-1\right)\right]_{r-r_0}(r-r_0)^2 \approx Ba^2(r-r_0)^2$$

This is of the form $\frac{1}{2}kx^2 = \frac{1}{2}k(r-r_0)^2$

for a simple harmonic oscillator with $k = 2Ba^2$

Then the molecule vibrates with frequency $f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} = \frac{a}{2\pi}\sqrt{\frac{2B}{\mu}} = \boxed{\frac{a}{\pi}\sqrt{\frac{B}{2\mu}}}$

(d) The zero-point energy is $\frac{1}{2}\hbar\omega = \frac{1}{2}hf = \frac{ha}{\pi}\sqrt{\frac{B}{8\mu}}$

Therefore, to dissociate the molecule in its ground state requires energy $\boxed{B - \frac{ha}{\pi}\sqrt{\frac{B}{8\mu}}}$.

P43.62

	$T=0$		$T=0.1T_F$		$T=0.2T_F$		$T=0.5T_F$	
$\frac{E}{E_F}$	$e^{[(E/E_F)-1](T_F/T)}$	$f(E)$	$e^{[(E/E_F)-1](T_F/T)}$	$f(E)$	$e^{[(E/E_F)-1](T_F/T)}$	$f(E)$	$e^{[(E/E_F)-1](T_F/T)}$	$f(E)$
0	$e^{-\infty}$	1.00	$e^{-10.0}$	1.000	$e^{-5.00}$	0.993	$e^{-2.00}$	0.881
0.500	$e^{-\infty}$	1.00	$e^{-5.00}$	0.993	$e^{-2.50}$	0.924	$e^{-1.00}$	0.731
0.600	$e^{-\infty}$	1.00	$e^{-4.00}$	0.982	$e^{-2.00}$	0.881	$e^{-0.800}$	0.690
0.700	$e^{-\infty}$	1.00	$e^{-3.00}$	0.953	$e^{-1.50}$	0.818	$e^{-0.600}$	0.646
0.800	$e^{-\infty}$	1.00	$e^{-2.00}$	0.881	$e^{-1.00}$	0.731	$e^{-0.400}$	0.599
0.900	$e^{-\infty}$	1.00	$e^{-1.00}$	0.731	$e^{-0.500}$	0.622	$e^{-0.200}$	0.550
1.00	e^{0}	0.500	e^{0}	0.500	e^{0}	0.500	e^{0}	0.500
1.10	$e^{+\infty}$	0.00	$e^{1.00}$	0.269	$e^{0.500}$	0.378	$e^{0.200}$	0.450
1.20	$e^{+\infty}$	0.00	$e^{2.00}$	0.119	$e^{1.00}$	0.269	$e^{0.400}$	0.401
1.30	$e^{+\infty}$	0.00	$e^{3.00}$	0.047 4	$e^{1.50}$	0.182	$e^{0.600}$	0.354
1.40	$e^{+\infty}$	0.00	$e^{4.00}$	0.018 0	$e^{2.00}$	0.119	$e^{0.800}$	0.310
1.50	$e^{+\infty}$	0.00	$e^{5.00}$	0.006 69	$e^{2.50}$	0.075 9	$e^{1.00}$	0.269

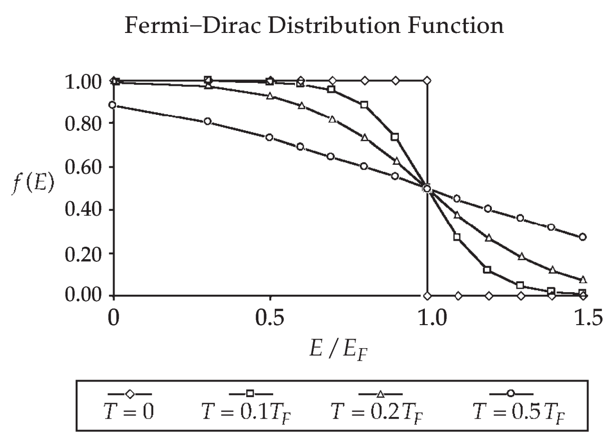


FIG. P43.62

ANSWERS TO EVEN PROBLEMS

P43.2 4.3 eV

P43.4 (a) 1.28 eV (b) $\sigma = 0.272$ nm, $\in = 4.65$ eV (c) 6.55 nN (d) 576 N/m

P43.6 (a) 0.0148 eV (b) 83.8 μm

P43.8 12.2×10^{-27} kg; 12.4×10^{-27} kg; They agree, because the small apparent difference can be attributed to uncertainty in the data.

P43.10 $1.46\times10^{-46}\ \text{kg}\cdot\text{m}^2$

P43.12 (a) $1.81\times10^{-45}\ \text{kg}\cdot\text{m}^2$ (b) 1.62 cm

P43.14 (a) 12.1 pm (b) 9.23 pm (c) HI is more loosely bound since it has the smaller k value.

P43.16 (a) 0, 364 μeV, 1.09 meV (b) 98.2 meV, 295 meV, 491 meV

P43.18 (a) 472 μm (b) 473 μm (c) 0.715 μm

P43.20 (a) $4.60\times10^{-48}\ \text{kg}\cdot\text{m}^2$ (b) 1.32×10^{14} Hz (c) 0.074 1 nm

P43.22 6.25×10^9

P43.24 (a) $\sim10^{17}$ (b) $\sim10^5\ \text{m}^3$

P43.26 (a) 0.444 nm, 0.628 nm, 0.769 nm

P43.28 (a) The Fermi energy is proportional to the spatial concentration of free electrons to the two-thirds power. (c) 6.06; copper by 3.33 times; it agrees with the equation because $6.06^{2/3} = 3.33$.

P43.30 0.938

P43.32 (a) 2.54×10^{28} electron/m^3 (b) 3.15 eV

P43.34 2%

P43.36 3.40×10^{17} electrons

P43.38 See the solution.

P43.40 All of the Balmer lines are absorbed, except for the red line at 656 nm, which is transmitted.

P43.42 1.91 eV

P43.44 –0.021 9 eV, 2.81 nm

P43.46 4.4 V

P43.48 (a) At $\Delta V = 0.200$ V, $I_D = I_W = 2.98$ mA, agreeing to three digits. (b) 67.1 Ω (c) 8.39 Ω

P43.50 (a) In the definition of resistance $\Delta V = IR$, if R is zero then $\Delta V = 0$ for any value of current. (b) The graph shows direct proportionality with resistance 0.023 2 Ω. (c) Expulsion of magnetic flux and therefore fewer current-carrying paths could explain the decrease in current.

P43.52 37

P43.54 5.23 J/g

P43.56 4.74 eV

P43.58 (c) 9.18 THz

P43.60 (a) $x_0 = \sqrt{\dfrac{3A}{B}}$ (b) $-2\sqrt{\dfrac{B^3}{27A}}$ (c) $-\dfrac{B^2}{12A}$

P43.62 See the solution.

44

Nuclear Structure

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q44.1 Because of electrostatic repulsion between the positively-charged nucleus and the $+2e$ alpha particle. To drive the α-particle into the nucleus would require extremely high kinetic energy.

***Q44.2** (a) X has a mass number less by 2 than the others. The ranking is W = Y = Z > X.

(b) Y has a greater atomic number, because a neutron in the parent nucleus has turned into a proton. X has an atomic number less by two than W, so the ranking is Y > W = Z > X.

(c) Y has one fewer neutron compared to the parent nucleus W, and X has two fewer neutrons than W. The ranking is W = Z > Y > X.

Q44.3 The nuclear force favors the formation of neutron-proton pairs, so a stable nucleus cannot be too far away from having equal numbers of protons and neutrons. This effect sets the upper boundary of the zone of stability on the neutron-proton diagram. All of the protons repel one another electrically, so a stable nucleus cannot have too many protons. This effect sets the lower boundary of the zone of stability.

Q44.4 Nuclei with more nucleons than bismuth-209 are unstable because the electrical repulsion forces among all of the protons is stronger than the nuclear attractive force between nucleons.

Q44.5 Nucleus *Y* will be more unstable. The nucleus with the higher binding energy requires more energy to be disassembled into its constituent parts.

Q44.6 Extra neutrons are required to overcome the increasing electrostatic repulsion of the protons. The neutrons participate in the net attractive effect of the nuclear force, but feel no Coulomb repulsion.

***Q44.7** (i) Answer (a). The liquid drop model gives a simpler account of a nuclear fission reaction, including the energy released and the probable fission product nuclei.

(ii) Answer (b). The shell model predicts magnetic moments by necessarily describing the spin and orbital angular momentum states of the nucleons.

(iii) Answer (b). Again, the shell model wins when it comes to predicting the spectrum of an excited nucleus, as the quantum model allows only quantized energy states, and thus only specific transitions.

Q44.8 The statement is false. Both patterns show monotonic decrease over time, but with very different shapes. For radioactive decay, maximum activity occurs at time zero. Cohorts of people now living will be dying most rapidly perhaps forty years from now. Everyone now living will be dead within less than two centuries, while the mathematical model of radioactive decay tails off exponentially forever. A radioactive nucleus never gets old. It has constant probability of decay however long it has existed.

***Q44.9** (i) Answer (b). Since the samples are of the same radioactive isotope, their half-lives are the same.

(ii) Answer (b). When prepared, sample G has twice the activity (number of radioactive decays per second) of sample H. After 5 half-lives, the activity of sample G is decreased by a factor of 2^5, and after 5 half-lives the activity of sample H is decreased by a factor of 2^5. So after 5 half-lives, the ratio of activities is still 2:1.

Q44.10 After one half-life, one half the radioactive atoms have decayed. After the second half-life, one half of the remaining atoms have decayed. Therefore $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$ of the original radioactive atoms have decayed after two half-lives.

Q44.11 Long-lived progenitors at the top of each of the three natural radioactive series are the sources of our radium. As an example, thorium-232 with a half-life of 14 Gyr produces radium-228 and radium-224 at stages in its series of decays, shown in Figure 44.17.

***Q44.12** Answer (d). A free neutron decays into a proton plus an electron and an antineutrino. This implies that a proton is more stable than a neutron, and in particular the proton has lower mass. Therefore a proton cannot decay into a neutron plus a positron and a neutrino. This reaction satisfies every conservation law except for energy conservation.

***Q44.13** The alpha particle and the daughter nucleus carry equal amounts of momentum in opposite directions. Since kinetic energy can be written as $\frac{p^2}{2m}$, the small-mass alpha particle has much more of the decay energy than the recoiling nucleus.

Q44.14 Yes. The daughter nucleus can be left in its ground state or sometimes in one of a set of excited states. If the energy carried by the alpha particle is mysteriously low, the daughter nucleus can quickly emit the missing energy in a gamma ray.

***Q44.15** Answer (d). The reaction energy is the amount of energy released as a result of a nuclear reaction. Equation 44.28 in the text implies that the reaction energy is (initial mass − final mass) c^2. The Q-value is taken as positive for an exothermic reaction.

***Q44.16** The samples would have started with more carbon-14 than we first thought. We would increase our estimates of their ages.

Q44.17 I_z may have 6 values for $I=\frac{5}{2}$, namely $\frac{5}{2}, \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$, and $-\frac{5}{2}$. Seven I_z values are possible for $I=3$.

***Q44.18** Answer (b). The frequency increases linearly with the magnetic field strength.

Q44.19 The decay of a radioactive nucleus at one particular moment instead of at another instant cannot be predicted and has no cause. Natural events are not just like a perfect clockworks. In history, the idea of a determinate mechanical Universe arose temporarily from an unwarranted wild extrapolation of Isaac Newton's account of planetary motion. Before Newton's time [really you can blame Pierre Simon de Laplace] and again now, no one thought of natural events as just like a perfect row of falling dominos. We can and do use the word "cause" more loosely to describe antecedent enabling events. One gear turning another is intelligible. So is the process of a hot dog getting toasted over a campfire, even though random molecular motion is at the essence of that process. In summary, we say that the future is not determinate. All natural events have causes in the ordinary sense of the word, but not necessarily in the contrived sense of a cause operating infallibly and predictably in a way that can be calculated. We have better reason now than ever before to think of the Universe as intelligible. First describing natural events, and second determining their causes form the basis of science, including physics but also scientific medicine and scientific bread-baking. The evidence alone of the past hundred years of discoveries in physics, finding causes of natural events from the photoelectric effect to x-rays and jets emitted by black holes, suggests that human intelligence is a good tool for figuring out how things go. Even without organized science, humans have always been searching for the causes of natural events, with explanations ranging from "the will of the gods" to Schrödinger's equation. We depend on the principle that things are intelligible as we make significant strides towards understanding the Universe. To hope that our search is not futile is the best part of human nature.

SOLUTIONS TO PROBLEMS

Section 44.1 **Some Properties of Nuclei**

P44.1 An iron nucleus (in hemoglobin) has a few more neutrons than protons, but in a typical water molecule there are eight neutrons and ten protons.
So protons and neutrons are nearly equally numerous in your body, each contributing mass (say) 35 kg:

$$35\text{ kg}\left(\frac{1\text{ nucleon}}{1.67\times10^{-27}\text{ kg}}\right)\boxed{\sim10^{28}\text{ protons}}$$

and $\boxed{\sim10^{28}\text{ neutrons}}$

The electron number is precisely equal to the proton number, $\boxed{\sim10^{28}\text{ electrons}}$.

P44.2 $\frac{1}{2}mv^2 = q\Delta V$ and $\frac{mv^2}{r} = qvB$

$2m\Delta V = qr^2B^2$: $r = \sqrt{\frac{2m\Delta V}{qB^2}} = \sqrt{\frac{2(1\ 000\text{ V})}{(1.60\times10^{-19}\text{ C})(0.200\text{ T})^2}}\sqrt{m}$

$r = \left(5.59\times10^{11}\text{ m}/\sqrt{\text{kg}}\right)\sqrt{m}$

(a) For ^{12}C, $m = 12$ u and $r = \left(5.59\times10^{11}\text{ m}/\sqrt{\text{kg}}\right)\sqrt{12\left(1.66\times10^{-27}\text{ kg}\right)}$

$r = 0.078\,9\text{ m} = \boxed{7.89\text{ cm}}$

For ^{13}C: $r = \left(5.59\times10^{11}\text{ m}/\sqrt{\text{kg}}\right)\sqrt{13\left(1.66\times10^{-27}\text{ kg}\right)}$

$r = 0.082\,1\text{ m} = \boxed{8.21\text{ cm}}$

(b) With $r_1 = \sqrt{\frac{2m_1\Delta V}{qB^2}}$ and $r_2 = \sqrt{\frac{2m_2\Delta V}{qB^2}}$

the ratio gives $\boxed{\frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}}}$

$$\frac{r_1}{r_2} = \frac{7.89\text{ cm}}{8.21\text{ cm}} = 0.961$$

and $\sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{12\text{ u}}{13\text{ u}}} = 0.961$

so they do agree.

***P44.3** (a) Let V represent the volume of the tank. The number of moles present is

$$n = \frac{PV}{RT} = \frac{1.013\times10^5\text{ N }V\text{ mol}\cdot\text{K}}{8.314\text{ N}\cdot\text{m m}^2\ \ 273\text{ K}} = 44.6\ V\text{ mol/m}^3$$

Then the number of molecules is $6.023\times10^{23}\times44.6\ V/\text{m}^3$.

The volume of one molecule is $2\frac{4}{3}\pi r^3 = \frac{8\pi}{3}\left(\frac{10^{-10}\text{ m}}{2}\right)^3 = 1.047\times10^{-30}\text{ m}^3$.

The volume of all the molecules is $2.69\times10^{25}\ V\,(1.047\times10^{-30}) = 2.81\times10^{-5}\ V$.
So the fraction of the volume occupied by the hydrogen molecules is $\boxed{2.81\times10^{-5}}$.
An atom is precisely one half of a molecule.

(b) $$\frac{\textit{nuclear volume}}{\textit{atomic volume}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi(d/2)^3} = \left(\frac{1.20\times10^{-15}\text{ m}}{0.5\times10^{-10}\text{ m}}\right)^3 = \boxed{1.38\times10^{-14}}$$

In linear dimension, the nucleus is small inside the atom in the way a fat strawberry is small inside the width of the Grand Canyon. In terms of volume, the nucleus is *really* small.

P44.4 $E_\alpha = 7.70\text{ MeV}$

(a) $d_{\min} = \frac{4k_eZe^2}{mv^2} = \frac{2k_eZe^2}{E_\alpha} = \frac{2(8.99\times10^9)(79)(1.60\times10^{-19})^2}{7.70(1.60\times10^{-13})} = 29.5\times10^{-15}\text{ m} = \boxed{29.5\text{ fm}}$

(b) The de Broglie wavelength of the α is

$$\lambda = \frac{h}{m_\alpha v_\alpha} = \frac{h}{\sqrt{2m_\alpha E_\alpha}} = \frac{6.626\times10^{-34}}{\sqrt{2(6.64\times10^{-27})7.70(1.60\times10^{-13})}} = 5.18\times10^{-15}\text{ m} = \boxed{5.18\text{ fm}}$$

(c) Since $\boxed{\lambda \text{ is much less than the distance of closest approach}}$, the α may be considered a particle.

P44.5 (a) The initial kinetic energy of the alpha particle must equal the electrostatic potential energy at the distance of closest approach.

$$K_i = U_f = \frac{k_eqQ}{r_{\min}}$$

$$r_{\min} = \frac{k_eqQ}{K_i} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2)(79)(1.60\times10^{-19}\text{ C})^2}{(0.500\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})} = \boxed{4.55\times10^{-13}\text{ m}}$$

(b) Since $K_i = \frac{1}{2}m_\alpha v_i^2 = \frac{k_eqQ}{r_{\min}}$,

$$v_i = \sqrt{\frac{2k_eqQ}{m_\alpha r_{\min}}} = \sqrt{\frac{2(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2)(79)(1.60\times10^{-19}\text{ C})^2}{(4.00\text{ u})(1.66\times10^{-27}\text{ kg/u})(3.00\times10^{-13}\text{ m})}} = \boxed{6.04\times10^6\text{ m/s}}$$

P44.6 (a) $r = r_0A^{1/3} = (1.20\times10^{-15}\text{ m})(4)^{1/3} = \boxed{1.90\times10^{-15}\text{ m}}$

(b) $r = r_0A^{1/3} = (1.20\times10^{-15}\text{ m})(238)^{1/3} = \boxed{7.44\times10^{-15}\text{ m}}$

P44.7 The number of nucleons in a star of two solar masses is

$$A = \frac{2(1.99\times10^{30}\text{ kg})}{1.67\times10^{-27}\text{ kg/nucleon}} = 2.38\times10^{57}\text{ nucleons}$$

Therefore $r = r_0A^{1/3} = (1.20\times10^{-15}\text{ m})(2.38\times10^{57})^{1/3} = \boxed{16.0\text{ km}}$

P44.8 $V = \frac{4}{3}\pi r^4 = \frac{4}{3}\pi(0.021\,5\text{ m})^3 = 4.16\times10^{-5}\text{ m}^3$

We take the nuclear density from Example 44.2

$$m = \rho V = (2.3\times10^{17}\text{ kg/m}^3)(4.16\times10^{-5}\text{ m}^3) = 9.57\times10^{12}\text{ kg}$$

and $F = G\frac{m_1m_2}{r^2} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(9.57\times10^{12}\text{ kg})^2}{(1.00\text{ m})^2}$

$F = \boxed{6.11\times10^{15}\text{ N}}$ toward the other ball.

Section 44.2 **Nuclear Binding Energy**

P44.9 Using atomic masses as given in the table in the text,

(a) For $^2_1$H: $\dfrac{-2.014\,102+1(1.008\,665)+1(1.007\,825)}{2}$

$$\frac{E_b}{A}=(0.001\,194\text{ u})\left(\frac{931.5\text{ MeV}}{\text{u}}\right)=\boxed{1.11\text{ MeV/nucleon}}$$

(b) For $^4_2$He: $\dfrac{2(1.008\,665)+2(1.007\,825)-4.002\,603}{4}$

$$\frac{E_b}{A}=0.007\,59\text{ u}c^2=\boxed{7.07\text{ MeV/nucleon}}$$

(c) For $^{56}_{26}$Fe: $30(1.008\,665)+26(1.007\,825)-55.934\,942=0.528\text{ u}$

$$\frac{E_b}{A}=\frac{0.528}{56}=0.009\,44\text{ u}c^2=\boxed{8.79\text{ MeV/nucleon}}$$

(d) For $^{238}_{92}$U: $146(1.008\,665)+92(1.007\,825)-238.050\,783=1.934\,2\text{ u}$

$$\frac{E_b}{A}=\frac{1.934\,2}{238}=0.008\,13\text{ u}c^2=\boxed{7.57\text{ MeV/nucleon}}$$

***P44.10** $\Delta M=Zm_{\text{H}}+Nm_{\text{n}}-M$ $\qquad \dfrac{E_b}{A}=\dfrac{\Delta M\,(931.5)}{A}$

Nuclei	Z	N	M in u	ΔM in u	$\frac{E_b}{A}$ in MeV
^{55}Mn	25	30	54.938 050	0.517 5	8.765
^{56}Fe	26	30	55.934 942	0.528 46	8.790
^{59}Co	27	32	58.933 200	0.555 35	8.768

$\therefore$ ^{56}Fe has a greater $\dfrac{E_b}{A}$ than its neighbors. This tells us finer detail than is shown in Figure 44.5.

P44.11 (a) The neutron-to-proton ratio $\dfrac{A-Z}{Z}$ is greatest for $\boxed{^{139}_{55}\text{Cs}}$ and is equal to 1.53.

(b) $\boxed{^{139}\text{La}}$ has the largest binding energy per nucleon of 8.378 MeV.

(c) ^{139}Cs with a mass of 138.913 u. We locate the nuclei carefully on Figure 44.4, the neutron–proton plot of stable nuclei. $\boxed{\text{Cesium}}$ appears to be farther from the center of the zone of stability. Its instability means extra energy and extra mass.

P44.12 Use Equation 44.2.

Then for $^{23}_{11}$Na, $\dfrac{E_b}{A}=8.11\text{ MeV/nucleon}$

and for $^{23}_{12}$Mg, $\dfrac{E_b}{A}=7.90\text{ MeV/nucleon}$

The binding energy per nucleon is greater for $^{23}_{11}$Na by $\boxed{0.210\text{ MeV}}$. There is less proton repulsion in Na^{23}. It is the more stable nucleus.

P44.13 The binding energy of a nucleus is $E_b(\text{MeV}) = \left[ZM(\text{H}) + Nm_n - M\left({}^A_Z\text{X}\right)\right](931.494 \text{ MeV/u})$

For ${}^{15}_{8}\text{O}$: $E_b = [8(1.007\,825 \text{ u}) + 7(1.008\,665 \text{ u}) - 15.003\,065 \text{ u}](931.494 \text{ MeV/u}) = 111.96 \text{ MeV}$

For ${}^{15}_{7}\text{N}$: $E_b = [7(1.007\,825 \text{ u}) + 8(1.008\,665 \text{ u}) - 15.000\,109 \text{ u}](931.494 \text{ MeV/u}) = 115.49 \text{ MeV}$

Therefore, $\boxed{\text{the binding energy of } {}^{15}_{7}\text{N is larger by 3.54 MeV}}$.

P44.14 (a) The radius of the ^{40}Ca nucleus is: $R = r_0 A^{1/3} = (1.20 \times 10^{-15} \text{ m})(40)^{1/3} = 4.10 \times 10^{-15} \text{ m}$

The energy required to overcome electrostatic repulsion is

$$U = \frac{3k_e Q^2}{5R} = \frac{3(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\left[20(1.60 \times 10^{-19} \text{ C})\right]^2}{5(4.10 \times 10^{-15} \text{ m})} = 1.35 \times 10^{-11} \text{ J} = \boxed{84.1 \text{ MeV}}$$

(b) The binding energy of ${}^{40}_{20}\text{Ca}$ is

$$E_b = [20(1.007\,825 \text{ u}) + 20(1.008\,665 \text{ u}) - 39.962\,591 \text{ u}](931.5 \text{ MeV/u}) = \boxed{342 \text{ MeV}}$$

(c) The nuclear force is so strong that the binding energy greatly exceeds the minimum energy needed to overcome electrostatic repulsion.

P44.15 Removal of a neutron from ${}^{43}_{20}\text{Ca}$ would result in the residual nucleus, ${}^{42}_{20}\text{Ca}$. If the required separation energy is S_n, the overall process can be described by

$$\text{mass}\left({}^{43}_{20}\text{Ca}\right) + S_n = \text{mass}\left({}^{42}_{20}\text{Ca}\right) + \text{mass}(\text{n})$$

$$S_n = (41.958\,618 + 1.008\,665 - 42.958\,767) \text{ u} = (0.008\,516 \text{ u})(931.5 \text{ MeV/u}) = \boxed{7.93 \text{ MeV}}$$

Section 44.3 **Nuclear Models**

P44.16 (a) The first term overstates the importance of volume and the second term *subtracts* this overstatement.

(b) For spherical volume $\dfrac{(4/3)\pi R^3}{4\pi R^2} = \boxed{\dfrac{R}{3}}$. For cubical volume $\dfrac{R^3}{6R^2} = \boxed{\dfrac{R}{6}}$.

The maximum binding energy or lowest state of energy is achieved by building "nearly" spherical nuclei.

P44.17 $\Delta E_b = E_{bf} - E_{bi}$

For $A = 200$, $\dfrac{E_b}{A} = 7.8 \text{ MeV}$

so $E_{bi} = 200(7.8 \text{ MeV}) = 1\,560 \text{ MeV}$

For $A \approx 100$, $\dfrac{E_b}{A} = 8.7 \text{ MeV}$

so $E_{bf} = 2(100)(8.7 \text{ MeV})$
$= 1\,740 \text{ MeV}$

$\Delta E_b = E_{bf} - E_{bi}$: $E_b = 1\,740 \text{ MeV} - 1\,560 \text{ MeV}$
$\approx \boxed{200 \text{ MeV}}$

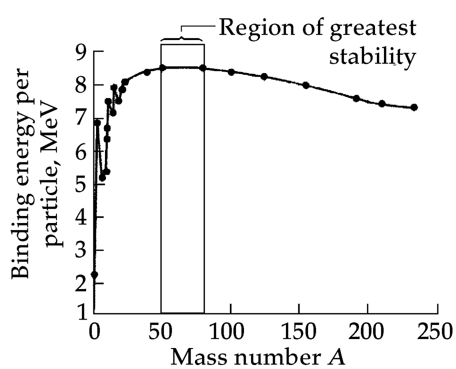


FIG. P44.17

P44.18 (a) "Volume" term: $E_1 = C_1 A = (15.7 \text{ MeV})(56) = 879 \text{ MeV}$

"Surface" term: $E_2 = -C_2 A^{2/3} = -(17.8 \text{ MeV})(56)^{2/3} = -260 \text{ MeV}$

"Coulomb" term: $E_3 = -C_3 \dfrac{Z(Z-1)}{A^{1/3}} = -(0.71 \text{ MeV})\dfrac{(26)(25)}{(56)^{1/3}} = -121 \text{ MeV}$

"Asymmetry" term: $E_4 = C_4 \dfrac{(A-2Z)^2}{A} = -(23.6 \text{ MeV})\dfrac{(56-52)^2}{56} = -6.74 \text{ MeV}$

$\boxed{E_b = 491 \text{ MeV}}$

(b) $\dfrac{E_1}{E_b} = 179\%$; $\dfrac{E_2}{E_b} = -53.0\%$; $\dfrac{E_3}{E_b} = -24.6\%$; $\dfrac{E_4}{E_b} = -1.37\%$

Section 44.4 **Radioactivity**

P44.19 $\dfrac{dN}{dt} = -\lambda N$

so $\lambda = \dfrac{1}{N}\left(-\dfrac{dN}{dt}\right) = \left(1.00\times10^{-15}\right)\left(6.00\times10^{11}\right) = 6.00\times10^{-4} \text{ s}^{-1}$

$T_{1/2} = \dfrac{\ln 2}{\lambda} = \boxed{1.16\times10^3 \text{ s}}\,(=19.3 \text{ min})$

P44.20 $R = R_0 e^{-\lambda t} = (6.40 \text{ mCi})e^{-(\ln 2/8.04 \text{ d})(40.2 \text{ d})} = (6.40 \text{ mCi})\left(e^{-\ln 2}\right)^5 = (6.40 \text{ mCi})\left(\dfrac{1}{2^5}\right) = \boxed{0.200 \text{ mCi}}$

P44.21 (a) From $R = R_0 e^{-\lambda t}$,

$$\lambda = \frac{1}{t}\ln\left(\frac{R_0}{R}\right) = \left(\frac{1}{4.00 \text{ h}}\right)\ln\left(\frac{10.0}{8.00}\right) = 5.58\times10^{-2} \text{ h}^{-1} = \boxed{1.55\times10^{-5} \text{ s}^{-1}}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \boxed{12.4 \text{ h}}$$

(b) $N_0 = \dfrac{R_0}{\lambda} = \dfrac{10.0\times10^{-3} \text{ Ci}}{1.55\times10^{-5}/\text{s}}\left(\dfrac{3.70\times10^{10}/\text{s}}{1 \text{ Ci}}\right) = \boxed{2.39\times10^{13} \text{ atoms}}$

(c) $R = R_0 e^{-\lambda t} = (10.0 \text{ mCi})\exp\left(-5.58\times10^{-2}\times30.0\right) = \boxed{1.88 \text{ mCi}}$

***P44.22** From the law of radioactive decay $N = N_0\, e^{-\lambda t}$ the decay rate is originally $R_0 = -dN/dt$ and decreases according to $R = -dN/dt = +N_0\, \lambda\, e^{-\lambda t}$ $\quad R = R_0\, e^{-\lambda t}$

Then algebra to isolate the decay constant gives $R_0/R = e^{\lambda t}$ $\quad \ln(R_0/R) = \lambda t$ $\quad \lambda = \dfrac{1}{t}\ln\left(\dfrac{R_0}{R}\right)$

Now $\lambda = \dfrac{\ln 2}{T_{1/2}}$ gives $\dfrac{\ln 2}{T_{1/2}} = \dfrac{1}{t}\ln\left(\dfrac{R_0}{R}\right)$ $\quad T_{1/2} = \dfrac{(\ln 2)t}{\ln(R_0/R)}$

where $t = \Delta t$ is the time interval during which the activity decreases from R_0 to R.

P44.23 The number of nuclei that decay during the interval will be $N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$.

First we find λ: $\quad \lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{0.693}{64.8\text{ h}} = 0.010\,7\text{ h}^{-1} = 2.97\times 10^{-6}\text{ s}^{-1}$

and $\quad N_0 = \dfrac{R_0}{\lambda} = \dfrac{(40.0\ \mu\text{Ci})(3.70\times 10^4\text{ s}^{-1}/\mu\text{Ci})}{2.97\times 10^{-6}\text{ s}^{-1}} = 4.98\times 10^{11}\text{ nuclei}$

Substituting these values, $\quad N_1 - N_2 = \left(4.98\times 10^{11}\right)\left[e^{-(0.010\,7\text{ h}^{-1})(10.0\text{ h})} - e^{-(0.010\,7\text{ h}^{-1})(12.0\text{ h})}\right]$

Hence, the number of nuclei decaying during the interval is $N_1 - N_2 = \boxed{9.47\times 10^9\text{ nuclei}}$.

P44.24 The number of nuclei that decay during the interval will be $N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$.

First we find λ: $\quad \lambda = \dfrac{\ln 2}{T_{1/2}}$

so $\quad e^{-\lambda t} = e^{\ln 2\left(-t/T_{1/2}\right)} = 2^{-t/T_{1/2}}$

and $\quad N_0 = \dfrac{R_0}{\lambda} = \dfrac{R_0 T_{1/2}}{\ln 2}$

Substituting in these values $\quad N_1 - N_2 = \dfrac{R_0 T_{1/2}}{\ln 2}\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right) = \boxed{\dfrac{R_0 T_{1/2}}{\ln 2}\left(2^{-t_1/T_{1/2}} - 2^{-t_2/T_{1/2}}\right)}$

***P44.25** The number remaining after time $\dfrac{T_{1/2}}{2} = \dfrac{\ln 2}{2\lambda}$

is $\quad N = N_0 e^{-\lambda t} = N_0\, e^{-\lambda \ln 2/2\lambda} = N_0\left(e^{-\ln 2}\right)^{1/2} = N_0\left(\dfrac{1}{2}\right)^{1/2} = 0.7071\, N_0$

The number decaying in this first half of the first half-life is $N_0 - 0.7071\, N_0 = 0.2929\, N_0$.
The number remaining after time $T_{1/2}$ is $0.500\, N_0$, so the number decaying in the second half of the first half-life is $0.7071\, N_0 - 0.500\, N_0 = 0.2071\, N_0$.
The ratio required is then $\quad 0.2929\, N_0/0.2071\, N_0 = \boxed{1.41}$

P44.26 (a) $\frac{dN_2}{dt}$ = rate of change of N_2

= rate of production of N_2 − rate of decay of N_2

= rate of decay of N_1 − rate of decay of N_2

$= \lambda_1 N_1 - \lambda_2 N_2$

(b) From the trial solution

$$N_2(t) = \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}\left(e^{-\lambda_2 t} - e^{-\lambda_1 t}\right)$$

$$\therefore \frac{dN_2}{dt} = \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}\left(-\lambda_2 e^{-\lambda_2 t} + \lambda_1 e^{-\lambda_1 t}\right) \qquad (1)$$

$$\therefore \frac{dN_2}{dt} + \lambda_2 N_2 = \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}\left(-\lambda_2 e^{-\lambda_2 t} + \lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}\right)$$

$$= \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}\left(\lambda_1 - \lambda_2\right)e^{-\lambda_1 t}$$

$$= \lambda_1 N_1$$

So $\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$ as required.

(c) The functions to be plotted are

$$N_1(t) = 1\,000 e^{-\left(0.223\,6\ \text{min}^{-1}\right)t}$$

$$N_2(t) = 1\,130.8\left[e^{-\left(0.223\,6\ \text{min}^{-1}\right)t} - e^{-\left(0.025\,9\ \text{min}^{-1}\right)t}\right]$$

From the graph: $t_m \approx \boxed{10.9\text{ min}}$

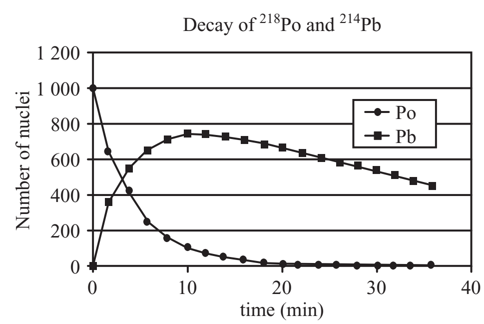


FIG. P44.26(c)

(d) From (1), $\frac{dN_2}{dt} = 0$ if $\lambda_2 e^{-\lambda_2 t} = \lambda_1 e^{-\lambda_1 t}$. $\therefore e^{(\lambda_1 - \lambda_2)t} = \frac{\lambda_1}{\lambda_2}$. Thus, $t = \boxed{t_m = \frac{\ln(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}}$.

With $\lambda_1 = 0.223\,6\text{ min}^{-1}$, $\lambda_2 = 0.025\,9\text{ min}^{-1}$, this formula gives $t_m = \boxed{10.9\text{ min}}$, in agreement with the result of part (c).

P44.27 We have all this information: $N_x(0) = 2.50 N_y(0)$

$$N_x(3\text{d}) = 4.20 N_y(3\text{d})$$

$$N_x(0)e^{-\lambda_x 3\text{d}} = 4.20 N_y(0)e^{-\lambda_y 3\text{d}} = 4.20\frac{N_x(0)}{2.50}e^{-\lambda_y 3\text{d}}$$

$$e^{3\text{d}\lambda_x} = \frac{2.5}{4.2}e^{3\text{d}\lambda_y}$$

$$3\text{d}\lambda_x = \ln\left(\frac{2.5}{4.2}\right) + 3\text{d}\lambda_y$$

$$3\text{d}\frac{0.693}{T_{1/2\,x}} = \ln\left(\frac{2.5}{4.2}\right) + 3\text{d}\frac{0.693}{1.60\text{ d}} = 0.781$$

$$T_{1/2\,x} = \boxed{2.66\text{ d}}$$

Section 44.5 **The Decay Processes**

P44.28 (a) A gamma ray has zero charge and it contains no protons or neutrons. So for a gamma ray $Z = 0$ and $A = 0$. Keeping the total values of Z and A for the system conserved then requires $Z = 28$ and $A = 65$ for X. With this atomic number it must be nickel, and the nucleus must be in an exited state, so it is ${}^{65}_{28}\text{Ni}^*$.

(b) $\alpha = {}^{4}_{2}\text{He}$ has $Z = 2$ and $A = 4$
so for X we require $Z = 84 - 2 = 82$
for Pb and $A = 215 - 4 = 211$, $X = {}^{211}_{82}\text{Pb}$

(c) A positron $e^+ = {}^{0}_{1}e$ has charge the same as a nucleus with $Z = 1$. A neutrino ${}^{0}_{0}\nu$ has no charge. Neither contains any protons or neutrons. So X must have by conservation $Z = 26 + 1 = 27$. It is Co. And $A = 55 + 0 = 55$. It is ${}^{55}_{27}\text{Co}$.

Similar reasoning about balancing the sums of Z and A across the reaction reveals:

(d) ${}^{0}_{-1}e$

(e) ${}^{1}_{1}\text{H}$ (or p). Note that this process is a nuclear reaction, rather than radioactive decay. We can solve it from the same principles, which are fundamentally conservation of charge and conservation of baryon number.

P44.29 $Q = \left(M_{\text{U-238}} - M_{\text{Th-234}} - M_{\text{He-4}}\right)(931.5\ \text{MeV/u})$

$$Q = (238.050\,783 - 234.043\,596 - 4.002\,603)\ \text{u}(931.5\ \text{MeV/u}) = \boxed{4.27\ \text{MeV}}$$

P44.30 $N_C = \left(\dfrac{0.021\,0\ \text{g}}{12.0\ \text{g/mol}}\right)\left(6.02\times 10^{23}\ \text{molecules/mol}\right)$

$\left(N_C = 1.05\times 10^{21}\ \text{carbon atoms}\right)$ of which 1 in 7.70×10^{11} is a ^{14}C atom

$$\left(N_0\right)_{\text{C-14}} = 1.37\times 10^{9},\qquad \lambda_{\text{C-14}} = \frac{\ln 2}{5\,730\ \text{yr}} = 1.21\times 10^{-4}\ \text{yr}^{-1} = 3.83\times 10^{-12}\ \text{s}^{-1}$$

$$R = \lambda N = \lambda N_0 e^{-\lambda t}$$

At $t = 0$,

$$R_0 = \lambda N_0 = \left(3.83\times 10^{-12}\ \text{s}^{-1}\right)\left(1.37\times 10^{9}\right)\left[\frac{7(86\,400\ \text{s})}{1\ \text{week}}\right]$$

$$= 3.17\times 10^{3}\ \text{decays/week}$$

At time t,

$$R = \frac{837}{0.88} = 951\ \text{decays/week}$$

Taking logarithms,

$$\ln\frac{R}{R_0} = -\lambda t \quad \text{so} \quad t = \frac{-1}{\lambda}\ln\left(\frac{R}{R_0}\right)$$

$$t = \frac{-1}{1.21\times 10^{-4}\ \text{yr}^{-1}}\ln\left(\frac{951}{3.17\times 10^{3}}\right) = \boxed{9.96\times 10^{3}\ \text{yr}}$$

***P44.31** (a) The decay constant is $\lambda = \ln 2/10\text{ h} = 0.0693/\text{h}$.
The number of parent nuclei is given by $10^6\, e^{-0.0693\,t}$ where t is in hours.
The number of daughter nuclei is equal to the number of missing parent nuclei,
$N_d = 10^6 - 10^6 e^{-0.0693t}$ $\boxed{N_d = 10^6(1 - e^{-0.0693t}) \text{ where } t \text{ is in hours}}$.

(b) The number of daughter nuclei starts from zero at $t = 0$. The number of stable product nuclei always increases with time and asymptotically approaches 1.00×10^6 as t increases without limit.

Its rate of increase is $\dfrac{dN_d}{dt} = 10^6(0 + 0.0693\, e^{-0.0693t}) = 6.93 \times 10^4 \dfrac{1}{\text{h}} e^{-0.0693t}$.

The number of daughter nuclei first increases most rapidly, at 6.93×10^4/h, and then more and more slowly. Its rate of change approaches zero in the far future.

P44.32 $^3_1\text{H nucleus} \rightarrow {}^3_2\text{He nucleus} + e^- + \bar{\nu}$

becomes $^3_1\text{H nucleus} + e^- \rightarrow {}^3_2\text{He nucleus} + 2e^- + \bar{\nu}$

Ignoring the slight difference in ionization energies,

we have $^3_1\text{H atom} \rightarrow {}^3_2\text{He atom} + \bar{\nu}$

$$3.016\,049 \text{ u} = 3.016\,029 \text{ u} + 0 + \frac{Q}{c^2}$$

$$Q = (3.016\,049 \text{ u} - 3.016\,029 \text{ u})(931.5 \text{ MeV/u}) = 0.018\,6 \text{ MeV} = \boxed{18.6 \text{ keV}}$$

P44.33 (a) $\boxed{e^- + p \rightarrow n + \nu}$

(b) For nuclei, $^{15}\text{O} + e^- \rightarrow {}^{15}\text{N} + \nu$

Add seven electrons to both sides to obtain $\boxed{^{15}_{8}\text{O atom} \rightarrow {}^{15}_{7}\text{N atom} + \nu}$

(c) From the table of isotopic masses in the chapter text, $m(^{15}\text{O}) = m(^{15}\text{N}) + \dfrac{Q}{c^2}$

$$\Delta m = 15.003\,065 \text{ u} - 15.000\,109 \text{ u} = 0.002\,956 \text{ u}$$

$$Q = (931.5 \text{ MeV/u})(0.002\,956 \text{ u}) = \boxed{2.75 \text{ MeV}}$$

P44.34 (a) For e^+ decay,

$$Q = (M_X - M_Y - 2m_e)c^2 = [39.962\,591 \text{ u} - 39.963\,999 \text{ u} - 2(0.000\,549 \text{ u})](931.5 \text{ MeV/u})$$

$Q = -2.33$ MeV

Since $Q < 0$, the decay $\boxed{\text{cannot occur}}$ spontaneously.

(b) For alpha decay,

$$Q = (M_X - M_\alpha - M_Y)c^2 = [97.905\,287 \text{ u} - 4.002\,603 \text{ u} - 93.905\,088 \text{ u}](931.5 \text{ MeV/u})$$

$Q = -2.24$ MeV

Since $Q < 0$, the decay $\boxed{\text{cannot occur}}$ spontaneously.

(c) For alpha decay,

$$Q = (M_X - M_\alpha - M_Y)c^2 = [143.910\,083 \text{ u} - 4.002\,603 \text{ u} - 139.905\,434 \text{ u}](931.5 \text{ MeV/u})$$

$Q = 1.91$ MeV

Since $Q > 0$, the decay $\boxed{\text{can occur}}$ spontaneously.

Section 44.6 **Natural Radioactivity**

P44.35

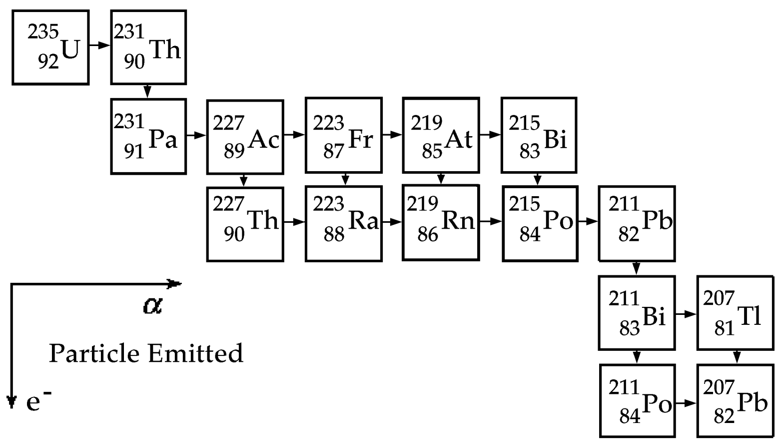


FIG. P44.35

P44.36 (a) Let N be the number of ^{238}U nuclei and N' be ^{206}Pb nuclei.

Then $N = N_0 e^{-\lambda t}$ and $N_0 = N + N'$ so $N = (N + N')e^{-\lambda t}$ or $e^{\lambda t} = 1 + \frac{N'}{N}$.

Taking logarithms, $\lambda t = \ln\left(1 + \frac{N'}{N}\right)$ where $\lambda = \frac{\ln 2}{T_{1/2}}$

Thus, $t = \left(\frac{T_{1/2}}{\ln 2}\right)\ln\left(1 + \frac{N'}{N}\right)$

If $\frac{N}{N'} = 1.164$ for the $^{238}\text{U} \to {}^{206}\text{Pb}$ chain with $T_{1/2} = 4.47 \times 10^9$ yr, the age is:

$$t = \left(\frac{4.47 \times 10^9 \text{ yr}}{\ln 2}\right)\ln\left(1 + \frac{1}{1.164}\right) = \boxed{4.00 \times 10^9 \text{ yr}}$$

(b) From above, $e^{\lambda t} = 1 + \frac{N'}{N}$. Solving for $\frac{N}{N'}$ gives $\frac{N}{N'} = \frac{e^{-\lambda t}}{1 - e^{-\lambda t}}$.

With $t = 4.00 \times 10^9$ yr and $T_{1/2} = 7.04 \times 10^8$ yr for the $^{235}\text{U} \to {}^{207}\text{Pb}$ chain,

$$\lambda t = \left(\frac{\ln 2}{T_{1/2}}\right)t = \frac{(\ln 2)(4.00 \times 10^9 \text{ yr})}{7.04 \times 10^8 \text{ yr}} = 3.938 \text{ and } \boxed{\frac{N}{N'} = 0.0199}$$

With $t = 4.00 \times 10^9$ yr and $T_{1/2} = 1.41 \times 10^{10}$ yr for the $^{232}\text{Th} \to {}^{208}\text{Pb}$ chain,

$$\lambda t = \frac{(\ln 2)(4.00 \times 10^9 \text{ yr})}{1.41 \times 10^{10} \text{ yr}} = 0.1966 \text{ and } \boxed{\frac{N}{N'} = 4.60}$$

P44.37 (a) $$4.00\ \text{pCi/L} = \left(\frac{4.00\times10^{-12}\ \text{Ci}}{1\ \text{L}}\right)\left(\frac{3.70\times10^{10}\ \text{Bq}}{1\ \text{Ci}}\right)\left(\frac{1.00\times10^{3}\ \text{L}}{1\ \text{m}^3}\right) = \boxed{148\ \text{Bq/m}^3}$$

(b) $$N = \frac{R}{\lambda} = R\left(\frac{T_{1/2}}{\ln 2}\right) = \left(148\ \text{Bq/m}^3\right)\left(\frac{3.82\ \text{d}}{\ln 2}\right)\left(\frac{86\,400\ \text{s}}{1\ \text{d}}\right) = \boxed{7.05\times10^{7}\ \text{atoms/m}^3}$$

(c) $$\text{mass} = \left(7.05\times10^{7}\ \text{atoms/m}^3\right)\left(\frac{1\ \text{mol}}{6.02\times10^{23}\ \text{atoms}}\right)\left(\frac{222\ \text{g}}{1\ \text{mol}}\right) = 2.60\times10^{-14}\ \text{g/m}^3$$

Since air has a density of $1.20\ \text{kg/m}^3$, the fraction consisting of radon is

$$\text{fraction} = \frac{2.60\times10^{-14}\ \text{g/m}^3}{1\,200\ \text{g/m}^3} = \boxed{2.17\times10^{-17}}$$

.

P44.38 Number remaining: $N = N_0 e^{-(\ln 2)t/T_{1/2}}$

Fraction remaining: $\frac{N}{N_0} = e^{-\lambda t} = e^{-(\ln 2)t/T_{1/2}}$

(a) With $T_{1/2} = 3.82$ d and $t = 7.00$ d, $\frac{N}{N_0} = e^{-(\ln 2)(7.00)/(3.82)} = \boxed{0.281}$

(b) When $t = 1.00\ \text{yr} = 365.25$ d, $\frac{N}{N_0} = e^{-(\ln 2)(365.25)/(3.82)} = \boxed{1.65\times10^{-29}}$

(c) $\boxed{\text{Radon is continuously created}}$ as one daughter in the series of decays starting from the long-lived isotope ^{238}U.

Section 44.7 **Nuclear Reactions**

P44.39 (a) For X, $A = 24 + 1 - 4 = 21$
and $Z = 12 + 0 - 2 = 10$, so X is $\boxed{^{21}_{10}\text{Ne}}$

(b) $A = 235 + 1 - 90 - 2 = 144$
and $Z = 92 + 0 - 38 - 0 = 54$, so X is $\boxed{^{144}_{54}\text{Xe}}$

(c) $A = 2 - 2 = 0$
and $Z = 2 - 1 = +1$, so X must be a positron.

As it is ejected, so is a neutrino: $\boxed{\text{X} = {}^{0}_{1}\text{e}^{+}}$ and $\boxed{\text{X}' = {}^{0}_{0}\nu}$

P44.40 (a) Add two electrons to both sides of the reaction to have it in neutral-atom terms:

$$4\,{}^{1}_{1}\text{H atom} \rightarrow {}^{4}_{2}\text{He atom} + Q \qquad Q = \Delta mc^2 = \left[4M_{{}^{1}_{1}\text{H}} - M_{{}^{4}_{2}\text{He}}\right]c^2$$

$$Q = \left[4(1.007\,825\text{ u}) - 4.002\,603\text{ u}\right](931.5\text{ MeV/u})\left(\frac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right) = \boxed{4.28\times10^{-12}\text{ J}}$$

(b) $$N = \frac{1.99\times10^{30}\text{ kg}}{1.67\times10^{-27}\text{ kg/atom}} = \boxed{1.19\times10^{57}\text{ atoms}} = 1.19\times10^{57}\text{ protons}$$

(c) The energy that could be created by this many protons in this reaction is:

$$\left(1.19\times10^{57}\text{ protons}\right)\left(\frac{4.28\times10^{-12}\text{ J}}{4\text{ protons}}\right) = 1.27\times10^{45}\text{ J}$$

$$\mathcal{P} = \frac{E}{\Delta t} \qquad \text{so} \qquad \Delta t = \frac{E}{\mathcal{P}} = \frac{1.27\times10^{45}\text{ J}}{3.85\times10^{26}\text{ W}} = 3.31\times10^{18}\text{ s} = \boxed{105\text{ billion years}}$$

P44.41 (a) $$\boxed{{}^{197}_{79}\text{Au} + {}^{1}_{0}\text{n} \rightarrow {}^{198}_{79}\text{Au}^* \rightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}\text{e} + \bar{\nu}}$$

(b) Consider adding 79 electrons:

$${}^{197}_{79}\text{Au atom} + {}^{1}_{0}\text{n} \rightarrow {}^{198}_{80}\text{Hg atom} + \bar{\nu} + Q$$

$$Q = \left[M_{{}^{197}\text{Au}} + m_n - M_{{}^{198}\text{Hg}}\right]c^2$$

$$Q = \left[196.966\,552 + 1.008\,665 - 197.966\,752\right]\text{ u}(931.5\text{ MeV/u}) = \boxed{7.89\text{ MeV}}$$

P44.42 Neglect recoil of product nucleus (i.e., do not require momentum conservation for the system of colliding particles). The energy balance gives $K_{\text{emerging}} = K_{\text{incident}} + Q$. To find Q:

$$Q = \left[(M_{\text{H}} + M_{\text{Al}}) - (M_{\text{Si}} + m_n)\right]c^2$$

$$Q = \left[(1.007\,825 + 26.981\,539) - (26.986\,705 + 1.008\,665)\right]\text{ u}(931.5\text{ MeV/u}) = -5.59\text{ MeV}$$

Thus, $K_{\text{emerging}} = 6.61\text{ MeV} - 5.59\text{ MeV} = \boxed{1.02\text{ MeV}}$.

P44.43 $${}^{9}_{4}\text{Be} + 1.665\text{ MeV} \rightarrow {}^{8}_{4}\text{Be} + {}^{1}_{0}\text{n},\ \text{so}\ M_{{}^{8}_{4}\text{Be}} = M_{{}^{9}_{4}\text{Be}} - \frac{Q}{c^2} - m_n$$

$$M_{{}^{8}_{4}\text{Be}} = 9.012\,182\text{ u} - \frac{(-1.665\text{ MeV})}{931.5\text{ MeV/u}} - 1.008\,665\text{ u} = \boxed{8.005\,3\text{ u}}$$

$${}^{9}_{4}\text{Be} + {}^{1}_{0}\text{n} \rightarrow {}^{10}_{4}\text{Be} + 6.812\text{ MeV},\ \text{so}\ M_{{}^{10}_{4}\text{Be}} = M_{{}^{9}_{4}\text{Be}} + m_n - \frac{Q}{c^2}$$

$$M_{{}^{10}_{4}\text{Be}} = 9.012\,182\text{ u} + 1.008\,665\text{ u} - \frac{6.812\text{ MeV}}{931.5\text{ MeV/u}} = \boxed{10.013\,5\text{ u}}$$

P44.44 (a) $${}^{10}_{5}\text{B} + {}^{4}_{2}\text{He} \rightarrow {}^{13}_{6}\text{C} + {}^{1}_{1}\text{H}$$

The product nucleus is $\boxed{{}^{13}_{6}\text{C}}$.

(b) $${}^{13}_{6}\text{C} + {}^{1}_{1}\text{H} \rightarrow {}^{10}_{5}\text{B} + {}^{4}_{2}\text{He}$$

The product nucleus is $\boxed{{}^{10}_{5}\text{B}}$.

Section 44.8 **Nuclear Magnetic Resonance and Magnetic Resonance Imagining**

P44.45 (a) $f_n = \frac{|2\mu B|}{h} = \frac{2(1.913\,5)(5.05\times10^{-27}\text{ J/T})(1.00\text{ T})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{29.2\text{ MHz}}$

(b) $f_p = \frac{2(2.792\,8)(5.05\times10^{-27}\text{ J/T})(1.00\text{ T})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{42.6\text{ MHz}}$

(c) In the Earth's magnetic field,

$$f_p = \frac{2(2.792\,8)(5.05\times10^{-27})(50.0\times10^{-6})}{6.626\times10^{-34}} = \boxed{2.13\text{ kHz}}$$

P44.46

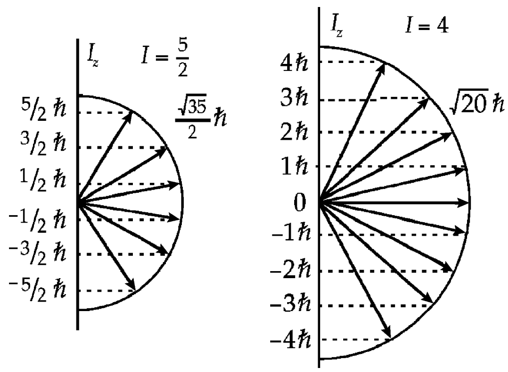


FIG. P44.46

Additional Problems

P44.47 (a) $Q = \left[M_{^9\text{Be}} + M_{^4\text{He}} - M_{^{12}\text{C}} - m_n\right]c^2$

$Q = [9.012\,182\text{ u} + 4.002\,603\text{ u} - 12.000\,000\text{ u} - 1.008\,665\text{ u}](931.5\text{ MeV/u}) = \boxed{5.70\text{ MeV}}$

(b) $Q = \left[2M_{^2\text{H}} - M_{^3\text{He}} - m_n\right]$

$Q = \left[2(2.014\,102) - 3.016\,029 - 1.008\,665\right]\text{u}(931.5\text{ MeV/u}) = \boxed{3.27\text{ MeV (exothermic)}}$

P44.48 (a) With m_n and v_n as the mass and speed of the neutrons, Equation 9.24 of the text becomes, after making appropriate notational changes, for the two collisions $v_1 = \left(\frac{2m_n}{m_n + m_1}\right)v_n$, and

$$v_2 = \left(\frac{2m_n}{m_n + m_2}\right)v_n$$

$$\therefore (m_n + m_2)v_2 = (m_n + m_1)v_1 = 2m_n v_n$$

$$\therefore m_n(v_2 - v_1) = m_1 v_1 - m_2 v_2$$

$$\therefore m_n = \frac{m_1 v_1 - m_2 v_2}{v_2 - v_1}$$

(b) $m_n = \frac{(1\text{ u})(3.30\times10^{7}\text{ m/s}) - (14\text{ u})(4.70\times10^{6}\text{ m/s})}{4.70\times10^{6}\text{ m/s} - 3.30\times10^{7}\text{ m/s}} = \boxed{1.16\text{ u}}$

P44.49 (a) At threshold, the particles have no kinetic energy relative to each other. That is, they move like two particles that have suffered a perfectly inelastic collision. Therefore, in order to calculate the reaction threshold energy, we can use the results of a perfectly inelastic collision. Initially, the projectile M_a moves with velocity v_a while the target M_X is at rest. We have from momentum conservation for the projectile-target system:

$$M_a v_a = (M_a + M_X) v_c$$

The initial energy is: $E_i = \frac{1}{2} M_a v_a^2$

The final kinetic energy is:

$$E_f = \frac{1}{2}(M_a + M_X) v_c^2 = \frac{1}{2}(M_a + M_X)\left[\frac{M_a v_a}{M_a + M_X}\right]^2 = \left[\frac{M_a}{M_a + M_X}\right] E_i$$

From this, we see that E_f is always less than E_i and the change in energy, $E_f - E_i$, is given by

$$E_f - E_i = \left[\frac{M_a}{M_a + M_X} - 1\right] E_i = -\left[\frac{M_X}{M_a + M_X}\right] E_i$$

This loss of kinetic energy in the isolated system corresponds to an increase in mass-energy during the reaction. Thus, the absolute value of this kinetic energy change is equal to $-Q$ (remember that Q is negative in an endothermic reaction). The initial kinetic energy E_i is the threshold energy E_{th}.

Therefore, $$-Q = \left[\frac{M_X}{M_a + M_X}\right] E_{th}$$

or $$E_{th} = -Q\left[\frac{M_X + M_a}{M_X}\right] = \boxed{-Q\left[1 + \frac{M_a}{M_X}\right]}$$

(b) First, calculate the Q-value for the reaction: $Q = \left[M_{\text{N-14}} + M_{\text{He-4}} - M_{\text{O-17}} - M_{\text{H-1}}\right] c^2$

$$Q = [14.003\,074 + 4.002\,603 - 16.999\,132 - 1.007\,825]\,\text{u}(931.5\ \text{MeV/u}) = -1.19\ \text{MeV}$$

Then, $E_{th} = -Q\left[\frac{M_X + M_a}{M_X}\right] = -(-1.19\ \text{MeV})\left[1 + \frac{4.002\,603}{14.003\,074}\right] = \boxed{1.53\ \text{MeV}}$.

***P44.50** (a) The system of a separated proton and electron puts out energy 13.606 eV to become a hydrogen atom in its ground state. This decrease in its rest energy appears also as a decrease in mass: the mass is $\boxed{\text{smaller}}$.

(b) $$|\Delta m| = \frac{|E|}{c^2} = \frac{13.6\ \text{eV}}{(3 \times 10^8\ \text{m/s})^2}\left(\frac{1.6 \times 10^{-19}\ \text{J}}{1\ \text{eV}}\right) = 2.42 \times 10^{-35}\ \text{kg}$$

$$= 2.42 \times 10^{-35}\ \text{kg}\left(\frac{1\ \text{u}}{1.66 \times 10^{-27}\ \text{kg}}\right) = \boxed{1.46 \times 10^{-8}\ \text{u}}$$

(c) $$\frac{1.46 \times 10^{-8}\ \text{u}}{1.007\,825\ \text{u}} = 1.45 \times 10^{-8} = \boxed{1.45 \times 10^{-6}\%}$$

(d) The textbook table lists 1.007 825 u as the atomic mass of hydrogen. This correction of 0.000 000 01 u is on the order of 100 times $\boxed{\text{too small}}$ to affect the values listed.

***P44.51** The problem statement can be: Find the reaction energy (Q value) of the reaction
${}^{10}_{5}\text{B} + {}^{4}_{2}\text{He} \rightarrow {}^{13}_{6}\text{C} + {}^{1}_{1}\text{H}$.
Solving gives $Q = (14.015\,54 - 14.011\,18)\text{ u }c^2(931.5\text{ MeV/u }c^2) = 4.06\text{ MeV}$ for the energy released by the reaction as it is converted from rest energy into other forms.

P44.52 (a) $$N_0 = \frac{\text{mass}}{\text{mass per atom}} = \frac{1.00\text{ kg}}{(239.05\text{ u})(1.66\times10^{-27}\text{ kg/u})} = \boxed{2.52\times10^{24}}$$

(b) $$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(2.412\times10^{4}\text{ yr})(3.156\times10^{7}\text{ s/yr})} = 9.106\times10^{-13}\text{ s}^{-1}$$
$$R_0 = \lambda N_0 = (9.106\times10^{-13}\text{ s}^{-1})(2.52\times10^{24}) = \boxed{2.29\times10^{12}\text{ Bq}}$$

(c) $$R = R_0 e^{-\lambda t},\text{ so } t = \frac{-1}{\lambda}\ln\left(\frac{R}{R_0}\right) = \frac{1}{\lambda}\ln\left(\frac{R_0}{R}\right)$$
$$t = \frac{1}{9.106\times10^{-13}\text{ s}^{-1}}\ln\left(\frac{2.29\times10^{12}\text{ Bq}}{0.100\text{ Bq}}\right) = 3.38\times10^{13}\text{ s}\left(\frac{1\text{ yr}}{3.156\times10^{7}\text{ s}}\right) = \boxed{1.07\times10^{6}\text{ yr}}$$

P44.53 (a) ${}^{57}_{27}\text{Co} \rightarrow {}^{57}_{26}\text{Fe} + {}^{0}_{+1}\text{e} + {}^{0}_{0}\nu$

The Q-value for this positron emission is $Q = \left[M_{{}^{57}\text{Co}} - M_{{}^{57}\text{Fe}} - 2m_e\right]c^2$.

$$Q = [56.936\,296 - 56.935\,399 - 2(0.000\,549)]\text{u}(931.5\text{ MeV/u}) = -0.187\text{ MeV}$$

Since $Q < 0$, this reaction $\boxed{\text{cannot spontaneously occur}}$.

(b) ${}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e} + {}^{0}_{0}\bar{\nu}$

The Q-value for this e^- decay is $Q = \left[M_{{}^{14}\text{C}} - M_{{}^{14}\text{N}}\right]c^2$.

$$Q = [14.003\,242 - 14.003\,074]\text{u}(931.5\text{ MeV/u}) = 0.156\text{ MeV} = 156\text{ keV}$$

Since $Q > 0$, the decay $\boxed{\text{can spontaneously occur}}$.

(c) The energy released in the reaction of (b) is shared by the electron and neutrino. Thus, $\boxed{K_e\text{ can range from zero to 156 keV}}$.

P44.54 (a) $r = r_0 A^{1/3} = 1.20\times10^{-15} A^{1/3}\text{ m}$

When $A = 12$, $r = \boxed{2.75\times10^{-15}\text{ m}}$

(b) $$F = \frac{k_e(Z-1)e^2}{r^2} = \frac{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(Z-1)(1.60\times10^{-19}\text{ C})^2}{r^2}$$

When $Z = 6$ and $r = 2.75\times10^{-15}\text{ m}$, $F = \boxed{152\text{ N}}$

(c) $$U = \frac{k_e q_1 q_2}{r} = \frac{k_e(Z-1)e^2}{r} = \frac{(8.99\times10^{9})(Z-1)(1.6\times10^{-19})^2}{r}$$

When $Z = 6$ and $r = 2.75\times10^{-15}\text{ m}$, $U = 4.19\times10^{-13}\text{ J} = \boxed{2.62\text{ MeV}}$

(d) $A = 238$; $Z = 92$, $r = \boxed{7.44\times10^{-15}\text{ m}}$ $F = \boxed{379\text{ N}}$

and $U = 2.82\times10^{-12}\text{ J} = \boxed{17.6\text{ MeV}}$

P44.55 (a) Because the reaction $p \rightarrow n + e^+ + \nu$ would violate the law of $\boxed{\text{conservation of energy}}$

$$m_p = 1.007\,276 \text{ u} \qquad m_n = 1.008\,665 \text{ u} \qquad m_{e^+} = 5.49 \times 10^{-4} \text{ u}$$

Note that $m_n + m_{e^+} > m_p$

(b) The $\boxed{\text{required energy can come from the electrostatic repulsion}}$ of protons in the parent nucleus.

(c) Add seven electrons to both sides of the reaction for nuclei ${}^{13}_{7}\text{N} \rightarrow {}^{13}_{6}\text{C} + e^+ + \nu$ to obtain the reaction for neutral atoms ${}^{13}_{7}\text{N atom} \rightarrow {}^{13}_{6}\text{C atom} + e^+ + e^- + \nu$

$$Q = c^2\left[m\left({}^{13}\text{N}\right) - m\left({}^{13}\text{C}\right) - m_{e^+} - m_{e^-} - m_\nu\right]$$

$$Q = (931.5 \text{ MeV/u})\left[13.005\,739 - 13.003\,355 - 2\left(5.49 \times 10^{-4}\right) - 0\right]\text{u}$$

$$Q = (931.5 \text{ MeV/u})\left(1.286 \times 10^{-3} \text{ u}\right) = \boxed{1.20 \text{ MeV}}$$

P44.56 (a) A least-square fit to the graph yields:

$$\lambda = -\text{slope} = -\left(-0.250 \text{ h}^{-1}\right) = 0.250 \text{ h}^{-1}$$

and

$$\ln(\text{cpm})\big|_{t=0} = \text{intercept} = 8.30$$

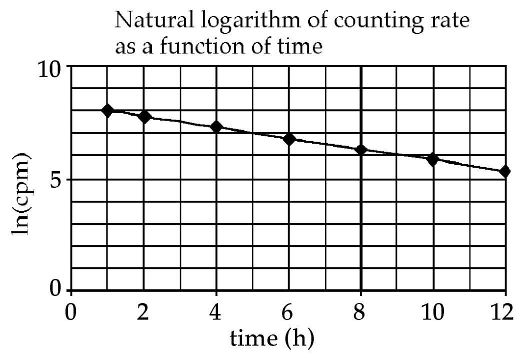


FIG. P44.56

(b) $\lambda = 0.250 \text{ h}^{-1}\left(\dfrac{1 \text{ h}}{60.0 \text{ min}}\right) = \boxed{4.17 \times 10^{-3} \text{ min}^{-1}}$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{4.17 \times 10^{-3} \text{ min}^{-1}}$$

$$= 166 \text{ min} = \boxed{2.77 \text{ h}}$$

(c) From (a), intercept $= \ln(\text{cpm})_0 = 8.30$

Thus, $(\text{cpm})_0 = e^{8.30}$ counts/min $= \boxed{4.02 \times 10^3 \text{ counts/min}}$

(d) $N_0 = \dfrac{R_0}{\lambda} = \dfrac{1}{\lambda}\dfrac{(\text{cpm})_0}{\text{Eff}} = \dfrac{4.02 \times 10^3 \text{ counts/min}}{\left(4.17 \times 10^{-3} \text{ min}^{-1}\right)(0.100)} = \boxed{9.65 \times 10^6 \text{ atoms}}$

P44.57 (a) One liter of milk contains this many ${}^{40}\text{K}$ nuclei:

$$N = (2.00 \text{ g})\left(\frac{6.02 \times 10^{23} \text{ nuclei/mol}}{39.1 \text{ g/mol}}\right)\left(\frac{0.011\,7}{100}\right) = 3.60 \times 10^{18} \text{ nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{1.28 \times 10^9 \text{ yr}}\left(\frac{1 \text{ yr}}{3.156 \times 10^7 \text{ s}}\right) = 1.72 \times 10^{-17} \text{ s}^{-1}$$

$$R = \lambda N = \left(1.72 \times 10^{-17} \text{ s}^{-1}\right)\left(3.60 \times 10^{18}\right) = \boxed{61.8 \text{ Bq}}$$

(b) For the iodine, $R = R_0 e^{-\lambda t}$ with $\lambda = \dfrac{\ln 2}{8.04 \text{ d}}$

$$t = \frac{1}{\lambda}\ln\left(\frac{R_0}{R}\right) = \frac{8.04 \text{ d}}{\ln 2}\ln\left(\frac{2\,000}{61.8}\right) = \boxed{40.3 \text{ d}}$$

P44.58 (a) If ΔE is the energy difference between the excited and ground states of the nucleus of mass M, and hf is the energy of the emitted photon, conservation of energy for the nucleus-photon system gives

$$\Delta E = hf + E_r \qquad (1)$$

Where E_r is the recoil energy of the nucleus, which can be expressed as

$$E_r = \frac{Mv^2}{2} = \frac{(Mv)^2}{2M} \qquad (2)$$

Since system momentum must also be conserved, we have

$$Mv = \frac{hf}{c} \qquad (3)$$

Hence, E_r can be expresses as $E_r = \dfrac{(hf)^2}{2Mc^2}$

When $hf << Mc^2$

we can make the approximation that $hf \approx \Delta E$

so $E_r \approx \boxed{\dfrac{(\Delta E)^2}{2Mc^2}}$

(b) $E_r = \dfrac{(\Delta E)^2}{2Mc^2}$ where $\Delta E = 0.014\,4$ MeV

and $Mc^2 = (57\text{ u})(931.5\text{ MeV/u}) = 5.31\times10^4\text{ MeV}$

Therefore, $E_r = \dfrac{\left(1.44\times10^{-2}\text{ MeV}\right)^2}{2\left(5.31\times10^4\text{ MeV}\right)} = \boxed{1.94\times10^{-3}\text{ eV}}$

P44.59 We have $N_{235} = N_{0,235}e^{-\lambda_{235}t}$

and $N_{238} = N_{0,238}e^{-\lambda_{238}t}$

$$\frac{N_{235}}{N_{238}} = 0.007\,25 = e^{\left(-(\ln 2)t/T_{h,235}+(\ln 2)t/T_{h,\,238}\right)}$$

Taking logarithms,

$$-4.93 = \left(-\frac{\ln 2}{0.704\times10^9\text{ yr}} + \frac{\ln 2}{4.47\times10^9\text{ yr}}\right)t$$

or

$$-4.93 = \left(-\frac{1}{0.704\times10^9\text{ yr}} + \frac{1}{4.47\times10^9\text{ yr}}\right)(\ln 2)t$$

$$t = \frac{-4.93}{\left(-1.20\times10^{-9}\text{ yr}^{-1}\right)\ln 2} = \boxed{5.94\times10^9\text{ yr}}$$

P44.60 (a) For cobalt-56,

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{77.1 \text{ d}}\left(\frac{365.25 \text{ d}}{1 \text{ yr}}\right) = 3.28 \text{ yr}^{-1}$$

The elapsed time from July 1054 to July 2007 is 953 yr.

$$R = R_0 e^{-\lambda t}$$

implies $$\frac{R}{R_0} = e^{-\lambda t} = e^{-\left(3.28 \text{ yr}^{-1}\right)(953 \text{ yr})} = e^{-3\,129} = e^{-(\ln 10)1\,359} = \boxed{\sim 10^{-1\,359}}$$

(b) For carbon-14,

$$\lambda = \frac{\ln 2}{5\,730 \text{ yr}} = 1.21 \times 10^{-4} \text{ yr}^{-1}$$

$$\frac{R}{R_0} = e^{-\lambda t} = e^{-\left(1.21\times 10^{-4} \text{ yr}^{-1}\right)(953 \text{ yr})} = e^{-0.115} = \boxed{0.891}$$

P44.61 $E = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}$ so the energies are $E_1 = +\mu B$ and $E_2 = -\mu B$

$\mu = 2.792\,8\mu_n$ and $\mu_n = 5.05 \times 10^{-27}$ J/T

$$\Delta E = 2\mu B = 2(2.792\,8)\left(5.05\times 10^{-27} \text{ J/T}\right)(12.5 \text{ T}) = 3.53\times 10^{-25} \text{ J} = \boxed{2.20 \times 10^{-6} \text{ eV}}$$

P44.62 $R = R_0 \exp(-\lambda t)$ lets us write $\ln R = \ln R_0 - \lambda t$ which is the equation of a straight line with $|\text{slope}| = \lambda$. The logarithmic plot shown in Figure P44.62 is fitted by $\ln R = 8.44 - 0.262t$. If t is measured in minutes, then decay constant λ is 0.262 per minute. The half–life is

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.262/\text{min}} = \boxed{2.64 \text{ min}}.$$

The reported half–life of ^{137}Ba is 2.55 min. The difference reflects experimental uncertainties.

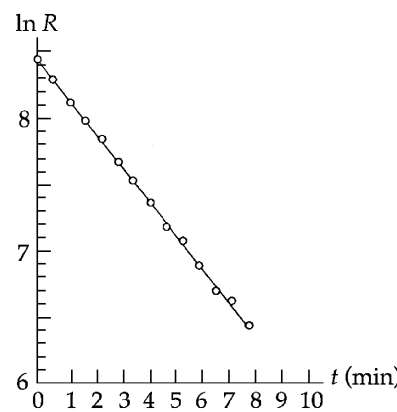


FIG. P44.62

P44.63 $K=\frac{1}{2}mv^2$, so $v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{2(0.040\,0\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{1.67\times10^{-27}\text{ kg}}}=2.77\times10^3\text{ m/s}$

The time for the trip is $t=\frac{x}{v}=\frac{1.00\times10^4\text{ m}}{2.77\times10^3\text{ m/s}}=3.61\text{ s}$.

The number of neutrons finishing the trip is given by $N=N_0e^{-\lambda t}$.

The fraction decaying is $1-\frac{N}{N_0}=1-e^{-(\ln 2)t/T_{1/2}}=1-e^{-(\ln 2)(3.61\text{ s}/624\text{ s})}=0.004\,00=\boxed{0.400\%}$.

P44.64 For an electric charge density $\rho=\frac{Ze}{(4/3)\pi R^3}$.

Using Gauss's Law inside the sphere,

$$E\cdot 4\pi r^2=\frac{(4/3)\pi r^3}{\epsilon_0}\frac{Ze}{(4/3)\pi R^3}: \qquad E=\frac{1}{4\pi\epsilon_0}\frac{Zer}{R^3} \qquad (r\le R)$$

$$E=\frac{1}{4\pi\epsilon_0}\frac{Ze}{r^2} \qquad (r\ge R)$$

We now find the electrostatic energy: $U=\int_{r=0}^{\infty}\frac{1}{2}\epsilon_0 E^2 4\pi r^2 dr$

$$U=\frac{1}{2}\epsilon_0\int_0^R\left(\frac{1}{4\pi\epsilon_0}\right)^2\frac{Z^2e^2r^2}{R^6}4\pi r^2dr+\frac{1}{2}\epsilon_0\int_R^\infty\left(\frac{1}{4\pi\epsilon_0}\right)^2\frac{Z^2e^2}{r^4}4\pi r^2dr=\frac{Z^2e^2}{8\pi\epsilon_0}\left[\frac{R^5}{5R^6}+\frac{1}{R}\right]$$

$$=\boxed{\frac{3}{20}\frac{Z^2e^2}{\pi\epsilon_0 R}}$$

P44.65 (a) If we assume all the ^{87}Sr came from ^{87}Rb,

then $N=N_0e^{-\lambda t}$

yields $t=\frac{-1}{\lambda}\ln\left(\frac{N}{N_0}\right)=\frac{T_{1/2}}{\ln 2}\ln\left(\frac{N_0}{N}\right)$

where $N=N_{\text{Rb-87}}$

and $N_0=N_{\text{Sr-87}}+N_{\text{Rb-87}}$

$$t=\frac{(4.75\times10^{10}\text{ yr})}{\ln 2}\ln\left(\frac{1.82\times10^{10}+1.07\times10^9}{1.82\times10^{10}}\right)=\boxed{3.91\times10^9\text{ yr}}$$

(b) It could be $\boxed{\text{no older}}$. The rock could be younger if some ^{87}Sr were originally present. We must make some assumption about the original quantity of radioactive material. In part (a) we assumed that the rock originally contained no strontium.

P44.66 (a) For the electron capture, $^{93}_{43}\text{Tc} + {}^{0}_{-1}\text{e} \rightarrow {}^{93}_{42}\text{Mo} + \gamma$

The disintegration energy is $Q = \left[M_{^{93}\text{Tc}} - M_{^{93}\text{Mo}}\right]c^2$.

$Q = [92.910\,2 - 92.906\,8]\text{u}(931.5\ \text{MeV/u}) = 3.17\ \text{MeV} > 2.44\ \text{MeV}$

Electron capture is allowed [to all specified excited states] in $^{93}_{42}\text{Mo}$.

For positron emission, $^{93}_{43}\text{Tc} \rightarrow {}^{93}_{42}\text{Mo} + {}^{0}_{+1}\text{e} + \gamma$

The disintegration energy is $Q' = \left[M_{^{93}\text{Tc}} - M_{^{93}\text{Mo}} - 2m_e\right]c^2$.

$Q' = [92.910\,2 - 92.906\,8 - 2(0.000\,549)]\text{u}(931.5\ \text{MeV/u}) = 2.14\ \text{MeV}$

Positron emission can reach [the 1.35, 1.48, and 2.03 MeV states] but there is insufficient energy to reach the 2.44 MeV state.

(b) The daughter nucleus in both forms of decay is [$^{93}_{42}\text{Mo}$].

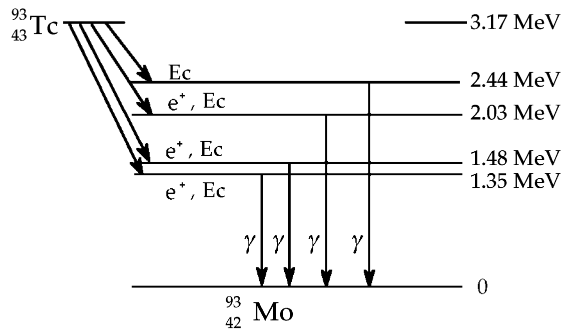


FIG. P44.66

ANSWERS TO EVEN PROBLEMS

P44.2 (a) 7.89 cm and 8.21 cm (b) See the solution.

P44.4 (a) 29.5 fm (b) 5.18 fm (c) See the solution.

P44.6 (a) 1.90 fm (b) 7.44 fm

P44.8 6.11 PN toward the other ball

P44.10 They agree with the figure.

P44.12 0.210 MeV greater for ^{23}Na because it has less proton repulsion

P44.14 (a) 84.1 MeV (b) 342 MeV (c) The nuclear force of attraction dominates over electrical repulsion.

P44.16 (a) See the solution. (b) $\dfrac{R}{3}$ and $\dfrac{R}{6}$; see the solution.

P44.18 (a) 491 MeV (b) 179%, −53.0%, −24.6%, −1.37%

P44.20 0.200 mCi

P44.22 See the solution.

P44.24 $\dfrac{R_0 T_{1/2}}{\ln 2}\left(2^{-t_1/T_{1/2}} - 2^{-t_2/T_{1/2}}\right)$

P44.26 (a) See the solution. (b) See the solution. (c) See the solution; 10.9 min.
(d) $t_m = \dfrac{\ln(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}$; yes

P44.28 (a) ${}^{65}_{28}\text{Ni}^*$ (b) ${}^{211}_{82}\text{Pb}$ (c) ${}^{55}_{27}\text{Co}$ (d) ${}^{0}_{-1}\text{e}$ (e) ${}^{1}_{1}\text{H}$

P44.30 9.96×10^3 yr

P44.32 ${}^{3}_{1}\text{H atom} \rightarrow {}^{3}_{2}\text{He atom} + \bar{\nu}$; 18.6 keV

P44.34 (a) cannot occur (b) cannot occur (c) can occur

P44.36 (a) 4.00 Gyr (b) 0.019 9 and 4.60

P44.38 (a) 0.281 (b) 1.65×10^{-29} (c) See the solution.

P44.40 (a) 4.28 pJ (b) 1.19×10^{57} atoms (c) 105 Gyr

P44.42 1.02 MeV

P44.44 (a) ${}^{13}_{6}\text{C}$ (b) ${}^{10}_{5}\text{B}$

P44.46 See the solution.

P44.48 (a) See the solution. (b) 1.16 u

P44.50 (a) smaller (b) 1.46×10^{-8} u (c) 1.45×10^{-6} % (d) no

P44.52 (a) 2.52×10^{24} (b) 2.29 TBq (c) 1.07 Myr

P44.54 (a) 2.75 fm (b) 152 N (c) 2.62 MeV (d) 7.44 fm, 379 N, 17.6 MeV

P44.56 (a) See the solution. (b) 4.17×10^{-3} min^{-1}; 2.77 h (c) 4.02×10^3 counts/min
(d) 9.65×10^6 atoms

P44.58 (a) See the solution. (b) 1.94 meV

P44.60 (a) $\sim 10^{-1\,359}$ (b) 0.891

P44.62 2.64 min

P44.64 See the solution.

P44.66 (a) electron capture to all; positron emission to the 1.35 MeV, 1.48 MeV, and 2.03 MeV states
(b) ${}^{93}_{42}\text{Mo}$; see the solution.

45

Applications of Nuclear Physics

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q45.1 The hydrogen nuclei in water molecules have mass similar to that of a neutron, so that they can efficiently rob a fast-moving neutron of kinetic energy as they scatter it. Once the neutron is slowed down, a hydrogen nucleus can absorb it in the reaction $n + {}_1^1\text{H} \rightarrow {}_1^2\text{H}$.

Q45.2 The excitation energy comes from the binding energy of the extra nucleon.

***Q45.3** Answer (c). The function of the moderator is to slow down the neutrons released by one fission so that they can efficiently cause more fissions.

Q45.4 The advantage of a fission reaction is that it can generate much more electrical energy per gram of fuel compared to fossil fuels. Also, fission reactors do not emit greenhouse gases as combustion byproducts like fossil fuels—the only necessary environmental discharge is heat. The cost involved in producing fissile material is comparable to the cost of pumping, transporting and refining fossil fuel.

The disadvantage is that some of the products of a fission reaction are radioactive—and some of those have long half-lives. The other problem is that there will be a point at which enough fuel is spent that the fuel rods do not supply power economically and need to be replaced. The fuel rods are still radioactive after removal. Both the waste and the “spent” fuel rods present serious health and environmental hazards that can last for tens of thousands of years. Accidents and sabotage involving nuclear reactors can be very serious, as can accidents and sabotage involving fossil fuels.

***Q45.5** Answer (b). In $m = E/c^2$ we estimate 5×10^{13} J$/9 \times 10^{16}$ m^2/s$^2 \approx 6 \times 10^{-4}$ kg = 0.6 g.

***Q45.6** Answer (d). We compute 235 + 1 – 137 – 96 = 3. All the protons that start out in the uranium nucleus end up in the fission product nuclei.

Q45.7 The products of fusion reactors are generally not themselves unstable, while fission reactions result in a chain of reactions which almost all have some unstable products.

Q45.8 For the deuterium nuclei to fuse, they must be close enough to each other for the nuclear forces to overcome the Coulomb repulsion of the protons—this is why the ion density is a factor. The more time that the nuclei in a sample spend in close proximity, the more nuclei will fuse—hence the confinement time is a factor.

Q45.9 Fusion of light nuclei to a heavier nucleus releases energy. Fission of a heavy nucleus to lighter nuclei releases energy. Both processes are steps towards greater stability on the curve of binding energy, Figure 44.5. The energy release per nucleon is typically greater for fusion, and this process is harder to control.

Q45.10 Advantages of fusion: high energy yield, no emission of greenhouse gases, fuel very easy to obtain, reactor cannot go supercritical like a fission reactor, low amounts of radioactive waste. Disadvantages: requires high energy input to sustain reaction, lithium and helium are scarce, neutrons released by reaction cause structural damage to reactor housing.

***Q45.11** Answer $Q_1 > Q_2 > Q_3 > 0$. Because all of the reactions involve 108 nucleons, we can look just at the change in binding-energy-per-nucleon as shown on the curve of binding energy. The jump from lithium to carbon is the biggest jump, and next the jump from $A = 27$ to $A = 54$, which is near the peak of the curve. The step up for fission from $A = 108$ to $A = 54$ is smallest. Both of the fusion reactions described and the fission reaction put out energy, so Q is positive for all. Imagine turning the curve of binding energy upside down so that it bends down like a cross-section of a bathtub. On such a curve of total energy per nucleon versus mass number it is easy to identify the fusion of small nuclei, the fission of large nuclei, and even the alpha decay of uranium, as exoenergetic processes. The most stable nucleus is at the drain of the bathtub, with minimum energy.

***Q45.12** Answer (d). The particles lose energy by collisions with nuclei in the bubble chamber to make their speed and their cyclotron radii $r = mv/qB$ decrease.

***Q45.13** Answer (d) only. The Geiger counter responds to an individual particle with a pulse whose size is determined by the tube power supply.

Q45.14 For each additional dynode, a larger applied voltage is needed, and hence a larger output from a power supply—"infinite" amplification would not be practical. Nor would it be desirable: the goal is to connect the tube output to a simple counter, so a massive pulse amplitude is not needed. If you made the detector sensitive to weaker and weaker signals, you would make it more and more sensitive to background noise.

***Q45.15** Answer (b). The cyclotron radius is given by $r = mv/qB = \sqrt{2\frac{1}{2}m^2v^2}/qB = \sqrt{2mK}/qB$. With the same K, $|q|$, and B for both particles, the electron with much smaller mass has smaller radius and is deflected more.

***Q45.16** Answer b > c > a > d. Dose (a) is 1 rem. Dose (b) is 10 rem. Doses (c) and (d) are 4 to 5 rem. If we assume that (a) and (b) as well as (c) were whole-body doses to many kilograms of tissue, we find the ranking stated.

Q45.17 Sometimes the references are indeed oblique. Some can serve for more than one form of energy or mode of transfer. Here is one list:

kinetic: ocean currents
rotational kinetic: Earth turning
gravitational: water lifted up; you on the perilous bridge
elastic: Elastic energy is necessary for sound, listed below.
Vibrational energy could be separately exemplified by the swaying bridge of land.
internal: lava in and from infernal volcanoes; or in forging a chain
chemical: corrosive smoke
sound: thunder
electrical transmission: lightning
electromagnetic radiation: heavens blazing; lightning
atomic electronic: In the blazing heavens, stars have different colors because of different predominant energy losses by atoms at their surfaces.
nuclear: The blaze of the heavens is produced by nuclear reactions in the cores of stars.

Remarkably, the word "energy" in this translation is an anachronism. Goethe wrote the song a few years before Thomas Young coined the term.

SOLUTIONS TO PROBLEMS

Section 45.1 **Interactions Involving Neutrons**

Section 45.2 **Nuclear Fission**

P45.1 The energy is

$$3.30\times10^{10}\text{ J}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right)\left(\frac{1\text{ U-235 nucleus}}{208\text{ MeV}}\right)\left(\frac{235\text{ g}}{6.02\times10^{23}\text{ nucleus}}\right)\left(\frac{\text{M}}{10^6}\right)$$

$$=\boxed{0.387\text{ g}}\text{ of U-235}$$

P45.2 $\Delta m=(m_n+M_{\text{U}})-(M_{\text{Zr}}+M_{\text{Te}}+3m_n)$

$$\Delta m=(1.008\,665\text{ u}+235.043\,923\text{ u})-(97.912\,7\text{ u}+134.916\,5\text{ u}+3(1.008\,665\text{ u}))$$

$$\Delta m=0.197\,39\text{ u}=3.28\times10^{-28}\text{ kg}\quad\text{so}\quad Q=\Delta mc^2=2.95\times10^{-11}\text{ J}=\boxed{184\text{ MeV}}$$

P45.3 Three different fission reactions are possible: ${}^{1}_{0}\text{n}+{}^{235}_{92}\text{U}\rightarrow{}^{90}_{38}\text{Sr}+{}^{144}_{54}\text{Xe}+2\,{}^{1}_{0}\text{n}$ $\boxed{{}^{144}_{54}\text{Xe}}$

${}^{1}_{0}\text{n}+{}^{235}_{92}\text{U}\rightarrow{}^{90}_{38}\text{Sr}+{}^{143}_{54}\text{Xe}+3\,{}^{1}_{0}\text{n}$ $\boxed{{}^{143}_{54}\text{Xe}}$ ${}^{1}_{0}\text{n}+{}^{235}_{92}\text{U}\rightarrow{}^{90}_{38}\text{Sr}+{}^{142}_{54}\text{Xe}+4\,{}^{1}_{0}\text{n}$ $\boxed{{}^{142}_{54}\text{Xe}}$

P45.4 (a) $Q=(\Delta m)c^2=\left[m_n+M_{\text{U235}}-M_{\text{Ba141}}-M_{\text{Kr92}}-3m_n\right]c^2$

$$\Delta m=\left[(1.008\,665+235.043\,923)-(140.914\,4+91.926\,2+3\times1.008\,665)\right]\text{u}=0.185\,993\text{ u}$$

$$Q=(0.185\,993\text{ u})(931.5\text{ MeV/u})=\boxed{173\text{ MeV}}$$

(b) $f=\dfrac{\Delta m}{m_i}=\dfrac{0.185993\text{ u}}{236.05\text{ u}}=7.88\times10^{-4}=\boxed{0.078\,8\%}$

P45.5 ${}^{1}_{0}\text{n}+{}^{232}_{90}\text{Th}\rightarrow{}^{233}_{90}\text{Th}\rightarrow{}^{233}_{91}\text{Pa}+\text{e}^-+\bar{\nu}$ ${}^{233}_{91}\text{Pa}\rightarrow{}^{233}_{92}\text{U}+\text{e}^-+\bar{\nu}$

P45.6 (a) The initial mass is $1.007\,825\text{ u}+11.009\,306\text{ u}=12.017\,131\text{ u}$. The final mass is $3(4.002\,603\text{ u})=12.007\,809\text{ u}$. The rest mass annihilated is $\Delta m=0.009\,322\text{ u}$. The energy created is $Q=\Delta mc^2=0.009\,322\text{ u}\left(\dfrac{931.5\text{ MeV}}{1\text{ u}}\right)=\boxed{8.68\text{ MeV}}$.

(b) The proton and the boron nucleus have positive charges. The colliding particles must have enough kinetic energy to approach very closely in spite of their electric repulsion.

P45.7 The available energy to do work is 0.200 times the energy content of the fuel.

$$(1.00\text{ kg fuel})(0.034\,0\ {}^{235}\text{U/fuel})\left(\frac{1\,000\text{ g}}{1\text{ kg}}\right)\left(\frac{1\text{ mol}}{235\text{ g}}\right)(6.02\times10^{23}/\text{mol})\left(\frac{(208)(1.60\times10^{-13}\text{ J})}{\text{fission}}\right)$$

$$=2.90\times10^{12}\text{ J}$$

$$(2.90\times10^{12}\text{ J})(0.200)=5.80\times10^{11}\text{ J}=(1.00\times10^{5}\text{ N})\Delta r$$

$$\Delta r=5.80\times10^{6}\text{ m}=\boxed{5.80\text{ Mm}}$$

P45.8 If the electrical power output of 1 000 MW is 40.0% of the power derived from fission reactions, the power output of the fission process is

$$\frac{1\,000 \text{ MW}}{0.400} = \left(2.50\times10^{9}\text{ J/s}\right)\left(8.64\times10^{4}\text{ s/d}\right) = 2.16\times10^{14}\text{ J/d}$$

The number of fissions per day is

$$\left(2.16\times10^{14}\text{ J/d}\right)\left(\frac{1\text{ fission}}{200\times10^{6}\text{ eV}}\right)\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 6.74\times10^{24}\text{ d}^{-1}$$

This also is the number of ^{235}U nuclei used, so the mass of ^{235}U used per day is

$$\left(6.74\times10^{24}\text{ nuclei/d}\right)\left(\frac{235\text{ g/mol}}{6.02\times10^{23}\text{ nuclei/mol}}\right) = 2.63\times10^{3}\text{ g/d} = \boxed{2.63\text{ kg/d}}$$

In contrast, a coal-burning steam plant producing the same electrical power uses more than 6×10^{6} kg/d of coal.

Section 45.3 **Nuclear Reactors**

P45.9 Mass of ^{235}U available $\approx (0.007)\left(10^{9}\text{ metric tons}\right)\left(\frac{10^{6}\text{ g}}{1\text{ metric ton}}\right) = 7\times10^{12}\text{ g}$

$$\text{number of nuclei} \approx \left(\frac{7\times10^{12}\text{ g}}{235\text{ g/mol}}\right)\left(6.02\times10^{23}\text{ nuclei/mol}\right) = 1.8\times10^{34}\text{ nuclei}$$

The energy available from fission (at 208 MeV/event) is

$$E \approx \left(1.8\times10^{34}\text{ events}\right)\left(208\text{ MeV/event}\right)\left(1.60\times10^{-13}\text{ J/MeV}\right) = 6.0\times10^{23}\text{ J}$$

This would last for a time interval of

$$\Delta t = \frac{E}{\mathcal{P}} \approx \frac{6.0\times10^{23}\text{ J}}{7.0\times10^{12}\text{ J/s}} = \left(8.6\times10^{10}\text{ s}\right)\left(\frac{1\text{ yr}}{3.16\times10^{7}\text{ s}}\right) \approx \boxed{3\,000\text{ yr}}$$

***P45.10** (a) For a sphere: $V = \frac{4}{3}\pi r^3$ and $r = \left(\frac{3V}{4\pi}\right)^{1/3}$ so $\frac{A}{V} = \frac{4\pi r^2}{(4/3)\pi r^3} = \boxed{4.84V^{-1/3}}$

(b) For a cube: $V = \ell^3$ and $\ell = V^{1/3}$ so $\frac{A}{V} = \frac{6\ell^2}{\ell^3} = \boxed{6V^{-1/3}}$

(c) For a parallelepiped: $V = 2a^3$ and $a = \left(\frac{V}{2}\right)^{1/3}$ so $\frac{A}{V} = \frac{\left(2a^2+8a^2\right)}{2a^3} = \boxed{6.30V^{-1/3}}$

(d) The answers show that the sphere has the smallest surface area for a given volume and the brick has the greatest surface area of the three. Therefore, the [sphere has the least leakage] and the [parallelepiped has the greatest leakage].

P45.11 In one minute there are $\frac{60.0\text{ s}}{1.20\text{ ms}} = 5.00\times10^4$ fissions.

So the rate increases by a factor of $(1.000\,25)^{50\,000} = \boxed{2.68\times10^5}$.

P45.12 $\mathcal{P} = 10.0\text{ MW} = 1.00\times10^7\text{ J/s}$. If each decay delivers $1.00\text{ MeV} = 1.60\times10^{-13}\text{ J}$, then the

number of decays/s $= \frac{10^7\text{ J/s}}{1.6\times10^{-13}\text{ J}} = \boxed{6.25\times10^{19}\text{ Bq}}$.

***P45.13** (a) Since $K = p^2/2m$, we have

$$p = \sqrt{2mK} = \sqrt{2m\tfrac{3}{2}k_BT} = \sqrt{3(1.67\times10^{-27}\text{ kg})(1.38\times10^{-23}\text{ J/K})\,300\text{ K}}$$

$$= \boxed{4.55\times10^{-24}\text{ kg}\cdot\text{m/s}}$$

(b) $\lambda = h/p = 6.63\times10^{-34}\text{ J}\cdot\text{s}/4.55\times10^{-24}\text{ kg}\cdot\text{m/s} = \boxed{0.146\text{ nm}}$. This size has the same order of magnitude as an atom's outer electron cloud, and is vastly larger than a nucleus.

***P45.14** We take the radii of the helium and gold nuclei as 1.20 fm $4^{1/3}$ = 1.90 fm and 1.20 fm $197^{1/3}$ = 6.98 fm. The center to center distance is then 8.89 fm and the electric potential energy is

$$U = qV = \frac{k_e q_1 q_2}{r} = \frac{8.99\times10^9\text{ N}\cdot\text{m}^2\ 2\times1.6\times10^{-19}\text{C}\ 79\,e}{\text{C}^2\ 8.89\times10^{-15}\text{ m}} = \boxed{25.6\text{ MeV}}$$

Section 45.4 **Nuclear Fusion**

P45.15 (a) The Q value for the D-T reaction is 17.59 MeV.
Specific energy content in fuel for D-T reaction:

$$\frac{(17.59\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{(5\text{ u})(1.66\times10^{-27}\text{ kg/u})} = 3.39\times10^{14}\text{ J/kg}$$

$$r_{\text{DT}} = \frac{(3.00\times10^9\text{ J/s})(3\,600\text{ s/hr})}{(3.39\times10^{14}\text{ J/kg})(10^{-3}\text{ kg/g})} = \boxed{31.9\text{ g/h burning of D and T}}$$

(b) Specific energy content in fuel for D-D reaction: $Q = \frac{1}{2}(3.27+4.03) = 3.65\text{ MeV}$ average of two Q values

$$\frac{(3.65\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{(4\text{ u})(1.66\times10^{-27}\text{ kg/u})} = 8.80\times10^{13}\text{ J/kg}$$

$$r_{\text{DD}} = \frac{(3.00\times10^9\text{ J/s})(3\,600\text{ s/hr})}{(8.80\times10^{13}\text{ J/kg})(10^{-3}\text{ kg/g})} = \boxed{122\text{ g/h burning of D}}$$

***P45.16** (a) We assume that the nuclei are stationary at closest approach, so that the electrostatic potential energy equals the total energy E.

$$U_f = \frac{k_e(Z_1e)(Z_2e)}{r_{\min}} = E$$

$$E = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2 Z_1Z_2}{1.00\times10^{-14}\ \text{m}} = \boxed{(2.30\times10^{-14}\ \text{J})Z_1Z_2} = (144\ \text{keV})\,Z_1Z_2$$

(b) The energy is proportional to each atomic number.

(c) Take $Z_1 = 1$ and $Z_2 = 59$ or vice versa. This choice minimizes the product $Z_1 Z_2$. If extra cleverness is allowed, take $Z_1 = 0$ and $Z_2 = 60$: use neutrons as the bombarding particles. A neutron is a nucleon but not an atomic nucleus.

(d) For both the D-D and the D-T reactions, $Z_1 = Z_2 = 1$. Thus, the minimum energy required in both cases is

$$E = (2.30\times10^{-14}\ \text{J})\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right) = \boxed{144\ \text{keV}}$$

Section 45.4 in the text gives more accurate values for the critical ignition temperatures, of about 52 keV for D-D fusion and 6 keV for D-T fusion. The nuclei can fuse by tunneling. A triton moves more slowly than a deuteron at a given temperature. Then D-T collisions last longer than D-D collisions and have much greater tunneling probabilities.

P45.17 (a) $r_f = r_\text{D} + r_\text{T} = (1.20\times10^{-15}\ \text{m})\left[(2)^{1/3} + (3)^{1/3}\right] = \boxed{3.24\times10^{-15}\ \text{m}}$

(b) $$U_f = \frac{k_e e^2}{r_f} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{3.24\times10^{-15}\ \text{m}} = 7.10\times10^{-14}\ \text{J} = \boxed{444\ \text{keV}}$$

(c) Conserving momentum, $m_\text{D}v_i = (m_\text{D} + m_\text{T})v_f$ or $v_f = \left(\frac{m_\text{D}}{m_\text{D}+m_\text{T}}\right)v_i = \boxed{\frac{2}{5}v_i}$

(d) $K_i + U_i = K_f + U_f$: $\quad K_i + 0 = \frac{1}{2}(m_\text{D}+m_\text{T})v_f^2 + U_f = \frac{1}{2}(m_\text{D}+m_\text{T})\left(\frac{m_\text{D}}{m_\text{D}+m_\text{T}}\right)^2 v_i^2 + U_f$

$$K_i + 0 = \left(\frac{m_\text{D}}{m_\text{D}+m_\text{T}}\right)\left(\frac{1}{2}m_\text{D}v_i^2\right) + U_f = \left(\frac{m_\text{D}}{m_\text{D}+m_\text{T}}\right)K_i + U_f$$

$\left(1 - \frac{m_\text{D}}{m_\text{D}+m_\text{T}}\right)K_i = U_f$: $\quad K_i = U_f\left(\frac{m_\text{D}+m_\text{T}}{m_\text{T}}\right) = \frac{5}{3}(444\ \text{keV}) = \boxed{740\ \text{keV}}$

(e) The nuclei can fuse by tunneling through the potential-energy barrier.

P45.18 (a) $V = \left(317\times10^{6}\ \text{mi}^3\right)\left(\dfrac{1\,609\ \text{m}}{1\ \text{mi}}\right)^3 = 1.32\times10^{18}\ \text{m}^3$

$$m_{\text{water}} = \rho V = \left(10^3\ \text{kg/m}^3\right)\left(1.32\times10^{18}\ \text{m}^3\right) = 1.32\times10^{21}\ \text{kg}$$

$$m_{H_2} = \left(\frac{M_{H_2}}{M_{H_2O}}\right)m_{H_2O} = \left(\frac{2.016}{18.015}\right)\left(1.32\times10^{21}\ \text{kg}\right) = 1.48\times10^{20}\ \text{kg}$$

$$m_{\text{Deuterium}} = (0.030\,0\%)\,m_{H_2} = \left(0.030\,0\times10^{-2}\right)\left(1.48\times10^{20}\ \text{kg}\right) = 4.43\times10^{16}\ \text{kg}$$

The number of deuterium nuclei in this mass is

$$N = \frac{m_{\text{Deuterium}}}{m_{\text{Deuteron}}} = \frac{4.43\times10^{16}\ \text{kg}}{(2.014\ \text{u})\left(1.66\times10^{-27}\ \text{kg/u}\right)} = 1.33\times10^{43}$$

Since two deuterium nuclei are used per fusion, ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^4_2\text{He} + Q/c^2$, the number of events is $\dfrac{N}{2} = 6.63\times10^{42}$.

The energy released per event is

$$Q = \left[M_{^2\text{H}} + M_{^2\text{H}} - M_{^4\text{He}}\right]c^2 = \left[2(2.014\,102) - 4.002\,603\right]\text{u}(931.5\ \text{MeV/u}) = 23.8\ \text{MeV}$$

The total energy available is then

$$E = \left(\frac{N}{2}\right)Q = \left(6.63\times10^{42}\right)(23.8\ \text{MeV})\left(\frac{1.60\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right) = \boxed{2.53\times10^{31}\ \text{J}}$$

(b) The time this energy could possibly meet world requirements is

$$\Delta t = \frac{E}{\mathcal{P}} = \frac{2.53\times10^{31}\ \text{J}}{100\left(7.00\times10^{12}\ \text{J/s}\right)} = \left(3.61\times10^{16}\ \text{s}\right)\left(\frac{1\ \text{yr}}{3.16\times10^{7}\ \text{s}}\right)$$

$$= \boxed{1.14\times10^{9}\ \text{yr}} \sim 1\ \text{billion years}$$

P45.19 (a) Average *KE* per particle is $\dfrac{3}{2}k_BT = \dfrac{1}{2}mv^2$.

Therefore, $v_{\text{rms}} = \sqrt{\dfrac{3k_BT}{m}} = \sqrt{\dfrac{3\left(1.38\times10^{-23}\ \text{J/K}\right)\left(4.00\times10^{8}\ \text{K}\right)}{2.014\left(1.661\times10^{-27}\ \text{kg}\right)}} = \boxed{2.22\times10^{6}\ \text{m/s}}$

(b) $t = \dfrac{x}{v} \sim \dfrac{0.1\ \text{m}}{10^6\ \text{m/s}} = \boxed{\sim10^{-7}\ \text{s}}$

P45.20 (a) Including both ions and electrons, the number of particles in the plasma is $N = 2nV$ where n is the ion density and V is the volume of the container. Application of Equation 21.6 gives the total energy as

$$E = \frac{3}{2}Nk_\mathrm{B}T = 3nVk_\mathrm{B}T$$

$$= 3\left(2.0\times10^{13}\ \mathrm{cm^{-3}}\right)\left[\left(50\ \mathrm{m^3}\right)\left(\frac{10^6\ \mathrm{cm^3}}{1\ \mathrm{m^3}}\right)\right]\left(1.38\times10^{-23}\ \mathrm{J/K}\right)\left(4.0\times10^8\ \mathrm{K}\right)$$

$$E = \boxed{1.7\times10^7\ \mathrm{J}}$$

(b) From Table 20.2, the heat of vaporization of water is $L_v = 2.26\times10^6\ \mathrm{J/kg}$. The mass of water that could be boiled away is

$$m = \frac{E}{L_v} = \frac{1.7\times10^7\ \mathrm{J}}{2.26\times10^6\ \mathrm{J/kg}} = \boxed{7.3\ \mathrm{kg}}$$

P45.21 (a) Lawson's criterion for the D-T reaction is $n\tau \geq 10^{14}\ \mathrm{s/cm^3}$. For a confinement time of $\tau = 1.00\ \mathrm{s}$, this requires a minimum ion density of $n = \boxed{10^{14}\ \mathrm{cm^{-3}}}$.

(b) At the ignition temperature of $T = 4.5\times10^7\ \mathrm{K}$ and the ion density found above, the plasma pressure is

$$P = 2nk_\mathrm{B}T = 2\left[\left(10^{14}\ \mathrm{cm^{-3}}\right)\left(\frac{10^6\ \mathrm{cm^3}}{1\ \mathrm{m^3}}\right)\right]\left(1.38\times10^{-23}\ \mathrm{J/K}\right)\left(4.5\times10^7\ \mathrm{K}\right)$$

$$= \boxed{1.24\times10^5\ \mathrm{J/m^3}}$$

(c) The required magnetic energy density is then

$$u_B = \frac{B^2}{2\mu_0} \geq 10P = 10\left(1.24\times10^5\ \mathrm{J/m^3}\right) = 1.24\times10^6\ \mathrm{J/m^3}$$

$$B \geq \sqrt{2\left(4\pi\times10^{-7}\ \mathrm{N/A^2}\right)\left(1.24\times10^6\ \mathrm{J/m^3}\right)} = \boxed{1.77\ \mathrm{T}}$$ This is a very strong field.

P45.22 The number of nuclei in 1.00 metric ton of trash is

$$N = 1\,000\ \mathrm{kg}\left(1\,000\ \mathrm{g/kg}\right)\frac{6.02\times10^{23}\ \mathrm{nuclei/mol}}{56.0\ \mathrm{g/mol}} = 1.08\times10^{28}\ \mathrm{nuclei}$$

At an average charge of 26.0 e/nucleus, $q = \left(1.08\times10^{28}\right)(26.0)\left(1.60\times10^{-19}\right) = 4.47\times10^{10}\ \mathrm{C}$

Therefore $t = \dfrac{q}{I} = \dfrac{4.47\times10^{10}}{1.00\times10^6} = 4.47\times10^4\ \mathrm{s} = \boxed{12.4\ \mathrm{h}}$

Section 45.5 **Radiation Damage**

P45.23 $N_0 = \dfrac{\text{mass present}}{\text{mass of nucleus}} = \dfrac{5.00 \text{ kg}}{(89.907\,7 \text{ u})(1.66\times10^{-27} \text{ kg/u})} = 3.35\times10^{25} \text{ nuclei}$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{29.1 \text{ yr}} = 2.38\times10^{-2} \text{ yr}^{-1} = 4.53\times10^{-8} \text{ min}^{-1}$$

$$R_0 = \lambda N_0 = (4.53\times10^{-8} \text{ min}^{-1})(3.35\times10^{25}) = 1.52\times10^{18} \text{ counts/min}$$

$$\frac{R}{R_0} = e^{-\lambda t} = \frac{10.0 \text{ counts/min}}{1.52\times10^{18} \text{ counts/min}} = 6.59\times10^{-18}$$

and $\lambda t = -\ln(6.59\times10^{-18}) = 39.6$

giving $t = \dfrac{39.6}{\lambda} = \dfrac{39.6}{2.38\times10^{-2} \text{ yr}^{-1}} = \boxed{1.66\times10^{3} \text{ yr}}$

P45.24 The source delivers 100 mrad of 2-MeV γ-rays/h at a 1.00-m distance.

(a) For γ-rays, dose in rem = dose in rad.
Thus a person would have to stand there $\boxed{\text{10.0 hours}}$ to receive 1.00 rem from a 100-mrad/h source.

(b) If the γ-radiation is emitted isotropically, the dosage rate falls off as $\dfrac{1}{r^2}$.

Thus a dosage 10.0 mrad/h would be received at a distance $r = \sqrt{10.0} \text{ m} = \boxed{3.16 \text{ m}}$.

P45.25 (a) The number of x-ray images made per year is

$n = (8 \text{ x-ray/d})(5 \text{ d/wk})(50 \text{ wk/yr}) = 2.0\times10^{3} \text{ x-ray/yr}$

The average dose per photograph is $\dfrac{5.0 \text{ rem/yr}}{2.0\times10^{3} \text{ x-ray/yr}} = \boxed{2.5\times10^{-3} \text{ rem/x-ray image}}$

(b) The technician receives low-level background radiation at a rate of 0.13 rem/yr. The dose of 5.0 rem/yr received as a result of the job is

$\dfrac{5.0 \text{ rem/yr}}{0.13 \text{ rem/yr}} = \boxed{\text{38 times the assumed background level}}$. The technician's occupational exposure is high compared to background radiation.

P45.26 (a) $I = I_0 e^{-\mu x}$, so $x = \dfrac{1}{\mu}\ln\left(\dfrac{I_0}{I}\right)$

With $\mu = 1.59 \text{ cm}^{-1}$, the thickness when $I = \dfrac{I_0}{2}$ is $x = \dfrac{1}{1.59 \text{ cm}^{-1}}\ln(2) = \boxed{0.436 \text{ cm}}$.

(b) When $\dfrac{I_0}{I} = 1.00\times10^{4}$, $x = \dfrac{1}{1.59 \text{ cm}^{-1}}\ln(1.00\times10^{4}) = \boxed{5.79 \text{ cm}}$.

P45.27 $1\text{ rad} = 10^{-2}\ \text{J/kg}$ $\quad Q = mc\Delta T \quad \mathcal{P}\Delta t = mc\Delta T$

$$\Delta t = \frac{mc\Delta T}{\mathcal{P}} = \frac{m(4\,186\ \text{J/kg}\cdot{}^{\circ}\text{C})(50.0^{\circ}\text{C})}{(10)(10^{-2}\ \text{J/kg}\cdot\text{s})(m)} = \boxed{2.09\times10^{6}\ \text{s}} \approx 24\ \text{days!}$$

Note that power is the product of dose rate and mass.

P45.28 $$\frac{Q}{m} = \frac{\text{absorbed energy}}{\text{unit mass}} = (1\,000\ \text{rad})\frac{10^{-2}\ \text{J/kg}}{1\ \text{rad}} = 10.0\ \text{J/kg}$$

The rise in body temperature is calculated from $Q = mc\Delta T$ where $c = 4\,186\ \text{J/kg}$ for water and the human body

$$\Delta T = \frac{Q}{mc} = (10.0\ \text{J/kg})\frac{1}{4\,186\ \text{J/kg}\cdot{}^{\circ}\text{C}} = \boxed{2.39\times10^{-3}\,{}^{\circ}\text{C}}$$ The temperature change is negligible.

P45.29 If half of the 0.140-MeV gamma rays are absorbed by the patient, the total energy absorbed is

$$E = \frac{(0.140\ \text{MeV})}{2}\left[\left(\frac{1.00\times10^{-8}\ \text{g}}{98.9\ \text{g/mol}}\right)\left(\frac{6.02\times10^{23}\ \text{nuclei}}{1\ \text{mol}}\right)\right] = 4.26\times10^{12}\ \text{MeV}$$

$$E = (4.26\times10^{12}\ \text{MeV})(1.60\times10^{-13}\ \text{J/MeV}) = 0.682\ \text{J}$$

Thus, the dose received is $\quad \text{Dose} = \dfrac{0.682\ \text{J}}{60.0\ \text{kg}}\left(\dfrac{1\ \text{rad}}{10^{-2}\ \text{J/kg}}\right) = \boxed{1.14\ \text{rad}}$

***P45.30** The decay constant is $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{17\ \text{d}} = 0.040\,8/\text{d}$. The number of nuclei remaining after 30 d is $N = N_0 e^{-\lambda T} = N_0 e^{-0.040\,8(30)} = 0.294 N_0$. The number decayed is $N_0 - N = N_0(1 - 0.294) = 0.706 N_0$. Then the energy release is

$$2.12\ \text{J} = 0.706 N_0 (21.0\times10^{3}\ \text{eV})\left(\frac{1.6\times10^{-19}\ \text{J}}{1\ \text{eV}}\right)$$

$$N_0 = \frac{2.12\ \text{J}}{2.37\times10^{-15}\ \text{J}} = 8.94\times10^{14}$$

(a) $\quad R_0 = \lambda N_0 = \dfrac{0.040\,8}{\text{d}}(8.94\times10^{14})\left(\dfrac{1\ \text{d}}{86\,400\ \text{s}}\right) = \boxed{4.22\times10^{8}\ \text{Bq}}$

(b) $\quad \text{original sample mass} = m = N_{\text{original}} m_{\text{one atom}} = 8.94\times10^{14}(103\ \text{u})\left(\dfrac{1.66\times10^{-27}\ \text{kg}}{1\ \text{u}}\right)$

$$= \boxed{1.53\times10^{-10}\ \text{kg}} = 1.53\times10^{-7}\ \text{g} = 153\ \text{ng}$$

P45.31 The nuclei initially absorbed are $N_0 = \left(1.00\times10^{-9}\text{ g}\right)\left(\dfrac{6.02\times10^{23}\text{ nuclei/mol}}{89.9\text{ g/mol}}\right) = 6.70\times10^{12}$

The number of decays in time t is $\Delta N = N_0 - N = N_0\left(1-e^{-\lambda t}\right) = N_0\left(1-e^{-(\ln 2)t/T_{1/2}}\right)$

At the end of 1 year, $\dfrac{t}{T_{1/2}} = \dfrac{1.00\text{ yr}}{29.1\text{ yr}} = 0.034\,4$

and $\Delta N = N_0 - N = \left(6.70\times10^{12}\right)\left(1-e^{-0.023\,8}\right) = 1.58\times10^{11}$

The energy deposited is $E = \left(1.58\times10^{11}\right)(1.10\text{ MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right) = 0.027\,7\text{ J}$

Thus, the dose received is $\text{Dose} = \left(\dfrac{0.027\,7\text{ J}}{70.0\text{ kg}}\right) = \boxed{3.96\times10^{-4}\text{ J/kg}} = 0.039\,6\text{ rad}$

Section 45.6 **Radiation Detectors**

P45.32 (a) $E_I = 10.0\text{ eV}$ is the energy required to liberate an electron from a dynode. Let n_i be the number of electrons incident upon a dynode, each having gained energy $e(\Delta V)$ as it was accelerated to this dynode. The number of electrons that will be freed from this dynode is $N_i = n_i e\dfrac{\Delta V}{E_I}$:

At the first dynode, $n_i = 1$ and $N_1 = \dfrac{(1)e(100\text{ V})}{10.0\text{ eV}} = \boxed{10^1\text{ electrons}}$

(b) For the second dynode, $n_i = N_1 = 10^1$, so $N_2 = \dfrac{(10^1)e(100\text{ V})}{10.0\text{ eV}} = 10^2$

At the third dynode, $n_i = N_2 = 10^2$ and $N_3 = \dfrac{(10^2)e(100\text{ V})}{10.0\text{ eV}} = 10^3$

Observing the developing pattern, we see that the number of electrons incident on the seventh and last dynode is $n_7 = N_6 = \boxed{10^6}$.

(c) The number of electrons incident on the last dynode is $n_7 = 10^6$. The total energy these electrons deliver to that dynode is given by

$$E = n_i e(\Delta V) = 10^6 e(700\text{ V} - 600\text{ V}) = \boxed{10^8\text{ eV}}$$

P45.33 (a) $$\frac{E}{E_\beta}=\frac{(1/2)C(\Delta V)^2}{0.500\text{ MeV}}=\frac{(1/2)\left(5.00\times10^{-12}\text{ F}\right)\left(1.00\times10^{3}\text{ V}\right)^2}{(0.500\text{ MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right)}=\boxed{3.12\times10^{7}}$$

(b) $$N=\frac{Q}{e}=\frac{C(\Delta V)}{e}=\frac{\left(5.00\times10^{-12}\text{ F}\right)\left(1.00\times10^{3}\text{ V}\right)}{1.60\times10^{-19}\text{ C}}=\boxed{3.12\times10^{10}\text{ electrons}}$$

P45.34 (a) The average time between slams is $60\text{ min}/38=1.6\text{ min}$. Sometimes, the actual interval is nearly zero. Perhaps about equally as often, it is $2\times1.6\text{ min}$. Perhaps about half as often, it is $4\times1.6\text{ min}$. Somewhere around $5\times1.6\text{ min}=\boxed{8.0\text{ min}}$, the chances of randomness producing so long a wait get slim, so such a long delay might likely be due to mischief.

(b) The midpoints of the time intervals are separated by 5.00 minutes. We use $R=R_0e^{-\lambda t}$. Subtracting the background counts,

$$337-5(15)=\left[372-5(15)\right]e^{-\left(\ln 2/T_{1/2}\right)(5.00\text{ min})}$$

or $$\ln\left(\frac{262}{297}\right)=\ln(0.882)=-3.47\text{ min}/T_{1/2}\text{ which yields }T_{1/2}=\boxed{27.6\text{ min}}$$

(c) As in the random events in part (a), we imagine a ±5 count counting uncertainty. The smallest likely value for the half-life is then given by

$$\ln\left(\frac{262-5}{297+5}\right)=-3.47\text{ min}/T_{1/2},\text{ or }\left(T_{1/2}\right)_{\min}=21.1\text{ min}$$

The largest credible value is found from

$$\ln\left(\frac{262+5}{297-5}\right)=-3.47\text{ min}/T_{1/2},\text{ yielding }\left(T_{1/2}\right)_{\max}=38.8\text{ min}$$

Thus, $$T_{1/2}=\left(\frac{38.8+21.1}{2}\right)\pm\left(\frac{38.8-21.1}{2}\right)\text{ min}=(30\pm9)\text{ min}=\boxed{30\text{ min}\pm30\%}$$

Section 45.7 **Uses of Radiation**

P45.35 The initial specific activity of ^{59}Fe in the steel,

$$(R/m)_0=\frac{20.0\ \mu\text{Ci}}{0.200\text{ kg}}=\frac{100\ \mu\text{Ci}}{\text{kg}}\left(\frac{3.70\times10^{4}\text{ Bq}}{1\ \mu\text{Ci}}\right)=3.70\times10^{6}\text{ Bq/kg}$$

After 1 000 h, $$\frac{R}{m}=\left(\frac{R}{m}\right)_0e^{-\lambda t}=\left(3.70\times10^{6}\text{ Bq/kg}\right)e^{-\left(6.40\times10^{-4}\text{ h}^{-1}\right)(1\,000\text{ h})}=1.95\times10^{6}\text{ Bq/kg}$$

The activity of the oil is $$R_{\text{oil}}=\left(\frac{800}{60.0}\text{ Bq/liter}\right)(6.50\text{ liters})=86.7\text{ Bq}$$

Therefore $$m_{\text{in oil}}=\frac{R_{\text{oil}}}{(R/m)}=\frac{86.7\text{ Bq}}{1.95\times10^{6}\text{ Bq/kg}}=4.45\times10^{-5}\text{ kg}$$

So that wear rate is $$\frac{4.45\times10^{-5}\text{ kg}}{1\,000\text{ h}}=\boxed{4.45\times10^{-8}\text{ kg/h}}$$

P45.36 (a) The number of photons is $\frac{10^4 \text{ MeV}}{1.04 \text{ MeV}} = 9.62 \times 10^3$. Since only 50% of the photons are detected, the number of ^{65}Cu nuclei decaying is twice this value, or 1.92×10^4. In two half-lives, three-fourths of the original nuclei decay, so $\frac{3}{4} N_0 = 1.92 \times 10^4$ and $N_0 = 2.56 \times 10^4$. This is 1% of the ^{65}Cu, so the number of ^{65}Cu is 2.56×10^6 $\boxed{\sim 10^6}$.

(b) Natural copper is 69.17% ^{63}Cu and 30.83% ^{63}Cu. Thus, if the sample contains N_{Cu} copper atoms, the number of atoms of each isotope is

$N_{63} = 0.691\,7 N_{\text{Cu}}$ and $N_{65} = 0.308\,3 N_{\text{Cu}}$

Therefore, $\frac{N_{63}}{N_{65}} = \frac{0.691\,7}{0.308\,3}$ or $N_{63} = \left(\frac{0.6917}{0.3083}\right) N_{65} = \left(\frac{0.6917}{0.3083}\right)\left(2.56 \times 10^6\right) = 5.75 \times 10^6$.

The total mass of copper present is then $m_{\text{Cu}} = (62.93 \text{ u}) N_{63} + (64.93 \text{ u}) N_{65}$:

$$m_{\text{Cu}} = \left[(62.93)\left(5.75 \times 10^6\right) + (64.93)\left(2.56 \times 10^6\right)\right] \text{u} \left(1.66 \times 10^{-24} \text{ g/u}\right)$$
$$= 8.77 \times 10^{-16} \text{ g} \quad \boxed{\sim 10^{-15} \text{ g}}$$

P45.37 (a) Starting with $N = 0$ radioactive atoms at $t = 0$, the rate of increase is (production – decay)

$$\frac{dN}{dt} = R - \lambda N \qquad \text{so} \qquad dN = (R - \lambda N)\,dt$$

The variables are separable.

$$\int_0^N \frac{dN}{R - \lambda N} = \int_0^t dt: \qquad -\frac{1}{\lambda} \ln\left(\frac{R - \lambda N}{R}\right) = t$$

so $$\ln\left(\frac{R - \lambda N}{R}\right) = -\lambda t \quad \text{and} \quad \left(\frac{R - \lambda N}{R}\right) = e^{-\lambda t}$$

Therefore $$1 - \frac{\lambda}{R} N = e^{-\lambda t} \qquad N = \boxed{\frac{R}{\lambda}\left(1 - e^{-\lambda t}\right)}$$

(b) The maximum number of radioactive nuclei would be $\boxed{\frac{R}{\lambda}}$.

***P45.38** (a) With $I(x) = \frac{1}{2} I_0$, $I(x) = I_0 e^{-\mu x}$ becomes

$$\frac{1}{2} I_0 = I_0 e^{-0.72x/\text{mm}} \qquad 2 = e^{+0.72x/\text{mm}}$$

$$\ln 2 = 0.72\,x/\text{mm} \qquad x = \frac{\ln 2 \text{ mm}}{0.72} = \boxed{0.963 \text{ mm}}$$

(b) $$I(0.8 \text{ mm}) = I_0 e^{-0.72(0.8)} = 0.562 I_0$$

$$I(0.7 \text{ mm}) = I_0 e^{-0.72(0.7)} = 0.604 I_0$$

$$\text{fractional change} = \frac{0.604 I_0 - 0.562 I_0}{0.562 I_0} = 0.074\ 7 = \boxed{7.47\%}$$

Additional Problems

P45.39 (a) At 6×10^8 K, the average kinetic energy of a carbon atom is

$$\frac{3}{2} k_B T = (1.5)(8.62 \times 10^{-5} \text{ eV/K})(6 \times 10^8 \text{ K}) = \boxed{8 \times 10^4 \text{ eV}}$$

Note that 6×10^8 K is about $6^2 = 36$ times larger than 1.5×10^7 K, the core temperature of the Sun. This factor corresponds to the higher potential-energy barrier to carbon fusion compared to hydrogen fusion. It could be misleading to compare it to the temperature $\sim 10^8$ K required for fusion in a low-density plasma in a fusion reactor.

(b) The energy released is

$$E = \left[2m\left(\text{C}^{12}\right) - m\left(\text{Ne}^{20}\right) - m\left(\text{He}^4\right)\right]c^2$$

$$E = (24.000\ 000 - 19.992\ 440 - 4.002\ 603)(931.5) \text{ MeV} = \boxed{4.62 \text{ MeV}}$$

In the second reaction,

$$E = \left[2m\left(\text{C}^{12}\right) - m\left(\text{Mg}^{24}\right)\right](931.5) \text{ MeV/u}$$

$$E = (24.000\ 000 - 23.985\ 042)(931.5) \text{ MeV} = \boxed{13.9 \text{ MeV}}$$

(c) The energy released is the energy of reaction of the number of carbon nuclei in a 2.00-kg sample, which corresponds to

$$\Delta E = \left(2.00 \times 10^3 \text{ g}\right)\left(\frac{6.02 \times 10^{23} \text{ atoms/mol}}{12.0 \text{ g/mol}}\right)\left(\frac{4.62 \text{ MeV/fusion event}}{2 \text{ nuclei/fusion event}}\right)\left(\frac{1 \text{ kWh}}{2.25 \times 10^{19} \text{ MeV}}\right)$$

$$\Delta E = \frac{\left(1.00 \times 10^{26}\right)(4.62)}{2\left(2.25 \times 10^{19}\right)} \text{kWh} = \boxed{1.03 \times 10^7 \text{ kWh}}$$

P45.40 To conserve momentum, the two fragments must move in opposite directions with speeds v_1 and v_2 such that

$$m_1 v_1 = m_2 v_2 \qquad \text{or} \qquad v_2 = \left(\frac{m_1}{m_2}\right) v_1$$

The kinetic energies after the break-up are then

$$K_1 = \frac{1}{2} m_1 v_1^2 \qquad \text{and} \qquad K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_2 \left(\frac{m_1}{m_2}\right)^2 v_1^2 = \left(\frac{m_1}{m_2}\right) K_1$$

The fraction of the total kinetic energy carried off by m_1 is

$$\frac{K_1}{K_1 + K_2} = \frac{K_1}{K_1 + (m_1/m_2) K_1} = \boxed{\frac{m_2}{m_1 + m_2}}$$

and the fraction carried off by m_2 is $1 - \frac{m_2}{m_1 + m_2} = \boxed{\frac{m_1}{m_1 + m_2}}$.

P45.41 (a) $Q = 236.045\ 562\text{u}\,c^2 - 86.920\ 711\text{u}\,c^2 - 148.934\ 370\text{u}\,c^2 = 0.190\ 481\text{u}\,c^2 = \boxed{177 \text{ MeV}}$

Immediately after fission, this Q-value is the total kinetic energy of the fission products.

(b) $K_{\text{Br}} = \left(\frac{m_{\text{La}}}{m_{\text{Br}} + m_{\text{La}}}\right) Q$, from Problem 45.40

$$= \left(\frac{149 \text{ u}}{87 \text{ u} + 149 \text{ u}}\right)(177.4 \text{ MeV}) = \boxed{112 \text{ MeV}}$$

$$K_{\text{La}} = Q - K_{\text{Br}} = 177.4 \text{ MeV} - 112.0 \text{ MeV} = \boxed{65.4 \text{ MeV}}$$

(c) $$v_{\text{Br}} = \sqrt{\frac{2K_{\text{Br}}}{m_{\text{Br}}}} = \sqrt{\frac{2(112 \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{(87 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})}} = \boxed{1.58 \times 10^7 \text{ m/s}}$$

$$v_{\text{La}} = \sqrt{\frac{2K_{\text{La}}}{m_{\text{La}}}} = \sqrt{\frac{2(65.4 \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{(149 \text{ u})(1.66 \times 10^{-27} \text{ kg/u})}} = \boxed{9.20 \times 10^6 \text{ m/s}}$$

P45.42 For a typical ^{235}U fission, $Q = 208 \text{ MeV}$ and the initial mass is 235 u. Thus, the fractional energy loss is

$$\frac{Q}{mc^2} = \frac{208 \text{ MeV}}{(235 \text{ u})(931.5 \text{ MeV/u})} = 9.50 \times 10^{-4} = \boxed{0.095\,0\%}$$

For the D-T fusion reaction, $Q = 17.6 \text{ MeV}$

The initial mass is $m = (2.014 \text{ u}) + (3.016 \text{ u}) = 5.03 \text{ u}$

The fractional loss in this reaction is $\frac{Q}{mc^2} = \frac{17.6 \text{ MeV}}{(5.03 \text{ u})(931.5 \text{ MeV/u})} = 3.75 \times 10^{-3} = \boxed{0.375\%}$

$\frac{0.375\%}{0.0950\%} = 3.95$ or $\boxed{\text{the fractional loss in D-T fusion is about 4 times that in } ^{235}\text{U fission}}$.

P45.43 The decay constant is $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(12.3\text{ yr})(3.16\times 10^{7}\text{ s/yr})} = 1.78\times 10^{-9}\text{ s}^{-1}$.

The tritium in the plasma decays at a rate of

$$R = \lambda N = (1.78\times 10^{-9}\text{ s}^{-1})\left[\left(\frac{2.00\times 10^{14}}{\text{cm}^3}\right)\left(\frac{10^6\text{ cm}^3}{1\text{ m}^3}\right)(50.0\text{ m}^3)\right]$$

$$R = 1.78\times 10^{13}\text{ Bq} = (1.78\times 10^{13}\text{ Bq})\left(\frac{1\text{ Ci}}{3.70\times 10^{10}\text{ Bq}}\right) = \boxed{482\text{ Ci}}$$

$\boxed{\text{The fission inventory is } \frac{4\times 10^{10}\text{ Ci}}{482\text{ Ci}} \sim 10^8 \text{ times greater}}$ than this amount.

***P45.44** The original activity per area is $\frac{5\times 10^6\text{ Ci}}{10^4\text{ km}^2}\left(\frac{1\text{ km}}{10^3\text{ m}}\right)^2 = 5\times 10^{-4}\text{ Ci/m}^2$.

The decay constant is $\lambda = \ln 2/27.7$ yr.
The decay law $N = N_0\, e^{-\lambda t}$ becomes the law of decrease of activity $R = R_0\, e^{-\lambda t}$. If the material is not transported, it describes the time evolution of activity per area $R/A = R_0/A\, e^{-\lambda t}$. Solving gives

$$e^{\lambda t} = \frac{R_0/A}{R/A} \qquad \lambda t = \ln\left(\frac{R_0/A}{R/A}\right) = \frac{\ln 2}{27.7\text{ yr}}t$$

$$t = \frac{27.7\text{ yr}}{\ln 2}\ln\left(\frac{R_0/A}{R/A}\right) = \frac{27.7\text{ yr}}{\ln 2}\ln\left(\frac{5\times 10^{-4}\text{ Ci/m}^2}{2\times 10^{-6}\text{ Ci/m}^2}\right) = \boxed{221\text{ yr}}$$

P45.45 The complete fissioning of 1.00 gram of U^{235} releases

$$Q = \frac{(1.00\text{ g})}{235\text{ grams/mol}}(6.02\times 10^{23}\text{ atoms/mol})(200\text{ MeV/fission})(1.60\times 10^{-13}\text{ J/MeV})$$
$$= 8.20\times 10^{10}\text{ J}$$

If all this energy could be utilized to convert m kilograms of 20.0°C water to 400°C steam (see Chapter 20 of text for values),

then $$Q = mc_w\,\Delta T + mL_v + mc_s\,\Delta T$$

$$Q = m\left[(4186\text{ J/kg °C})(80.0\text{ °C}) + 2.26\times 10^6\text{ J/kg} + (2010\text{ J/kg °C})(300\text{ °C})\right]$$

Therefore $$m = \frac{8.20\times 10^{10}\text{ J}}{3.20\times 10^6\text{ J/kg}} = \boxed{2.56\times 10^4\text{ kg}}$$

P45.46 When mass m of ^{235}U undergoes complete fission, releasing 200 MeV per fission event, the total energy released is:

$$Q=\left(\frac{m}{235\ \text{g/mol}}\right)N_{\text{A}}\left(200\ \text{MeV}\right)\ \text{where}\ N_{\text{A}}\ \text{is Avogadro's number.}$$

If all this energy could be utilized to convert a mass m_w of liquid water at T_c into steam at T_h, then $Q=m_w\left[c_w\left(100°\text{C}-T_c\right)+L_v+c_s\left(T_h-100°\text{C}\right)\right]$
where c_w is the specific heat of liquid water, L_v is the latent heat of vaporization, and c_s is the specific heat of steam. Solving for the mass of water converted gives

$$m_w=\frac{Q}{\left[c_w\left(100°\text{C}-T_c\right)+L_v+c_s\left(T_h-100°\text{C}\right)\right]}$$

$$=\boxed{\frac{mN_{\text{A}}\left(200\ \text{MeV}\right)}{\left(235\ \text{g/mol}\right)\left[c_w\left(100°\text{C}-T_c\right)+L_v+c_s\left(T_h-100°\text{C}\right)\right]}}.$$

***P45.47** (a) We have 0.9928 kg of ^{238}U, comprising
N = 0.9928 kg (6.02×10^{23} atoms/mol)/(0.238 kg/mol) = 2.51×10^{24} nuclei, with activity
$R=\lambda N=(\ln 2/T_{1/2})N=2.51\times10^{24}(\ln 2/4.47\times10^{9}\ \text{yr})(1\ \text{yr}/3.16\times10^{7}\ \text{s})(1\ \text{Ci}/3.7\times10^{10}/\text{s})$
$=\boxed{333\ \mu\text{Ci}}$

We have 0.0072 kg of ^{235}U, comprising
N = 0.0072 kg (6.02×10^{23} atoms/mol)/(0.235 kg/mol) = 1.84×10^{22} nuclei, with activity
$R=\lambda N=(\ln 2/T_{1/2})N=1.84\times10^{22}(\ln 2/7.04\times10^{8}\ \text{yr})(1\ \text{yr}/3.16\times10^{7}\ \text{s})(1\ \text{Ci}/3.7\times10^{10}/\text{s})$
$=\boxed{15.5\ \mu\text{Ci}}$

We have 0.00005 kg of ^{234}U, comprising
$N=5\times10^{-5}$ kg (6.02×10^{23} atoms/mol)/(0.23404 kg/mol) = 1.29×10^{20} nuclei, with activity
$R=\lambda N=(\ln 2/T_{1/2})N=1.29\times10^{20}(\ln 2/2.44\times10^{5}\ \text{yr})(1\ \text{yr}/3.16\times10^{7}\ \text{s})(1\ \text{Ci}/3.7\times10^{10}/\text{s})$
$=\boxed{312\ \mu\text{Ci}}$

(b) The total activity is (333 + 15.5 + 312) μCi = 661 μCi, so the fractional contributions are respectively 333/661 = $\boxed{50.4\%}$ 15.5/661 = $\boxed{2.35\%}$ and 312/661 = $\boxed{47.3\%}$

(c) It is potentially dangerous, notably if the material is inhaled as a powder. With precautions to minimize human contact, microcurie sources are routinely used in laboratories.

P45.48 (a) $\Delta V=4\pi r^2\Delta r=4\pi\left(14.0\times10^{3}\ \text{m}\right)^2\left(0.05\ \text{m}\right)=1.23\times10^{8}\ \text{m}^3\ \boxed{\sim10^{8}\ \text{m}^3}$

(b) The force on the next layer is determined by atmospheric pressure.

$W=P\Delta V=\left(1.013\times10^{5}\ \text{N/m}^2\right)\left(1.23\times10^{8}\ \text{m}^3\right)=1.25\times10^{13}\ \text{J}\ \boxed{\sim10^{13}\ \text{J}}$

(c) $1.25\times10^{13}\ \text{J}=\frac{1}{10}\left(\text{yield}\right),\ \text{so yield}=1.25\times10^{14}\ \text{J}\ \boxed{\sim10^{14}\ \text{J}}$

(d) $\frac{1.25\times10^{14}\ \text{J}}{4.2\times10^{9}\ \text{J/ton TNT}}=2.97\times10^{4}\ \text{ton TNT}\sim10^{4}\ \text{ton TNT}$

or $\boxed{\sim 10\ \text{kilotons}}$

P45.49 (a) The number of molecules in 1.00 liter of water (mass = 1 000 g) is

$$N=\left(\frac{1.00\times10^{3}\ \text{g}}{18.0\ \text{g/mol}}\right)\left(6.02\times10^{23}\ \text{molecules/mol}\right)=3.34\times10^{25}\ \text{molecules.}$$

The number of deuterium nuclei contained in these molecules is

$$N'=\left(3.34\times10^{25}\ \text{molecules}\right)\left(\frac{1\ \text{deuteron}}{3300\ \text{molecules}}\right)=1.01\times10^{22}\ \text{deuterons.}$$

Since 2 deuterons are consumed per fusion event, the number of events possible is

$\frac{N'}{2}=5.07\times10^{21}$ reactions, and the energy released is

$$E_{\text{fusion}}=\left(5.07\times10^{21}\ \text{reactions}\right)\left(3.27\ \text{MeV/reaction}\right)=1.66\times10^{22}\ \text{MeV}$$

$$E_{\text{fusion}}=\left(1.66\times10^{22}\ \text{MeV}\right)\left(1.60\times10^{-13}\ \text{J/MeV}\right)=\boxed{2.65\times10^{9}\ \text{J}}.$$

(b) In comparison to burning 1.00 liter of gasoline, the energy from the fusion of deuterium is

$$\frac{E_{\text{fusion}}}{E_{\text{gasoline}}}=\frac{2.65\times10^{9}\ \text{J}}{3.40\times10^{7}\ \text{J}}=\boxed{78.0\ \text{times larger}}.$$

P45.50 Momentum conservation: $0=m_{\text{Li}}\vec{\mathbf{v}}_{\text{Li}}+m_{\alpha}\vec{\mathbf{v}}_{\alpha}$ or $m_{\text{Li}}v_{\text{Li}}=m_{\alpha}v_{\alpha}$

Thus,

$$K_{\text{Li}}=\frac{1}{2}m_{\text{Li}}v_{\text{Li}}^{2}=\frac{1}{2}\frac{\left(m_{\text{Li}}v_{\text{Li}}\right)^{2}}{m_{\text{Li}}}=\frac{\left(m_{\alpha}v_{\alpha}\right)^{2}}{2m_{\text{Li}}}=\left(\frac{m_{\alpha}^{2}}{2m_{\text{Li}}}\right)v_{\alpha}^{2}$$

$$K_{\text{Li}}=\left(\frac{\left(4.002\ 6\ \text{u}\right)^{2}}{2\left(7.016\ 0\ \text{u}\right)}\right)\left(9.25\times10^{6}\ \text{m/s}\right)^{2}=\left(1.14\ \text{u}\right)\left(9.25\times10^{6}\ \text{m/s}\right)^{2}$$

$$K_{\text{Li}}=1.14\left(1.66\times10^{-27}\ \text{kg}\right)\left(9.25\times10^{6}\ \text{m/s}\right)^{2}=1.62\times10^{-13}\ \text{J}=\boxed{1.01\ \text{MeV}}.$$

P45.51 (a) The thermal power transferred to the water is $\mathcal{P}_{w}=0.970(\text{waste heat})$

$$\mathcal{P}_{w}=0.970\left(3\,065-1\,000\right)\text{MW}=2.00\times10^{9}\ \text{J/s}$$

r_{w} is the mass of water heated per hour:

$$r_{w}=\frac{\mathcal{P}_{w}}{c(\Delta T)}=\frac{\left(2.00\times10^{9}\ \text{J/s}\right)\left(3600\ \text{s/h}\right)}{\left(4186\ \text{J/kg}\cdot{}^{\circ}\text{C}\right)\left(3.50\ {}^{\circ}\text{C}\right)}=\boxed{4.91\times10^{8}\ \text{kg/h}}$$

The volume used per hour is $\frac{4.91\times10^{8}\ \text{kg/h}}{1.00\times10^{3}\ \text{kg/m}^{3}}=\boxed{4.91\times10^{5}\ \text{m}^{3}/\text{h}}$

(b) The ^{235}U fuel is consumed at a rate

$$r_{f}=\left(\frac{3\,065\times10^{6}\ \text{J/s}}{7.80\times10^{10}\ \text{J/g}}\right)\left(\frac{1\ \text{kg}}{1\,000\ \text{g}}\right)\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right)=\boxed{0.141\ \text{kg/h}}$$

P45.52 The number of nuclei in 0.155 kg of ^{210}Po is

$$N_0 = \left(\frac{155 \text{ g}}{209.98 \text{ g/mol}}\right)\left(6.02 \times 10^{23} \text{ nuclei/mol}\right) = 4.44 \times 10^{23} \text{ nuclei}$$

The half-life of ^{210}Po is 138.38 days, so the decay constant is given by

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(138.38 \text{ d})\left(8.64 \times 10^4 \text{ s/d}\right)} = 5.80 \times 10^{-8} \text{ s}^{-1}$$

The initial activity is

$$R_0 = \lambda N_0 = \left(5.80 \times 10^{-8} \text{ s}^{-1}\right)\left(4.44 \times 10^{23} \text{ nuclei}\right) = 2.58 \times 10^{16} \text{ Bq}$$

The energy released in each $^{210}_{84}\text{Po} \to {}^{206}_{82}\text{Pb} + {}^{4}_{2}\text{He}$ reaction is

$$Q = \left[M_{^{210}_{84}\text{Po}} - M_{^{206}_{82}\text{Pb}} - M_{^{4}_{2}\text{He}}\right]c^2:$$

$$Q = [209.982\,857 - 205.974\,449 - 4.002\,603]\text{u}(931.5 \text{ MeV/u}) = 5.41 \text{ MeV}$$

Thus, assuming a conversion efficiency of 1.00%, the initial power output of the battery is

$$\mathcal{P} = (0.010\,0)R_0 Q = (0.010\,0)\left(2.58 \times 10^{16} \text{ decays/s}\right)(5.41 \text{ MeV/decay})\left(1.60 \times 10^{-13} \text{ J/MeV}\right)$$

$$= \boxed{223 \text{ W}}$$

P45.53 (a) $V = \ell^3 = \frac{m}{\rho}$, so $\ell = \left(\frac{m}{\rho}\right)^{1/3} = \left(\frac{70.0 \text{ kg}}{18.7 \times 10^3 \text{ kg/m}^3}\right)^{1/3} = \boxed{0.155 \text{ m}}$

(b) Add 92 electrons to both sides of the given nuclear reaction. Then it becomes

$$^{238}_{92}\text{U atom} \to 8\,{}^{4}_{2}\text{He atom} + {}^{206}_{82}\text{Pb atom} + Q_{\text{net}}/c^2$$

$$Q_{\text{net}} = \left[M_{^{238}_{92}\text{U}} - 8M_{^{4}_{2}\text{He}} - M_{^{206}_{82}\text{Pb}}\right]c^2$$

$$= \left[238.050\,783 - 8(4.002\,603) - 205.974\,449\right] \text{u } (931.5 \text{ MeV/u})$$

$$Q_{\text{net}} = \boxed{51.7 \text{ MeV}}$$

(c) If there is a single step of decay, the number of decays per time is the decay rate R and the energy released in each decay is Q. Then the energy released per time is $\boxed{\mathcal{P} = QR}$. If there is a series of decays in steady state, the equation is still true, with Q representing the net decay energy.

(d) The decay rate for all steps in the radioactive series in steady state is set by the parent uranium:

$$N = \left(\frac{7.00 \times 10^4 \text{ g}}{238 \text{ g/mol}}\right)\left(6.02 \times 10^{23} \text{ nuclei/mol}\right) = 1.77 \times 10^{26} \text{ nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{4.47 \times 10^9 \text{ yr}} = 1.55 \times 10^{-10} \frac{1}{\text{yr}}$$

$$R = \lambda N = \left(1.55 \times 10^{-10} \frac{1}{\text{yr}}\right)\left(1.77 \times 10^{26} \text{ nuclei}\right) = 2.75 \times 10^{16} \text{ decays/yr}$$

so $$\mathcal{P} = QR = (51.7 \text{ MeV})\left(2.75 \times 10^{16} \frac{1}{\text{yr}}\right)\left(1.60 \times 10^{-13} \text{ J/MeV}\right) = \boxed{2.27 \times 10^5 \text{ J/yr}}$$

continued on next page

(e) dose in rem = dose in rad × RBE

$$5.00 \text{ rem/yr} = (\text{dose in rad/yr})1.10, \text{ giving } (\text{dose in rad/yr}) = 4.55 \text{ rad/yr}$$

The allowed whole-body dose is then $(70.0 \text{ kg})(4.55 \text{ rad/yr})\left(\dfrac{10^{-2} \text{ J/kg}}{1 \text{ rad}}\right) = \boxed{3.18 \text{ J/yr}}$.

P45.54 $E_T \equiv E(\text{thermal}) = \dfrac{3}{2}k_B T = 0.039 \text{ eV}$

$E_T = \left(\dfrac{1}{2}\right)^n E$ where $n \equiv$ number of collisions, and $0.039 = \left(\dfrac{1}{2}\right)^n (2.0 \times 10^6)$

Therefore $n = 25.6 = \boxed{26 \text{ collisions}}$

P45.55 Conservation of linear momentum and energy can be applied to find the kinetic energy of the neutron. We first suppose the particles are moving nonrelativistically.
The momentum of the alpha particle and that of the neutron must add to zero, so their velocities must be in opposite directions with magnitudes related by

$$m_n\vec{\mathbf{v}}_n + m_\alpha \vec{\mathbf{v}}_\alpha = 0 \qquad \text{or} \qquad (1.008\,7 \text{ u})v_n = (4.002\,6 \text{ u})v_\alpha$$

At the same time, their kinetic energies must add to 17.6 MeV

$$E = \frac{1}{2}m_n v_n^2 + \frac{1}{2}m_\alpha v_\alpha^2 = \frac{1}{2}(1.008\,7 \text{ u})v_n^2 + \frac{1}{2}(4.002\,6)v_\alpha^2 = 17.6 \text{ MeV}$$

Substitute $v_\alpha = 0.252\,0 v_n$: $E = (0.504\,35 \text{ u})v_n^2 + (0.127\,10 \text{ u})v_n^2 = 17.6 \text{ MeV}\left(\dfrac{1 \text{ u}}{931.494 \text{ MeV}/c^2}\right)$

$$v_n = \sqrt{\frac{0.018\,9c^2}{0.631\,45}} = 0.173c = 5.19 \times 10^7 \text{ m/s}$$

Since this speed is not too much greater than $0.1c$, we can get a reasonable estimate of the kinetic energy of the neutron from the classical equation,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.008\,7 \text{ u})(0.173c)^2\left(\frac{931.494 \text{ MeV}/c^2}{\text{u}}\right) = 14.1 \text{ MeV}$$

For a more accurate calculation of the kinetic energy, we should use relativistic expressions. Conservation of momentum gives

$$\gamma_n m_n \vec{\mathbf{v}}_n + \gamma_\alpha m_\alpha \vec{\mathbf{v}}_\alpha = 0 \qquad 1.008\,7\frac{v_n}{\sqrt{1 - v_n^2/c^2}} = 4.002\,6\frac{v_\alpha}{\sqrt{1 - v_\alpha^2/c^2}}$$

yielding $\quad \dfrac{v_\alpha^2}{c^2} = \dfrac{v_n^2}{15.746c^2 - 14.746v_n^2}$

Then $\quad (\gamma_n - 1)m_n c^2 + (\gamma_\alpha - 1)m_\alpha c^2 = 17.6 \text{ MeV}$

and $\quad v_n = 0.171c$ implying that $(\gamma_n - 1)m_n c^2 = \boxed{14.0 \text{ MeV}}$

P45.56 From the table of isotopic masses in Chapter 44, the half-life of ^{32}P is 14.26 d. Thus, the decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{14.26\text{ d}} = 0.048\,6\text{ d}^{-1} = 5.63\times 10^{-7}\text{ s}^{-1}$$

$$N_0 = \frac{R_0}{\lambda} = \frac{5.22\times 10^6\text{ decay/s}}{5.63\times 10^{-7}\text{ s}^{-1}} = 9.28\times 10^{12}\text{ nuclei}$$

At $t = 10.0$ days, the number remaining is

$$N = N_0 e^{-\lambda t} = \left(9.28\times 10^{12}\text{ nuclei}\right)e^{-\left(0.048\,6\text{ d}^{-1}\right)(10.0\text{ d})} = 5.71\times 10^{12}\text{ nuclei}$$

so the number of decays has been $N_0 - N = 3.57\times 10^{12}$ and the energy released is

$$E = \left(3.57\times 10^{12}\right)(700\text{ keV})\left(1.60\times 10^{-16}\text{ J/keV}\right) = 0.400\text{ J}$$

If this energy is absorbed by 100 g of tissue, the absorbed dose is

$$\text{Dose} = \left(\frac{0.400\text{ J}}{0.100\text{ kg}}\right)\left(\frac{1\text{ rad}}{10^{-2}\text{ J/kg}}\right) = \boxed{400\text{ rad}}$$

P45.57 (a) The number of Pu nuclei in $1.00\text{ kg} = \dfrac{6.02\times 10^{23}\text{ nuclei/mol}}{239.05\text{ g/mol}}(1\,000\text{ g})$.

The total energy $= \left(25.2\times 10^{23}\text{ nuclei}\right)(200\text{ MeV}) = 5.04\times 10^{26}\text{ MeV}$

$$E = \left(5.04\times 10^{26}\text{ MeV}\right)\left(4.44\times 10^{-20}\text{ kWh/MeV}\right) = \boxed{2.24\times 10^{7}\text{ kWh}}$$

or 22 million kWh

(b) $E = \Delta m c^2 = (3.016\,049\text{ u} + 2.014\,102\text{ u} - 4.002\,603\text{ u} - 1.008\,665\text{ u})(931.5\text{ MeV/u})$

$E = \boxed{17.6\text{ MeV for each D-T fusion}}$

(c) $E_n = (\text{Total number of D nuclei})(17.6)\left(4.44\times 10^{-20}\right)$

$$E_n = \left(6.02\times 10^{23}\right)\left(\frac{1\,000}{2.014}\right)(17.6)\left(4.44\times 10^{-20}\right) = \boxed{2.34\times 10^{8}\text{ kWh}}$$

(d) $E_n = \text{the number of C atoms in } 1.00\text{ kg} \times 4.20\text{ eV}$

$$E_n = \left(\frac{6.02\times 10^{26}}{12}\right)\left(4.20\times 10^{-6}\text{ MeV}\right)\left(4.44\times 10^{-20}\right) = \boxed{9.36\text{ kWh}}$$

(e) Coal is cheap at this moment in human history. We hope that safety and waste disposal problems can be solved so that nuclear energy can be affordable before scarcity drives up the price of fossil fuels. Burning coal in the open puts carbon dioxide into the atmosphere, worsening global warming. Plutonium is a very dangerous material to have sitting around.

P45.58 Add two electrons to both sides of the given reaction.

Then $4\,{}^1_1\text{H atom} \rightarrow {}^4_2\text{He atom} + Q/c^2$

where $Q = (\Delta m)c^2 = [4(1.007\,825) - 4.002\,603]\text{u}(931.5\ \text{MeV/u}) = 26.7\ \text{MeV}$

or $Q = (26.7\ \text{MeV})(1.60\times 10^{-13}\ \text{J/MeV}) = 4.28\times 10^{-12}\ \text{J}$

The proton fusion rate is then

$$\text{rate} = \frac{\text{power output}}{\text{energy per proton}} = \frac{3.85\times 10^{26}\ \text{J/s}}{(4.28\times 10^{-12}\ \text{J})/(4\ \text{protons})} = \boxed{3.60\times 10^{38}\ \text{protons/s}}$$

P45.59 (a) $Q_{\text{I}} = [M_{\text{A}} + M_{\text{B}} - M_{\text{C}} - M_{\text{E}}]c^2$, and $Q_{\text{II}} = [M_{\text{C}} + M_{\text{D}} - M_{\text{F}} - M_{\text{G}}]c^2$

$$Q_{\text{net}} = Q_{\text{I}} + Q_{\text{II}} = [M_{\text{A}} + M_{\text{B}} - M_{\text{C}} - M_{\text{E}} + M_{\text{C}} + M_{\text{D}} - M_{\text{F}} - M_{\text{G}}]c^2$$

$$Q_{\text{net}} = Q_{\text{I}} + Q_{\text{II}} = [M_{\text{A}} + M_{\text{B}} + M_{\text{D}} - M_{\text{E}} - M_{\text{F}} - M_{\text{G}}]c^2$$

Thus, reactions may be added. Any product like C used in a subsequent reaction does not contribute to the energy balance.

(b) Adding all five reactions gives

$${}^1_1\text{H} + {}^1_1\text{H} + {}^{\ 0}_{-1}\text{e} + {}^1_1\text{H} + {}^1_1\text{H} + {}^{\ 0}_{-1}\text{e} \rightarrow {}^4_2\text{He} + 2\nu + Q_{\text{net}}/c^2$$

or $4\,{}^1_1\text{H} + 2\,{}^{\ 0}_{-1}\text{e} \rightarrow {}^4_2\text{He} + 2\nu + Q_{\text{net}}/c^2$

Adding two electrons to each side gives $4\,{}^1_1\text{H atom} \rightarrow {}^4_2\text{He atom} + Q_{\text{net}}/c^2$

Thus $Q_{\text{net}} = \left[4M_{{}^1_1\text{H}} - M_{{}^4_2\text{He}}\right]c^2 = [4(1.007\,825) - 4.002\,603]\text{u}(931.5\ \text{MeV/u})$

$= \boxed{26.7\ \text{MeV}}$

P45.60 (a) The mass of the pellet is $m = \rho V = (0.200\ \text{g/cm}^3)\left[\frac{4\pi}{3}\left(\frac{1.50\times 10^{-2}\ \text{cm}}{2}\right)^3\right] = 3.53\times 10^{-7}\ \text{g}$

The pellet consists of equal numbers of ${}^2\text{H}$ and ${}^3\text{H}$ atoms, so the average molar mass is 2.50 and the total number of atoms is

$$N = \left(\frac{3.53\times 10^{-7}\ \text{g}}{2.50\ \text{g/mol}}\right)(6.02\times 10^{23}\ \text{atoms/mol}) = 8.51\times 10^{16}\ \text{atoms}$$

When the pellet is vaporized, the plasma will consist of $2N$ particles (N nuclei and N electrons). The total energy delivered to the plasma is 1.00% of 200 kJ or 2.00 kJ. The temperature of the plasma is found from $E = (2N)\left(\frac{3}{2}k_{\text{B}}T\right)$ as

$$T = \frac{E}{3Nk_{\text{B}}} = \frac{2.00\times 10^3\ \text{J}}{3(8.51\times 10^{16})(1.38\times 10^{-23}\ \text{J/K})} = \boxed{5.68\times 10^8\ \text{K}}$$

(b) Each fusion event uses 2 nuclei, so $\frac{N}{2}$ events will occur. The energy released will be

$$E = \left(\frac{N}{2}\right)Q = \left(\frac{8.51\times 10^{16}}{2}\right)(17.59\ \text{MeV})(1.60\times 10^{-13}\ \text{J/MeV}) = 1.20\times 10^5\ \text{J} = \boxed{120\ \text{kJ}}$$

P45.61 (a) The solar-core temperature of 15 MK gives particles enough kinetic energy to overcome the Coulomb-repulsion barrier to ${}^1_1\text{H} + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + \text{e}^+ + \nu$, estimated as $\dfrac{k_e(e)(2e)}{r}$. The Coulomb barrier to Bethe's fifth and eight reactions is like $\dfrac{k_e(e)(7e)}{r}$, larger by $\dfrac{7}{2}$ times, so the required temperature can be estimated as $\dfrac{7}{2}\left(15\times10^6\ \text{K}\right) \approx \boxed{5\times10^7\ \text{K}}$.

(b) For ${}^{12}\text{C} + {}^1\text{H} \rightarrow {}^{13}\text{N} + Q$,

$$Q_1 = (12.000\,000 + 1.007\,825 - 13.005\,739)(931.5\text{ MeV}) = \boxed{1.94\text{ MeV}}$$

For the second step, add seven electrons to both sides to have:

$${}^{13}\text{N atom} \rightarrow {}^{13}\text{C atom} + \text{e}^+ + \text{e}^- + Q$$

$$Q_2 = \left[13.005\,739 - 13.003\,355 - 2(0.000\,549)\right](931.5\text{ MeV}) = \boxed{1.20\text{ MeV}}$$

$$Q_3 = Q_7 = 2(0.000\,549)(931.5\text{ MeV}) = \boxed{1.02\text{ MeV}}$$

$$Q_4 = [13.003\,355 + 1.007\,825 - 14.003\,074](931.5\text{ MeV}) = \boxed{7.55\text{ MeV}}$$

$$Q_5 = [14.003\,074 + 1.007\,825 - 15.003\,065](931.5\text{ MeV}) = \boxed{7.30\text{ MeV}}$$

$$Q_6 = \left[15.003\,065 - 15.000\,109 - 2(0.000\,549)\right](931.5\text{ MeV}) = \boxed{1.73\text{ MeV}}$$

$$Q_8 = [15.000\,109 + 1.007\,825 - 12 - 4.002\,603](931.5\text{ MeV}) = \boxed{4.97\text{ MeV}}$$

The sum is $\boxed{26.7\text{ MeV}}$, the same as for the proton-proton cycle.

(c) Not quite all of the energy released appears as internal energy in the star. When a neutrino is created, it will likely fly directly out of the star without interacting with any other particle.

P45.62 (a) $\dfrac{I_2}{I_1} = \dfrac{I_0 e^{-\mu_2 x}}{I_0 e^{-\mu_1 x}} = \boxed{e^{-(\mu_2-\mu_1)x}}$

(b) $\dfrac{I_{50}}{I_{100}} = e^{-(5.40-41.0)(0.100)} = e^{3.56} = \boxed{35.2}$

(c) $\dfrac{I_{50}}{I_{100}} = e^{-(5.40-41.0)(1.00)} = e^{35.6} = \boxed{2.89\times10^{15}}$

Thus, a 1.00-cm-thick aluminum plate has essentially removed the long-wavelength x-rays from the beam.

P45.63 (a) The number of fissions occurring in the zeroth, first, second, . . . nth generation is

$$N_0,\ N_0K,\ N_0K^2,\ \ldots,\ N_0K^n$$

The total number of fissions that have occurred up to and including the nth generation is

$$N = N_0 + N_0K + N_0K^2 + \cdots + N_0K^n = N_0\left(1 + K + K^2 + \cdots + K^n\right)$$

Note that the factoring of the difference of two squares, $a^2 - 1 = (a+1)(a-1)$, can be generalized to a difference of two quantities to any power,

$$a^3 - 1 = \left(a^2 + a + 1\right)(a-1)$$
$$a^{n+1} - 1 = \left(a^n + a^{n-1} + \cdots + a^2 + a + 1\right)(a-1)$$

Thus $$K^n + K^{n-1} + \cdots + K^2 + K + 1 = \frac{K^{n+1}-1}{K-1}$$

and $$\boxed{N = N_0\frac{K^{n+1}-1}{K-1}}$$

(b) The number of U-235 nuclei is

$$N = 5.50\text{ kg}\left(\frac{1\text{ atom}}{235\text{ u}}\right)\left(\frac{1\text{ u}}{1.66\times10^{-27}\text{ kg}}\right) = 1.41\times10^{25}\text{ nuclei}$$

We solve the equation from part (a) for n, the number of generations:

$$\frac{N}{N_0}(K-1) = K^{n+1} - 1$$

$$\frac{N}{N_0}(K-1) + 1 = K^n(K)$$

$$n\ln K = \ln\left(\frac{N(K-1)/N_0 + 1}{K}\right) = \ln\left(\frac{N(K-1)}{N_0} + 1\right) - \ln K$$

$$n = \frac{\ln\left(N(K-1)/N_0 + 1\right)}{\ln K} - 1 = \frac{\ln\left(1.41\times10^{25}(0.1)/10^{20} + 1\right)}{\ln 1.1} - 1 = 99.2$$

Therefore time must be allotted for 100 generations:

$$\Delta t_b = 100\left(10\times10^{-9}\text{ s}\right) = \boxed{1.00\times10^{-6}\text{ s}}$$

(c) $$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{150\times10^9\text{ N/m}^2}{18.7\times10^3\text{ kg/m}^3}} = \boxed{2.83\times10^3\text{ m/s}}$$

(d) $$V = \frac{4}{3}\pi r^3 = \frac{m}{\rho}$$

$$r = \left(\frac{3m}{4\pi\rho}\right)^{1/3} = \left(\frac{3(5.5\text{ kg})}{4\pi\left(18.7\times10^3\text{ kg/m}^3\right)}\right)^{1/3} = 4.13\times10^{-2}\text{ m}$$

$$\Delta t_d = \frac{r}{v} = \frac{4.13\times10^{-2}\text{ m}}{2.83\times10^3\text{ m/s}} = \boxed{1.46\times10^{-5}\text{ s}}$$

continued on next page

(e) $14.6\ \mu s$ is greater than $1\ \mu s$, so the entire bomb can fission. The destructive energy released is

$$1.41\times10^{25}\text{ nuclei}\left(\frac{200\times10^{6}\text{ eV}}{\text{fissioning nucleus}}\right)\left(\frac{1.6\times10^{-19}\text{ J}}{1\text{ eV}}\right)=4.51\times10^{14}\text{ J}$$

$$=4.51\times10^{14}\text{ J}\left(\frac{1\text{ ton TNT}}{4.2\times10^{9}\text{ J}}\right)$$

$$=1.07\times10^{5}\text{ ton TNT}$$

$$=\boxed{107\text{ kilotons of TNT}}$$

What if? If the bomb did not have an "initiator" to inject 10^{20} neutrons at the moment when the critical mass is assembled, the number of generations would be

$$n=\frac{\ln\left(1.41\times10^{25}(0.1)/1+1\right)}{\ln 1.1}-1=582\text{ requiring }583\left(10\times10^{-9}\text{ s}\right)=5.83\ \mu\text{s}$$

This time is not very short compared with $14.6\ \mu s$, so this bomb would likely release much less energy.

ANSWERS TO EVEN PROBLEMS

P45.2 184 MeV

P45.4 (a) 173 MeV (b) 0.078 8%

P45.6 (a) 8.68 MeV (b) The proton and the boron nucleus have positive charges. The colliding particles must have enough kinetic energy to approach very closely in spite of their electric repulsion.

P45.8 2.63 kg/d

P45.10 (a) $4.84V^{-1/3}$ (b) $6V^{-1/3}$ (c) $6.30V^{-1/3}$ (d) The sphere has minimum loss and the parallelepiped maximum.

P45.12 6.25×10^{19} Bq

P45.14 25.6 MeV

P45.16 (a) (144 keV) $Z_1 Z_2$ (b) The energy is proportional to each atomic number. (c) Take $Z_1=1$ and $Z_2=59$ or vice versa. This choice minimizes the product $Z_1 Z_2$. (d) 144 keV for both, according to this model

P45.18 (a) 2.53×10^{31} J (b) 1.14×10^{9} yr

P45.20 (a) 1.7×10^{7} J (b) 7.3 kg

P45.22 12.4 h

P45.24 (a) 10.0 h (b) 3.16 m

P45.26 (a) 0.436 cm (b) 5.79 cm

P45.28 $2.39\times10^{-3}\,°\mathrm{C}$

P45.30 (a) 422 MBq (b) 153 ng

P45.32 (a) 10 (b) 10^6 (c) 10^8 eV

P45.34 (a) about 8 min (b) 27.6 min (c) 30 min ± 30%

P45.36 (a) $\sim10^6$ (b) $\sim10^{-15}$ g

P45.38 (a) 0.963 mm (b) It increases by 7.47%.

P45.40 See the solution.

P45.42 The fractional loss in D-T is about 4 times that in ^{235}U fission.

P45.44 221 yr

P45.46 $$\frac{mN_A(200\text{ MeV})}{(235\text{ g/mol})\left[c_w(100°\mathrm{C}-T_c)+L_v+c_s(T_h-100°\mathrm{C})\right]}$$

P45.48 (a) $\sim10^8\ \mathrm{m}^3$ (b) $\sim10^{13}$ J (c) $\sim10^{14}$ J (d) ~10 kilotons

P45.50 1.01 MeV

P45.52 223 W

P45.54 26 collisions

P45.56 400 rad

P45.58 3.60×10^{38} protons/s

P45.60 (a) 5.68×10^8 K (b) 120 kJ

P45.62 (a) See the solution. (b) 35.2 (c) 2.89×10^{15}

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Particle Physics and Cosmology

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q46.1 Strong Force—Mediated by gluons.
Electromagnetic Force—Mediated by photons.
Weak Force—Mediated by W^+, W^-, and Z^0 bosons.
Gravitational Force—Mediated by gravitons.

***Q46.2** Answer (b). The electron and positron together have very little momentum. A 1.02-MeV photon has a definite chunk of momentum. Production of a single gamma ray could not satisfy the law of conservation of momentum, which must hold true in this—and every—interaction.

Q46.3 Hadrons are massive particles with structure and size. There are two classes of hadrons: mesons and baryons. Hadrons are composed of quarks. Hadrons interact via the strong force. Leptons are light particles with no structure or size. It is believed that leptons are fundamental particles. Leptons interact via the weak force.

Q46.4 Baryons are heavy hadrons with spin $\frac{1}{2}$ or $\frac{3}{2}$ composed of three quarks. Mesons are light hadrons with spin 0 or 1 composed of a quark and an antiquark.

***Q46.5** Answer (c). The muon has much more rest energy than the electron and the neutrinos together. The missing rest energy goes into kinetic energy.

***Q46.6** Answer (b). The z component of its angular momentum must be 3/2, 1/2, –1/2, or –3/2, in units of $\hbar$.

Q46.7 The baryon number of a proton or neutron is one. Since baryon number is conserved, the baryon number of the kaon must be zero.

Q46.8 Decays by the weak interaction typically take 10^{-10} s or longer to occur. This is slow in particle physics.

***Q46.9** Answer a, b, c, and d. Protons feel all these forces; but within a nucleus the strong interaction predominates, followed by the electromagnetic interaction.

Q46.10 You can think of a conservation law as a superficial regularity which we happen to notice, as a person who does not know the rules of chess might observe that one player's two bishops are always on squares of opposite colors. Alternatively, you can think of a conservation law as identifying some stuff of which the universe is made. In classical physics one can think of both matter and energy as fundamental constituents of the world. We buy and sell both of them. In classical physics you can also think of linear momentum, angular momentum, and electric charge as basic stuffs of which the universe is made. In relativity we learn that matter and energy are not conserved separately, but are both aspects of the conserved quantity *relativistic total energy*. Discovered more recently, four conservation laws appear equally general and thus equally fundamental: Conservation of baryon number, conservation of electron-lepton number, conservation of tau-lepton number, and conservation of muon-lepton number. Processes involving the strong force and the electromagnetic force follow conservation of strangeness, charm, bottomness, and topness, while the weak interaction can alter the total S, C, B, and T quantum numbers of an isolated system.

Q46.11 No. Antibaryons have baryon number –1, mesons have baryon number 0, and baryons have baryon number +1. The reaction cannot occur because it would not conserve baryon number, unless so much energy is available that a baryon-antibaryon pair is produced.

Q46.12 The Standard Model consists of quantum chromodynamics (to describe the strong interaction) and the electroweak theory (to describe the electromagnetic and weak interactions). The Standard Model is our most comprehensive description of nature. It fails to unify the two theories it includes, and fails to include the gravitational force. It pictures matter as made of six quarks and six leptons, interacting by exchanging gluons, photons, and W and Z bosons.

Q46.13 (a) and (b) All baryons and antibaryons consist of three quarks. (c) and (d) All mesons and antimesons consist of two quarks. Since quarks have spin quantum number $\frac{1}{2}$ and can be spin-up or spin-down, it follows that the three-quark baryons must have a half-integer spin, while the two-quark mesons must have spin 0 or 1.

Q46.14 Each flavor of quark can have colors, designated as red, green, and blue. Antiquarks are colored antired, antigreen, and antiblue. A baryon consists of three quarks, each having a different color. By analogy to additive color mixing we call it colorless. A meson consists of a quark of one color and antiquark with the corresponding anticolor, making it colorless as a whole.

Q46.15 The electroweak theory of Glashow, Salam, and Weinberg predicted the W^+, W^-, and Z particles. Their discovery in 1983 confirmed the electroweak theory.

***Q46.16** Answer (e). Both trials conserve momentum. In the first trial all of the kinetic energy $K_1 + K_2 = 2K_1$ is converted into internal energy. In the second trial we end up with a glob of twice the mass moving at half the speed, so it has half the kinetic energy of one original clay ball, $K_1/2$. Energy $K_1/2$ is converted into internal energy, one-quarter of that converted in trial one.

Q46.17 Hubble determined experimentally that all galaxies outside the Local Group are moving away from us, with speed directly proportional to the distance of the galaxy from us.

***Q46.18** Answer c, b, d, e, a, f, g. The temperature corresponding to b is on the order of 10^{13} K. That for hydrogen fusion d is on the order of 10^7 K. A fully ionized plasma can be at 10^4 K. Neutral atoms can exist at on the order of 3 000 K, molecules at 1 000 K, and solids at on the order of 500 K.

Q46.19 Before that time, the Universe was too hot for the electrons to remain in any sort of stable orbit around protons. The thermal motion of both protons and electrons was too rapid for them to be in close enough proximity for the Coulomb force to dominate.

***Q46.20** Answer (a). The vast gulfs not just between stars but between galaxies and especially between clusters, empty of ordinary matter, are important to bring down the average density of the Universe. We can estimate the average density defined for the Solar System as the mass of the sun divided by the volume of a lightyear-size sphere around it:

$\frac{2\times10^{30}\ \text{kg}}{\frac{4}{3}\pi(2\times10^{16}\ \text{m})^3}=6\times10^{-20}\ \text{kg/m}^3=6\times10^{-23}\ \text{g/cm}^3$, ten million times larger than the critical density $3H^2/8\pi G = 6\times10^{-30}\ \text{g/cm}^3$.

Q46.21 The Universe is vast and could on its own terms get along very well without us. But as the cosmos is immense, life appears to be immensely scarce, and therefore precious. We must do our work, growing corn to feed the hungry while preserving our planet for future generations and preserving future possibilities for the universe. One person has singular abilities and opportunities for effort, faithfulness, generosity, honor, curiosity, understanding, and wonder. His or her place is to use those abilities and opportunities, unique in all the Universe.

SOLUTIONS TO PROBLEMS

Section 46.1 The Fundamental Forces in Nature

Section 46.2 Positrons and Other Antiparticles

P46.1 Assuming that the proton and antiproton are left nearly at rest after they are produced, the energy E of the photon must be

$$E=2E_0=2(938.3\ \text{MeV})=1\,876.6\ \text{MeV}=3.00\times10^{-10}\ \text{J}$$

Thus, $E=hf=3.00\times10^{-10}\ \text{J}$

$$f=\frac{3.00\times10^{-10}\ \text{J}}{6.626\times10^{-34}\ \text{J}\cdot\text{s}}=\boxed{4.53\times10^{23}\ \text{Hz}}$$

$$\lambda=\frac{c}{f}=\frac{3.00\times10^{8}\ \text{m/s}}{4.53\times10^{23}\ \text{Hz}}=\boxed{6.62\times10^{-16}\ \text{m}}$$

P46.2 The half-life of ^{14}O is 70.6 s, so the decay constant is $\lambda=\frac{\ln 2}{T_{1/2}}=\frac{\ln 2}{70.6\ \text{s}}=0.009\,82\ \text{s}^{-1}$.

The number of ^{14}O nuclei remaining after five minutes is

$$N=N_0e^{-\lambda t}=(10^{10})e^{-(0.009\,82\ \text{s}^{-1})(300\ \text{s})}=5.26\times10^{8}$$

The number of these in one cubic centimeter of blood is

$$N'=N\left(\frac{1.00\ \text{cm}^3}{\text{total volume of blood}}\right)=(5.26\times10^{8})\left(\frac{1.00\ \text{cm}^3}{2000\ \text{cm}^3}\right)=2.63\times10^{5}$$

and their activity is $R=\lambda N'=(0.009\ 82\ \text{s}^{-1})(2.63\times10^{5})=2.58\times10^{3}\ \text{Bq}\ \boxed{\sim10^{3}\ \text{Bq}}$

***P46.3** (a) The rest energy of a total of 6.20 g of material is converted into energy of electromagnetic radiation:

$$E=mc^2=6.20\times10^{-3}\ \text{kg}(3\times10^{8}\ \text{m/s})^2=\boxed{5.58\times10^{14}\ \text{J}}$$

(b) $5.58\times10^{14}\ \text{J}=5.58\times10^{14}\ \text{J}\left(\frac{\$0.14}{\text{kWh}}\right)\left(\frac{\text{k}}{1\ 000}\right)\left(\frac{\text{W}}{\text{J/s}}\right)\left(\frac{1\ \text{h}}{3\ 600\ \text{s}}\right)=\boxed{\$2.17\times10^{7}}$

All from two cents' worth of stuff.

P46.4 The minimum energy is released, and hence the minimum frequency photons are produced, when the proton and antiproton are at rest when they annihilate.

That is, $E = E_0$ and $K = 0$. To conserve momentum, each photon must carry away one-half the energy.

Thus $$E_{\min} = \frac{2E_0}{2} = E_0 = 938.3\text{ MeV} = hf_{\min}$$

Thus, $$f_{\min} = \frac{(938.3\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{(6.626\times10^{-34}\text{ J}\cdot\text{s})} = \boxed{2.27\times10^{23}\text{ Hz}}$$

$$\lambda = \frac{c}{f_{\min}} = \frac{3.00\times10^{8}\text{ m/s}}{2.27\times10^{23}\text{ Hz}} = \boxed{1.32\times10^{-15}\text{ m}}$$

P46.5 In $\gamma \to p^+ + p^-$,

we start with energy 2.09 GeV
we end with energy 938.3 MeV + 938.3 MeV + 95.0 MeV + K_2
where K_2 is the kinetic energy of the second proton.

Conservation of energy for the creation process gives $\boxed{K_2 = 118\text{ MeV}}$

Section 46.3 **Mesons and the Beginning of Particle Physics**

P46.6 The reaction is $\mu^+ + e^- \to \nu + \nu$
muon-lepton number before reaction: $(-1)+(0) = -1$
electron-lepton number before reaction: $(0)+(1) = 1$

Therefore, after the reaction, the muon-lepton number must be –1. Thus, one of the neutrinos must be the anti-neutrino associated with muons, and one of the neutrinos must be the neutrino associated with electrons:

$\boxed{\bar{\nu}_\mu}$ and $\boxed{\nu_e}$

Then $\mu^+ + e^- \to \bar{\nu}_\mu + \nu_e$

P46.7 The creation of a virtual Z^0 boson is an energy fluctuation $\Delta E = 91\times10^{9}$ eV. It can last no longer than $\Delta t = \frac{\hbar}{2\Delta E}$ and move no farther than

$$c(\Delta t) = \frac{hc}{4\pi\Delta E} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{4\pi(91\times10^{9}\text{ eV})}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right)$$

$$= 1.06\times10^{-18}\text{ m} = \boxed{\sim10^{-18}\text{ m}}$$

***P46.8** (a) The particle's rest energy is mc^2. The time interval during which a virtual particle of this mass could exist is at most Δt in $\Delta E \Delta t = \frac{\hbar}{2} = mc^2 \Delta t$. The distance it could move is at most

$$c\Delta t = \frac{\hbar c}{2mc^2} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{4\pi mc^2(1.602\times10^{-19}\ \text{J/eV})} = \frac{1.240\times10^{-6}\ \text{eV}\cdot\text{m}}{4\pi mc^2} = \frac{1240\ \text{eV}\cdot\text{nm}}{4\pi mc^2}$$

$$= \frac{98.7\ \text{eV}\cdot\text{nm}}{mc^2}$$

According to Yukawa's line of reasoning, this distance is the range of a force that could be associated with the exchange of virtual particles of this mass.

(b) The range is inversely proportional to the mass of the field particle.

(c) Our rule describes the electromagnetic, weak, and gravitational interactions.
For the electromagnetic and gravitational interactions, we take the limiting form of the rule with infinite range and zero mass for the field particle. For the weak interaction, 98.7 eV · nm/90 GeV ≈ 10^{-18} m = 10^{-3} fm, in agreement with the tabulated information. For the strong interaction, we do not have a separately measured mass for a gluon, so we cannot say that this rule defines the range.

(d) 98.7 eV·nm/938.3 MeV ≈ 10^{-1-9-6} m $\boxed{\sim 10^{-16}\ \text{m}}$

P46.9 From Table 46.2 in the chapter text $M_{\pi^0} = 135\ \text{MeV}/c^2$.

Therefore, $E_\gamma = \boxed{67.5\ \text{MeV}}$ for each photon

$$p = \frac{E_\gamma}{c} = \boxed{67.5\ \text{MeV}/c}$$

and $$f = \frac{E_\gamma}{h} = \boxed{1.63\times10^{22}\ \text{Hz}}$$

P46.10 The time interval for a particle traveling with the speed of light to travel a distance of 3×10^{-15} m is

$$\Delta t = \frac{d}{v} = \frac{3\times10^{-15}\ \text{m}}{3\times10^{8}\ \text{m/s}} = \boxed{\sim 10^{-23}\ \text{s}}$$

P46.11 (a) $\Delta E = \left(m_n - m_p - m_e\right)c^2$

From the table of isotopic masses in Chapter 44,

$\Delta E = (1.008\ 665 - 1.007\ 825)(931.5) = \boxed{0.782\text{ MeV}}$

(b) Assuming the neutron at rest, momentum conservation for the decay process implies $p_p = p_e$. Relativistic energy for the system is conserved

$$\sqrt{\left(m_p c^2\right)^2 + p_p^2 c^2} + \sqrt{\left(m_e c^2\right)^2 + p_e^2 c^2} = m_n c^2$$

Since $p_p = p_e$, $\sqrt{(938.3)^2 + (pc)^2} + \sqrt{(0.511)^2 + (pc)^2} = 939.6\text{ MeV}$

Solving the algebra gives $pc = 1.19\text{ MeV}$

If $p_e c = \gamma m_e v_e c = 1.19\text{ MeV}$, then $\dfrac{\gamma v_e}{c} = \dfrac{1.19\text{ MeV}}{0.511\text{ MeV}} = \dfrac{x}{\sqrt{1-x^2}} = 2.33$ where $x = \dfrac{v_e}{c}$

Solving, $x^2 = \left(1 - x^2\right)5.43$ and $x = \dfrac{v_e}{c} = 0.919$

$$\boxed{v_e = 0.919c}$$

Then $m_p v_p = \gamma_e m_e v_e$: $v_p = \dfrac{\gamma_e m_e v_e c}{m_p c} = \dfrac{(1.19\text{ MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right)}{\left(1.67\times10^{-27}\right)\left(3.00\times10^{8}\text{ m/s}\right)}$

$$v_p = 3.80\times10^5\text{ m/s} = \boxed{380\text{ km/s}}$$

(c) $\boxed{\text{The electron is relativistic; the proton is not.}}$ Our criterion for answers accurate to three significant digits is that the electron is moving at more than one-tenth the speed of light and the proton at less than one-tenth the speed of light.

Section 46.4 **Classification of Particles**

P46.12 In $? + \text{p}^+ \rightarrow \text{n} + \mu^+$, charge conservation requires the unknown particle to be neutral. Baryon number conservation requires baryon number = 0. The muon-lepton number of ? must be –1.

So the unknown particle must be $\boxed{\bar{\nu}_\mu}$.

Section 46.5 Conservation Laws

P46.13 (a) $p+\bar{p}\to\mu^{+}+e^{-}$ $\boxed{L_e}$ $0+0\to 0+1$

and $\boxed{L_\mu}$ $0+0\to -1+0$

(b) $\pi^{-}+p\to p+\pi^{+}$ $\boxed{\text{charge}}$ $-1+1\to +1+1$

(c) $p+p\to p+\pi^{+}$ $\boxed{\text{baryon number}}$: $1+1\to 1+0$

(d) $p+p\to p+p+n$ $\boxed{\text{baryon number}}$: $1+1\to 1+1+1$

(e) $\gamma+p\to n+\pi^{0}$ $\boxed{\text{charge}}$ $0+1\to 0+0$

P46.14 (a) Baryon number and charge are conserved, with respective values of $0+1=0+1$ and $1+1=1+1$ in both reactions.

(b) $\boxed{\text{Strangeness is } \textit{not} \text{ conserved}}$ in the second reaction.

P46.15 (a) $\pi^{-}\to\mu^{-}+\boxed{\bar{\nu}_\mu}$ L_μ: $0\to 1-1$

(b) $K^{+}\to\mu^{+}+\boxed{\nu_\mu}$ L_μ: $0\to -1+1$

(c) $\boxed{\bar{\nu}_e}+p^{+}\to n+e^{+}$ L_e: $-1+0\to 0-1$

(d) $\boxed{\nu_e}+n\to p^{+}+e^{-}$ L_e: $1+0\to 0+1$

(e) $\boxed{\nu_\mu}+n\to p^{+}+\mu^{-}$ L_μ: $1+0\to 0+1$

(f) $\mu^{-}\to e^{-}+\boxed{\bar{\nu}_e}+\boxed{\nu_\mu}$ L_μ: $1\to 0+0+1$ and L_e: $0\to 1-1+0$

P46.16 Baryon number conservation $\boxed{\text{allows the first and forbids the second}}$.

P46.17 (a) $p^{+}\to\pi^{+}+\pi^{0}$ $\boxed{\text{Baryon number}}$ conservation is violated: $1\to 0+0$

(b) $p^{+}+p^{+}\to p^{+}+p^{+}+\pi^{0}$ This reaction $\boxed{\text{can occur}}$.

(c) $p^{+}+p^{+}\to p^{+}+\pi^{+}$ $\boxed{\text{Baryon number}}$ is violated: $1+1\to 1+0$

(d) $\pi^{+}\to\mu^{+}+\nu_\mu$ This reaction $\boxed{\text{can occur}}$.

(e) $n^{0}\to p^{+}+e^{-}+\bar{\nu}_e$ This reaction $\boxed{\text{can occur}}$.

(f) $\pi^{+}\to\mu^{+}+n$ Violates $\boxed{\text{baryon number}}$: $0\to 0+1$

Violates $\boxed{\text{muon-lepton number}}$: $0\to -1+0$

P46.18 Momentum conservation for the decay requires the pions to have equal speeds.

The total energy of each is $\dfrac{497.7\text{ MeV}}{2}$

so $E^2 = p^2c^2 + \left(mc^2\right)^2$ gives

$$(248.8\text{ MeV})^2 = (pc)^2 + (139.6\text{ MeV})^2$$

Solving, $pc = 206\text{ MeV} = \gamma mvc = \dfrac{mc^2}{\sqrt{1-(v/c)^2}}\left(\dfrac{v}{c}\right)$

$$\frac{pc}{mc^2} = \frac{206\text{ MeV}}{139.6\text{ MeV}} = \frac{1}{\sqrt{1-(v/c)^2}}\left(\frac{v}{c}\right) = 1.48$$

$$\frac{v}{c} = 1.48\sqrt{1-\left(\frac{v}{c}\right)^2}$$

and $\left(\dfrac{v}{c}\right)^2 = 2.18\left[1-\left(\dfrac{v}{c}\right)^2\right] = 2.18 - 2.18\left(\dfrac{v}{c}\right)^2$

$$3.18\left(\frac{v}{c}\right)^2 = 2.18$$

so $\dfrac{v}{c} = \sqrt{\dfrac{2.18}{3.18}} = 0.828$

and $\boxed{v = 0.828c}$

P46.19 (a) In the suggested reaction $\text{p} \rightarrow \text{e}^+ + \gamma$

We would have for baryon numbers $+1 \rightarrow 0 + 0$

$\Delta B \neq 0$, so baryon number conservation would be violated.

(b) From conservation of momentum for the decay: $p_e = p_\gamma$

Then, for the positron, $E_e^2 = \left(p_e c\right)^2 + E_{0,e}^2$

becomes $E_e^2 = \left(p_\gamma c\right)^2 + E_{0,e}^2 = E_\gamma^2 + E_{0,e}^2$

From conservation of energy for the system: $E_{0,p} = E_e + E_\gamma$

or $E_e = E_{0,p} - E_\gamma$

so $E_e^2 = E_{0,p}^2 - 2E_{0,p}E_\gamma + E_\gamma^2$

Equating this to the result from above gives $E_\gamma^2 + E_{0,e}^2 = E_{0,p}^2 - 2E_{0,p}E_\gamma + E_\gamma^2$

or $E_\gamma = \dfrac{E_{0,p}^2 - E_{0,e}^2}{2E_{0,p}} = \dfrac{(938.3\text{ MeV})^2 - (0.511\text{ MeV})^2}{2(938.3\text{ MeV})} = \boxed{469\text{ MeV}}$

Thus $E_e = E_{0,p} - E_\gamma = 938.3\text{ MeV} - 469\text{ MeV} = \boxed{469\text{ MeV}}$

Also, $p_\gamma = \dfrac{E_\gamma}{c} = \boxed{\dfrac{469\text{ MeV}}{c}}$

and $p_e = p_\gamma = \boxed{\dfrac{469\text{ MeV}}{c}}$

continued on next page

(c) The total energy of the positron is $E_e = 469 \text{ MeV}$

But, $E_e = \gamma E_{0,e} = \dfrac{E_{0,e}}{\sqrt{1-(v/c)^2}}$

so $\sqrt{1-\left(\dfrac{v}{c}\right)^2} = \dfrac{E_{0,e}}{E_e} = \dfrac{0.511 \text{ MeV}}{469 \text{ MeV}} = 1.09 \times 10^{-3}$

which yields $\boxed{v = 0.999\,999\,4c}$

P46.20 The relevant conservation laws are $\Delta L_e = 0$

$\Delta L_\mu = 0$

and $\Delta L_\tau = 0$

(a) $\pi^+ \to \pi^0 + e^+ + ?$ $\quad L_e: \; 0 \to 0 - 1 + L_e$ $\quad$ implies $L_e = 1$ $\quad$ and we have a $\boxed{\nu_e}$

(b) $? + p \to \mu^- + p + \pi^+$ $\quad L_\mu: \; L_\mu + 0 \to +1 + 0 + 0$ $\quad$ implies $L_\mu = 1$ $\quad$ and we have a $\boxed{\nu_\mu}$

(c) $\Lambda^0 \to p + \mu^- + ?$ $\quad L_\mu: \; 0 \to 0 + 1 + L_\mu$ $\quad$ implies $L_\mu = -1$ $\quad$ and we have a $\boxed{\bar{\nu}_\mu}$

(d) $\tau^+ \to \mu^+ + ? + ?$ $\quad L_\mu: \; 0 \to -1 + L_\mu$ $\quad$ implies $L_\mu = 1$ $\quad$ and we have a $\boxed{\nu_\mu}$

$L_\tau: \; -1 \to 0 + L_\tau$ $\quad$ implies $L_\tau = -1$ $\quad$ and we have a $\boxed{\bar{\nu}_\tau}$

Conclusion for (d): $L_\mu = 1$ for one particle, and $L_\tau = -1$ for the other particle.

We have $\boxed{\nu_\mu}$

and $\boxed{\bar{\nu}_\tau}$

Section 46.6 **Strange Particles and Strangeness**

P46.21 (a) $\Lambda^0 \to p + \pi^-$ $\quad$ Strangeness: $-1 \to 0 + 0$ $\quad$ (strangeness is $\boxed{\text{not conserved}}$)

(b) $\pi^- + p \to \Lambda^0 + K^0$ $\quad$ Strangeness: $0 + 0 \to -1 + 1$ $\quad$ ($0 = 0$ and strangeness is $\boxed{\text{conserved}}$)

(c) $\bar{p} + p \to \bar{\Lambda}^0 + \Lambda^0$ $\quad$ Strangeness: $0 + 0 \to +1 - 1$ $\quad$ ($0 = 0$ and strangeness is $\boxed{\text{conserved}}$)

(d) $\pi^- + p \to \pi^- + \Sigma^+$ $\quad$ Strangeness: $0 + 0 \to 0 - 1$ $\quad$ ($0 \neq -1$: strangeness is $\boxed{\text{not conserved}}$)

(e) $\Xi^- \to \Lambda^0 + \pi^-$ $\quad$ Strangeness: $-2 \to -1 + 0$ $\quad$ ($-2 \neq -1$ so strangeness is $\boxed{\text{not conserved}}$)

(f) $\Xi^0 \to p + \pi^-$ $\quad$ Strangeness: $-2 \to 0 + 0$ $\quad$ ($-2 \neq 0$ so strangeness is $\boxed{\text{not conserved}}$)

P46.22 The $\rho^0 \to \pi^+ + \pi^-$ decay must occur via the strong interaction.
The $K_S^0 \to \pi^+ + \pi^-$ decay must occur via the weak interaction.

P46.23 (a) $\mu^- \to e^- + \gamma$ L_e: $0 \to 1+0$,

and L_μ: $1 \to 0$

(b) $n \to p + e^- + \nu_e$ L_e: $0 \to 0+1+1$

(c) $\Lambda^0 \to p + \pi^0$ Strangeness: $-1 \to 0+0$,

and charge: $0 \to +1+0$

(d) $p \to e^+ + \pi^0$ Baryon number: $+1 \to 0+0$

(e) $\Xi^0 \to n + \pi^0$ Strangeness: $-2 \to 0+0$

P46.24 (a) $\pi^- + p \to 2\eta$ violates conservation of baryon number as $0+1 \to 0$, $\boxed{\text{not allowed}}$.

(b) $K^- + n \to \Lambda^0 + \pi^-$

Baryon number, $0+1 \to 1+0$
Charge, $-1+0 \to 0-1$
Strangeness, $-1+0 \to -1+0$

Lepton number, $0 \to 0$
The interaction may occur via the $\boxed{\text{strong interaction}}$ since all are conserved.

(c) $K^- \to \pi^- + \pi^0$

Strangeness, $-1 \to 0+0$
Baryon number, $0 \to 0$
Lepton number, $0 \to 0$
Charge, $-1 \to -1+0$
Strangeness conservation is violated by one unit, but everything else is conserved. Thus, the reaction can occur via the $\boxed{\text{weak interaction}}$, but not the strong or electromagnetic interaction.

(d) $\Omega^- \to \Xi^- + \pi^0$

Baryon number, $1 \to 1+0$
Lepton number, $0 \to 0$
Charge, $-1 \to -1+0$
Strangeness, $-3 \to -2+0$

May occur by $\boxed{\text{weak interaction}}$, but not by strong or electromagnetic.

(e) $\eta \to 2\gamma$

Baryon number, $0 \to 0$
Lepton number, $0 \to 0$
Charge, $0 \to 0$
Strangeness, $0 \to 0$
No conservation laws are violated, but photons are the mediators of the electromagnetic interaction. Also, the lifetime of the η is consistent with the $\boxed{\text{electromagnetic interaction}}$.

P46.25 (a) $\Xi^- \to \Lambda^0 + \mu^- + \nu_\mu$

Baryon number:	$+1 \to +1+0+0$	Charge:	$-1 \to 0-1+0$
L_e:	$0 \to 0+0+0$	L_μ:	$0 \to 0+1+1$
L_τ:	$0 \to 0+0+0$	Strangeness:	$-2 \to -1+0+0$

Conserved quantities are: $\boxed{B,\ \text{charge},\ L_e,\ \text{and}\ L_\tau}$

(b) $K_S^0 \to 2\pi^0$

Baryon number:	$0 \to 0$	Charge:	$0 \to 0$
L_e:	$0 \to 0$	L_μ:	$0 \to 0$
L_τ:	$0 \to 0$	Strangeness:	$+1 \to 0$

Conserved quantities are: $\boxed{B,\ \text{charge},\ L_e,\ L_\mu,\ \text{and}\ L_\tau}$

(c) $K^- + p \to \Sigma^0 + n$

Baryon number:	$0+1 \to 1+1$	Charge:	$-1+1 \to 0+0$
L_e:	$0+0 \to 0+0$	L_μ:	$0+0 \to 0+0$
L_τ:	$0+0 \to 0+0$	Strangeness:	$-1+0 \to -1+0$

Conserved quantities are: $\boxed{S,\ \text{charge},\ L_e,\ L_\mu,\ \text{and}\ L_\tau}$

(d) $\Sigma^0 + \Lambda^0 + \gamma$

Baryon number:	$+1 \to 1+0$	Charge:	$0 \to 0$
L_e:	$0 \to 0+0$	L_μ:	$0 \to 0+0$
L_τ:	$0 \to 0+0$	Strangeness:	$-1 \to -1+0$

Conserved quantities are: $\boxed{B,\ S,\ \text{charge},\ L_e,\ L_\mu,\ \text{and}\ L_\tau}$

(e) $e^+ + e^- \to \mu^+ + \mu^-$

Baryon number:	$0+0 \to 0+0$	Charge:	$+1-1 \to +1-1$
L_e:	$-1+1 \to 0+0$	L_μ:	$0+0 \to +1-1$
L_τ:	$0+0 \to 0+0$	Strangeness:	$0+0 \to 0+0$

Conserved quantities are: $\boxed{B,\ S,\ \text{charge},\ L_e,\ L_\mu,\ \text{and}\ L_\tau}$

(f) $\bar{p} + n \to \bar{\Lambda}^0 + \Sigma^-$

Baryon number:	$-1+1 \to -1+1$	Charge:	$-1+0 \to 0-1$
L_e:	$0+0 \to 0+0$	L_μ:	$0+0 \to 0+0$
L_τ:	$0+0 \to 0+0$	Strangeness:	$0+0 \to +1-1$

Conserved quantities are: $\boxed{B,\ S,\ \text{charge},\ L_e,\ L_\mu,\ \text{and}\ L_\tau}$

P46.26 (a) $K^+ + p \to \underline{?} + p$

The strong interaction conserves everything.

Baryon number,	$0+1 \to B+1$	so	$B=0$
Charge,	$+1+1 \to Q+1$	so	$Q=+1$
Lepton numbers,	$0+0 \to L+0$	so	$L_e = L_\mu = L_\tau = 0$
Strangeness,	$+1+0 \to S+0$	so	$S=1$

The conclusion is that the particle must be positively charged, a non-baryon, with strangeness of +1. Of particles in Table 46.2, it can only be the $\boxed{K^+}$. Thus, this is an elastic scattering process.

The weak interaction conserves all but strangeness, and $\Delta S = \pm 1$.

(b) $\Omega^- \to \underline{?} + \pi^-$

Baryon number,	$+1 \to B+0$	so	$B=1$
Charge,	$-1 \to Q-1$	so	$Q=0$
Lepton numbers,	$0 \to L+0$	so	$L_e = L_\mu = L_\tau = 0$
Strangeness,	$-3 \to S+0$	so	$\Delta S = 1$: $S=-2$

The particle must be a neutral baryon with strangeness of –2. Thus, it is the $\boxed{\Xi^0}$.

(c) $K^+ \to \underline{?} + \mu^+ + \nu_\mu$

Baryon number,	$0 \to B+0+0$	so	$B=0$
Charge,	$+1 \to Q+1+0$	so	$Q=0$
Lepton numbers,	L_e, $0 \to L_e+0+0$	so	$L_e=0$
	L_μ, $0 \to L_\mu-1+1$	so	$L_\mu=0$
	L_τ, $0 \to L_\tau+0+0$	so	$L_\tau=0$
Strangeness,	$1 \to S+0+0$	so	$\Delta S = \pm 1$
			(for weak interaction): $S=0$

The particle must be a neutral meson with strangeness $=0 \Rightarrow \boxed{\pi^0}$.

P46.27 Time-dilated lifetime:

$$T = \gamma T_0 = \frac{0.900\times10^{-10}\ \text{s}}{\sqrt{1-v^2/c^2}} = \frac{0.900\times10^{-10}\ \text{s}}{\sqrt{1-(0.960)^2}} = 3.214\times10^{-10}\ \text{s}$$

$$\text{distance} = 0.960\left(3.00\times10^8\ \text{m/s}\right)\left(3.214\times10^{-10}\ \text{s}\right) = \boxed{9.26\ \text{cm}}$$

P46.28 (a) $$p_{\Sigma^+} = eBr_{\Sigma^+} = \frac{(1.602\,177\times10^{-19}\text{ C})(1.15\text{ T})(1.99\text{ m})}{5.344\,288\times10^{-22}\,(\text{kg}\cdot\text{m/s})/(\text{MeV}/c)} = \boxed{\frac{686\text{ MeV}}{c}}$$

$$p_{\pi^+} = eBr_{\pi^+} = \frac{(1.602\,177\times10^{-19}\text{ C})(1.15\text{ T})(0.580\text{ m})}{5.344\,288\times10^{-22}\,(\text{kg}\cdot\text{m/s})/(\text{MeV}/c)} = \boxed{\frac{200\text{ MeV}}{c}}$$

(b) Let ϕ be the angle made by the neutron's path with the path of the Σ^+ at the moment of decay. By conservation of momentum:

$$p_n\cos\phi + (199.961\,581\text{ MeV}/c)\cos 64.5° = 686.075\,081\text{ MeV}/c$$

$$\therefore p_n\cos\phi = 599.989\,401\text{ MeV}/c \qquad (1)$$

$$p_n\sin\phi = (199.961\,581\text{ MeV}/c)\sin 64.5° = 180.482\,380\text{ MeV}/c \qquad (2)$$

From (1) and (2): $$p_n = \sqrt{(599.989\,401\text{ MeV}/c)^2 + (180.482\,380\text{ MeV}/c)^2} = \boxed{627\text{ MeV}/c}$$

(c) $$E_{\pi^+} = \sqrt{(p_{\pi^+}c)^2 + (m_{\pi^+}c^2)^2} = \sqrt{(199.961\,581\text{ MeV})^2 + (139.6\text{ MeV})^2} = \boxed{244\text{ MeV}}$$

$$E_n = \sqrt{(p_nc)^2 + (m_nc^2)^2} = \sqrt{(626.547\,022\text{ MeV})^2 + (939.6\text{ MeV})^2} = \boxed{1\,130\text{ MeV}}$$

$$E_{\Sigma^+} = E_{\pi^+} + E_n = 243.870\,445\text{ MeV} + 1\,129.340\,219\text{ MeV} = \boxed{1\,370\text{ MeV}}$$

(d) $$m_{\Sigma^+}c^2 = \sqrt{E_{\Sigma^+}^2 - (p_{\Sigma^+}c)^2} = \sqrt{(1\,373.210\,664\text{ MeV})^2 - (686.075\,081\text{ MeV})^2} = 1\,190\text{ MeV}$$

$$\therefore m_{\Sigma^+} = \boxed{1\,190\text{ MeV}/c^2}$$

$$E_{\Sigma^+} = \gamma m_{\Sigma^+}c^2,\text{ where } \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \frac{1\,373.210\,664\text{ MeV}}{1\,189.541\,303\text{ MeV}} = 1.154\,4$$

Solving for v, we find $v = \boxed{0.500c}$.

P46.29 (a) Let $E_{\min}$ be the minimum total energy of the bombarding particle that is needed to induce the reaction. At this energy the product particles all move with the same velocity. The product particles are then equivalent to a single particle having mass equal to the total mass of the product particles, moving with the same velocity as each product particle. By conservation of energy:

$$E_{\min} + m_2c^2 = \sqrt{\left(m_3c^2\right)^2 + \left(p_3c\right)^2} \qquad (1)$$

By conservation of momentum: $p_3 = p_1$

$$\therefore \left(p_3c\right)^2 = \left(p_1c\right)^2 = E_{\min}^2 - \left(m_1c^2\right)^2 \qquad (2)$$

Substitute (2) in (1): $E_{\min} + m_2c^2 = \sqrt{\left(m_3c^2\right)^2 + E_{\min}^2 - \left(m_1c^2\right)^2}$

Square both sides:

$$E_{\min}^2 + 2E_{\min}m_2c^2 + \left(m_2c^2\right)^2 = \left(m_3c^2\right)^2 + E_{\min}^2 - \left(m_1c^2\right)^2$$

$$\therefore E_{\min} = \frac{\left(m_3^2 - m_1^2 - m_2^2\right)c^2}{2m_2}$$

$$\therefore K_{\min} = E_{\min} - m_1c^2 = \frac{\left(m_3^2 - m_1^2 - m_2^2 - 2m_1m_2\right)c^2}{2m_2} = \frac{\left[m_3^2 - \left(m_1 + m_2\right)^2\right]c^2}{2m_2}$$

Refer to Table 46.2 for the particle masses.

(b) $$K_{\min} = \frac{\left[4(938.3)\right]^2 \text{ MeV}^2/c^2 - \left[2(938.3)\right]^2 \text{ MeV}^2/c^2}{2\left(938.3 \text{ MeV}/c^2\right)} = \boxed{5.63 \text{ GeV}}$$

(c) $$K_{\min} = \frac{(497.7 + 1115.6)^2 \text{ MeV}^2/c^2 - (139.6 + 938.3)^2 \text{ MeV}^2/c^2}{2(938.3) \text{ MeV}/c^2} = \boxed{768 \text{ MeV}}$$

(d) $$K_{\min} = \frac{\left[2(938.3) + 135\right]^2 \text{ MeV}^2/c^2 - \left[2(938.3)\right]^2 \text{ MeV}^2/c^2}{2(938.3) \text{ MeV}/c^2} = \boxed{280 \text{ MeV}}$$

(e) $$K_{\min} = \frac{\left[\left(91.2\times10^3\right)^2 - \left[(938.3 + 938.3)^2\right] \text{ MeV}^2/c^2\right]}{2(938.3) \text{ MeV}/c^2} = \boxed{4.43 \text{ TeV}}$$

Section 46.7 Finding Patterns in the Particles

Section 46.8 Quarks

Section 46.9 Multicolored Quarks

Section 46.10 The Standard Model

P46.30 (a) The number of protons

$$N_p = 1\,000\text{ g}\left(\frac{6.02\times10^{23}\text{ molecules}}{18.0\text{ g}}\right)\left(\frac{10\text{ protons}}{\text{molecule}}\right) = 3.34\times10^{26}\text{ protons}$$

and there are $$N_n = 1\,000\text{ g}\left(\frac{6.02\times10^{23}\text{ molecules}}{18.0\text{ g}}\right)\left(\frac{8\text{ neutrons}}{\text{molecule}}\right) = 2.68\times10^{26}\text{ neutrons}$$

So there are for electric neutrality $\boxed{3.34\times10^{26}\text{ electrons}}$

The up quarks have number $2\left(3.34\times10^{26}\right)+2.68\times10^{26} = \boxed{9.36\times10^{26}\text{ up quarks}}$

and there are $2\left(2.68\times10^{26}\right)+3.34\times10^{26} = \boxed{8.70\times10^{26}\text{ down quarks}}$

(b) Model yourself as 65 kg of water. Then you contain:

$$65\left(3.34\times10^{26}\right)\boxed{\sim10^{28}\text{ electrons}}$$

$$65\left(9.36\times10^{26}\right)\boxed{\sim10^{29}\text{ up quarks}}$$

$$65\left(8.70\times10^{26}\right)\boxed{\sim10^{29}\text{ down quarks}}$$

Only these fundamental particles form your body. You have no strangeness, charm, topness, or bottomness.

P46.31 (a)

	proton	u	u	d	total
strangeness	0	0	0	0	0
baryon number	1	1/3	1/3	1/3	1
charge	e	$2e/3$	$2e/3$	$-e/3$	e

(b)

	neutron	u	d	d	total
strangeness	0	0	0	0	0
baryon number	1	1/3	1/3	1/3	1
charge	0	$2e/3$	$-e/3$	$-e/3$	0

P46.32 Quark composition of proton = uud and of neutron = udd.
Thus, if we neglect binding energies, we may write

$$m_p = 2m_u + m_d \tag{1}$$

and

$$m_n = m_u + 2m_d \tag{2}$$

Solving simultaneously,

we find $$m_u = \frac{1}{3}(2m_p - m_n) = \frac{1}{3}\left[2(938\ \text{MeV}/c^2) - 939.6\ \text{meV}/c^2\right] = \boxed{312\ \text{MeV}/c^2}$$

and from either (1) or (2), $m_d = \boxed{314\ \text{MeV}/c^2}$.

P46.33 (a)

	K^0	d	$\bar{s}$	total
strangeness	1	0	1	1
baryon number	0	1/3	–1/3	0
charge	0	$-e/3$	$e/3$	0

(b)

	Λ^0	u	d	s	total
strangeness	–1	0	0	–1	–1
baryon number	1	1/3	1/3	1/3	1
charge	0	$2e/3$	$-e/3$	$-e/3$	0

P46.34 In the first reaction, $\pi^- + p \to K^0 + \Lambda^0$, the quarks in the particles are $\bar{u}d + uud \to d\bar{s} + uds$. There is a net of 1 up quark both before and after the reaction, a net of 2 down quarks both before and after, and a net of zero strange quarks both before and after. Thus, the reaction conserves the net number of each type of quark.

In the second reaction, $\pi^- + p \to K^0 + n$, the quarks in the particles are $\bar{u}d + uud \to d\bar{s} + udd$. In this case, there is a net of 1 up and 2 down quarks before the reaction but a net of 1 up, 3 down, and 1 anti-strange quark after the reaction. Thus, the reaction does not conserve the net number of each type of quark.

P46.35 (a) $\pi^- + p \to K^0 + \Lambda^0$

In terms of constituent quarks: $\boxed{\bar{u}d + uud \to d\bar{s} + uds}$

up quarks:	$-1+2 \to 0+1$,	or	$1 \to 1$
down quarks:	$1+1 \to 1+1$,	or	$2 \to 2$
strange quarks:	$0+0 \to -1+1$,	or	$0 \to 0$

(b) $\pi^+ + p \to K^+ + \Sigma^+$ $\boxed{\bar{d}u + uud \to u\bar{s} + uus}$

up quarks:	$1+2 \to 1+2$,	or	$3 \to 3$
down quarks:	$-1+1 \to 0+0$,	or	$0 \to 0$
strange quarks:	$0+0 \to -1+1$,	or	$0 \to 0$

(c) $K^- + p \to K^+ + K^0 + \Omega^-$ $\boxed{\bar{u}s + uud \to u\bar{s} + d\bar{s} + sss}$

up quarks:	$-1+2 \to 1+0+0$,	or	$1 \to 1$
down quarks:	$0+1 \to 0+1+0$,	or	$1 \to 1$
strange quarks:	$1+0 \to -1-1+3$,	or	$1 \to 1$

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(d) $p + p \to K^0 + p + \pi^+ + \underline{?}$ $\qquad$ $uud + uud \to d\bar{s} + uud + u\bar{d} + \underline{?}$

The quark combination of ? must be such as to balance the last equation for up, down, and strange quarks.

up quarks:	$2+2=0+2+1+?$	(has 1 u quark)
down quarks:	$1+1=1+1-1+?$	(has 1 d quark)
strange quarks:	$0+0=-1+0+0+?$	(has 1 s quark)

quark composition $= uds = \boxed{\Lambda^0 \text{ or } \Sigma^0}$

P46.36 $\Sigma^0 + p \to \Sigma^+ + \gamma + X$

$dds + uud \to uds + 0 + ?$

The left side has a net 3d, 2u, and 1s. The right-hand side has 1d, 1u, and 1s leaving 2d and 1u missing.

$\boxed{\text{The unknown particle is a neutron, udd.}}$

Baryon and strangeness numbers are conserved.

P46.37 Compare the given quark states to the entries in Tables 46.4 and 46.5:

(a) $suu = \boxed{\Sigma^+}$

(b) $\bar{u}d = \boxed{\pi^-}$

(c) $\bar{s}d = \boxed{K^0}$

(d) $ssd = \boxed{\Xi^-}$

P46.38 (a) $\bar{u}\bar{u}\bar{d}$: $\quad$ charge $= \left(-\frac{2}{3}e\right) + \left(-\frac{2}{3}e\right) + \left(\frac{1}{3}e\right) = \boxed{-e}$. This is the $\boxed{\text{antiproton}}$.

(b) $\bar{u}\bar{d}\bar{d}$: $\quad$ charge $= \left(-\frac{2}{3}e\right) + \left(\frac{1}{3}e\right) + \left(\frac{1}{3}e\right) = \boxed{0}$. This is the $\boxed{\text{antineutron}}$.

Section 46.11 **The Cosmic Connection**

***P46.39** We let r in Hubble's law represent any distance.

(a) $$v = Hr = 17\times10^{-3}\ \frac{\text{m}}{\text{s}\cdot\text{ly}}1.85\ \text{m}\left(\frac{1\ \text{ly}}{c\cdot 1\ \text{yr}}\right)\left(\frac{c}{3\times10^{8}\ \text{m/s}}\right)\left(\frac{1\ \text{yr}}{3.156\times10^{7}\ \text{s}}\right)$$

$$= 17\times10^{-3}\ \frac{\text{m}}{\text{s}\cdot 9.47\times10^{15}\ \text{m}}1.85\ \text{m} = 1.80\times10^{-18}\ \frac{1}{\text{s}}1.85\ \text{m} = \boxed{3.32\times10^{-18}\ \text{m/s}}$$

This is unobservably small.

(b) $v = Hr = 1.80\times10^{-18}\ \frac{1}{\text{s}}3.84\times10^{8}\ \text{m} = \boxed{6.90\times10^{-10}\ \text{m/s}}$ again too small to measure

P46.40 Section 39.4 says $f_{\text{observer}} = f_{\text{source}}\sqrt{\dfrac{1+v_a/c}{1-v_a/c}}$

The velocity of approach, v_a, is the negative of the velocity of mutual recession: $v_a = -v$.

Then, $\dfrac{c}{\lambda'} = \dfrac{c}{\lambda}\sqrt{\dfrac{1-v/c}{1+v/c}}$ and $\boxed{\lambda' = \lambda\sqrt{\dfrac{1+v/c}{1-v/c}}}$

P46.41 (a) $\lambda' = \lambda\sqrt{\dfrac{1+v/c}{1-v/c}}$ $\quad 510 \text{ nm} = 434 \text{ nm}\sqrt{\dfrac{1+v/c}{1-v/c}}$

$$1.18^2 = \frac{1+v/c}{1-v/c} = 1.381$$

$$1+\frac{v}{c} = 1.381 - 1.381\frac{v}{c} \qquad 2.38\frac{v}{c} = 0.381$$

$$\frac{v}{c} = 0.160 \quad \text{or} \quad v = \boxed{0.160c} = 4.80\times10^{7} \text{ m/s}$$

(b) $v = HR$: $\quad R = \dfrac{v}{H} = \dfrac{4.80\times10^{7} \text{ m/s}}{17\times10^{-3} \text{ m/s}\cdot\text{ly}} = \boxed{2.82\times10^{9} \text{ ly}}$

P46.42 (a) $\lambda_n' = \lambda_n\sqrt{\dfrac{1+v/c}{1-v/c}} = (Z+1)\lambda_n$ $\quad \dfrac{1+v/c}{1-v/c} = (Z+1)^2$

$$1+\frac{v}{c} = (Z+1)^2 - \left(\frac{v}{c}\right)(Z+1)^2 \qquad \left(\frac{v}{c}\right)(Z^2+2Z+2) = Z^2+2Z$$

$$v = \boxed{c\left(\frac{Z^2+2Z}{Z^2+2Z+2}\right)}$$

(b) $R = \dfrac{v}{H} = \boxed{\dfrac{c}{H}\left(\dfrac{Z^2+2Z}{Z^2+2Z+2}\right)}$

P46.43 $v = HR$ $\quad H = \dfrac{(1.7\times10^{-2} \text{ m/s})}{\text{ly}}$

(a) $v(2.00\times10^{6} \text{ ly}) = 3.4\times10^{4} \text{ m/s}$ $\quad \lambda' = \lambda\sqrt{\dfrac{1+v/c}{1-v/c}} = 590(1.000\,113\,3) = \boxed{590.07 \text{ nm}}$

(b) $v(2.00\times10^{8} \text{ ly}) = 3.4\times10^{6} \text{ m/s}$ $\quad \lambda' = 590\sqrt{\dfrac{1+0.011\,33}{1-0.011\,33}} = \boxed{597 \text{ nm}}$

(c) $v(2.00\times10^{9} \text{ ly}) = 3.4\times10^{7} \text{ m/s}$ $\quad \lambda' = 590\sqrt{\dfrac{1+0.113\,3}{1-0.113\,3}} = \boxed{661 \text{ nm}}$

***P46.44** (a) What we can see is limited by the finite age of the Universe and by the finite speed of light. We can see out only to a look-back time equal to a bit less than the age of the Universe. Every year on your birthday the Universe also gets a year older, and light now in transit from still more distant objects arrives at Earth. So the radius of the visible Universe expands at the speed of light, which is $dr/dt = c = 1$ ly/yr.

(b) The volume of the visible section of the Universe is $(4/3)\pi r^3$ where $r = 13.7$ billion light-years. The rate of volume increase is

$$\frac{dV}{dt} = \frac{d\ \frac{4}{3}\pi r^3}{dt} = \frac{4}{3}\pi 3r^2\frac{dr}{dt} = 4\pi r^2 c = 4\pi\left(13.7\times10^9\ \text{ly}\frac{3\times10^8\ \frac{\text{m}}{\text{s}}\ 3.156\times10^7\ \text{s}}{1\ \text{ly}}\right)^2 3\times10^8\ \frac{\text{m}}{\text{s}}$$

$$= \boxed{6.34\times10^{61}\ \text{m}^3/\text{s}}$$

***P46.45** (a) The volume of the sphere bounded by the Earth's orbit is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(1.496\times10^{11}\ \text{m}\right)^3 = 1.40\times10^{34}\ \text{m}^3$$

$$m = \rho V = 6\times10^{-28}\ \text{kg/m}^3\ 1.40\times10^{34}\ \text{m}^3 = \boxed{8.41\times10^6\ \text{kg}}$$

(b) By Gauss's law, the dark matter would create a gravitational field acting on the Earth to accelerate it toward the Sun. It would shorten the duration of the year in the same way that 8.41×10^6 kg of extra material in the Sun would. This has the fractional effect of

$\frac{8.41\times10^6\ \text{kg}}{1.99\times10^{30}\ \text{kg}} = 4.23\times10^{-24}$ of the mass of the Sun. It is $\boxed{\text{immeasurably small}}$.

P46.46 (a) Wien's law: $\lambda_{\text{max}}T = 2.898\times10^{-3}\ \text{m}\cdot\text{K}$

Thus, $$\lambda_{\text{max}} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{2.73\ \text{K}} = 1.06\times10^{-3}\ \text{m} = \boxed{1.06\ \text{mm}}.$$

(b) This is a $\boxed{\text{microwave}}$.

P46.47 We assume that the fireball of the Big Bang is a black body.

$$I = e\sigma T^4 = (1)\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)(2.73\ \text{K})^4 = \boxed{3.15\times10^{-6}\ \text{W/m}^2}$$

As a bonus, we can find the current power of direct radiation from the Big Bang in the portion of the Universe observable to us. If it is fourteen billion years old, the fireball is a perfect sphere of radius fourteen billion light years, centered at the point halfway between your eyes:

$$\mathcal{P} = IA = I(4\pi r^2) = \left(3.15\times10^{-6}\ \text{W/m}^2\right)(4\pi)\left(14\times10^9\ \text{ly}\right)^2\left(\frac{3\times10^8\ \text{m/s}}{1\ \text{ly/yr}}\right)^2\left(3.156\times10^7\ \text{s/yr}\right)^2$$

$$\mathcal{P} = 7\times10^{47}\ \text{W}$$

P46.48 The density of the Universe is

$$\rho = 1.20\rho_c = 1.20\left(\frac{3H^2}{8\pi G}\right)$$

Consider a remote galaxy at distance r. The mass interior to the sphere below it is

$$M = \rho\left(\frac{4}{3}\pi r^3\right) = 1.20\left(\frac{3H^2}{8\pi G}\right)\left(\frac{4}{3}\pi r^3\right) = \frac{0.600H^2r^3}{G}$$

both now and in the future when it has slowed to rest from its current speed $v = Hr$. The energy of this galaxy-sphere system is constant as the galaxy moves to apogee distance R:

$$\frac{1}{2}mv^2 - \frac{GmM}{r} = 0 - \frac{GmM}{R} \qquad \text{so} \qquad \frac{1}{2}mH^2r^2 - \frac{Gm}{r}\left(\frac{0.600H^2r^3}{G}\right) = 0 - \frac{Gm}{R}\left(\frac{0.600H^2r^3}{G}\right)$$

$$-0.100 = -0.600\frac{r}{R} \qquad \text{so} \qquad R = 6.00r$$

The Universe will expand by a factor of $\boxed{6.00}$ from its current dimensions.

P46.49 (a) $k_B T \approx 2m_p c^2$

$$\text{so} \qquad T \approx \frac{2m_p c^2}{k_B} = \frac{2(938.3\text{ MeV})}{(1.38\times10^{-23}\text{ J/K})}\left(\frac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right)\boxed{\sim 10^{13}\text{ K}}$$

(b) $k_B T \approx 2m_e c^2$

$$\text{so} \qquad T \approx \frac{2m_e c^2}{k_B} = \frac{2(0.511\text{ MeV})}{(1.38\times10^{-23}\text{ J/K})}\left(\frac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right)\boxed{\sim 10^{10}\text{ K}}$$

P46.50 (a) The Hubble constant is defined in $v = HR$. The distance R between any two far-separated objects opens at constant speed according to $R = vt$. Then the time t since the Big Bang is found from

$$v = H\,vt \qquad 1 = Ht \qquad t = \frac{1}{H}$$

(b) $$\frac{1}{H} = \frac{1}{17\times10^{-3}\text{ m/s}\cdot\text{ly}}\left(\frac{3\times10^8\text{ m/s}}{1\text{ ly/yr}}\right) = \boxed{1.76\times10^{10}\text{ yr}} = 17.6\text{ billion years}$$

P46.51 (a) Consider a sphere around us of radius R large compared to the size of galaxy clusters. If the matter M inside the sphere has the critical density, then a galaxy of mass m at the surface of the sphere is moving just at escape speed v according to

$$K + U_g = 0 \qquad \frac{1}{2}mv^2 - \frac{GMm}{R} = 0$$

The energy of the galaxy-sphere system is conserved, so this equation is true throughout the history of the Universe after the Big Bang, where $v = \frac{dR}{dt}$. Then

$$\left(\frac{dR}{dt}\right)^2 = \frac{2GM}{R} \qquad \frac{dR}{dt} = R^{-1/2}\sqrt{2GM} \qquad \int_0^R \sqrt{R}\,dR = \sqrt{2GM}\int_0^T dt$$

$$\left.\frac{R^{3/2}}{3/2}\right|_0^R = \sqrt{2GM}\,t\Big|_0^T \qquad \frac{2}{3}R^{3/2} = \sqrt{2GM}\,T$$

$$T = \frac{2}{3}\frac{R^{3/2}}{\sqrt{2GM}} = \frac{2}{3}\frac{R}{\sqrt{2GM/R}}$$

From above, $\sqrt{\frac{2GM}{R}} = v$

so $T = \frac{2}{3}\frac{R}{v}$

Now Hubble's law says $v = HR$

So $T = \frac{2}{3}\frac{R}{HR} = \frac{2}{3H}$

(b) $$T = \frac{2}{3\left(17\times10^{-3}\ \text{m/s}\cdot\text{ly}\right)}\left(\frac{3\times10^{8}\ \text{m/s}}{1\ \text{ly/yr}}\right) = \boxed{1.18\times10^{10}\ \text{yr}} = 11.8 \text{ billion years}$$

P46.52 In our frame of reference, Hubble's law is exemplified by $\vec{\mathbf{v}}_1 = H\vec{\mathbf{R}}_1$ and $\vec{\mathbf{v}}_2 = H\vec{\mathbf{R}}_2$. From these we may form the equations $-\vec{\mathbf{v}}_1 = -H\vec{\mathbf{R}}_1$ and $\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 = H\left(\vec{\mathbf{R}}_2 - \vec{\mathbf{R}}_1\right)$. These equations express Hubble's law as seen by the observer in the first galaxy cluster, as she looks at us to find $-\vec{\mathbf{v}}_1 = H\left(-\vec{\mathbf{R}}_1\right)$ and as she looks at cluster two to find $\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 = H\left(\vec{\mathbf{R}}_2 - \vec{\mathbf{R}}_1\right)$.

Section 46.12 **Problems and Perspectives**

***P46.53** (a) $$L = \sqrt{\frac{\hbar G}{c^3}} = \sqrt{\frac{\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)}{\left(3.00\times10^{8}\ \text{m/s}\right)^3}} = \boxed{1.61\times10^{-35}\ \text{m}}$$

(b) The Planck time is given as $T = \frac{L}{c} = \frac{1.61\times10^{-35}\ \text{m}}{3.00\times10^{8}\ \text{m/s}} = \boxed{5.38\times10^{-44}\ \text{s}}$, which is approximately equal to the duration of the ultra-hot epoch.

(c) Yes. The uncertainty principle foils any attempt at making observations of things when the age of the Universe was less than the Planck time. The opaque fireball of the Big Bang, measured as the cosmic microwave background radiation, prevents us from receiving visible light from things before the Universe was a few hundred thousand years old. Walls of more profound fire hide all information from still earlier times.

Additional Problems

P46.54 We find the number N of neutrinos:

$$10^{46}\ \text{J} = N(6\ \text{MeV}) = N\left(6\times1.60\times10^{-13}\ \text{J}\right)$$

$$N = 1.0\times10^{58}\ \text{neutrinos}$$

The intensity at our location is

$$\frac{N}{A} = \frac{N}{4\pi r^2} = \frac{1.0\times10^{58}}{4\pi\left(1.7\times10^{5}\ \text{ly}\right)^2}\left(\frac{1\ \text{ly}}{\left(3.00\times10^{8}\ \text{m/s}\right)\left(3.16\times10^{7}\ \text{s}\right)}\right)^2 = 3.1\times10^{14}\ \text{m}^{-2}$$

The number passing through a body presenting $5\,000\ \text{cm}^2 = 0.50\ \text{m}^2$

is then $\left(3.1\times10^{14}\ \frac{1}{\text{m}^2}\right)\left(0.50\ \text{m}^2\right) = 1.5\times10^{14}$

or $\boxed{\sim 10^{14}}$

P46.55 A photon travels the distance from the Large Magellanic Cloud to us in 170 000 years. The hypothetical massive neutrino travels the same distance in 170 000 years plus 10 seconds:

$$c(170\,000\ \text{yr}) = v(170\,000\ \text{yr} + 10\ \text{s})$$

$$\frac{v}{c} = \frac{170\,000\ \text{yr}}{170\,000\ \text{yr} + 10\ \text{s}} = \frac{1}{1+\left\{10\ \text{s}/\left[\left(1.7\times10^{5}\ \text{yr}\right)\left(3.156\times10^{7}\ \text{s/yr}\right)\right]\right\}} = \frac{1}{1+1.86\times10^{-12}}$$

For the neutrino we want to evaluate mc^2 in $E = \gamma mc^2$:

$$mc^2 = \frac{E}{\gamma} = E\sqrt{1-\frac{v^2}{c^2}} = 10\ \text{MeV}\sqrt{1-\frac{1}{\left(1+1.86\times10^{-12}\right)^2}} = 10\ \text{MeV}\sqrt{\frac{\left(1+1.86\times10^{-12}\right)^2-1}{\left(1+1.86\times10^{-12}\right)^2}}$$

$$mc^2 \approx 10\ \text{MeV}\sqrt{\frac{2\left(1.86\times10^{-12}\right)}{1}} = 10\ \text{MeV}\left(1.93\times10^{-6}\right) = 19\ \text{eV}$$

Then the upper limit on the mass is

$$m = \boxed{\frac{19\ \text{eV}}{c^2}} \qquad \text{or} \qquad m = \frac{19\ \text{eV}}{c^2}\left(\frac{\text{u}}{931.5\times10^{6}\ \text{eV}/c^2}\right) = 2.1\times10^{-8}\ \text{u}$$

P46.56 (a) $\pi^- + \text{p} \to \Sigma^+ + \pi^0$ is forbidden by $\boxed{\text{charge conservation}}$

(b) $\mu^- \to \pi^- + \nu_e$ is forbidden by $\boxed{\text{energy conservation}}$

(c) $\text{p} \to \pi^+ + \pi^+ + \pi^-$ is forbidden by $\boxed{\text{baryon number conservation}}$

P46.57 The total energy in neutrinos emitted per second by the Sun is:

$$(0.4)\left[4\pi\left(1.5\times10^{11}\right)^2\right]\text{W} = 1.1\times10^{23}\ \text{W}$$

Over 10^9 years, the Sun emits 3.6×10^{39} J in neutrinos. This represents an annihilated mass

$$mc^2 = 3.6\times10^{39}\ \text{J}$$

$$m = 4.0\times10^{22}\ \text{kg}$$

About $\boxed{\text{1 part in 50 000 000}}$ of the Sun's mass, over 10^9 years, has been lost to neutrinos.

P46.58 $p + p \to p + \pi^+ + X$

The protons each have 70.4 MeV of kinetic energy. In accord with conservation of momentum for the collision, particle X has zero momentum and thus zero kinetic energy. Conservation of system energy then requires

$$M_p c^2 + M_\pi c^2 + M_X c^2 = \left(M_p c^2 + K_p\right) + \left(M_p c^2 + K_p\right)$$

$$M_X c^2 = M_p c^2 + 2K_p - M_\pi c^2 = 938.3 \text{ MeV} + 2(70.4 \text{ MeV}) - 139.6 \text{ MeV} = 939.5 \text{ MeV}$$

X must be a neutral baryon of rest energy 939.5 MeV. Thus X is a $\boxed{\text{neutron}}$.

P46.59 (a) If $2N$ particles are annihilated, the energy released is $2Nmc^2$. The resulting photon momentum is $p = \frac{E}{c} = \frac{2Nmc^2}{c} = 2Nmc$. Since the momentum of the system is conserved, the rocket will have momentum $2Nmc$ directed opposite the photon momentum.

$$\boxed{p = 2Nmc}$$

(b) Consider a particle that is annihilated and gives up its rest energy mc^2 to another particle which also has initial rest energy mc^2 (but no momentum initially).

$$E^2 = p^2c^2 + \left(mc^2\right)^2$$

Thus $\left(2mc^2\right)^2 = p^2c^2 + \left(mc^2\right)^2$

where p is the momentum the second particle acquires as a result of the annihilation of the first particle. Thus $4\left(mc^2\right)^2 = p^2c^2 + \left(mc^2\right)^2$, $p^2 = 3\left(mc^2\right)^2$. So $p = \sqrt{3}mc$.

This process is repeated N times (annihilate $\frac{N}{2}$ protons and $\frac{N}{2}$ antiprotons). Thus the total momentum acquired by the ejected particles is $\sqrt{3}Nmc$, and this momentum is imparted to the rocket.

$$\boxed{p = \sqrt{3}Nmc}$$

(c) Method (a) produces greater speed since $2Nmc > \sqrt{3}Nmc$.

P46.60 By relativistic energy conservation in the reaction, $$E_\gamma + m_e c^2 = \frac{3m_e c^2}{\sqrt{1 - v^2/c^2}} \qquad (1)$$

By relativistic momentum conservation for the system, $$\frac{E_\gamma}{c} = \frac{3m_e v}{\sqrt{1 - v^2/c^2}} \qquad (2)$$

Dividing (2) by (1), $$X = \frac{E_\gamma}{E_\gamma + m_e c^2} = \frac{v}{c}$$

Subtracting (2) from (1), $$m_e c^2 = \frac{3m_e c^2}{\sqrt{1 - X^2}} - \frac{3m_e c^2 X}{\sqrt{1 - X^2}}$$

Solving, $1 = \frac{3 - 3X}{\sqrt{1 - X^2}}$ and $X = \frac{4}{5}$ so $E_\gamma = 4m_e c^2 = \boxed{2.04 \text{ MeV}}$

P46.61 $m_\Lambda c^2 = 1115.6$ MeV $\qquad \Lambda^0 \to p + \pi^-$

$m_p c^2 = 938.3$ MeV $\qquad m_\pi c^2 = 139.6$ MeV

The difference between starting rest energy and final rest energy is the kinetic energy of the products.

$$K_p + K_\pi = 37.7 \text{ MeV} \qquad \text{and} \qquad p_p = p_\pi = p$$

Applying conservation of relativistic energy to the decay process, we have

$$\left[\sqrt{(938.3)^2 + p^2c^2} - 938.3\right] + \left[\sqrt{(139.6)^2 + p^2c^2} - 139.6\right] = 37.7 \text{ MeV}$$

Solving the algebra yields

$$p_\pi c = p_p c = 100.4 \text{ MeV}$$

Then, $K_p = \sqrt{\left(m_p c^2\right)^2 + (100.4)^2} - m_p c^2 = \boxed{5.35 \text{ MeV}}$

$$K_\pi = \sqrt{(139.6)^2 + (100.4)^2} - 139.6 = \boxed{32.3 \text{ MeV}}$$

P46.62 $p + p \to p + n + \pi^+$

The total momentum is zero before the reaction. Thus, all three particles present after the reaction may be at rest and still conserve system momentum. This will be the case when the incident protons have minimum kinetic energy. Under these conditions, conservation of energy for the reaction gives

$$2\left(m_p c^2 + K_p\right) = m_p c^2 + m_n c^2 + m_\pi c^2$$

so the kinetic energy of each of the incident protons is

$$K_p = \frac{m_n c^2 + m_\pi c^2 - m_p c^2}{2} = \frac{(939.6 + 139.6 - 938.3) \text{ MeV}}{2} = \boxed{70.4 \text{ MeV}}$$

P46.63 $\Sigma^0 \to \Lambda^0 + \gamma$

From Table 46.2, $m_\Sigma = 1\,192.5 \text{ MeV}/c^2$ and $m_\Lambda = 1115.6 \text{ MeV}/c^2$

Conservation of energy in the decay requires

$E_{0,\Sigma} = \left(E_{o,\Lambda} + K_\Lambda\right) + E_\gamma$ or $1\,192.5 \text{ MeV} = \left(1\,115.6 \text{ MeV} + \dfrac{p_\Lambda^{\,2}}{2m_\Lambda}\right) + E_\gamma$

System momentum conservation gives $|p_\Lambda| = |p_\gamma|$, so the last result may be written as

$$1\,192.5 \text{ MeV} = \left(1\,115.6 \text{ MeV} + \frac{p_\gamma^{\,2}}{2m_\Lambda}\right) + E_\gamma$$

or $1\,192.5 \text{ MeV} = \left(1\,115.6 \text{ MeV} + \dfrac{p_\gamma^{\,2} c^2}{2m_\Lambda c^2}\right) + E_\gamma$

Recognizing that $m_\Lambda c^2 = 1\,115.6$ MeV and $p_\gamma c = E_\gamma$

we now have $1\,192.5 \text{ MeV} = 1\,115.6 \text{ MeV} + \dfrac{E_\gamma^{\,2}}{2(1\,115.6 \text{ MeV})} + E_\gamma$

Solving this quadratic equation gives $E_\gamma = \boxed{74.4 \text{ MeV}}$

P46.64 The momentum of the proton is $qBr = (1.60\times10^{-19}\ \text{C})(0.250\ \text{kg/C}\cdot\text{s})(1.33\ \text{m})$

$p_p = 5.32\times10^{-20}\ \text{kg}\cdot\text{m/s}$ $\quad cp_p = 1.60\times10^{-11}\ \text{kg}\cdot\text{m}^2/\text{s}^2 = 1.60\times10^{-11}\ \text{J} = 99.8\ \text{MeV}$

Therefore $p_p = 99.8\ \text{MeV}/c$

The total energy of the proton is $E_p = \sqrt{E_0^2 + (cp)^2} = \sqrt{(938.3)^2 + (99.8)^2} = 944\ \text{MeV}$

For the pion, the momentum qBr is the same (as it must be from conservation of momentum in a 2-particle decay).

$p_\pi = 99.8\ \text{MeV}/c$ $\quad E_{0\pi} = 139.6\ \text{MeV}$

$$E_\pi = \sqrt{E_0^2 + (cp)^2} = \sqrt{(139.6)^2 + (99.8)^2} = 172\ \text{MeV}$$

Thus $E_{\text{total after}} = E_{\text{total before}} = \text{Rest energy}$

Rest energy of unknown particle $= 944\ \text{MeV} + 172\ \text{MeV} = 1116\ \text{MeV}$ (This is a Λ^0 particle!)

Mass $= \boxed{1116\ \text{MeV}/c^2}$.

P46.65 $\pi^- \to \mu^- + \bar{\nu}_\mu$: From the conservation laws for the decay,

$$m_\pi c^2 = 139.6\ \text{MeV} = E_\mu + E_\nu \qquad [1]$$

and $p_\mu = p_\nu,\ E_\nu = p_\nu c$: $\quad E_\mu^2 = (p_\mu c)^2 + (105.7\ \text{MeV})^2 = (p_\nu c)^2 + (105.7\ \text{MeV})^2$

or $$E_\mu^2 - E_\nu^2 = (105.7\ \text{MeV})^2 \qquad [2]$$

Since $$E_\mu + E_\nu = 139.6\ \text{MeV} \qquad [1]$$

and $$(E_\mu + E_\nu)(E_\mu - E_\nu) = (105.7\ \text{MeV})^2 \qquad [2]$$

then $$E_\mu - E_\nu = \frac{(105.7\ \text{MeV})^2}{139.6\ \text{MeV}} = 80.0 \qquad [3]$$

Subtracting [3] from [1], $\quad 2E_\nu = 59.6\ \text{MeV}$ and $\boxed{E_\nu = 29.8\ \text{MeV}}$

P46.66 The expression $e^{-E/k_BT}dE$ gives the fraction of the photons that have energy between E and $E + dE$. The fraction that have energy between E and infinity is

$$\frac{\int_E^\infty e^{-E/k_BT}dE}{\int_0^\infty e^{-E/k_BT}dE} = \frac{\int_E^\infty e^{-E/k_BT}(-dE/k_BT)}{\int_0^\infty e^{-E/k_BT}(-dE/k_BT)} = \frac{e^{-E/k_BT}\Big|_E^\infty}{e^{-E/k_BT}\Big|_0^\infty} = e^{-E/k_BT}$$

We require T when this fraction has a value of 0.0100 (i.e., 1.00%)

and $E = 1.00\ \text{eV} = 1.60\times10^{-19}\ \text{J}$

Thus, $0.010\,0 = e^{-(1.60\times10^{-19}\ \text{J})/(1.38\times10^{-23}\ \text{J/K})T}$

or $\ln(0.010\,0) = -\dfrac{1.60\times10^{-19}\ \text{J}}{(1.38\times10^{-23}\ \text{J/K})T} = -\dfrac{1.16\times10^4\ \text{K}}{T}$ giving $T = \boxed{2.52\times10^3\ \text{K}}$

P46.67 (a) This diagram represents the annihilation of an electron and an antielectron. From charge and lepton-number conservation at either vertex, the exchanged particle must be an electron, $\boxed{e^-}$.

(b) This is the tough one. A neutrino collides with a neutron, changing it into a proton with release of a muon. This is a weak interaction. The exchanged particle has charge $+e$ and is a $\boxed{W^+}$.

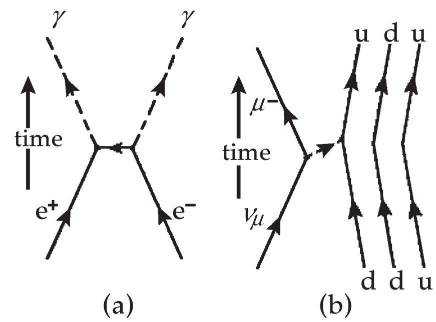

FIG. P46.67

P46.68 (a) The mediator of this weak interaction is a $\boxed{Z^0 \text{ boson}}$.

(b) The Feynman diagram shows a down quark and its antiparticle annihilating each other. They can produce a particle carrying energy, momentum, and angular momentum, but zero charge, zero baryon number, and, it may be, no color charge. In this case the product particle is a $\boxed{\text{photon}}$.

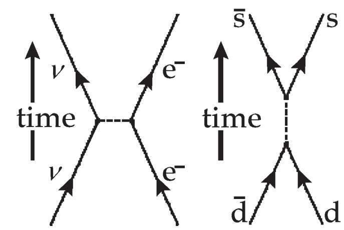

FIG. P46.68

For conservation of both energy and momentum in the collision we would expect two photons; but momentum need not be strictly conserved, according to the uncertainty principle, if the photon travels a sufficiently short distance before producing another matter-antimatter pair of particles, as shown in Figure P46.68. Depending on the color charges of the d and $\bar{\text{d}}$ quarks, the ephemeral particle could also be a $\boxed{\text{gluon}}$, as suggested in the discussion of Figure 46.13(b).

P46.69 (a) At threshold, we consider a photon and a proton colliding head-on to produce a proton and a pion at rest, according to $\text{p} + \gamma \to \text{p} + \pi^0$. Energy conservation gives

$$\frac{m_p c^2}{\sqrt{1-u^2/c^2}} + E_\gamma = m_p c^2 + m_\pi c^2$$

Momentum conservation gives $\dfrac{m_p u}{\sqrt{1-u^2/c^2}} - \dfrac{E_\gamma}{c} = 0.$

Combining the equations, we have

$$\frac{m_p c^2}{\sqrt{1-u^2/c^2}} + \frac{m_p c^2}{\sqrt{1-u^2/c^2}}\frac{u}{c} = m_p c^2 + m_\pi c^2$$

$$\frac{938.3 \text{ MeV}(1+u/c)}{\sqrt{(1-u/c)(1+u/c)}} = 938.3 \text{ MeV} + 135.0 \text{ MeV}$$

so $\dfrac{u}{c} = 0.134$

and $E_\gamma = \boxed{127 \text{ MeV}}$

(b) $\lambda_{max} T = 2.898 \text{ mm} \cdot \text{K}$

$$\lambda_{max} = \frac{2.898 \text{ mm} \cdot \text{K}}{2.73 \text{ K}} = \boxed{1.06 \text{ mm}}$$

continued on next page

(c) $E_\gamma = hf = \frac{hc}{\lambda} = \frac{1\,240 \text{ eV} \cdot 10^{-9} \text{ m}}{1.06 \times 10^{-3} \text{ m}} = \boxed{1.17 \times 10^{-3} \text{ eV}}$

(d) In the primed reference frame, the proton is moving to the right at $\frac{u'}{c} = 0.134$ and the photon is moving to the left with $hf' = 1.27 \times 10^{8}$ eV. In the unprimed frame, $hf = 1.17 \times 10^{-3}$ eV. Using the Doppler effect equation from Section 39.4, we have for the speed of the primed frame

$$1.27 \times 10^{8} = \sqrt{\frac{1 + v/c}{1 - v/c}}\, 1.17 \times 10^{-3}$$

$$\frac{v}{c} = 1 - 1.71 \times 10^{-22}$$

Then the speed of the proton is given by

$$\frac{u}{c} = \frac{u'/c + v/c}{1 + u'v/c^2} = \frac{0.134 + 1 - 1.71 \times 10^{-22}}{1 + 0.134\left(1 - 1.71 \times 10^{-22}\right)} = 1 - 1.30 \times 10^{-22}$$

And the energy of the proton is

$$\frac{m_p c^2}{\sqrt{1 - u^2/c^2}} = \frac{938.3 \text{ MeV}}{\sqrt{1 - \left(1 - 1.30 \times 10^{-22}\right)^2}} = 6.19 \times 10^{10} \times 938.3 \times 10^{6} \text{ eV} = \boxed{5.81 \times 10^{19} \text{ eV}}$$

ANSWERS TO EVEN PROBLEMS

P46.2 ~10^3 Bq

P46.4 2.27×10^{23} Hz; 1.32 fm

P46.6 $\bar{\nu}_\mu$ and ν_e

P46.8 (b) The range is inversely proportional to the mass of the field particle. (c) Our rule describes the electromagnetic, weak, and gravitational interactions. For the electromagnetic and gravitational interactions, we take the limiting form of the rule with infinite range and zero mass for the field particle. For the weak interaction, 98.7 eV · nm/90 GeV ≈ 10^{-18} m = 10^{-3} fm, in agreement with the tabulated information. (d) ~10^{-16} m

P46.10 ~10^{-23} s

P46.12 $\bar{\nu}_\mu$

P46.14 (b) The second violates strangeness conservation.

P46.16 The second violates conservation of baryon number.

P46.18 $0.828c$

P46.20 (a) $\bar{\nu}_e$ (b) $\bar{\nu}_\mu$ (c) $\bar{\nu}_\mu$ (d) $\nu_\mu + \bar{\nu}_\tau$

P46.22 See the solution.

P46.24 (a) not allowed; violates conservation of baryon number (b) strong interaction (c) weak interaction (d) weak interaction (e) electromagnetic interaction

P46.26 (a) K^+ (b) Ξ^0 (c) π^0

P46.28 (a) $\frac{686\text{ MeV}}{c}$ and $\frac{200\text{ MeV}}{c}$ (b) $627\ \text{MeV}/c$ (c) 244 MeV, 1 130 MeV, 1 370 MeV (d) $1190\ \text{MeV}/c^2,\ 0.500c$

P46.30 (a) 3.34×10^{26} e^-, 9.36×10^{26} u, 8.70×10^{26} d (b) $\sim 10^{28}$ e^-, $\sim 10^{29}$ u, $\sim 10^{29}$ d. I have zero strangeness, charm, topness, and bottomness.

P46.32 $m_u = 312\ \text{MeV}/c^2$ $m_d = 314\ \text{MeV}/c^2$

P46.34 See the solution.

P46.36 a neutron, udd

P46.38 (a) $-e$, antiproton b) 0, antineutron

P46.40 See the solution.

P46.42 (a) $v = c\left(\frac{Z^2 + 2Z}{Z^2 + 2Z + 2}\right)$ (b) $\frac{c}{H}\left(\frac{Z^2 + 2Z}{Z^2 + 2Z + 2}\right)$

P46.44 (a) What we can see is limited by the finite age of the Universe and by the finite speed of light. We can see out only to a look-back time equal to a bit less than the age of the Universe. Every year on your birthday the Universe also gets a year older, and light now in transit from still more distant objects arrives at Earth. So the radius of the visible Universe expands at the speed of light, which is 1 ly/yr. (b) $6.34 \times 10^{61}\ \text{m}^3/\text{s}$

P46.46 (a) 1.06 mm (b) microwave

P46.48 6.00

P46.50 (a) See the solution. (b) 17.6 Gyr

P46.52 See the solution.

P46.54 $\sim 10^{14}$

P46.56 (a) charge (b) energy (c) baryon number

P46.58 neutron

P46.60 2.04 MeV

P46.62 70.4 MeV

P46.64 $1\,116\ \text{MeV}/c^2$

P46.66 2.52×10^3 K

P46.68 (a) Z^0 boson (b) gluon or photon